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A Method of *m*-Point Sinusoidal Signal Amplitude Estimation

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The paper presents a new and original method of *m*-point estimation of sinusoidal signal amplitude. In this method, an *m*-point estimator is calculated on the basis of *m* initial signal samples. The way the estimator is constructed is explained. It is shown that the starting point for constructing the estimator is two initial signal samples. Next, in order to determine the estimator general form, three and *m* subsequent initial signal samples appearing in a signal period are used. Some special cases of an estimator are considered. Such an estimator is compared with a four-point estimator proposed by Vizireanu and Halunga. It is shown that the *m*-point estimator makes it possible to estimate the signal amplitude more accurately.

Keywords: Sinusoidal signal, amplitude, three and four samples, estimator, maximum error.

1. INTRODUCTION

Signal estimation methods can be divided into two main groups. The first group includes methods for parameter estimation in the time domain (least squares method, correlation methods) [1]. The second group includes methods for parameter estimation in the frequency domain (FFT and DFT) [2]. Each of these methods is characterized by a specific level of computational complexity. This level depends on the amount of data or on the number of the operations which lead to determining the parameters. Generally, an increasing amount of data or operations results in an increased level of the method accuracy, but also in extended calculation time. In measurement systems with a digital measurement algorithm in which, besides the accuracy, also the cost and calculation time count, it is suggested that methods making it possible to estimate signal parameters on the basis of the least possible number of samples should be used. Such methods include point methods making it possible to estimate signal parameters in the time domain on the basis of signal samples.

The subject matter of the paper focuses on the issue of sinusoidal signal amplitude. A new and original method of *m*-point amplitude estimation is presented. In this method, an *m*-point amplitude estimator is calculated on the basis of *m* initial signal samples. The way the estimator is constructed is described. It is shown that the starting point for constructing the estimator is two initial signal samples. Next, in order to determine the estimator general form, three and *m* subsequent initial sinusoidal signal samples occurring in the signal period are used. In the literature, point estimators of sinusoidal signal parameters are amply represented and find many practical applications [3]-[5]. Especially noteworthy are the estimators proposed by Vizireanu and Halunga, among which a four-point signal amplitude estimator ought to be singled out [6]. Particular cases of an *m*-point estimator are considered. Such an estimator is compared with the four-point Vizireanu and Halunga estimator. In order to compare the estimators, simulations and a measurement experiment have been carried out consisting of the acquisition of samples of the sinusoidal signal. Maximum errors of the estimators are determined. Based on this, it is shown that the *m*-point estimator makes it possible to estimate the signal amplitude more accurately.

2. SINUSOIDAL SIGNAL AND ITS SAMPLES

Let $x(t)$, $t\in\mathbf{R}$ be a sinusoidal signal with the amplitude $A\in\mathbf{R}_+\backslash\{0\}$, the DC component $A_0\in\mathbf{R}$, the period $T\in\mathbf{R}_+\backslash\{0\}$ and the initial phase $\varphi\in\mathbf{R}$. Then

$$x(t) = A_0 + A\sin\left(\frac{2\pi}{T}t + \varphi\right) \tag{1}$$

Let us assume that the signal $x(t)$ is uniformly sampled with the number of samples $M\in\mathbf{N}\backslash\{0, 1\}$. Then the samples

$$\mathrm{x}[i] = A_0 + A\sin\left(2\pi\frac{f}{f_s}i + \varphi\right),\ i = 0,1,\ldots,N\cdot M - 1 \tag{2}$$

where $f = 1/T$, f_s, and $N\in\mathbf{N}\backslash\{0\}$ are, respectively, the frequency, the sampling frequency, and the number of signal periods. Synchronous sampling being the case, the frequency f_s is an integer M-multiple of the basic frequency f. It means that

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$$x[i] = A_0 + A\sin\left(\frac{2\pi}{M}i + \varphi\right) \quad (3)$$

3. METHODS OF POINT ESTIMATION OF SINUSOIDAL SIGNAL AMPLITUDE

A. Three-point method

In the three-point method, an estimator enabling to estimate amplitude A on the basis of three initial samples $x[i]$ of signal $x(t)$ is constructed. The starting point for constructing the estimator is two initial signal samples.

Let $2 \le m < M$, $m \in \mathbf{N}\backslash\{0, 1\}$ be the number of equations in an equation system which can be defined on the basis of (3). Let us assume $m = 2$ and $A_0 = 0$. Then $M \ge 3$ and

$$\begin{aligned} x[0] &= A\sin(\varphi) \\ x[1] &= A\sin\left(\frac{2\pi}{M} + \varphi\right) \end{aligned} \quad (4)$$

Solving system of equations (4) with respect to A, we obtain

$$A_1^{(2p)} = \frac{x[0] + x[1]}{\sin(\varphi)\left(1 + \cos\left(\frac{2\pi}{M}\right)\right) + \cos(\varphi)\sin\left(\frac{2\pi}{M}\right)} \quad (5)$$

Assuming

$$\begin{aligned} Z_1^{(2p)} &= 1 + \cos\left(\frac{2\pi}{M}\right) \\ Z_2^{(2p)} &= \sin\left(\frac{2\pi}{M}\right) \\ Z_3^{(2p)} &= x[0] + x[1] \end{aligned} \quad (6)$$

then on the basis of (4)-(6), we obtain the equation

$$A_1^{(2p)} = \frac{Z_3^{(2p)}}{\frac{x[0]}{A} Z_1^{(2p)} + \operatorname{sign}(\cos(\varphi))\sqrt{1 - \left(\frac{x[0]}{A}\right)^2}\, Z_2^{(2p)}} \quad (7)$$

where $\operatorname{sign}(z)$, $z \in \mathbf{R}$ is the signum function [7]. Substituting amplitude A for $A_1^{(2p)}$ and solving equation (7) with respect to A, we obtain that

$$A_1^{(2p)} = \left(x^2[0]\left(\left(\frac{Z_1^{(2p)}}{Z_2^{(2p)}}\right)^2 + 1\right) - 2x[0]\frac{Z_1^{(2p)} Z_3^{(2p)}}{\left(Z_2^{(2p)}\right)^2} + \left(\frac{Z_3^{(2p)}}{Z_2^{(2p)}}\right)^2 \right)^{\frac{1}{2}} \quad (8)$$

We proceed similarly in the case when $m = 3$ and $A_0 = 0$. Then $M \ge 4$, and the system determined on the basis of (3) assumes the form

$$\begin{aligned} x[0] &= A\sin(\varphi) \\ x[1] &= A\sin\left(\frac{2\pi}{M} + \varphi\right) \\ x[2] &= A\sin\left(\frac{4\pi}{M} + \varphi\right) \end{aligned} \quad (9)$$

