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Statistical modeling of copper losses in the silicate slag of the sulfide concentrate smelting process

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This article presents the results of the statistical modeling of copper losses in the silicate slag of the sulfide concentrates smelting process. The aim of this study was to define the correlation dependence of the degree of copper losses in the silicate slag on the following parameters of technological processes: SiO_2, FeO, Fe_3O_4, CaO and Al_2O_3 content in the slag and copper content in the matte. Multiple linear regression analysis (MLRA), artificial neural networks (ANNs) and adaptive network based fuzzy inference system (ANFIS) were used as tools for mathematical analysis of the indicated problem. The best correlation coefficient (R^2 = 0.719) of the final model was obtained using the ANFIS modeling approach.

Keywords: modeling, copper concentrate, smelting process, copper losses, slag, ANFIS.

INTRODUCTION

In modern pyrometallurgical process of obtaining copper by smelting of sulfide concentrates two intermediate phases are created: silicate slag and copper matte. These phases are separated by gravity because of the difference in specific weight. The precipitated copper matte is further processed, while the slag is removed from the system[1].

In various processes of melting sulfidic copper concentrates, the copper content of the slag varies within the limits of 0.5 to 2%. In order to reduce the copper content in the silicate slag, different strategies of managing the melting process are being employed: a) minimizing the amount of generated slag, b) minimizing the percentage of copper in the slag and c) processing the slag to recover as much as possible Cu[1, 2].

Slag produced in the autogenous smelting process (Flash, INCO, Isasmelt, etc.), with a copper content of about 2%, is further treated in the flotation process[2] or processed in the electric furnace, in order to lower the copper content in the slag to the value of 0.5–0.6% as in the stationary conditions of smelting in the reverberatory furnace. The final slag with the copper content of 0.5–0.6% is deposited in tailing pounds or used in the production of some types of building materials[3, 4, 5].

The loss of copper with the slag is a global problem in the pyrometallurgy of copper because it has a significant impact on the economics of copper production[2]. Copper is present in the silicate slag melt due to its solubility in sulfide, oxide and elemental form, as well as in the form of residual non-precipitated droplets of copper matte[6–8]. The dissolved Cu is associated with either O^{2-} ions (Cu_2O), or with S^{2-} ions (Cu_2S). The Cu_2O becomes the dominant form of dissolved Cu at matte grades above 70% Cu[9] due to the increased activity of Cu_2S in the matte. Higher Cu_2S activity pushes the reaction:

$$Cu_2S_{(matte)} + FeO_{(slag)} \rightarrow Cu_2O_{(slag)} + FeS_{(matte)} \quad (1)$$

to the right. The solubility of sulfur in slag is also lower in contact with higher-grade mattes[10].

Numerous studies have shown that the copper content in the slag can be reduced by reducing the viscosity through regulating its composition, especially the content of: SiO_2, FeO, Fe_3O_4, CaO, Al_2O_3 as well as the copper content in the copper matte[11, 12].

Goni and Sanchez[11] and Živković et al.[10] have shown that the slag composition has the largest influence on the copper loss in it. Sulfide copper concentrates smelting slag predominantly consists of fayalite ($2FeO \cdot SiO_2$) which is contained with more than 70%. It has been found that a high content of SiO_2 in the slag lowers its copper content due to the reduction in the viscosity and the oxidation potential of the slag[8].

Magnetite (Fe_3O_4) which is partially formed in the process of melting and added with the return converter slag increases oxidation potential and viscosity of the slag, which leads to an increase in its copper content due to the reaction in the slag – copper matte interphase layer[46]:

$$Cu_2S_{(matte)} + 3Fe_3O_{4(slag)} \rightarrow Cu_2O_{(slag)} + 9FeO_{(slag)} + SO_{2(gas)} \quad (2)$$

Matte droplets can become suspended in smelter slag by several other mechanisms. Some are carried upwards from the molten matte layer by gas bubbles generated during the reaction (2) and the following reaction:

$$3Fe_3O_{4(slag)} + FeS_{(matte)} \rightarrow 10FeO_{(slag)} + SO_2(gas) \quad (3)$$

Other components contained within the slag: FeO, CaO and Al_2O_3 each, in their own way, affect the basicity of the slag[13] on which the distribution coefficients between the slag and the copper matte for Cu and accessory metals depend. Furthermore, the copper content of the copper matte affects the distribution of Cu and accessory metals between the slag and the copper matte[12]. Despite the fact that reverberatory furnace is largely outdated and replaced with more up to date processes, there are many results published that can be find in contemporary literature describing the research in this kind of furnace[3, 8, 14, 15]. This way, according to the reference sources, large numbers of researches were conducted under controlled conditions in the reverberatory smelting furnace, in order to define mathematical models for the prediction of copper content in the waste slag, which have a satisfactory level of R^2 for the given conditions[13].

