

## **THE PRACTICAL ASPECTS OF IMPLEMENTATION OF THE THRUST ALLOCATION PROCEDURE FOR A MULTI-PROPULSOR UNDERWATER ROBOT**

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### **ABSTRACT**

This article addresses the practical aspects of the synthesis of an automatic control system for the thrust allocation strategy in the propulsion system of an unmanned underwater vehicle. The vehicle under consideration is a robot submarine equipped with a multi-propulsion system providing four degrees of freedom of movement. The power distribution algorithms are based on limited optimisation methods that allow the determination, on the basis of generalised torques and forces, of how much thrust is required to be produced by individual propulsors. Considering the issue of power distribution as a task of square and linear programming, two algorithms of thrust allocation were proposed and compared. The conducted model tests made it possible to evaluate their quality and efficiency in relation to speed and computational complexity.

**Keywords:** underwater vehicle, propulsion system, power distribution.

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### ARTICLE INFO

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PolHypRes 2018 Vol. 65 Issue 4 pp. 39 - 48

**ISSN:** 1734-7009 **eISSN:** 2084-0535

**DOI:** 10.2478/phr-2018-0022

**Pages:** 10, **figures:** 0, **tables:** 0

**page www of the periodical:** [www.phr.net.pl](http://www.phr.net.pl)

**Original article**

**Submission date:** 24.08.2018 r.

**Acceptance for print:** 19.10.2018 r.

**Publisher**

Polish Hyperbaric Medicine and Technology Society

## INTRODUCTION

Unmanned underwater vehicles (UUVs) play a significant role among the various technical devices used to penetrate the seas and oceans. With their propulsors and manoeuvrability, they are designed to perform tasks in the depths ranging from several to several thousand meters. Most often they play the role of floating platforms where various sensors and tools are mounted necessary to perform specific tasks and missions. These devices fall into the following categories:

- Remotely Operated Vehicles (ROVs), usually connected to the carrier vessel by means of a tethered cable (the umbilical) through which the entire communication takes place; drag on the umbilical, as a result of underwater currents for example, affects the movement of the vessel and can result in significant interference and loss of energy, steerability and manoeuvrability;
- Autonomous Underwater Vehicles (AUVs), self-propelled vessels with their own power sources and fully automated controls.

The development of ROVs led to the creation in the 1980s of a device called an Underwater Robotic Vehicle (URV). Usually it has a frame structure, and its standard equipment includes hydroacoustic observation devices, navigation systems and devices, manipulators, vision sensors and reflectors. They can penetrate the bottom in search of lost objects, inspect underwater parts of hydrotechnical structures, record underwater works and research as well as participate in search and rescue activities. Underwater robots have found a wide range of applications, both civil and military, in particular where human activity is impractical or undesirable.

## DESCRIPTION OF THE AUTOMATIC CONTROL SYSTEM

The movement of an URV with six-axis motion controllers will be described by means of the following vectors [1,2]:

$$\begin{aligned}\boldsymbol{\eta} &= [x, y, z, \phi, \theta, \psi]^T \\ \mathbf{v} &= [u, v, w, p, q, r]^T \\ \boldsymbol{\tau} &= [X, Y, Z, K, M, N]^T\end{aligned}\quad (1)$$

where:

$\boldsymbol{\eta}$  – vector of position and orientation of the robot in space,  
 $x, y, z$  – position coordinates,  
 $\phi, \theta, \psi$  – orientation coordinates,  
 $\mathbf{v}$  – linear and angular velocity vector,  
 $u, v, w$  – linear velocities along longitudinal, transversal and vertical axes,  
 $p, q, r$  – angular velocities about longitudinal, transversal and vertical axes,  
 $\boldsymbol{\tau}$  – vector of forces and torques of the forces acting on the robot,  
 $X, Y, Z$  – forces about longitudinal, transversal and vertical axes,  
 $K, M, N$  – torques along longitudinal, transversal and vertical axes.