Solving system of equations (9) with respect to A, we obtain

$$A_1^{(3p)} = \frac{x[0] + x[1] + x[2]}{D} \quad (10)$$

Where

$$\begin{aligned} D = {} & \sin(\varphi)\left(1 + \cos\left(\frac{2\pi}{M}\right) + \cos\left(\frac{4\pi}{M}\right)\right) \\ & + \cos(\varphi)\left(\sin\left(\frac{2\pi}{M}\right) + \sin\left(\frac{4\pi}{M}\right)\right) \end{aligned} \quad (11)$$

Assuming

$$\begin{aligned} Z_1^{(3p)} &= 1 + \cos\left(\frac{2\pi}{M}\right) + \cos\left(\frac{4\pi}{M}\right) \\ Z_2^{(3p)} &= \sin\left(\frac{2\pi}{M}\right) + \sin\left(\frac{4\pi}{M}\right) \\ Z_3^{(3p)} &= x[0] + x[1] + x[2] \end{aligned} \quad (12)$$

then on the basis of (9)-(12), we obtain the equation

$$A_1^{(3p)} = \frac{Z_3^{(3p)}}{\frac{x[0]}{A} Z_1^{(3p)} + \operatorname{sign}(\cos(\varphi))\sqrt{1 - \left(\frac{x[0]}{A}\right)^2}\, Z_2^{(3p)}} \quad (13)$$

Substituting amplitude A for $A_1^{(3p)}$ and solving equation (13) with respect to A, we obtain that

$$A_1^{(3p)} = \left(x^2[0]\left(\left(\frac{Z_1^{(3p)}}{Z_2^{(3p)}}\right)^2 + 1\right) - 2x[0]\frac{Z_1^{(3p)} Z_3^{(3p)}}{\left(Z_2^{(3p)}\right)^2} + \left(\frac{Z_3^{(3p)}}{Z_2^{(3p)}}\right)^2 \right)^{\frac{1}{2}} \quad (14)$$

Since functions $\sin(2\pi/M)$, $\cos(2\pi/M)$, $\sin(4\pi/M)$, and $\cos(4\pi/M)$ in equations (6) and (12) are not represented by samples $x[i]$ of signal $x(t)$, then quantities (8) and (14) do not yet constitute descriptions of point estimators. Note though that the sample

$$\mathrm{x}[2] = A\sin\left(\frac{4\pi}{M}+\varphi\right) = 2\cos\left(\frac{2\pi}{M}\right)\mathrm{x}[1]-\mathrm{x}[0] \tag{15}$$

From the above equation it follows that [8]

$$\cos\left(\frac{2\pi}{M}\right) = \frac{\mathrm{x}[0]+\mathrm{x}[2]}{2\mathrm{x}[1]} \tag{16}$$

Since for $M \geq 3$ the function $\sin(2\pi/M) > 0$, then based on (16) we obtain

$$\begin{aligned}
\sin\left(\frac{2\pi}{M}\right) &= \sqrt{1-\frac{(\mathrm{x}[0]+\mathrm{x}[2])^2}{4\mathrm{x}^2[1]}} \\
\sin\left(\frac{4\pi}{M}\right) &= \frac{\mathrm{x}[0]+\mathrm{x}[2]}{\mathrm{x}[1]}\sqrt{1-\frac{(\mathrm{x}[0]+\mathrm{x}[2])^2}{4\mathrm{x}^2[1]}} \\
\cos\left(\frac{4\pi}{M}\right) &= \frac{(\mathrm{x}[0]+\mathrm{x}[2])^2}{2\mathrm{x}^2[1]}-1
\end{aligned} \tag{17}$$

Substituting the expressions in equation (17) for the expressions in equation (6), we obtain

$$\begin{aligned}
Z_1^{(2p)} &= 1+\frac{\mathrm{x}[0]+\mathrm{x}[2]}{2\mathrm{x}[1]} \\
Z_2^{(2p)} &= \sqrt{1-\frac{(\mathrm{x}[0]+\mathrm{x}[2])^2}{4\mathrm{x}^2[1]}} \\
Z_3^{(2p)} &= \mathrm{x}[0]+\mathrm{x}[1]
\end{aligned} \tag{18}$$

We proceed similarly with equation (12). Substituting the expressions in equation (17) for the expressions in equation (12), we obtain

$$\begin{aligned}
Z_1^{(3p)} &= \frac{\mathrm{x}[0]+\mathrm{x}[1]+\mathrm{x}[2]}{2\mathrm{x}^2[1]}(\mathrm{x}[0]+\mathrm{x}[2]) \\
Z_2^{(3p)} &= \frac{\mathrm{x}[0]+\mathrm{x}[1]+\mathrm{x}[2]}{\mathrm{x}[1]}\sqrt{1-\frac{(\mathrm{x}[0]+\mathrm{x}[2])^2}{4\mathrm{x}^2[1]}} \\
Z_3^{(3p)} &= \mathrm{x}[0]+\mathrm{x}[1]+\mathrm{x}[2]
\end{aligned} \tag{19}$$

In this way, quantities (8) and (14) become point estimators of amplitude A of signal $x(t)$. One of the initial conditions for calculating a three-point estimator is the assumption that the sample $\mathrm{x}[1] \neq 0$. Moreover, situations ought to be excluded from the calculations in which, due to the estimator and signal sample construction, it is not possible to carry out the mathematical operations of extraction of roots and division. Fixing the initial conditions makes it possible to calculate the estimator correctly.

At this point, some clarification is called for. Equation (8) does not, contrary to what its name suggests, constitute a description of a two-point estimator, but of a three-point estimator. It follows from the fact that the functions in equation (17) are calculated on the basis of three samples $\mathrm{x}[i]$ of signal $x(t)$. Furthermore, in this paper, such an estimator is denoted by the symbol $A_1^{(3p)*}$ and is called three-point estimator. Let us take note that the following theorem holds true.

Theorem 1. If $M \geq 4$, then $A_1^{(3p)} = A_1^{(3p)*}$.