When it comes to more advanced copper smelter processes, Liu and his associates[16, 17] defined the dynamic model for the prediction of the copper content in the waste slag in the flash smelting process. These studies have confirmed the earlier results of Zivkovic et al.[8, 10] about the influence of the content of SiO_2, CaO, FeO in the slag and Cu in the matte. However, validation of

such defined models in a real industrial process conditions did not provide satisfactory results. Parameters of the smelting process are pre-defined and do not provide the option of affecting the copper losses in the slag with their variation.

For the modeling of technological processes in order to predict the output of the process based on the change of the process input, under the real process conditions, the main problem is the uncertainty of the modeling parameters, despite the fact that this uncertainty is limited. When linear statistical models (LSM) do not give satisfactory results, a non-linear models based on artificial intelligence are being employed. These models have the potential for solving complex technological applications that involve a large number of independent parameters in a nonlinear relationship[18, 19]. Models based on artificial neural networks (ANN) can be found in the literature over a long period of time[13, 20–22]. However, in cases where the parameters of the technological process used for modeling are in a state of great entropy, the uncertainty is large and the potential of ANN is not able to adequately connect these parameters in non-linear relationship. In recent times, adaptive neuro-fuzzy inference system (ANFIS) is being used, which represents a combination of Artificial Neural Network (ANN) and Fuzzy Inference System (FIS) in a way that the neural network learning algorithm is used to determine the parameters of the FIS[23]. ANFIS in many cases enables modeling of parameters of the technological process under conditions of significant uncertainty, with a greater degree of determination in relation to all the other models[24–29].

The motive for the research, whose results are presented in this paper, is to define a satisfactory mathematical model to predict the copper content in the waste slag of sulfide concentrates smelting in the reverberatory furnace, depending on the input parameters of the technological process which can be controlled during the process. The process of smelting of sulfide concentrates under the real industrial conditions is carried out under conditions of considerable uncertainty, therefore the definition of an adequate model for process management poses a challenge that is of both theoretical and practical importance. Defined model, with satisfactory R^2 value, allows for the better management of the sulphide copper concentrates smelting process with the aim to reduce the content of copper in the waste slag, which improves the economics of the process.

MODELING THE COPPER LOSSES IN THE SILICATE SLAG

Experimental data

The research, whose results are presented in this paper, was performed in the copper smelting company RTB Bor, Serbia in which the smelting process is carried out in a reverberatory furnace and the resulting silicate slag, with the copper content within the range of 0.5–0.6%, is the final waste material, which is deposited directly to the waste yard near to the smelter plant.

During the year 2012 the sampling was conducted at each discharge of the slag from the reverberatory furnaces in the company RTB Bor, Serbia, and the composite sample was chemically analyzed each day for the contents of the following components: Cu, SiO_2, FeO, Fe_3O_4, CaO, Al_2O_3, as well as the Cu content in the copper matte. Details considering the sampling procedure and methodology of samples preparation are presented in our previous publications[8, 10, 12, 13]. For quantitative determination of the elements, with the concentration above 0.1%, optical emission spectrograph OFS JARRELL – ASH 70.000 (SAD) was used, while for the elements below 0.1%, mass spectrograph JOEL JMS – 0.1 MB (Japan) with ionic optical system Matauh-Herzog, was used.

While monitoring the process of sulfide copper concentrates smelting during the days of sampling for analysis, technological process parameters (temperature, volume, dynamics,...) had been within the defined limits, which presents a guarantee that the analyzed samples of the slag and the matte represent the real conditions of the technological process.

In this way the statistical sample of 356 measurements during the 2012 was formed, with measured contents of all components. Obtained results are shown in Table 1.

