Modeling of Cash Flows from Nonperforming Loans in a Commercial Bank

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Abstract

The purpose of this paper is to derive a model for calculation of maturities and volumes of repayments that a bank may expect from nonretail nonperforming loans (hereafter NPLs). Expected inflows from nonretail NPLs follow a probability distribution, defined by size and timing of historic repayments of NPLs. Empirical analysis has shown that probability distribution of expected inflows from nonretail NPLs considerably deviates from symmetric distribution and is asymmetric to the right. Accuracy of derived model depends upon available data in banks about NPLs by corporate sectors and recovery rates by time intervals. The model in this paper is in interest of any bank and in particular of banks with a higher fraction of NPLs in their loan portfolio. Contribution of this paper to the added value in the area of liquidity risk management in banks is high because the remaining literature does not deliver other models for the same purpose.

Keywords: bank, liquidity risk, cash flow modeling, credit risk, non-performing loans

Introduction

An NPL is a loan that is subject to late repayment or is unlikely to be repaid by the borrower. Each NPL not only reduces profitability of the bank but deteriorates its liquidity situation. If the NPL portfolio increases, the bank needs to increase loan loss provisions. Loan loss provisions reduce bank profits. If enlargement of loan loss provisions due to the NPL portfolio increase is greater than operating profit, the bank ends up with a loss. And a loss over a number of subsequent time horizons requires owners to increase capital of the bank. Therefore, appearance of each new NPL in the loan portfolio of a bank reduces equity value of bank owners. More about equity value and goal of corporate operations can be found in Levy and Sarnat (1977).

Business environments with low interest rates are particularly demanding for banks, as low interest rates reduce interest income and consequently operating profit of a bank. The lower operating profit of a bank, the sooner will the bank end up with a loss at a given level of loan loss provisions. Enlargement of the NPL portfolio will additionally worsen a bank's position as appearance of an NPL stops contractual interest payments. Interest income of the bank will therefore additionally decline. The larger the NPL portfolio of a bank, the lower is the bank's interest income. This paper is organized as follows. We will first review available literature, relevant to our research. Then, we will build a model for calculation of maturities and volumes

of repayments, which a bank may expect from NPLs. Based on empirical data about nonretail NPLs from a sample bank, we will then estimate probability distribution of expected inflows from NPLs. In the following section, we will review results and discuss them. In the last section, we will provide a conclusion, which also shows model limitations and recommendations for further research.

Literature Review

Absence of interest payments deteriorates a bank's liquidity. But this is not the only reason why a bank's liquidity deteriorates in the case the NPL portfolio of the bank increases. At the moment of NPL identification become contractual maturities and amounts of repayments random variables. Cutoff times and corresponding amounts of repayments in the future are linked to probabilities, which vary. A bank can steer the level of probabilities with the quality of its internal workout process. Guo, Jarrow, and Zeng (2009) explained that the earlier a bank identifies an NPL, the higher is the expected recovery rate. The authors assumed that default triggers the recovery rate process, which, if triggered, depends on asset value of the firm. If the debt matures before the firm becomes insolvent (defined as the firm's asset value falling below an insolvency barrier), then the debt is paid in full or at some fractional level. The fractional recovery when the firm is solvent exceeds the amount that would be paid if the firm becomes insolvent and enters bankruptcy. Guo, Jarrow, and Zeng (2009) also showed that, if the firm's asset value is below an insolvency threshold at the time of default, then the default and bankruptcy intensity are equal. However, if the asset value is above or equal to this critical level, then the default and bankruptcy intensities are distinct, and default does not necessarily lead to immediate bankruptcy.

Carey and Gordy (2007) offered a model and evidence that private debtholders play a key role in setting the endogenous asset value threshold below which corporations declare bankruptcy. The model, in the spirit of Black and Cox (1976), implies that the recovery rate at emergence from bankruptcy on all of a firm's debt is related to the pre-bankruptcy share of private debt in all of a firm's debt.