Proof. Based on the components of equation (18), we obtain

$$\begin{aligned}
\frac{Z_1^{(2p)}}{Z_2^{(2p)}} &= \frac{\mathrm{x}[0]+2\mathrm{x}[1]+\mathrm{x}[2]}{2\mathrm{x}[1]\sqrt{1-\frac{(\mathrm{x}[0]+\mathrm{x}[2])^2}{4\mathrm{x}^2[1]}}} \\
\frac{Z_3^{(2p)}}{Z_2^{(2p)}} &= \frac{\mathrm{x}[0]+\mathrm{x}[1]}{\sqrt{1-\frac{(\mathrm{x}[0]+\mathrm{x}[2])^2}{4\mathrm{x}^2[1]}}} \\
\frac{Z_1^{(2p)}Z_3^{(2p)}}{\left(Z_2^{(2p)}\right)^2} &= -\frac{2\mathrm{x}[1](\mathrm{x}[0]+\mathrm{x}[1])}{\mathrm{x}[0]-2\mathrm{x}[1]+\mathrm{x}[2]}
\end{aligned} \tag{20}$$

If $M \geq 4$, then estimator (8) assumes the form

$$\begin{aligned}
A_1^{(3p)*} = A_1^{(2p)} &= \left(\mathrm{x}^2[0]\left(-\frac{4\mathrm{x}[1]}{\mathrm{x}[0]-2\mathrm{x}[1]+\mathrm{x}[2]}\right)\right. \\
&\left.+\frac{4\mathrm{x}[1]\mathrm{x}[0](\mathrm{x}[0]+\mathrm{x}[1])}{\mathrm{x}[0]-2\mathrm{x}[1]+\mathrm{x}[2]}-\frac{4\mathrm{x}^2[1](\mathrm{x}[0]+\mathrm{x}[1])^2}{(\mathrm{x}[0]+\mathrm{x}[2])^2-4\mathrm{x}^2[1]}\right)^{\frac{1}{2}} \\
&= \sqrt{\frac{4\mathrm{x}^2[1](\mathrm{x}[0]\mathrm{x}[2]-\mathrm{x}^2[1])}{(\mathrm{x}[0]+\mathrm{x}[2])^2-4\mathrm{x}^2[1]}}.
\end{aligned} \tag{21}$$

On the other hand, based on the components of equation (19), we obtain

$$\begin{aligned}
\frac{Z_1^{(3p)}}{Z_2^{(3p)}} &= \frac{\mathrm{x}[0]+\mathrm{x}[2]}{2\mathrm{x}[1]\sqrt{1-\frac{(\mathrm{x}[0]+\mathrm{x}[2])^2}{4\mathrm{x}^2[1]}}} \\
\frac{Z_3^{(3p)}}{Z_2^{(3p)}} &= \frac{\mathrm{x}[1]}{\sqrt{1-\frac{(\mathrm{x}[0]+\mathrm{x}[2])^2}{4\mathrm{x}^2[1]}}} \\
\frac{Z_1^{(3p)}Z_3^{(3p)}}{\left(Z_2^{(3p)}\right)^2} &= -\frac{2\mathrm{x}^2[1](\mathrm{x}[0]+\mathrm{x}[2])}{(\mathrm{x}[0]+\mathrm{x}[2])^2-4\mathrm{x}^2[1]}
\end{aligned} \tag{22}$$

Hence for $M \geq 4$, estimator (14) assumes the following form

$$A_1^{(3p)} = \left(\mathrm{x}^2[0]\left(-\frac{4\mathrm{x}^4[1]}{(\mathrm{x}[0]+\mathrm{x}[2])^2 - 4\mathrm{x}^2[1]} - \frac{4\mathrm{x}^4[1]}{(\mathrm{x}[0]+\mathrm{x}[2])^2 - 4\mathrm{x}^2[1]} + \frac{4\mathrm{x}[0]\mathrm{x}^2[1](\mathrm{x}[0]+\mathrm{x}[2])}{(\mathrm{x}[0]+\mathrm{x}[2])^2 - 4\mathrm{x}^2[1]} \right) \right)^{\frac{1}{2}} = \sqrt{\frac{4\mathrm{x}^2[1](\mathrm{x}[0]\mathrm{x}[2]-\mathrm{x}^2[1])}{(\mathrm{x}[0]+\mathrm{x}[2])^2 - 4\mathrm{x}^2[1]}} \tag{23}$$

Thus $A_1^{(3p)} = A_1^{(3p)*}$.

A serious drawback of the estimators $A_1^{(3p)}$ and $A_1^{(3p)*}$ is that in calculations they do not include the DC component A_0 of signal $x(t)$. It turns out, however, that in certain special cases, estimator $A_1^{(3p)*}$ can allow for such a component. It takes place when initial phase φ of signal $x(t)$ assumes the value, e.g., $\varphi = 0$ or $\varphi = \pi/2$. Let us take into account estimator $A_1^{(3p)*}$. Let us assume $m = 2$, $A_0 \in \mathbf{R}$, and $\varphi = k\pi$, $k \in \mathbf{Z}$. Then $M \geq 3$ and

$$\mathrm{x}[0] = A_0 + A\sin(k\pi) = A_0 \tag{24}$$

Based on (5) and (24), we obtain

$$A_1^{(3p)*} = \frac{\mathrm{x}[0]+\mathrm{x}[1]-2A_0}{\sin(\varphi)+\sin\left(\frac{2\pi}{M}+\varphi\right)} = \frac{\mathrm{x}[1]-\mathrm{x}[0]}{\sin\left(\frac{2\pi}{M}\right)} \tag{25}$$

Since the functions in equations (16) and (17) assume the form

$$\cos\left(\frac{2\pi}{M}\right) = \frac{\mathrm{x}[0]+\mathrm{x}[2]-2A_0}{2(\mathrm{x}[1]-A_0)} = \frac{\mathrm{x}[2]-\mathrm{x}[0]}{2(\mathrm{x}[1]-\mathrm{x}[0])}$$
$$\sin\left(\frac{2\pi}{M}\right) = \sqrt{1-\frac{(\mathrm{x}[2]-\mathrm{x}[0])^2}{4(\mathrm{x}[1]-\mathrm{x}[0])^2}} \tag{26}$$

then

$$A_1^{(3p)*} = \frac{\mathrm{x}[1]-\mathrm{x}[0]}{\sqrt{1-\frac{(\mathrm{x}[2]-\mathrm{x}[0])^2}{4(\mathrm{x}[1]-\mathrm{x}[0])^2}}} = \frac{2(\mathrm{x}[1]-\mathrm{x}[0])^2}{\sqrt{4(\mathrm{x}[1]-\mathrm{x}[0])^2-(\mathrm{x}[2]-\mathrm{x}[0])^2}} \tag{27}$$

Let us assume that $m = 2$, $A_0 \in \mathbf{R}$, and $\varphi = \pi/2 + 2k\pi$. Then $M \geq 3$ and

$$\mathrm{x}[0] = A_0 + A\sin\left(\frac{\pi}{2}+2k\pi\right) = A_0 + A \tag{28}$$

Based on (5), (24) and (26), we obtain

$$A_1^{(3p)*} = \frac{\mathrm{x}[0]+\mathrm{x}[1]-2A_0}{\sin(\varphi)+\sin\left(\frac{2\pi}{M}+\varphi\right)} = \frac{\mathrm{x}[1]-\mathrm{x}[0]+2A}{1+\cos\left(\frac{2\pi}{M}\right)} = \frac{\mathrm{x}[1]-\mathrm{x}[0]+2A}{1+\frac{\mathrm{x}[2]-\mathrm{x}[0]}{2(\mathrm{x}[1]-\mathrm{x}[0])}} \tag{29}$$

Substituting amplitude A for $A_1^{(3p)}$ and solving equation (29) with respect to A, we obtain that

$$A_1^{(3p)*} = \frac{2(\mathrm{x}[1]-\mathrm{x}[0])^2}{3\mathrm{x}[0]-4\mathrm{x}[1]+\mathrm{x}[2]} \tag{30}$$

If $M \geq 4$, then according to Theorem 1, equations (27) and (30) can be applied to calculate estimator $A_1^{(3p)}$, when $\varphi = k\pi$ and $\varphi = \pi/2 + 2k\pi$, respectively.