The data for the research presented in this paper were collected, during the year 2012, by measuring input and output process parameters, under stable operation mode of the production line. All 356 data sets were collected this way, comprising the following:

Input parameters of the process: SiO_2 content in the slag (mass pct.) = X1; FeO content in the slag (mass pct.) = X2; Fe_3O_4 content in the slag (mass pct.) = X3; CaO content in the slag (mass pct.) = X4; Al_2O_3 content in the slag (mass pct.) = X5; Copper content in the matte (mass pct.) = X6.

Output parameter of the process: copper content in the slag (mass pct) = Y.

The values of the measured input parameters of the technological process (X1–X6) and the process quality indicator – output of the process (Y), are presented in Table 1 in the form of descriptive statistics results.

Table 1. Values of the Input (X_i) and the Output (Y) Variables of the industrial sulfidic copper concentrates smelting process – Descriptive Statistics of 356 Data Sets

	X1	X2	X3	X4	X5	X6	Y
Std. Error of Mean	0.060194	0.041463	0.075148	0.027346	0.023295	0.127423	0.002777
Std. Deviation	1.135744	0.782328	1.417890	0.515970	0.439529	2.404218	0.052402
Variance	1.290	0.612	2.010	0.266	0.193	5.780	0.003
Range	7.460	5.640	8.610	4.070	2.390	13.800	0.700
Minimum	29.790	4.880	37.950	1.950	4.580	35.420	0.490
Maximum	37.250	10.520	46.560	6.020	6.970	49.220	1.190

It should be noted that output variable – Y has a small variance (Table 1). However, it presents the copper content in the slag that is one of the most important parameters of the industrial sulfidic copper concentrates smelting process; therefore it cannot be omitted in the analysis. A small change in Y leads to a considerable change of the technical parameters and the economy of the investigated process.

Multilinear Regression Analysis

In order to define the correlation dependence in the form: output of the process (Y) = f input of the process (X1– X6), a bivariate correlation analysis was performed. As the result of this analysis, Pearson correlation (PC) coefficients, with responding statistical significance, were calculated (Table 2).

To define the dependence of the output parameter as the function of the input parameters, using the multiple linear regression analysis (MLRA) with acceptable level of fitting (strong correlation), it is necessary that the value of PC is near 0.5, with statistical significance ($p \leq 0.05$)[13, 19, 30]. An analysis of the data presented in the Table 2 reveals that statistical significance is attained in the following cases: $Y \rightarrow X_3$; PC = –0.227 ($p < 0.001$); $Y \rightarrow X_2$; PC = 0.26 ($p < 0.001$); $Y \rightarrow X_6$; PC = 0.312 ($p < 0.001$). This was also the case for the following interdependence between the predictors of the process: $X_1 \rightarrow X_2$; PC = –0.67 ($p < 0.001$); $X_1 \rightarrow X_3$; PC = –0.66 ($p < 0.001$); $X_3 \rightarrow X_4$; PC = –0.57 ($p < 0.001$) and $X_3 \rightarrow X_5$; PC = –0.52 ($p < 0.001$). However, there were no cases with strong linear correlation. Low values of PC and the satisfactory level of statistical significance ($p < 0.001$) indicate that the application of MLRA probably cannot provide a satisfactory coefficient of determination (R^2) of the linear model. To further test the linear regression analysis applicability, ANOVA analysis of the MLRA model was performed. For the purpose of MLRA analysis, the assembly of 356 input and output data sets was divided into two groups. The first group consisted of 243 (68 pct) randomly selected data lines, and it was used for training of the model, whereas the second group consisted of 113 (32 pct) remaining data lines from the starting data base, and it was used for testing of the model.

Linear dependence of the copper content in the slag (Y) on influencing parameters of the technological process (X1–X6) was obtained using SPSS software application Version 17.0 (SPSS Inc., Chicago, IL). The results of the ANOVA tests of developed model are presented in Table 3. Significant F statistics (Table 3) indicate that using the model is better than guessing the mean. Also, the significance value of the F statistic is less than 0.05, which means that the variations explained by the model are not caused by chance. However, the ratio of regression to residual is 19 pct: 81 pct, advocating that only 19 pct of the dependent variable (Y) values are explained by the model. Where "Regression" displays information about the variation accounted for by the model, and "residual" displays information about the variation that is not accounted for by the model[17]. This is a clear indicator that the MLRA model of investigated data set would not result in adequate enough accuracy. This was additionally proven with the low level of the coefficient of determination (R^2), for the training phase of the MLRA modeling approach, which is presented in the Table 4.

