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Optimal assignment of workers to supporting services in a hospital

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Supporting services play an important role in health care institutions such as hospitals. This paper presents an application of operations research model for optimal allocation of workers among supporting services in a public hospital. The services include logistics, inventory management, financial management, operations management, medical analysis, etc. The optimality criterion of the problem is to minimize operations costs of supporting services subject to some specific constraints. The constraints represent specific conditions for resource allocation in a hospital. The overall problem is formulated as an integer program in the literature known as the assignment problem, where the decision variables represent the assignment of people to various jobs. The results of some computational experiments modeled on a real data from a selected Polish hospital are reported.

Key words: assignment problem, integer programming, services operations management, health care planning.

Introduction

The assignment of service positions plays an important role in health care institutions. Poorly assigned positions in hospital departments or over-employment may result in increased expenses and/or degraded customer service. If too many workers are assigned,

Table 1. Number of workers and service jobs in hospital departments before optimization

Supporting Service Departments	Number of workers	Number of jobs
Central Heating Department	16	5
Power Department	15	3
Medical Bottled Gases Department	6	2
Ventilation and Air-condition Department	8	4
Heating and Air-condition Department	11	4
Distribution Department	6	3
Medical Equipment Department	8	4
Technical Department	11	5
Economy Department	21	5
Hospital Pharmacy	20	11
Sterilization Department	27	5
Material Monitoring Department	13	5
Information Department	7	4
Business Executive Department	8	5
Technical Executive Department	4	4
Law Regulation Department	7	3
Attorneys-at-law Department	4	2
Number of workers in all departments	192	–

capital costs are likely to exceed the desirable value [2]. The supporting services have a strong impact on performance of health care institutions such as hospitals. In hospital departments, the supporting services include financial management, logistics, inventory management, analytic laboratories, etc. This paper presents an application of operations research model for optimal supporting service jobs allocation in a public health care institution. The optimality criterion of the problem is to minimize operations costs of supporting service subject to some specific constraints. The constraints represent specific conditions for resource allocation in a hospital [6]. The overall problem is formulated as an integer program in the literature known as the assignment problem [1, 3, 5]. The binary decision variables represent the assignment of people to various services. This paper shows practical usefulness of mathematical programming approach to optimization of supporting services in health care institutions. The results of some computational experiments modeled after a real data from a selected Polish hospital are reported.

Data used for computations

The real data from a selected Polish public health care institution was used for computations, for a one month period. The data include 17 supporting service hospital departments, in which there are 74 types of supporting service jobs [7]. Supporting service departments in the hospital consist in total of 187.25 permanent positions with 192 workers employed before optimization. Specific data consists of average salaries for selected jobs in departments defined as costs of assignment of workers to jobs. Furthermore, average amount of money paid monthly for services in each department was used. Additional parameters include the number of permanent positions in each department and size of permanent positions (i.e. 0.25, 0.50, 0.75, 1.00) for each job defined as partial or full time. In addition, the minimum number of permanent positions for each job in each department was given, and the maximal number of positions a single worker can be assigned.

Table 1 presents the number of workers and service jobs in hospital departments and the total number of workers in all departments before the optimization.

Table 2 shows the number of permanent positions and maximum amount of money paid for services in hospital departments before optimization.

Table 2. Number of permanent positions and maximum amount of money paid for services in hospital departments before optimization

Supporting Service Departments	Number of permanent positions	Amount of money paid for services
Central Heating Department	5	29250
Power Department	3	31050
Medical Bottled Gases Department	2	11400
Ventilation and Air-condition Department	4	16650
Heating and Air-condition Department	4	21200
Distribution Department	3	13600
Medical Equipment Department	4	17500
Technical Department	5	20950
Economy Department	5	31360
Hospital Pharmacy	11	43400
Sterilization Department	5	41500
Material Monitoring Department	5	27150
Information Department	4	16100
Business Executive Department	5	15450
Technical Executive Department	4	7150
Law Regulation Department	7	16100
Attorneys-at-law Department	2.5	7950
Money paid for services in all departments	–	367760

Problem formulation

Mathematical programming approach deals with optimization problems of maximizing or minimizing a function of many variables subject to inequality and equality constraints and integrality restrictions on some or all of the variables. In particular, 0-1 variables represent binary choice. Therefore, the model presented in this paper is defined as an integer programming problem.