B. m-Point method

Proceeding with the previous analysis, an m-point estimator of amplitude A of signal $x(t)$ can be devised.

Let $2 \leq m < M$ be the number of equations in an equations system which can be defined based on (3). Let us assume $A_0 = 0$. Then

$$\begin{aligned}
\mathrm{x}[0] &= A\sin(\varphi) \\
\mathrm{x}[1] &= A\sin\left(\frac{2\pi}{M}+\varphi\right) \\
&\vdots \\
\mathrm{x}[m-1] &= A\sin\left(\frac{2\pi}{M}(m-1)+\varphi\right)
\end{aligned} \tag{31}$$

Solving system (31) with respect to A, we obtain

$$A_1^{(mp)} = \frac{\mathrm{x}[0]+\mathrm{x}[1]+\ldots+\mathrm{x}[m-1]}{D} \tag{32}$$

where

$$D = \sin(\varphi)\left(1+\cos\left(\frac{2\pi}{M}\right)+\ldots.+\cos\left(\frac{2\pi}{M}(m-1)\right)\right) + \cos(\varphi)\left(\sin\left(\frac{2\pi}{M}\right)+\ldots+\sin\left(\frac{2\pi}{M}(m-1)\right)\right) \tag{33}$$

Assuming

$$Z_1^{(mp)} = \sum_{r=0}^{m-1} \cos\left(\frac{2\pi}{M} r\right)$$
$$Z_2^{(mp)} = \sum_{r=0}^{m-1} \sin\left(\frac{2\pi}{M} r\right) \tag{34}$$
$$Z_3^{(mp)} = \sum_{r=0}^{m-1} \mathrm{x}[r]$$

then on the basis of (31)-(34), we obtain the equation

$$A_1^{(mp)} = \frac{Z_3^{(mp)}}{\frac{\mathrm{x}[0]}{A} Z_1^{(mp)} + \mathrm{sign}(\cos(\varphi))\sqrt{1-\left(\frac{\mathrm{x}[0]}{A}\right)^2} Z_2^{(mp)}} \tag{35}$$

Substituting amplitude A for $A_1^{(mp)}$ and solving equation (35) with respect to A, we obtain that

$$A_1^{(mp)} = \left(\mathrm{x}^2[0]\left(\left(\frac{Z_1^{(mp)}}{Z_2^{(mp)}}\right)^2 + 1\right) - 2\mathrm{x}[0]\frac{Z_1^{(mp)} Z_3^{(mp)}}{\left(Z_2^{(mp)}\right)^2} + \left(\frac{Z_3^{(mp)}}{Z_2^{(mp)}}\right)^2 \right)^{\frac{1}{2}} \tag{36}$$

Quantity (36) is not yet a point estimator of amplitude A of signal $x(t)$. Let us take into account that for $r \in \mathbf{N}$ the functions [7]

$$\sin(rx) = \sum_{k=0}^{r} \binom{r}{k} \cos^k(x) \sin^{r-k}(x) \sin\left(\frac{\pi}{2}(r-k)\right)$$
$$\cos(rx) = \sum_{k=0}^{r} \binom{r}{k} \cos^k(x) \sin^{r-k}(x) \cos\left(\frac{\pi}{2}(r-k)\right) \tag{37}$$

Hence

$$Z_1^{(mp)} = \sum_{r=0}^{m-1}\sum_{k=0}^{r}\left(\binom{r}{k}\cos^k\left(\frac{2\pi}{M}\right)\sin^{r-k}\left(\frac{2\pi}{M}\right) \cdot \cos\left(\frac{\pi}{2}(r-k)\right)\right)$$
$$Z_2^{(mp)} = \sum_{r=0}^{m-1}\sum_{k=0}^{r}\left(\binom{r}{k}\cos^k\left(\frac{2\pi}{M}\right)\sin^{r-k}\left(\frac{2\pi}{M}\right) \cdot \sin\left(\frac{\pi}{2}(r-k)\right)\right) \tag{38}$$

Utilizing equations (16) and (17), we finally obtain that

$$Z_1^{(mp)} = \sum_{r=0}^{m-1}\sum_{k=0}^{r}\left(\binom{r}{k}\left(\frac{\mathrm{x}[0]+\mathrm{x}[2]}{2\mathrm{x}[1]}\right)^k \cdot \left(1-\frac{(\mathrm{x}[0]+\mathrm{x}[2])^2}{4\mathrm{x}^2[1]}\right)^{\frac{r-k}{2}} \cos\left(\frac{\pi}{2}(r-k)\right)\right)$$
$$Z_2^{(mp)} = \sum_{r=0}^{m-1}\sum_{k=0}^{r}\left(\binom{r}{k}\left(\frac{\mathrm{x}[0]+\mathrm{x}[2]}{2\mathrm{x}[1]}\right)^k \cdot \left(1-\frac{(\mathrm{x}[0]+\mathrm{x}[2])^2}{4\mathrm{x}^2[1]}\right)^{\frac{r-k}{2}} \sin\left(\frac{\pi}{2}(r-k)\right)\right) \tag{39}$$

In this way, quantity (36) becomes an m-point estimator of amplitude A of signal $x(t)$. The initial conditions for the calculations of the estimator are the same as they are in the case of a three-point estimator.

Similarly to the case of estimator $A_1^{(3p)*}$, a situation can be considered in which the DC component A_0 is taken into account while calculating estimator $A_1^{(mp)}$. We assume that $A_0 \neq 0$, $M \geq 3$ and $\varphi = k\pi$, $k \in \mathbf{Z}$. Then, based on (24) and (36), we obtain

$$A_1^{(mp)} = \left((\mathrm{x}[0]-A_0)^2\left(\left(\frac{Z_1^{(mp)}}{Z_2^{(mp)}}\right)^2 + 1\right) - 2(\mathrm{x}[0]-A_0)\frac{Z_1^{(mp)} Z_3^{(mp)}}{\left(Z_2^{(mp)}\right)^2} + \left(\frac{Z_3^{(mp)}}{Z_2^{(mp)}}\right)^2 \right)^{\frac{1}{2}} = \frac{Z_3^{(mp)}}{Z_2^{(mp)}} \tag{40}$$