The multiple correlation coefficient (R) presents the linear correlation between the observed and model predicted values of the dependent variable. Its small value (0.432) indicates a weak relationship. R^2, which is the coefficient of determination, is the squared value of the multiple correlation coefficient. It shows that approximately 19 pct of variation in Y is explained by the model, as is indicated already by the regression to residual ratio[17]. Accordingly, it can be concluded that the MLRA approach would not be the appropriate tool for modeling of the investigated process, because of low values of all adequacy tests indicators.

Artificial Neural Networks (ANNs)

Artificial neural networks represent a class of tools that can facilitate the exploration of large systems in ways not previously possible. These methods have shown explosive growth in the last decade and are still being developed at a breathtaking pace. In many ways, neural networks

Table 2. Correlation Matrix for the input ($X_1 - X_6$) and the Output (Y) variables of the industrial copper concentrate smelting process (Number of Data Points for each variable is 356)

		X1	X2	X3	X4	X5	X6	Y
X1	Pearson Correlation	1						
	Sig. (2-tailed)							
X2	Pearson Correlation	-.670**	1					
	Sig. (2-tailed)	.000						
X3	Pearson Correlation	-.661**	.207**	1				
	Sig. (2-tailed)	.000	.000					
X4	Pearson Correlation	.030	-.100	-.569**	1			
	Sig. (2-tailed)	.571	.060	.000				
X5	Pearson Correlation	.169**	-.059	-.515**	.342**	1		
	Sig. (2-tailed)	.001	.269	.000	.000			
X6	Pearson Correlation	-.057	.017	-.067	.011	.280**	1	
	Sig. (2-tailed)	.287	.751	.208	.830	.000		
Y	Pearson Correlation	-.227**	.260**	.067	-.082	.049	.312**	1
	Sig. (2-tailed)	.000	.000	.204	.120	.359	.000	

**. Correlation is significant at the 0.01 level (2-tailed).

Table 3. Results of ANOVA[a, b] Test Performed During Training of the Model

	Sum of Squares	df	Mean Square	F	Sig.
Regression	0.182	6	0.030	13.337	0.000[a]
Residual	0.793	349	0.002		
Total	0.975	355			

a. Predictors: (Constant), X6, X4, X1, X5, X2, X3
b. Dependent Variable: Y

Table 4. MLRA Summary[a,b] of the Model Developed During Training Phase

Model	R	R Square	Adjusted R^2	Std. Error of the Estimate
1	0.432[a]	0.187	0.173	0.047668

a. Predictors: (Constant), X6, X4, X1, X5, X2, X3
b. Dependent Variable: Y

can be viewed as nonlinear approaches to multivariate statistical methods, not bounded by assumptions of normality or linearity.

Although neural networks have originated outside the field of statistics and have even been viewed as an alternative to statistical methods in some circles, some signs indicate that this viewpoint is making way for an appreciation of the ways in which neural networks complement classic statistics[19, 31]. Details about ANN procedure description are discussed in details in the references[18, 24].

However, the demerit of the ANNs based models, lays in the fact that it is based on only one rule describing the behavior of input variables. This way, it is difficult to develop accurate enough model for modeling the set of input variables with wide range (like variables X_1, X_3 and X_6 in Table 1), because all values of this variables will enter the model in one wide scope. Also, it is difficult to use the same model on validation of new set of input parameters, obtained from the same process in subsequent time intervals, if the range of the input parameters is too broad.

ANN methodology was applied for modeling the copper losses in the silicate slag of the sulfide concentrates smelting process, using available data, whose descriptive statistic is presented in Table 1. The same as in the MLRA procedure, the assembly of 356 input and output data sets was divided into two groups. The first group consisted of 243 (68 pct) of randomly selected data lines, and it was used for training of the network, whereas the second group consisted of 113 (32 pct) of remaining data lines from the starting database, and it was used for testing of the network.

For the development of relational ANN configuration, we used previously defined input parameters X1–X6 and output parameter Y (copper content in the slag), as the elements of the network architecture. For such a network, the obtained coefficient of determination was only $R^2 = 0.452$ for the training phase. During the ANN testing phase, the calculated coefficient of determination (R^2) was additionally decreased in comparison with the testing phase, and now it equals 0.368.