Suppose there are m people and p jobs, where $m \neq p$. Each job must be done by at least one person; also, each person can do at least, one job. The cost of person i doing job k is c_{ik}. The problem objective is to assign the people to the jobs so as to minimize the total cost of completing all of the jobs [5].

The optimality criterion of the defined problem is to minimize operations costs of supporting service subject to some specific constraints. The constraints represent specific conditions for resource allocation in a hospital. The overall problem is formulated as a modified assignment problem. The decision variables represent the assignment of people among various services [6].

Table 3. Notations

Indices		
i	—	worker, $i \in I = \{1, \ldots, m\}$
j	—	supporting service hospital department, $j \in J = \{1, \ldots, n\}$
k	—	type of supporting service job, $k \in K = \{1, \ldots, p\}$
Input parameters		
c_{ik}	—	cost of assignment of worker i to job k (i.e. monthly salary)
C_j	—	amount of money paid regularly for services in department j
e_k	—	size of permanent (partial or full) time position for job k (i.e. e_k = 0.25 or 0.50 or 0.75 or 1.00)
E_j	—	number of permanent positions in department j
f_{jk}	—	minimum number of permanent positions for job k in department j
A	—	big positive number
Decision variables		
x_{ijk}		1 if worker i is assigned to job k in department j, 0 otherwise
y_i		1 if worker i is assigned to any job in any department, 0 otherwise

Optimization models

The problem of optimal assignment is formulated as a single objective integer program, which allows commercially available software (e.g. AMPL/CPLEX [4]) to be applied for solving medium size, yet practical instances [6].
Minimize

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} c_{ik} x_{ijk} \tag{1}$$

subject to

$$\sum_{i=1}^{m}\sum_{k=1}^{p} c_{ik} x_{ijk} \le C_j, \quad j \in J \tag{2}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{p} e_k x_{ijk} \le E_j, \quad j \in J \tag{3}$$

$$\sum_{j=1}^{n}\sum_{k=1}^{p} e_k x_{ijk} \le 2, \quad i \in I \tag{4}$$

$$\sum_{i=1}^{m} x_{ijk} \ge f_{jk}, \quad j \in J, \quad k \in K \tag{5}$$

$$y_i = \begin{cases} 1 & \text{if} \quad \sum_{j=1}^{n}\sum_{k=1}^{p} x_{ijk} > 0 \\ 0 & \text{if} \quad \sum_{j=1}^{n}\sum_{k=1}^{p} x_{ijk} = 0 \end{cases}, \quad i \in I \tag{6}$$

$$\frac{\left(\sum_{j=1}^{n}\sum_{k=1}^{p} x_{ijk}\right)}{A} \le y_i \le \left(\sum_{j=1}^{n}\sum_{k=1}^{p} x_{ijk}\right), \quad i \in I \tag{7}$$

$$x_{ijk} \in \{0,1\}, \quad i \in I, \quad j \in J, \quad k \in K \tag{8}$$

$$y_i \in \{0,1\}, \quad i \in I \tag{9}$$

In the optimization model constraint (7) can be replaced by alternative constraint (10):
Minimize (1)
subject to (2), (3), (4), (5), (6), (8) and (9)