Underwater robot control is usually executed from the control panel on the carrier ship. However, the multidimensionality of the controlled unit makes it difficult to train the operators and reduces the effectiveness of their work. For this reason, it is desirable to introduce an increasing use of automatic control, which would allow the performance of a number of routine

tasks without human intervention, such as: moving to a given point in tidal waters with limitations of the motion path, positioning at a given work place, performing simple operations using a manipulator (such as cutting a rope, mounting of a load, etc.). This is achieved by means of a control system, in which the role of the operator is limited to general supervision of the course of the mission and entering such commands as: coordinates of points of return, options for the tasks performed, commands to discontinue activities, etc.

The basic modules of the control system are shown in Figure 1. The basic element is the autopilot, which by comparing the current position of the controlled device with the set values determines the forces and torques  $\tau_z$  that must be produced by the propulsion system in order for the robot's behaviour to be in accordance with the assumed one. The corresponding thrust vector  $\mathbf{f}$  is calculated in the thrust distribution module and transmitted as a manipulated variable to the propulsion system.

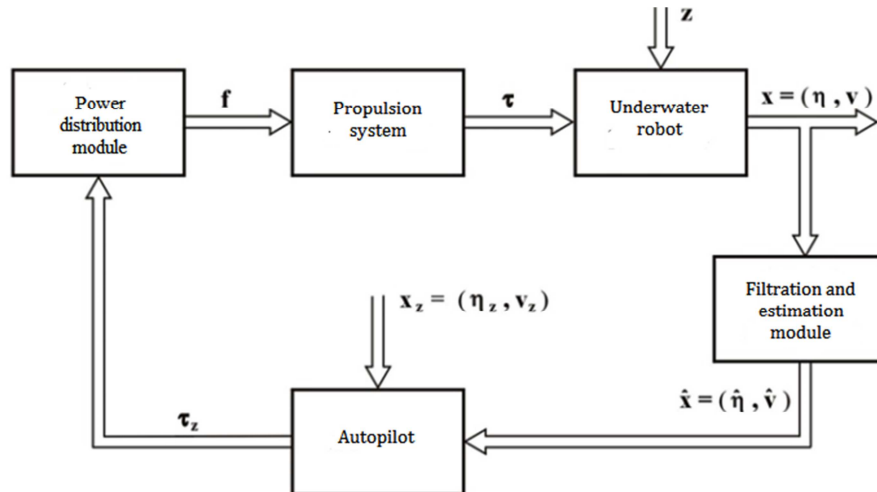


Fig. 1 General overview of an underwater robot control system.

### THRUST ALLOCATION PROCEDURE

The propulsors are a strong source of non-linearity of the dynamics of underwater robots, due to their mutual interaction during manoeuvres, which makes it difficult to maintain high quality control [3,4]. Therefore, from the point of view of both the design of the robot and its subsequent operation, issues related to the selection of propulsor configurations and the determination of the power distribution rules for individual propulsors are particularly important. For most conventional underwater robots, an accepted solution is a construction possessing longitudinal and transverse metacentric stability, which ensures motion with small angles the pitch and roll. Hence, the primary motion of this type of structure is horizontal movement with changes in the depth of submersion, i.e. four-axis motion.

The considerations contained in this work refer to a robot with a propulsion system consisting of six propulsors with the configuration shown in Figure 2. Such

a structure of the propulsion system enables its division into two independent subsystems, i.e.:

- a horizontal motion subsystem consisting of four propulsors arranged diagonally in relation to the longitudinal and transverse axes of symmetry, ensuring progressive motion along these axes and rotational motion around the normal axis,
- a vertical motion subsystem consisting of one or two propulsors arranged vertically and performing a forward motion along the normal axis.