Recovery rates depend on general activity of business subjects in the economy. The number of defaulting firms in recessions rises and average recovery rate decreases (Bruche & Gonzales-Aguado, 2010). Trück *et al.* (2005) also explain that recoveries in recessions are much lower in comparison with times of economic expansion. Carey (1998) found that, especially for risky loans, recessions have an enormous impact on the distribution of recovery rates. According to his findings, this is especially true for the tails of the loss distribution.

While for investment-grade loans, the cyclical effect is rather small; thus, the author found that loss rates for subinvestment grade loans during a recession are more than 50% higher than during an expansion of the economy. Deshpande and Iyer (2009) explored correlations between loss rates and proposed a model for measurement of credit concentration risk and for calculation of the portfolio loss distribution.

Despite abundant literature on the determinants of default on loans and other debt instruments, relatively little is known about the factors that influence bank recoveries following default. However, recovery is critical to bank performance as well as to the proper measure of the capital needed to buffer against risk. Therefore, Khieu *et al.* (2012) identified determinants of bank loan recovery rates. The authors show that loan characteristics are, in general, more significant determinants of recovery rates than borrower characteristics prior to default. A variety of loan contract features are strongly related to the ultimate payoff for creditors. Secured loans have higher recoveries, especially when the collateral takes the form of inventories and accounts receivable. Loans to borrowers with prior defaults yield higher recoveries than first-time defaults, and arranging a prepackaged bankruptcy increases recoveries. Loan recoveries vary significantly and nonlinearly with the length of time to emerge. Macroeconomic conditions significantly affect recovery prospects, but the probability of default at the time of loan origination is unrelated to ultimate recoveries.

In an annual default study, Moody's (2013) explained the corporate default and recovery rates on a time horizon between 1920 and 2012. Measured by post-default trading prices, the average recovery rate for senior unsecured bonds in 2012 rose to 43,4% from 39,7% in 2011.

If the size of the NPL portfolio has raised rapidly, a liquidity crisis becomes inevitable for a bank (Sohaimi, 2013). Appearance of a new NPL deteriorates a bank's liquidity because cutoff times and amounts of future repayments for this loan deviate from contractually agreed maturities and amounts of repayments. As NPL appears due to inability of the obligor to pay on time, we can expect future repayments to occur later than contractually agreed. This reduces liquidity surplus of a bank in the short-term and increases the probability of illiquidity. The impact on probability is lower if an NPL is an amortizing loan and not a bullet loan. Consequently, it holds that, among two completely identical banks, which differ among themselves only in type of loans in the loan portfolio, where the first bank only has amortizing loans and the second bank only has bullet loans, exposure to liquidity risk is lower in the first bank.

We can conclude that the timing and size of future repayments of a new NPL based upon the timing and size of

historic repayments of NPLs, which have either been already repaid or written off by the bank (historic NPLs). We are going to show a possible approach in this paper.

The Model

Assume a bank maintains a database, in which all NPLs are included. Also assume for each NPL the following data are available: exposure at default, default start date, default end date, cut-off date, and amount of each payment between default start and default end date, and exposure at default end date. If an NPL has a default end date, then recovery process for this particular loan was finished. This happened either because the counterparty fully repaid its debt or because the outstanding debt was written off. In the first case, exposure at the default end date was zero; in the second case, exposure at default end date was positive, but the bank wrote off the remaining debt, as it expected no additional repayment. On the other hand, if an NPL does not have a default end date in the database, then this NPL is a pending NPL. For the NPL cash flow modeling, pending NPLs in the database cannot be considered, as the default end date for pending NPLs is unknown. Only historic NPLs can therefore be used for modeling of cash flows from NPLs.

BIS (2008) defined a set of principles for sound liquidity risk management and supervision. Principle 5 defines that a bank should have a sound process for identifying, measuring, monitoring, and controlling liquidity risk. This process should include a robust framework for projecting cash flows arising from assets, liabilities, and off-balance sheet items over an appropriate set of time horizons. The number of time horizons should correspond to a variety of factors. These include vulnerabilities to changes in liquidity needs and funding capacity on an intraday basis, day-to-day liquidity needs and funding capacity over short – and medium-term horizons up to one year, longer-term liquidity needs over one year and vulnerabilities to events, activities, and strategies that can put a significant strain on internal cash generation capability.