Where

$$Z_2^{(mp)} = \sum_{r=0}^{m-1}\sum_{k=0}^{r}\left(\binom{r}{k}\left(\frac{\mathrm{x}[0]+\mathrm{x}[2]-2A_0}{2(\mathrm{x}[1]-A_0)}\right)^k \cdot \left(1-\frac{(\mathrm{x}[0]+\mathrm{x}[2]-2A_0)^2}{4(\mathrm{x}[1]-A_0)^2}\right)^{\frac{r-k}{2}} \sin\left(\frac{\pi}{2}(r-k)\right)\right)$$
$$= \sum_{r=0}^{m-1}\sum_{k=0}^{r}\left(\binom{r}{k}\left(\frac{\mathrm{x}[2]-\mathrm{x}[0]}{2(\mathrm{x}[1]-\mathrm{x}[0])}\right)^k \cdot \left(1-\frac{(\mathrm{x}[2]-\mathrm{x}[0])^2}{4(\mathrm{x}[1]-\mathrm{x}[0])^2}\right)^{\frac{r-k}{2}} \sin\left(\frac{\pi}{2}(r-k)\right)\right) \tag{41}$$
$$Z_3^{(mp)} = \sum_{r=0}^{m-1}(\mathrm{x}[r]-A_0) = \sum_{r=0}^{m-1}\mathrm{x}[r] - m\mathrm{x}[0]$$

C. Four-point method

In a four-point method, the Vizireanu and Halunga estimator is used [6]. Such an estimator is calculated based on four initial samples x[i] of the sinusoidal signal $x(t)$. If all four signal samples assume different values, then

$$A_1^{(4p)} = \frac{\sqrt{\left(\mathrm{x}[2]-\mathrm{x}[1]\right)^2 - \left(\mathrm{x}[1]-\mathrm{x}[0]\right)\left(\mathrm{x}[3]-\mathrm{x}[2]\right)}}{D} \tag{42}$$

Where

$$D = \sqrt{2}\left(1-\frac{\mathrm{x}[1]-\mathrm{x}[0]+\mathrm{x}[3]-\mathrm{x}[2]}{2\left(\mathrm{x}[2]-\mathrm{x}[1]\right)}\right) \cdot\sqrt{1+\frac{\mathrm{x}[1]-\mathrm{x}[0]+\mathrm{x}[3]-\mathrm{x}[2]}{2\left(\mathrm{x}[2]-\mathrm{x}[1]\right)}} \tag{43}$$

is a four-point estimator of amplitude A of signal $x(t)$. The main advantage of estimator $A_1^{(4p)}$ is the fact that it allows to estimate amplitude A on the basis of the samples of which we only know that they are samples of a sinusoidal signal. Such an estimator also allows for DC component A_0 of signal $x(t)$. Apart from the said advantages, estimator $A_1^{(4p)}$ also has some drawbacks. The most serious one is due to the initial condition (x[0] ≠ x[1] ≠ x[2] ≠ x[4]) imposed on the estimator. For example, the estimator value cannot be known when $x(t)$ is a theoretical sinusoidal signal with initial phase $\varphi = 0$ sampled with the number of samples $M = 6$. In the literature, other point estimators have been described. The three- and five-point amplitude A estimators proposed by Wu and Hong can serve as examples [9]. Vizireanu and Halunga have shown that the Wu and Hong three-point estimator is in fact a special case of their own four-point estimator [10].

4. SIMULATION RESULTS

We shall assume that the estimation of amplitude A is carried out when the sinusoidal signal $x(t)$ is disturbed by the Gaussian noise $n(t)$ with the standard deviation $\sigma_n \in \mathbf{R}_+$. As a result of this operation, we obtain the signal $y(t) = x(t) + n(t)$ whose samples

$$\mathrm{y}[i] = \mathrm{x}[i] + \mathrm{n}[i] \tag{44}$$

Let us define by

$$\mathrm{SNR} = 10\log\left(\frac{A^2}{2}\frac{1}{\sigma_n^2}\right) \tag{45}$$

and expressed in decibels signal-to-noise ratio [11]. Then

$$\sigma_n = \sqrt{10^{\log\left(\frac{A^2}{2}\right)-\frac{1}{10}\mathrm{SNR}}} \tag{46}$$

Adding noise to a signal aims at simulating situations in which the result of amplitude A estimation is affected by disturbances occurring in a real measurement channel.

Let us assume that the samples y[i] of signal $y(t)$ are quantized in an ideal roundoff A/D converter with the step $q = 2A/2^B$, where B is the converter resolution, while $\mathrm{Q}(\pm z) = q\cdot\mathrm{round}(\pm z/q \pm 0.5)$, where round($w$) is a roundoff function rounding off the number $w \in \mathbf{R}$ to the nearest integer [12], [13]. In this way, quantized samples

$$\mathrm{y}_q[i] = \mathrm{Q}\left(\mathrm{x}[i]+\mathrm{n}[i]\right) \tag{47}$$

are obtained, constituting the basis for the estimation of amplitude A of signal $x(t)$.

Let us assume that amplitude A estimation is carried out based on the point methods described in this paper. The accuracy of the methods has been evaluated. Evaluation accuracy is about determining the maximum error

$$\delta = \max\left\{\delta_A\right\} \tag{48}$$

Where

$$\delta_A = \frac{\left|A_1 - A\right|}{A}100\% \tag{49}$$

is the relative error calculated in each period based on the result A_1 of the amplitude A estimation.

In order to compare with each other the results of the estimation of amplitude A, the following evaluation criteria of accuracy of the results were assumed. If $\delta < 1$ %, such result is considered as accurate. If $1\ \% < \delta < 10\ \%$, such result is conditionally considered as accurate. If $\delta > 10$ %, this result is considered as inaccurate and to be rejected.

In the first place a simulation study was carried out, consisting of the estimation of amplitude A using the three-point estimator and the four-point Vizireanu and Halunga estimator. The evaluation of the accuracy of the results of estimation was based on the calculation of errors $\delta^{(3p)*}$ and $\delta^{(4p)}$ for varying SNR and M = 4, 8, 12, 16. During the simulation it is assumed that A = 1V, f = 20 Hz, $A_0 = \varphi = 0$, B = 16, N = 100. Fig.1. presents the results of errors $\delta^{(3p)*}$ and $\delta^{(4p)}$ as a function of SNR.

The results obtained show that with increase of M and SNR, the value of errors $\delta^{(3p)*}$ and $\delta^{(4p)}$ increase as well. In the case of three-point estimator, the cause of increase in error $\delta^{(3p)*}$ is caused by increase of M, in the formula of estimator $A_1^{(3p)*}$ the value of the coefficient $Z_2^{(2p)}$ decreases, but faster than the value of the coefficient $Z_1^{(2p)}$ increases. In case of a four-point estimator, the cause of the increase in error $\delta^{(4p)}$ of the result is thus, that along with the increase in M, present in the denominator of the formula for estimator $A_1^{(4p)}$, the difference between the two initial samples decreases, but faster than the expression of the nominator in this formula increases its value.