This was the clear indicator that „one rule“ non-linear statistical modeling approach, based on ANN approach, is also inadequate tool for the modeling of the investigated process.

Adaptive Network Based Fuzzy Inference System (ANFIS)

Considering that the ANFIS procedure can allocate the values of each input variable in more than one scale, it can be used for modeling the set of input variables with relatively wide range; such are the variables X_1, X_3 and X_6, in Table 1. ANFIS inference system was proposed by Jang[23]. Although the ANN and fuzzy logic models are the main areas of artificial intelligence, ANFIS combines the two methods by using their strengths and thus eliminates some of the shortcomings of their individual application and as such is a reliable technique for solving complex nonlinear problems[32, 33].

The mechanism of fuzzy reasoning utilized by ANFIS is schematically shown in Figure 1.

For simplicity, it is assumed that the fuzzy inference has two inputs x_1 and x_2 and one output variable y and presumably that the rule base conditions two fuzzy if--then rules of Takagi and Sugeno's type, following rule equations can be defined[25, 34]:

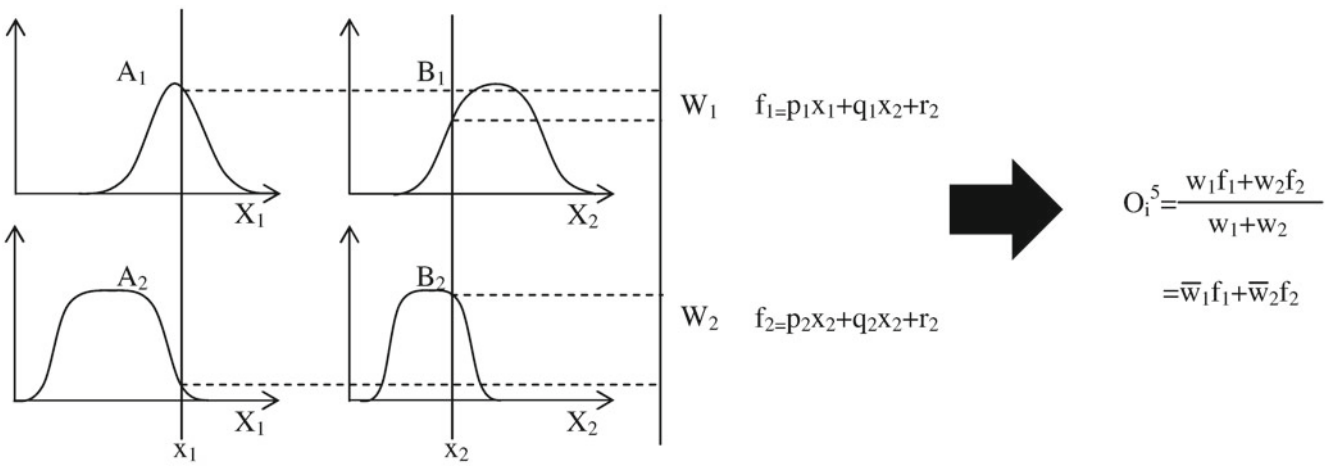

Figure 1. Fuzzy reasoning

Rule 1: If x_1 is A_1 and x_2 is B_1 then $f_1 = p_1 .x_1 + q_1.x_2 + r_1$
Rule 2: If x_1 is A_2 and x_2 is B_2 then $f_2 = p_2.x_1 + q_2.x_2 + r_2$

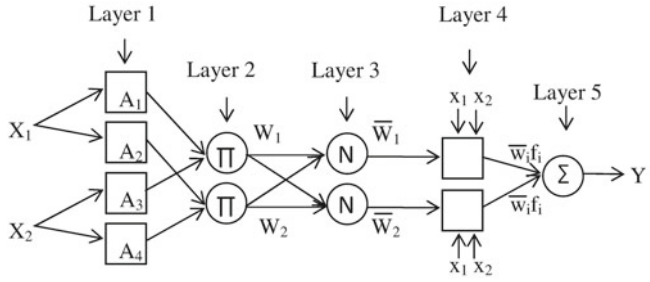

Figure 2. ANFIS architecture

The general ANFIS architecture is shown in the Figure 2.