Assume that a bank is in line with principles for sound liquidity risk management and supervision calculates net cash flows over a number of predefined time intervals, which are in general defined as $(t_{i-1}, t_i]$, $i \in \{1, 2, …, n\}$.

For each historic NPL, a bank should first calculate the number of all payments in a time horizon between default start date and default end date. Each payment is an inflow for the bank and reduces outstanding exposure to the counterparty in default. Then, cut-off dates of payments should be observed, and the time difference between cutoff date of each payment and default start date should be calculated. Let m be the number of all payments between the default start date and default end date. Time difference δ_j between cutoff date c_j of payment p_j and default start date d_s is in general defined as $\delta_j = c_j - d_s$, where $j \in \{1, 2, …, m\}$. Calculated δ_j indicate into which time interval payment p_j of an NPL should be mapped. Assume the bank defined n time intervals. Then, payment p_j should be mapped to time interval i, where $i \in \{1, 2, …, n\}$ such that $t_{i-1} < \delta_j \leq t_i$. All m payments p_j of an NPL should be mapped to corresponding time intervals the bank has defined for management of liquidity risk and for calculation of net cash flows. Then, payments p_j should be aggregated by time intervals such that only one payment per time interval exists. Let l_i be the number of all payments p_j, $j \in \{1, 2, …, m\}$, which meet the criteria $t_{i-1} < \delta_j \leq t_i$. Aggregated payment p_i for time interval $i \in \{1, 2, …, n\}$ is defined as the sum $p_i = \sum_{k=1}^{l_i} p_{j_k}$, where $j_k \in \{1, 2, …, m\}$ and $t_{i-1} < \delta_{j_k} \leq t_i$ for every $k \in \{1, 2, …, 1_i\}$.

If w is the number of all NPLs in the bank, then the sum of all payments s in time interval $i \in \{1, 2, …, n\}$ equals $s_i = \sum_{k=1}^{w} p_{ik}$. Consequently, $\sum_{i=1}^{n} s_i$ equals total recovery value, which, in comparison with the NPL portfolio value, gives an expected total recovery rate g for an NPL in the bank. For modeling of cash flows from NPLs, expected recovery rate r_i per time interval $i \in \{1, 2, …, n\}$ should be calculated.

Expected recovery rate r_i, $i \in \{1, 2, …, n\}$ is defined with the equation

$$r_i = \frac{\sum_{k=1}^{w} p_{ik}}{\sum_{i=1}^{n} s_i} \cdot g = \frac{s_i}{\sum_{i=1}^{n} s_i} \cdot g = \frac{g \cdot s_i}{\sum_{i=1}^{n} s_i} .$$

Payments p_{ik} for each $k \in \{1, 2, …, w\}$ in the NPL portfolio of the bank should be distributed over all $i \in \{1, 2, …, n\}$ time intervals. Therefore, in case of full recovery, it holds $\sum_{i=1}^{n} r_i = 1$, as are expected write-offs equal to zero; therefore, is $g = 1$. In case expected write-offs differ from zero and holds $g < 1$, consequently, it also holds $\sum_{i=1}^{n} r_i = g$.

Recovery rate r_i is an inflow from each cash unit, which is expected from an NPL in time interval $i \in \{1, 2, …, n\}$.

EU Commission adopted the Basel 3 framework and in the *Official Journal of the European Union* (L 176, 2013) published a corresponding framework for the supervision of credit institutions, investment firms, and their parent companies in all member states of the European Union and the EEA.

This framework is capital requirements directives CRD 4, which replace capital requirements directives 2006/48 and 2006/49. They are composed of the capital requirements directive (hereafter CRD) and of the capital requirements regulation (hereafter CRR) (Ernst & Young, 2011).

Default of a nonretail obligor is defined in Article 178 of the CRR. A default of a particular obligor shall be considered to have occurred when the institution considers that the obligor is unlikely to pay its credit obligations to the institution, the parent undertaking, or any of its subsidiaries in full, without recourse by the institution to actions such as realizing security, or when the obligor is past due more than 90 days on any material credit obligation to the institution, the parent undertaking or any of its subsidiaries.