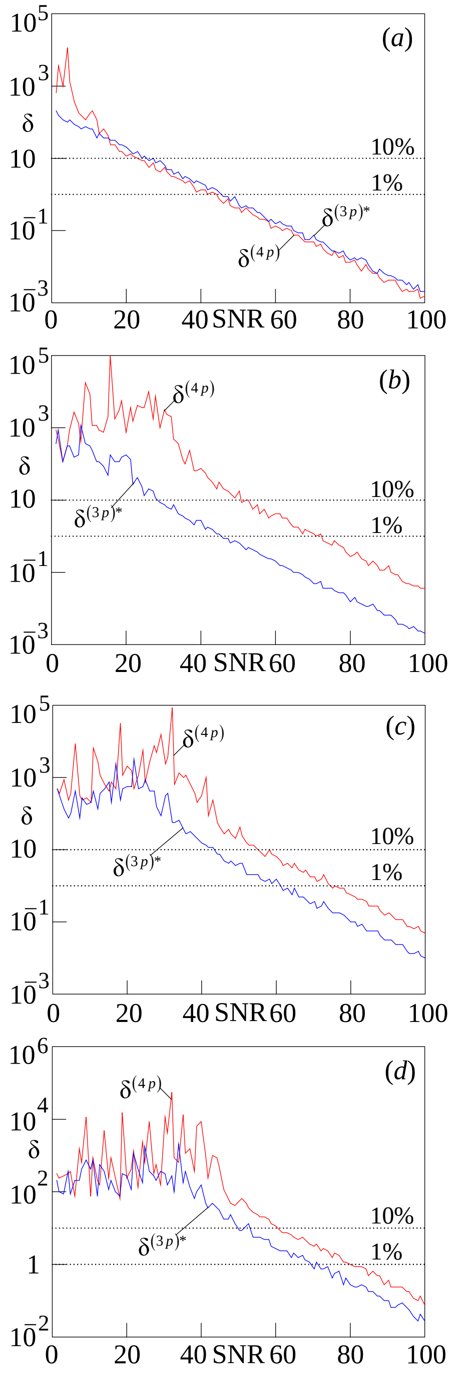

Fig.1. Results of errors $\delta^{(mp)}$ and $\delta^{(4p)}$ as a function of SNR, *a*) M = 4, *b*) M = 8, *c*) M = 12, *d*) M = 16.

However, even though the increase of M is followed by the reduction in the accuracy of the results, error $\delta^{(3p)*}$ can further on take values below 1 %. It happens, for example, when M = 16 and SNR > 70 dB (Fig.1.d)). In the following part of this article is shown that increase in M may be necessarily caused by the fact that for the low number of samples M, the m-point estimator may not give any benefits.

It remains to consider the question of the influence of the number of periods N of the signal on the value of errors $\delta^{(3p)*}$ and $\delta^{(4p)}$. It turns out that the increase in the number of periods leads to the increase of errors. This is due to the increase in the number of results $A_1^{(3p)*}$ and $A_1^{(4p)}$, which are taken into account when estimating amplitude A, while the random component of the results cannot be averaged to a sufficient degree. Table 1. shows the results of errors $\delta^{(3p)*}$ and $\delta^{(4p)}$ for SNR = 70 dB, M = 4, 8, 12, 16 and a selected number of periods N.

Table 1. Results of errors $\delta^{(3p)*}$ and $\delta^{(4p)}$ for a selected number of periods N.

SNR=70 dB	$\delta^{(3p)*}$		
	N=10	N=100	N=1 000
M=4	0.034%	0.060%	0.087%
M=8	0.044%	0.055%	0.070%
M=12	0.22%	0.40%	0.49%
M=16	0.86%	1.0%	1.2%
SNR=70 dB	$\delta^{(4p)}$		
	N=10	N=100	N=1 000
M=4	0.042%	0.059%	0.052%
M=8	0.54%	0.99%	1.7%
M=12	1.2%	2.1%	2.8%
M=16	2.8%	4.1%	3.9%

Note that due to the assumed number of periods N = 100, a new, lower number of periods gives in the most cases a lower error value, while larger number of periods gives a higher error value.

In the next step is checked whether in the same conditions of the simulation, m-point estimator will allow to estimate amplitude A with greater accuracy than the three-point estimator and four-point estimator. For this purpose, it was assumed SNR = 70 dB and simulations consisting of amplitude estimation using the m-point estimator were conducted. Accuracy evaluation of the results was based on the determination of an error $\delta^{(mp)}$ for M = 4, 8, 12, 16 and $2 \leq m < M$. Fig.2. presents the results of errors $\delta^{(mp)}$ and $\delta^{(4p)}$ as a function of m.

Table 2. shows selected results of errors $\delta^{(mp)}$ and $\delta^{(4p)}$ corresponding with those, presented in Fig.2.

The results show that in the case of a particularly small number of samples M, increase in m does not provide significant benefits, although in most cases, error $\delta^{(mp)} <$ 1 %. Therefore, the application of the m-point estimator will be seen in a situation where $M > 8$. For a smaller number of samples M what is proposed by the author of three-point estimator can be used. Let us note that the increase in the accuracy of the results of estimation of amplitude A, as a result of the increase in the number of samples m, can be observed when M = 12 and M = 16, but only for M = 12, and any number of samples $2 \leq m < M$, error $\delta^{(mp)} <$ 10 %. Furthermore, if M = 12 and $m < 7$, then $\delta^{(mp)} <$ 1 %, and for m = 5 we get the smallest error value, equal to $\min\{\delta^{(5p)}\}$= 0.043 %. It can be also noted that if m = 2, then $\delta^{(2p)} = \delta^{(3p)*}$ = 0.34 %. At the same time $\delta^{(4p)}$ = 1.8 %. This means that the four-point estimator is less accurate than the three- and m-point estimator, respectively, by an order (m = 2) and two orders of magnitude (m = 5). The reason why in the m-point estimator an increase of error value $\delta^{(mp)}$ is caused by the increase in number of samples m, is similar to that indicated earlier for the increase in error value $\delta^{(3p)*}$.