As can be seen in Figure 2, ANFIS architecture can be presented with five layers. Where X_1 and X_2 are inputs to nodes in layer 1, A_i and B_i are the linguistic label of the ranges of input variables (small, large, etc), associated with the node function. The node functions in the same layer are of the same function family as described below[24, 25, 29, 32]:

Layer 1. Membership functions of nodes located in layer 1 ($O_i^1 = \mu A_i(X_i)$ or $O_i^2 = \mu B_i(X_i)$) specify the degree to which the given X_i satisfies the quantifier A_i, B_i, etc. Usually, membership functions are either bell (shaped with maximum equal to 1 and minimum equal to 0), or a Gaussian function.

Layer 2. Nodes located in the layer 2 are multipliers, which are multiplying the signals exiting the layer 1 nodes. For example $O_i^2 = W_i = \mu A_i(X_i) \cdot \mu B_i(X_i)$, i = 1, 2, etc. Output of each node is representing the firing strength of a rule.

Layer 3. The i-th node of layer 3 calculates the ratio of i-th rules firing strength to sum of all rules firing strengths. This way $O_i^3 = \overline{W_i} = W_i /(W_1 + W_2 + \ldots)$, i = 1, 2, …

Layer 4. Every node i in the layer 4 has a node function of following type: $O_i^4 = \overline{W_i} \cdot f_1 = \overline{W_i} \cdot (p_i x_1 + q_i x_2 + r_i)$, where p_i, q_i and r_i will be referred to as consequent parameters.

Layer 5. The single node of layer 5 is the node that computes the overall output as the summation of all incoming signals i.e., $O_i^5 = \sum_i \overline{W_i f_i} = \frac{\sum_i W_i f_i}{\sum_i W_i}$

Training of the parameters in the ANFIS structure is accommodated according to the hybrid learning rule algorithm which is the integration of the gradient descent method and the least square methods. In the forward pass of the algorithm, functional signals go forward until layer 4 and the consequent parameters are identified by the least squares method to minimize the measured error. In the back propagation pass, the premise parameters are updated by the gradient descent method[13, 19, 32, 34].

The reason to use the Adaptive Network Based Fuzzy Inference System (ANFIS) for modeling the copper losses in the silicate slag of the sulfide concentrates smelting process, presented in this paper, can be found in the fact that the MLRA approach resulted with low accuracy of obtained model (R^2 = 0.187), as well as the ANNs approach which resulted with the accuracy of R^2 = 0.368.

For that reason, it was decided to develop the ANFIS based numerical model, which is based on more than one rule to describe the behavior of input variables. Considering that the ANFIS procedure can allocate the values of each input variable in more than one scale, it can be used for modeling the set of input variables with wide range; such are the variables X_1, X_3, and X_6, in Table 1.

According to the values and the ranges of input variables, presented in Table 1, it was decided that the two rules ANFIS network should be applied, to obtain adequate prediction results of the final model. Selected membership function, for all variables, was Gaussian