Particularly interesting is the horizontal motion system, in which the required propelling forces and torques, i.e. forces  $X$  and  $Y$  and torque  $N$ , are linear combinations of the thrust forces generated by the four propulsors. Therefore, the task of determining the vector of thrust  $f$  can be formulated as an optimisation problem, in which the criterion is to obtain the given forces and torque  $\tau_z$  with the minimum values of thrust generated by the propulsors.

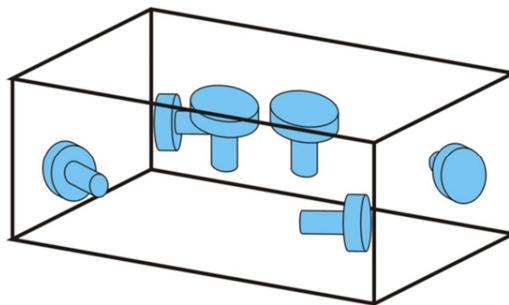


Fig. 2 The structure of a propulsion system with six propulsors.

This paper focuses on the problem of comparison of two algorithms of power distribution in the propulsion system of an underwater robot vehicle realising horizontal three-axis motion, considering the issue of thrust allocation as a task of square and linear programming.

### PROPULSOR DYNAMICS DESCRIPTION

The location of the URV's propulsors is determined in relation to the centre of its mass (Figure 3). In general, the relationship between the force and torque vector  $\tau$  and the thrust vector  $f$  of the propulsors is a combined non-linear function depending on, inter alia, the robot velocity vector, rotational speed of the propeller screw, water density [5].

In the case of flat robot motion, a commonly accepted simplification is the representation of the forces and torques as a function of the thrust generated by the propulsors using the following relation [1,6]:

$$\tau = T f \quad (2)$$

where:

$\tau = [\tau_X, \tau_Y, \tau_Z]^T$  – vector of forces and torque ( $\tau_X$  force along axis X,

$\tau_Y$  – force along axis Y,  $\tau_Z$  torque around axis Z),

$f = [f_1, f_2, f_3, f_4]^T$  – vector of thrust generated by propulsors,

$T$  – propulsor configuration matrix,

$$T = \begin{bmatrix} \cos(\alpha_1) & \cos(\alpha_2) & \cos(\alpha_3) & \cos(\alpha_4) \\ \sin(\alpha_1) & \sin(\alpha_2) & \sin(\alpha_3) & \sin(\alpha_4) \\ d_1 \sin(\alpha_1 - \varphi_1) & d_2 \sin(\alpha_2 - \varphi_2) & d_3 \sin(\alpha_3 - \varphi_3) & d_4 \sin(\alpha_4 - \varphi_4) \end{bmatrix}$$

$\alpha_i$  – the angle between the longitudinal axis of the robot and the direction of the thrust force of the  $i$ -th propulsor,

$d_i$  – distance of  $i$ -th propulsor from the robot's centre of mass,

$\varphi_i$  – the angle between the longitudinal axis of the robot and the line connecting the centre of mass with the centre of the axis of the  $i$ -th propulsor,  $i = 1, 4$ .

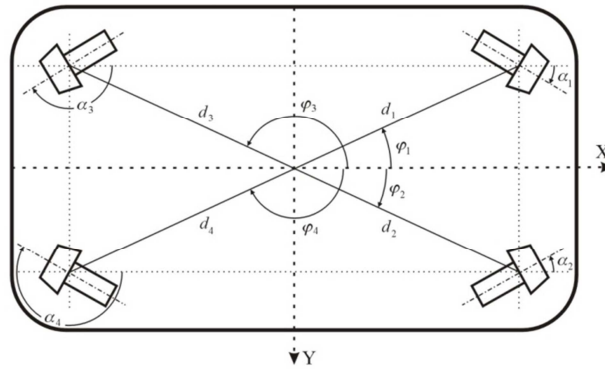


Fig. 3 The structure of propulsion system with six propulsors.