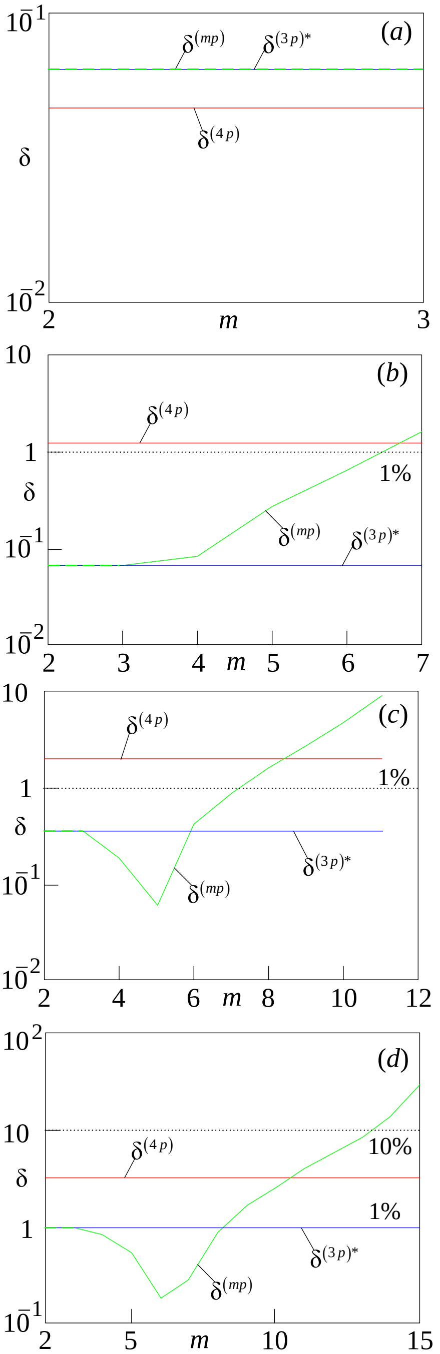

Fig.2. Results of errors $\delta^{(mp)}$ and $\delta^{(4p)}$ as a function of m, *a*) M=4, *b*) M=8, *c*) M=12, *d*) M=16.

Table 2. Results of errors $\delta^{(mp)}$ and $\delta^{(4p)}$ for a selected number of samples m.

SNR=70 dB	$\delta^{(mp)}<1\%$				$\delta^{(4p)}$
N=100	m=2	m=4	m=5	m=6	
M=4	0.062%	-	-	-	0.040%
M=8	0.058%	0.081%	0.27%	0.64%	0.97%
M=12	0.34%	0.19%	0.043%	0.35%	1.8%
M=16	0.95%	0.75%	0.50%	0.17%	3.8%

In practice, it is not always possible to provide measurement conditions, where SNR = 70 dB. Therefore, the author has prepared Table 3. and Table 4. of the guidelines, which can help in the application of the m-point estimator in the conditions where 60 dB < SNR < 80 dB. In the tables are shown only the parameters of the method and the signal processing parameters, which by the mode of simulation allow to obtain errors $\delta^{(mp)}$ at a level not exceeding 1 %. The tables show the results of error checking $\delta^{(4p)}$ for a four-point estimator. In order to corroborate the results, the process of estimating amplitude A was repeated $K = 10^4$ for each assumed number of samples M.

Table 3. Parameters of m-point method (N=10).

N=10	$\delta^{(mp)}<1\%$	$\min\{\delta^{(mp)}\}<1\%$	$\delta^{(4p)}$
M	SNR=60 dB		
4	$2\le m\le 3$	m=3	<1%
5	$2\le m\le 4$		<10%
6		$\min\{\delta^{(mp)}\vert_{m=3,4}\}$	$<10^4$%
8			<10%
10	$2\le m\le 5$	$\min\{\delta^{(mp)}\vert_{m=3,4,5}\}$	
12	$4\le m\le 5$	m=5	<100%
14	m=6	m=6	
M	SNR=70 dB		
4	$2\le m\le 3$	m=3	<1%
5	$2\le m\le 4$		
6	$2\le m\le 5$	$\min\{\delta^{(mp)}\vert_{m=3,4}\}$	$<10^4$%
8			<10%
10	$2\le m\le 6$	$\min\{\delta^{(mp)}\vert_{m=3,4,5}\}$	
12		$\min\{\delta^{(mp)}\vert_{m=4,5}\}$	
14	$4\le m\le 6$	$\min\{\delta^{(mp)}\vert_{m=5,6}\}$	
16	$6\le m\le 7$	$\min\{\delta^{(mp)}\vert_{m=6,7}\}$	
18	m=7	m=7	
20	m=8	m=8	<100%
22	m=9	m=9	
M	SNR=80 dB		
4	$2\le m\le 3$	m=3	<1%
8	$2\le m\le 7$	$\min\{\delta^{(mp)}\vert_{m=3,4}\}$	
12	$2\le m\le 8$	$\min\{\delta^{(mp)}\vert_{m=4,5}\}$	
16	$2\le m\le 9$	$\min\{\delta^{(mp)}\vert_{m=6,7}\}$	<10%
20	$4\le m\le 9$	m=8	
24	$8\le m\le 10$	m=9	
28	$10\le m\le 11$	m=11	
32	$12\le m\le 13$	$\min\{\delta^{(mp)}\vert_{m=12,13}\}$	<100%
36	m=14	m=14	

According to the author, the use of tables is very simple. Assuming that as the result of the measurement is obtained for N = 100 periods each comprising M = 8 samples $y_q[i]$. Assuming also that the reference value of amplitude A is known. On the basis of the samples obtained, SNR ≈ 70 dB. According to Table 4. amplitude A can be estimated with an error not exceeding 1 %, assuming in the m-point estimator any number of samples m between $2 \le m \le 5$. After the calculation of $A_1^{(mp)}$, the value of error $\delta^{(mp)}$ should be determined to check whether error $\delta^{(mp)} < 1$ %. If the estimate of amplitude A is to be made with the least possible error $\delta^{(mp)}$ not exceeding 1 %, then the m-point estimator should be used in consequence for m = 3, 4 and amplitude $A_1^{(mp)}$ calculated. Based on the results obtained and the reference value of amplitude A, calculate errors $\delta^{(mp)}$ and indicate the result of $A_1^{(mp)}$ for which error $\delta^{(mp)}$ takes the smallest value. Data from Table 3. and Table 4. should be considered only as a guide to help use the m-point estimator in practice. It should be remembered that in the real situation of measurement, the error of results may significantly differ from that obtained by the simulation mode.

Table 4. Parameters of m-point method (N=100).

N=100	$\delta^{(mp)}$<1%	min{$\delta^{(mp)}$}<1%	$\delta^{(4p)}$
M	SNR=60 dB		
4	2≤m≤3	m=3	<1%
5			<10%
6	2≤m≤4		<10^4%
8		min{$\delta^{(mp)}\|_{m=3,4}$}	<10%
10	2≤m≤4	m=4	
12	m=5	m=5	
14	m=6	m=6	<100%
M	SNR=70 dB		
4	2≤m≤3	m=3	<1%
5			
6	2≤m≤5		<10^3%
8		min{$\delta^{(mp)}\|_{m=3,4}$}	<10%
10	2≤m≤6	m=4	
12		m=5	
14	4≤m≤6	min{$\delta^{(mp)}\|_{m=5,6}$}	
16	5≤m≤7	min{$\delta^{(mp)}\|_{m=6,7}$}	
18	m=7	m=7	
20	m=8	m=8	<100%
22	m=9	m=9	
M	SNR=80 dB		
4	2≤m≤3	m=3	<1%
8	2≤m≤7	min{$\delta^{(mp)}\|_{m=3,4}$}	
12	2≤m≤8	m=5	<10%
16	2≤m≤9	m=6	
20	5≤m≤9	m=8	
24	8≤m≤10	m=9	
28	10≤m≤11	m=11	
32	m=12	m=12	<100%
36	m=14	m=14	

Taking into account all the presented simulation results, it is concluded that:

I. The three-point estimator allows an estimation of amplitude A with greater accuracy than the four-point Vizireanu and Halunga estimator.