Empirical Data and Analysis

Probability distribution of expected inflows from NPLs will now be estimated empirically. For this purpose, we are going to use data about NPLs from the database of a sample bank. A database with NPLs of the bank only includes nonretail NPLs; therefore, probability distribution of expected inflows from nonretail NPLs will be estimated. For estimation of probability distribution, we need for each NPL recovery rate and data about size and timing of all repayments. Recovery rate is only available for historic NPLs. The same is true for data about size and timing of all repayments. Consequently, we should only use data about historic NPLs from database of the bank.

Average recovery rate and average repayment speed depend on general activity of business subjects in the economy (Bruche & Gonzales-Aguado, 2010). The higher general activity of business subjects in the economy, the higher is the average recovery rate and average repayment speed. In order to estimate time invariable probability distribution of expected inflows from NPLs, historic NPLs with default start dates on time horizon over at least one economic cycle should be considered.

Modeling of inflows from NPLs connects liquidity with a bank's credit risk management. Connection exists in the area of models for loss given default (hereafter LGD) estimation. Therefore, the minimal spread between maximum and minimum default start dates of historic NPLs is defined with CRR requirements for one's own LGD estimates. CRR requires that banks estimate LGDs by facility grade or pool on the basis of the average realized LGDs by facility grade or pool using all observed defaults within the data sources (default weighted average). For exposures to corporates, institutions, and central governments and central banks, own LGD estimates shall be based on data over a minimum of five years (The European Parliament, 2013).

Consequently, we conclude that the difference between maximum and minimum default start date of historic NPLs should be at least five years. However, five years might not be enough to calculate time invariable probability distribution of inflows from NPLs—particularly not in times with high volatility of GDP in the economy. In the case of one's own LGD estimations, CRR requires that banks use LGD estimates, which are appropriate for an economic downturn if those are more conservative than the long-run average. Consequently, the same also holds for differences between default end date and default start date at historic NPLs.

High volatility of GDP since 2007 helps to conclude that historic NPLs with default start dates over last five years do not cover a sufficiently long-time horizon for estimation of time invariable probability distribution of expected inflows from NPLs. Minimum historic observation period should therefore be much longer than five years.

Assume that the bank defined time intervals for calculation of liquidity position as follows: one day, two days, three days, four days, five days, six days, seven days, eight days, nine days, 10, days, 11 days, 12 days, 13 days, 14, days, 15 days, 30 days, two months, three months, six months, one year, two years, three years, four years, five years, seven years, 10 years, and more than 10 years. Because we assume that a time interval for liquidity positions with remaining maturities over 10 years exists, also NPLs with difference between default end dates and default start dates of more than 10 years should be considered. If the last condition is not met, then it is not possible to estimate the probability distribution of expected inflows from NPLs on the time horizon beyond maximum difference between default end date and default start date.

Lower bound of the open interval is important because it defines maximum tenor of a liquidity position, which a bank would like to include in the management of liquidity risk. The higher the lower bound of the open interval, the higher is precision of the bank with management of the structural liquidity risk.

Results and Discussion

For each historic NPL in the database of the bank, we are now going to calculate the difference between the default end date and default start date. Consequently, obtained results are values of the variable X, i.e., the difference between default end date and default start date, which we can describe with the following descriptive statistics.

Maximum difference between default end date and default start date is 2948 days, which is $\frac{2948}{365} = 8{,}1$ years. This

Table 1. Descriptive Statistics for Difference Between Default End Date and Default Start Date

Statistics		
difference between default end date and default start date		
N	Valid	375
	Missing	0
Mean		829,55
Median		727,00
Mode		422
Std. Deviation		607,333
Variance		368853,938
Skewness		,626
Std. Error of Skewness		,126
Minimum		1
Malximum		2948

Source: Author's calculation.

measure of dispersion explains that we will not be able to estimate probability distribution of inflows from NPLs over a time horizon with a minimum tenor of over 10 years. It also explains that calculated probability distribution of expected inflows from NPLs will correspond to economic downturn and will therefore not be time invariable, however conservative and therefore appropriate for estimation of expected inflows from NPLs.