### POWER DISTRIBUTION AS A SQUARE PROGRAMMING TASK

Both literature studies [1,2,3] and own experiments [6] show that in practice the problem of thrust allocation in the propulsion system of an underwater robot vehicle is most often formulated as a square programming task in the following form:

determine

$$J = \min_f \frac{1}{2} f^T H f \quad (3)$$

with the limitations

$$\begin{aligned} \tau_z - T f &= 0 \\ -f &\leq -f_{\min} \\ f &\leq f_{\max} \end{aligned} \quad (4)$$

where:

$H$  – 4x4 positive definite diagonal matrix,

$$f_{\min} = [f_{1\min}, f_{2\min}, f_{3\min}, f_{4\min}]^T$$

$$f_{\max} = [f_{1\max}, f_{2\max}, f_{3\max}, f_{4\max}]^T$$

The main reason for considering this issue as a square programming task is the square representation of the power/thrust relationship. Under certain assumptions, this relationship can also be approximated by the linear function, which allows one to consider this optimisation problem as a linear programming task, whose computational complexity is significantly lower.

### POWER DISTRIBUTION AS A LINEAR PROGRAMMING TASK

The components of vector  $f$  have both positive and negative values, and therefore this has to be taken into account in the linear programming task, thus taking the following form:

determine

$$J = \min_f c^T |f| \quad (5)$$

with the limitations

$$\begin{aligned}\tau_z - \mathbf{T}\mathbf{f} &= \mathbf{0} \\ -\mathbf{f} &\leq -\mathbf{f}_{\min} \\ \mathbf{f} &\leq \mathbf{f}_{\max}\end{aligned}\quad (6)$$

where  $\mathbf{c}$  is a vector of non-negative components.

Since the cost function (5) contains absolute values, in order to apply the Simplex algorithm to solve the above optimisation problem it is necessary to transform the dependencies (5-6) to the following form: determine

$$J = \min_{\mathbf{u}} \mathbf{c}^T \mathbf{u} \quad (7)$$

with the limitations

$$\begin{aligned}\tau_z - \mathbf{T}\mathbf{f} &= \mathbf{0} \\ -\mathbf{f} &\leq \mathbf{u} \\ \mathbf{f} &\leq \mathbf{u} \\ -\mathbf{f} &\leq -\mathbf{f}_{\min} \\ \mathbf{f} &\leq \mathbf{f}_{\max}\end{aligned}\quad (8)$$

In the matrix denotation, the above linear programming task can be formulated as follows: determine

$$\min_{\mathbf{f}, \mathbf{u}} \begin{bmatrix} \mathbf{0}_{(1,4)} & \mathbf{c}^T \end{bmatrix} \begin{bmatrix} \mathbf{f} \\ \mathbf{u} \end{bmatrix} \quad (9)$$

with the limitations:

$$\begin{bmatrix} \mathbf{T} & \mathbf{0}_{(3,4)} \end{bmatrix} \begin{bmatrix} \mathbf{f} \\ \mathbf{u} \end{bmatrix} = \tau_d$$

$$\begin{bmatrix} -\mathbf{I}_{(4,4)} & -\mathbf{I}_{(4,4)} \\ \mathbf{I}_{(4,4)} & -\mathbf{I}_{(4,4)} \\ -\mathbf{I}_{(4,4)} & \mathbf{0}_{(4,4)} \\ \mathbf{I}_{(4,4)} & \mathbf{0}_{(4,4)} \end{bmatrix} \begin{bmatrix} \mathbf{f} \\ \mathbf{u} \end{bmatrix} \leq \begin{bmatrix} \mathbf{0}_{(4,1)} \\ \mathbf{0}_{(4,1)} \\ -\mathbf{f}_{\min} \\ \mathbf{f}_{\max} \end{bmatrix} \quad (10)$$

where  $\mathbf{I}_{(a \times b)}$  i  $\mathbf{0}_{(a \times b)}$  are respective identity and zero matrices of the dimension  $a \times b$ .