II. The m-point estimator allows to estimate amplitude A with greater accuracy than the three-point estimator. The condition for a more accurate estimation of the amplitude is to choose the number of samples m in such a way that error $\delta^{(mp)}$ takes the smallest possible value (min{$\delta^{(mp)}$} from Table 4. and Table 5.).

III. The m-point estimator should be used when the number of samples M > 8. Application of the estimator for a smaller number of samples does not bring significant benefits and may be replaced with the three-point estimator.

5. Experimental verification

Taking into account the simulation results, a measurement experiment has been carried out consisting of the acquisition of samples of the sinusoidal signal $x(t)$ with amplitude A = 1 V, frequency f = 20 Hz, and initial phase φ = 0. Signal $x(t)$ has been generated by an Agilent 33220A function generator. The signal sampling has been carried out by an Agilent 3458A voltmeter [14]. Prior to carrying out the measurements, appropriate corrections of signal frequency f and sampling frequency f_s have been made to ensure coherence of the samples collected without a break in subsequent periods. As a result, three series ($s1$, $s2$, $s3$) have been obtained, each containing N = 100 periods of the samples $u_q[i]$ of sinusoidal voltage. The number of samples in each period is M = 12. The measurement resolution is B = 16 and it results from the integration time t_i = 2 µs preprogrammed in an Agilent 3458A voltmeter. On account of the way of voltage conversion by the voltmeter, the obtained samples $u_q[i]$ require correction of their values. As a result of this operation, the samples

$$y_q[i] = \frac{1}{\operatorname{sinc}(2\pi f t_i)} u_q[i] \tag{50}$$

are obtained, on whose basis amplitude A of signal $x(t)$ is estimated. To estimate amplitude A, the m- and four-point methods presented in the paper are used.

The evaluation of the accuracy of the results of amplitude estimation is conducted based on the maximum error (48) determined on the basis of the relative error

$$\delta_A = \frac{|A_1 - A'|}{A'} 100\% = \frac{\left| A_1 - \sqrt{2}\frac{1}{n}\sum_{r=0}^{n-1}\mathrm{RMS}[r] \right|}{\sqrt{2}\frac{1}{n}\sum_{r=0}^{n-1}\mathrm{RMS}[r]} 100\% \tag{51}$$

calculated in each period based on the result A_1 of the amplitude A estimation, as well as on amplitude A'. Amplitude A' is calculated on the basis of an average of n = 10 values of the root mean square (RMS) voltage, measured by an Agilent 3458A voltmeter in the "Synchronous Sub-sample" mode. It is assumed that amplitude A' obtained in this way is the reference value of the results A_1 of amplitude A estimation. To justify this procedure, it ought to be emphasized that the relative error δ_{RMS} of the averaged result of RMS measurement calculated on the basis of the device specification amounted to 0.064 %, whereas error $\delta_{A'}$ of amplitude A' is equal to 0.048 %. Taking into account the collected series of samples, as well as the employed estimators $A_1^{(mp)}$ and $A_1^{(4p)}$, error $\delta_{A'}$ proves to be significantly smaller than errors δ, and in a prevailing number of cases smaller than errors δ_A. Fig.3. presents the results of error δ determined for the collected series of samples.

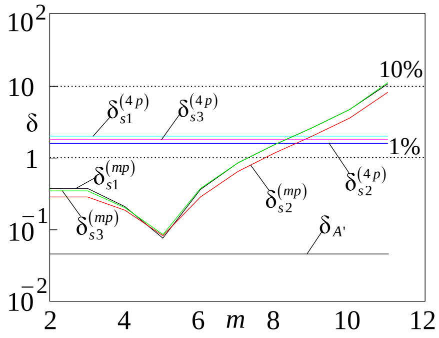

Fig.3. Results of errors $\delta^{(mp)}$ and $\delta^{(4p)}$ as a function of m.

Table 5. shows the results of errors $\delta^{(mp)}$ and $\delta^{(4p)}$ for a selected number of samples m and the results of calculating SNR for the collected series of samples $y_q[i]$.

Table 5. Results of errors $\delta^{(mp)}$ and $\delta^{(4p)}$ for a selected number of samples m.

	Series 1		Series 2		Series 3	
SNR	70.76 dB		70.55 dB		70.59 dB	
M=12	$\delta_{s1}^{(mp)}$	$\delta_{s1}^{(4p)}$	$\delta_{s2}^{(mp)}$	$\delta_{s2}^{(4p)}$	$\delta^{(mp)}$	$\delta_{s1}^{(4p)}$
m=2	0.38%	2.0%	0.29%	1.6%	0.35%	1.8%
m=5	0.077%		0.082%		0.084%	

The results of experimental studies are consistent with the simulation results. The conditions of measurement are obtained, where SNR is at the level of 70 dB. The three-point estimator has given the errors at the level not higher than 1 %, and the m-point estimator has allowed a significant increase in the accuracy of the estimation results of amplitude A.

In comparison with the four-point Vizireanu and Halunga estimator, the m-point estimator makes it possible to estimate amplitude A with higher accuracy. The author has made the measurement results available for independent verification of the results of amplitude estimation [15].

6. CONCLUSION

In the paper, an m-point method of estimating the amplitude of a sinusoidal signal has been presented. In this method, an estimator calculated on the basis of m initial signal samples is used. It has been shown that such an estimator is suitable for amplitude estimation when m is very small. The special cases of an estimator have been examined. Such an estimator has been compared with the four-point Vizireanu and Halunga estimator. It has been shown that a m-point estimator makes it possible to estimate amplitude with higher accuracy. Such an estimator still has all the drawbacks of estimators of this type. In its case, the accuracy of amplitude estimation is decided by the greatest number of samples in a period. In practice, the number of samples ought to be the smallest possible. Apart from the pointed out drawbacks, a m-point estimator also has several important advantages. It is very efficient as far as its calculation speed is concerned, has a simple structure, does not require sample acquisition within a full signal period. If a measurement system does not require substantial accuracy of the measurement of sinusoidal voltage signal amplitude, and simultaneously the sampling is performed with a small number of samples per period, then a m-point estimator can be an effective tool for amplitude estimation.

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