Mean explains average difference between default end date and default start date. On average, the bank in the last 8,1 years needed $\frac{829,55}{365} = 2,3$ years to reach default end date after identification of a NPL.

We can show frequency distribution of differences between default end date and default start date also with a histogram.

From Figure 1, we can see that the distribution of differences between default end date and default start date is asymmetric to the right. The skewness statistics, as defined by Campbell, Lo, and MacKinlay (1997) for a population with mean μ and standard deviation δ, is

$$S = \frac{1}{(n-1) \cdot \sigma^3} \cdot \sum_{i=1}^{n}(y_i - \mu)^3 = 0,626.$$

Result shows explicit deviation from symmetric distributions, for which holds $S = 0$. This is in line with expectations, as, on the one hand, some NPLs are simple in complexity and can be therefore repaid quickly; on the other hand, some NPLs are complex and require more time until final repayment.

Figure 1. Frequency Distribution of Differences Between Default End Date and Default Start Date
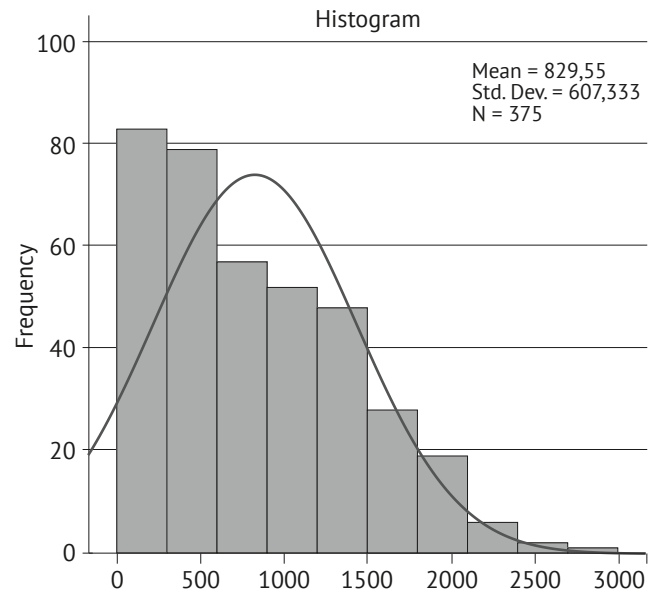
Difference between default end date and defaulst start date
Source: Author's calculation.

Assume that expected total recovery rate for an NPL is $g = 1$. With a mathematical model for calculation of expected recovery rates r_i by time intervals, we can now estimate probability distribution of expected inflows from nonretail NPLs on time intervals, which were defined by the bank for calculation of liquidity positions. Results are shown in Figure 2.

Figure 2. Probability Distribution of Expected Inflows from Nonretail NPLs by Time Intervals of the Bank
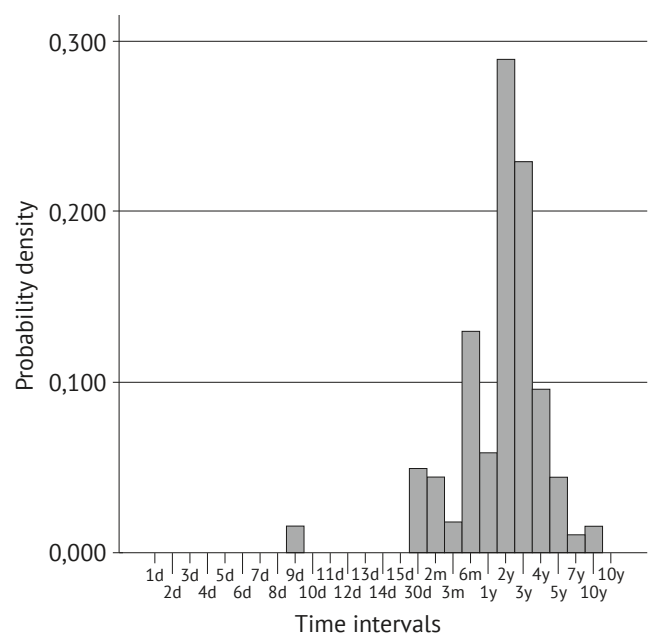
Source: Author's calculation.