The use of the simplex algorithm to solve the above optimisation task, due to its small size, allows for the development of a fast calculation procedure for the allocation of thrust in a multi-propulsion system, which is particularly important from the point of view of its practical application.

## MODEL TESTS

Comparative studies of both power distribution algorithms were carried out for an underwater robot equipped with a propulsion system as shown in Figure 4.

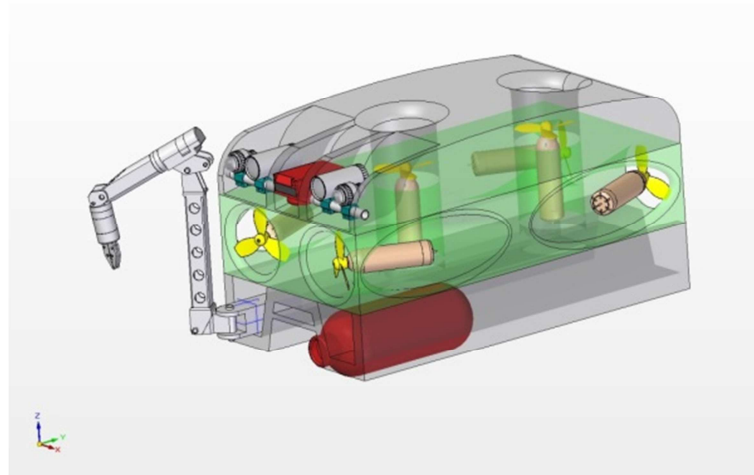


Fig. 4 Virtual image of an underwater robot.

The motion of the robot in the horizontal plane is carried out by four identical propulsors placed symmetrically in relation to the centre of mass and generating thrust up to  $\pm 1000$  N. The matrix of the configuration of the robot's propulsors  $\mathbf{T}$  is as follows:

$$\mathbf{T} = \begin{bmatrix} 0.875 & 0.875 & -0.875 & -0.875 \\ 0.485 & -0.485 & 0.485 & -0.485 \\ 0.332 & -0.332 & -0.332 & 0.332 \end{bmatrix} \quad (11)$$

Simulation studies were conducted in the MATLAB environment, using *linprog* functions for the linear programming task and *quadprog* functions for the square programming task [7]. Fig. 6 presents the values of the thrust vector  $\mathbf{f} = [f_1, f_2, f_3, f_4]^T$  determined using the *linprog* and *quadprog* functions for specified values of forces and torque  $\boldsymbol{\tau}_z = [\tau_{xz}, \tau_{yz}, \tau_{zz}]^T$  which have been illustrated in fig. 5. Based on the analysis of the calculated values of pressure vectors  $\mathbf{f}$  for both optimisation tasks, it was found that they are very similar.

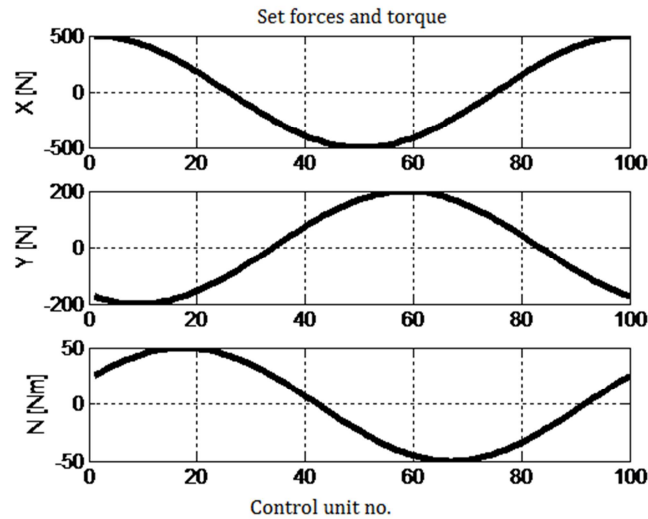


Fig. 5 The curves of the set forces  $X$  and  $Y$  and the torque  $N$ .