$$\sum_{j=1}^{n}\sum_{k=1}^{p} x_{ijk} = y_i, \quad i \in I \tag{10}$$

The optimality criterion (1) is to minimize operational costs of the supporting services. Constraint (2) imposes that the cost of assignment of workers to service jobs in each department must be less than or equal to maximum amount of money paid regularly for services in the department. Constraint (3) ensures that total size of permanent (partial or full) time positions for each job (i.e. 0.25 or 0.50 or 0.75 or 1.00) in each department must be less than or equal to the number of permanent positions in this department. Constraint (4) ensures that each worker can be assigned to maximum two full time positions in parallel. Constraint (5) is responsible for assignment of workers on at least minimal level requirements, e.g. the number of shifts on a selected service job. Constraint (6) defines the relation between decision variables x_{ijk} and y_i. If a worker i is assigned to some job at some department, then $y_i = 1$, otherwise $y_i = 0$. Constraint (7) defines the relation between binary decision variables x_{ijk} and y_i according to equation (6). Constraints (8) and (9) define binary decision variables. The first binary decision variable x_{ijk} denotes whether or not worker i is assigned to job k in department j. The second binary variable y_i denotes whether or not worker i is selected for assignment to any job in any department. Constraint (10) ensures that each worker can be assigned to at most one job. Equations (7) and (10) can be used alternatively, these constraints impose relation defined in equation (6) between binary decision variables x_{ijk} and y_i.

Computational results

In this section numerical examples and some computational results are presented to illustrate possible applications of the proposed formulations of integer programming of optimal assignment of service positions. Selected problem instances with the examples are modeled on a real data from a Polish hospital.

In the computational experiments the historical data is considered. Computational time range is from a few seconds to minutes or even hours. The computational experiments have been performed using AMPL with solver CPLEX 9.1 on computer Compaq Presario 1830 with Pentium III; RAM 512MB.

Table 4 shows the comparison of CPU time requirement for finding optimal solution with the use of constraint (7) or (10).

In this table, column 'IP simplex iteration' shows the number of integer programming simplex iterations until the solution is presented.

Table 4. Comparison of computational results with alternative constraints

Scenario	Assignment cost	Number of assigned workers	IP simplex iteration	CPU	Constraint
A	153251.0	77	2	10.49	(7)
A	153251.0	77	0	12.25	(10)
B	209751.5	108	3	12.46	(7)
B	209751.5	108	0	12.08	(10)
C	248951.5	131	3	9.17	(7)
C	248951.5	131	0	12.80	(10)
D	311651.5	166	4	7.47	(7)
D	311651.5	166	0	7.47	(10)

*CPU seconds for proving optimality on Pentium III, RAM 512MB /CPLEX 9.1

In Table 5 the number of workers assigned to supporting service hospital departments is presented.

As it has been recommended by the hospital managers four different scenarios of the assignment have been implemented. In scenario A, minimal number of people is employed in each supporting service department so that each type of a job has at least one worker assigned. In this case no shifts are considered. In scenario B at least two workers were assigned to each job. The scenario C secured the level of supporting service workers. In each department there are two shifts with at least two workers assigned to each job, but for some special cases, more than two workers are assigned to each job. Finally, scenario D presents optimal assignment of workers to jobs with a high service level with all currently employed workers. The results obtained have indicated the problem of over-employment in the hospital.

Table 5. Workers assigned in departments

Supporting Service Departments	Assignment of workers in departments according to scenario:			
	A	B	C	D
Central Heating Department	5	9	11	14
Power Department	3	5	8	13
Medical Bottled Gases Department	2	3	4	5
Ventilation and Air-condition Department	4	6	6	7
Heating and Air-condition Department	5	5	7	10
Distribution Department	5	5	5	5
Medical Equipment Department	4	6	6	7
Technical Department	5	8	8	9
Economy Department	5	8	13	18
Hospital Pharmacy	11	15	17	19
Sterilization Department	5	8	15	23
Material Monitoring Department	5	7	8	11
Information Department	4	5	5	6
Business Executive Department	5	6	6	7
Technical Executive Department	4	4	4	4
Law Regulation Department	3	5	5	5
Attorneys-at-law Department	2	3	3	3
Number of employees in all departments	77	108	131	166

*Scenarios A, B, C and D considered subject to hospital authority requirements

Conclusions

Operations research techniques, tools and theories have long been applied to a wide range of issues and problems in health care. This paper proves practical usefulness of mathematical programming approach to optimization of supporting service in a hospital. The results of some computational experiments modeled after a real data indicate that the number of hired workers can be reduced in all departments of the hospital. The proposed modified assignment problem can be easily implemented for management of supporting services in another health care institution.

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