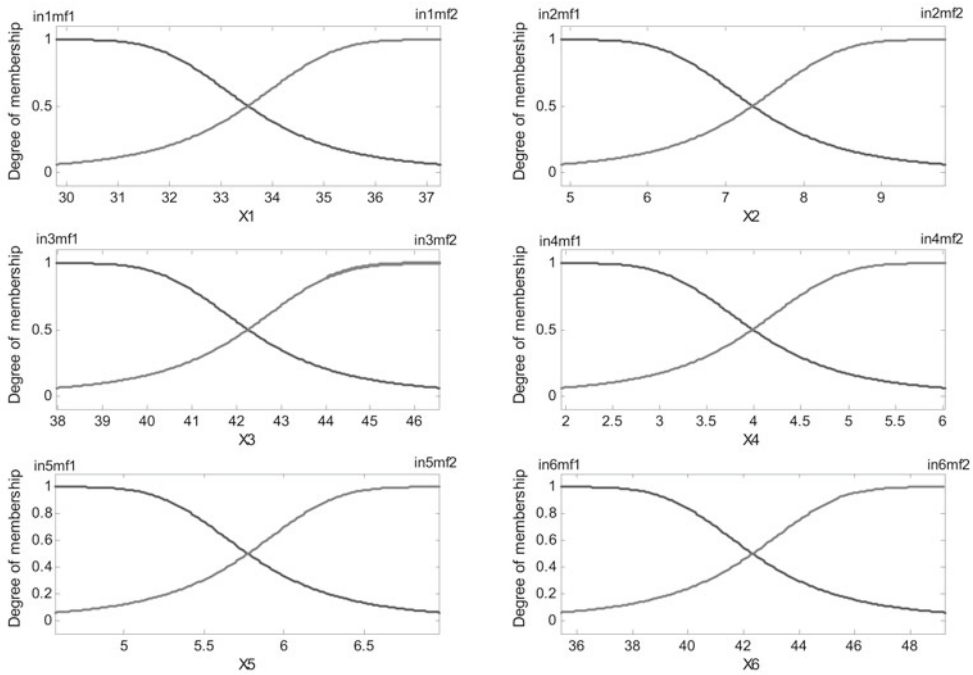

Figure 3. Membership functions of input variables (X_1 to X_6)

one. Representation of the membership functions for all 6 input variables is presented in Figure 3.

Same as in case of MLRA and the ANN, in order to apply the ANFIS methodology for the modeling of the copper losses in the silicate slag of the sulfide concentrates smelting process, presented in this paper, the assembly of 356 input and output samples was divided into two groups. The first group consisted of 243 (68 pct) of randomly selected samples, and it was used for training of the model, whereas the second group consisted of 113 (32 pct) of the remaining samples from the starting data set, and it was used for testing the model. The selection of the variables for these two stages was performed by using random number generator based on Bernoulli distribution.

During the training phase, the correction of the weighted parameters (p_i, q_i, r_i, etc) of the connections, presented in the Figure 1, was achieved through the necessary number of iterations, until the mean squared error between the calculated and measured outputs of the ANFIS network, was minimal. During the second phase, the remaining 32% of the data was used for testing of the "trained" network. In this phase, the network used the weighted parameters which were determined during the first phase. These new data, excluded during the network training stage, were then incorporated as the new input values (X_i) which were then transformed into the new outputs (Y). For calculation presented in this paper MATLAB ANFIS editor was used[35].

In the network training phase, the necessary number of iterations was performed until the error (RMSE) between the measured output of the copper content in the slag Y and the calculated values wasn't minimized and remained constant (Figure 4) . In the case of the investigation presented in this paper, optimal number of iterations (epochs) was 10.

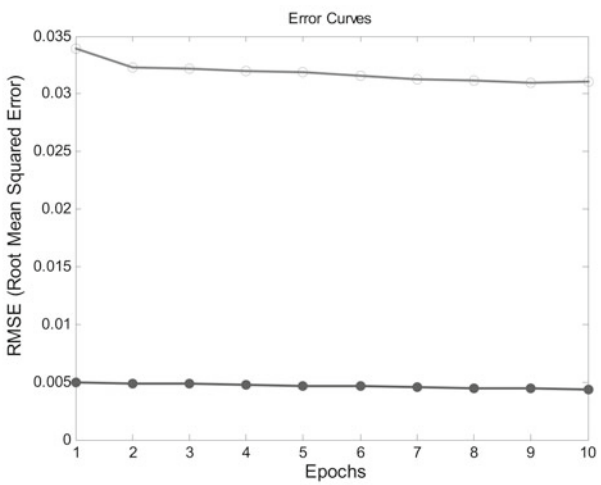

Figure 4. RMSE of the ANFIS model during the training (○) and the testing (●) stage

In the testing phase, remaining data lines were used to test the trained ANFIS network. Final fidelity of the obtained ANFIS model was assessed by coefficient of determination (R^2) between measured and model predicted values of the copper losses in the silicate slag of the sulfide concentrates smelting process (Y), in the training and the testing stage. The R^2 values are presented in Figure 5, and Figure 6, for the training and the testing stage, respectively.