In order to compare the energy expenditure, both methods were evaluated using the following expression:

$$E = \sum_i (f_{1i}^2 + f_{2i}^2 + f_{3i}^2 + f_{4i}^2) \quad (12)$$

The performed calculations showed that for the method based on the linear programming task the  $E$  value is about 3÷5% lower than in the square programming task. The curve representing the thrust generated by the propulsors on the basis of linear programming is not as smooth as that obtained from square programming, as shown in Figure 6.

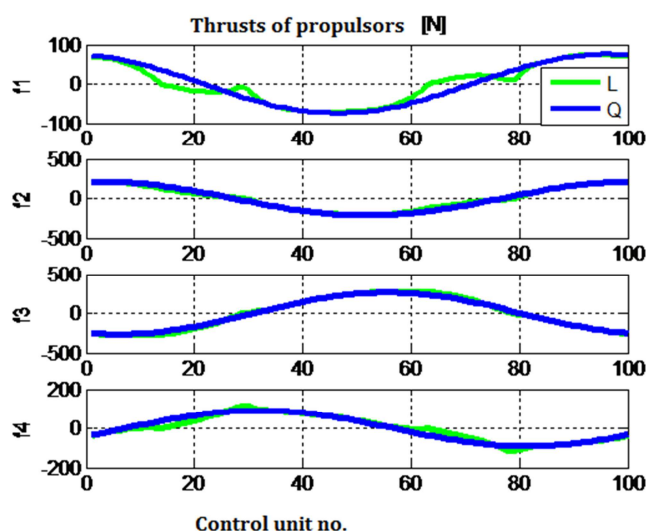


Fig. 6 Diagrams illustrating thrust generated by propellers on the basis of linear (L) and square (Q) programming tasks.

The conducted research has therefore shown that in the power distribution module of the URV's motion control system, an optimisation procedure based not only on square programming, but also on linear programming can be applied in relation to thrust allocation tasks. This is particularly important for small, but now very common, low-cost underwater robots (LC URV - Low Cost URV) where the on-board computer is of limited computing power.

## CONCLUSIONS

The proposed algorithms for the allocation of thrust in the multi-propeller propulsion system of an underwater robot are a continuation of research aimed at

developing a reliable and efficient system for controlling its motion.

The optimisation algorithms considered in the work, based on square and linear programming, allow for optimal power distribution in the underwater robot vehicle's propulsion system.

From the practical point of view of the implementation of the considered algorithms in an on-board computer with limited computing power, it seems more convenient to apply an allocation algorithm using linear programming, as it allows for the development of a simple and fast procedure for determining the vector of thrust at comparable control costs.

## REFERENCES

1. T. I. Fossen. *Guidance and control of ocean vehicles*. Wiley and sons, Chichester 1994;
2. T. I. Fossen. *Handbook of marine craft hydrodynamics and motion control*. Wiley and sons, Chichester 2011;
3. T. I. Fossen, T. A. Johansen, T. Perez. A Survey of Control Allocation Methods for Underwater Vehicles. In: *Underwater Vehicles* (A. V. Inzartsef, Ed.), In-Tech Education and Publishing, Vienna 2009, pp. 109-128;
4. J. Małecki. Model of Propeller for the Precision Control of Marine Vehicle. *Solid State Phenomena*, 2012, vol. 180, pp. 323-330;
5. A. Charchalis. *Thrust and propulsion systems in warships*. Akademia Marynarki Wojennej, Gdynia 2001;
6. J. Garus. Optimization of thrust allocation in propulsion system of underwater vehicle. *International Journal of Applied Mathematics and Computer Science*, 2004, vol. 14, no. 4, pp. 461-467;
7. B. Mrozek, Z. Mrozek. *MATLAB and Simulink*. Helion, Gliwice 2010.

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