If write-offs differ from zero and total recovery rate for a NPL is $g < 1$, then expected recovery rate r_i per time interval $i \in \{1, 2, \ldots, n\}$ reduces to $\sum_{i=1}^{n} r_i = g < 1$ and by definition also

$$r_i = \frac{\sum_{k=1}^{w} p_{ik}}{\sum_{i=1}^{n} s_i} \cdot g = \frac{\sum_{k=1}^{w} p_{ik}}{\sum_{i=1}^{n} s_i} \cdot \sum_{i=1}^{n} r_i \Rightarrow \sum_{k=1}^{w} p_{ik} = s_i = \frac{r_i \cdot \sum_{i=1}^{n} s_i}{\sum_{i=1}^{n} r_i} \quad .$$

However, the probability distribution of expected inflows from nonretail NPLs by time intervals of the bank remains unchanged.

Conclusion

Modeling of inflows from NPLs connects liquidity with a bank's credit risk management. Data requirements for modeling of inflows from NPLs are therefore implicitly defined with CRR requirements for one's own LGD estimates. Based on sample data about historic nonretail NPLs of a bank, we found that probability distribution of expected inflows from nonretail NPLs considerably deviates from symmetric distribution and is asymmetric to the right. The result is in line with expectations, as, on the one hand, some nonretail NPLs are simple in complexity and can be repaid quickly; on the other hand, some nonretail NPLs are complex and require more time until final repayment and default end date.

The first limitation of the model presented in this paper is linked to available data in banks about NPLs. Banks were not collecting data about NPLs and, therefore, are available time series of data short. Only recent regulations in the area of credit risk management, which offers to banks a possibility for construction of their own models for credit risk measurement, introduce new requirements for data collection. A longer data series, especially data series over a few economic cycles, will enable derivation of a more stable and time invariable probability distribution of expected inflows from NPLs. The second limitation of this model is its generality. There is only one model for all corporate sectors, but recovery rates and repayment possibilities by corporate sectors may be different. Consequently, this is one recommendation for future research and development. The third limitation of this model is equal recovery rate by time intervals. Various studies have shown that recovery rates reduce as time passes after default start date. Guo, Jarrow, and Zeng (2009) explain that the earlier a bank identifies an NPL, the higher is the expected recovery rate. Consequently, the key impact on model quality has identification of an NPL and hence definition of a default start date, which reflects the quality of credit risk management function in a bank. Finally, a similar model could be developed for a retail NPL, which is another recommendation for future research and development.

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Modeliranje denarnih tokov iz slabih posojil v komercialni banki

Izvleček

Namen tega članka je izpeljati model za izračun zapadlosti in obsega odplačil, ki jih lahko banka pričakuje iz nepotrošniških slabih posojil (NPL). Pričakovani prilivi iz nepotrošniških NPL-jev sledijo verjetnostni porazdelitvi, opredeljeni z velikostjo in izbiro pravih trenutkov zgodovinskih odplačil NPL-jev. Empirična analiza je pokazala, da verjetnostna porazdelitev pričakovanih vplačil nepotrošniških NPL-jev znatno odstopa od simetrične porazdelitve in je asimetrična v desno. Natančnost izpeljanega modela je odvisna od razpoložljivih bančnih podatkov o NPL-jih korporativnih sektorjev in stopnjah vračil po časovnih intervalih. V tem članku izoblikovan model je v interesu katerekoli banke, še posebej bank z višjimi deleži NPL-jev v njihovem posojilnem portfelju. Dodana vrednost tega članka se kaže na področju upravljanja tveganja likvidnosti v bankah, saj v preostali literaturi ni drugega modela za isti namen.

Ključne besede: banka, tveganje likvidnosti, modeliranje denarnega toka, kreditno tveganje, slaba posojila