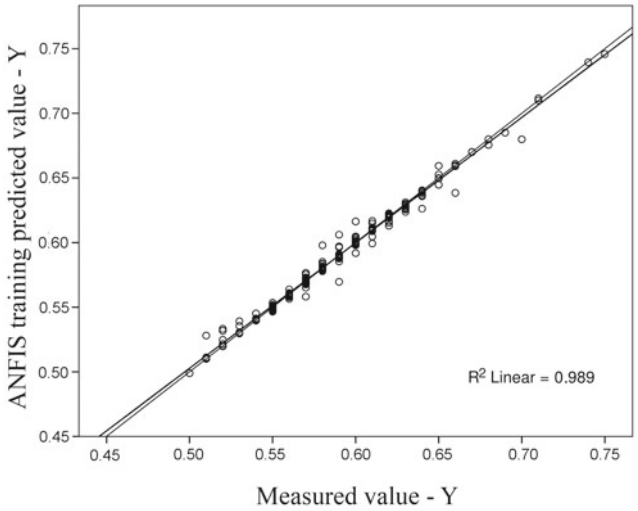

Figure 5. The copper losses in the silicate slag of the sulfide concentrates smelting process predicted by ANFIS vs. the actual measured value in the training stage

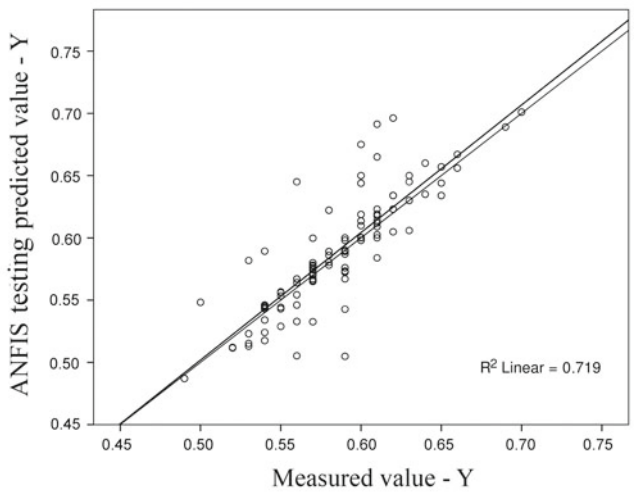

Figure 6. The copper losses in the silicate slag of the sulfide concentrates smelting process predicted by ANFIS vs. actual measured value in the testing stage

The ANFIS modeling approach, predicted the copper losses in the silicate slag of the sulfide concentrates smelting process with a determination coefficient $R^2 = 0.989$ (Fig. 5) and $R^2 = 0.719$ (Fig. 6), in the training stage and the testing stage – respectively, which represents very large significance. This means that the copper losses in the silicate slag of the sulfide concentrates smelting process can be predicted with accuracy near 72%, based on known input parameters (X_1 to X_6), using the ANFIS model described in this paper.

To further sustain the applicability of proposed ANFIS model, validation vas performed with the data collected during the year 2013. For that purpose, additional set of 113 data lines was recorded at the same production line, during the period January – May 2013. This was the same number of data, as previously used in the testing stage. With this new data base, validation of the model was performed. The ANFIS model responded with

expected high accuracy, resulting with the coefficient of determination $R^2 = 0.708$.

Limitations of the obtained results are reflected in the fact that the ANFIS model cannot show the strength of the influence of the individual input parameters (X_1–X_6) on the copper content in the waste slag (Y), which is the fundamental disadvantage of ANN and ANFIS methodology in comparison to the MLRA. However, the achieved $R^2 = 0.989$ in the model defining phase and $R^2 = 0.719$ in the model testing phase provide the possibility of predicting the copper content in the waste slag by changing the composition of input charge, with a significant probability (composition of the input charge in the smelting process defines the composition of the slag).

CONCLUSIONS

Results presented in this paper indicate that complex industrial systems can be modeled using nonlinear statistical approach based on ANFIS methodology, even in those cases where MLRA and ANNs do not achieve adequate accuracy.

Values of the correlation analysis of the degree of copper losses in the silicate slag of the sulfide concentrates smelting process, under industrial conditions in the factory RTB Bor, Bor (Serbia) were determined using the ANFIS methodology. The selected ANFIS structure consisted of 243 (68 pct) samples for training and 113 (32 pct) samples for testing.

The values of the coefficient of determination (R^2) were 0.989 and 0.719, in the training and the testing stage – respectively. These results indicated a highly acceptable degree of fitting of the dependence $Y = f(X_1–X_6)$, obtained using ANFIS procedure as part of the Math Lab software application, version 7.1 (2007). The results presented in this paper indicated that the defined elements of the ANFIS structure can be applied in order to accurately predict the copper losses in the silicate slag of the sulfide concentrates smelting process in the investigated factory. This claim was sustained by validation of the model on completely new data set, obtained in the same factory, during 2013.

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