

Research on Air Flow Measurement and Optimization of Control Algorithm in Air Disinfection System

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As the air flow control system has the characteristics of delay and uncertainty, this research designed and achieved a practical air flow control system by using the hydrodynamic theory and the modern control theory. Firstly, the mathematical model of the air flow distribution of the system is analyzed from the hydrodynamics perspective. Then the model of the system is transformed into a lumped parameter state space expression by using the Galerkin method. Finally, the air flow is distributed more evenly through the estimation of the system state and optimal control. The simulation results show that this algorithm has good robustness and anti-interference ability.

Keywords: Wind surface control, hydrodynamics, Galerkin method, optimal control

1. INTRODUCTION

RECENT YEARS, friendly living environment and fresh air are being pursued more and more urgently. Therefore, the development and application of air purification disinfection systems draws increasing public attention, and its market demand continues to grow. In such systems, the control of its internal air flow directly affects the effect of air purification. Previous studies have mainly focused on the control of the average wind speed. For example, Li Yin made a more systematic summary of the traditional PID algorithm-based fan control [1]; Li Ke, who designed an expert-fuzzy PID control system, has introduced the intelligent control and made some innovations [2]; B. R. Sorensen, who takes into full consideration low-carbon, environmental and other factors, designed a more energy-efficient fan control strategy, and made the fan more energy efficient through the control [3].

However, owing to the uneven distribution of the wind speed in the space, the results obtained by the average wind speed are, in fact, valid only for some local area of the system. When the overall performance of the system is considered, the control of gas flow of surface uniformity is very important, which is rarely seen in the previous researches. In the field of industrial control, wind power, aerospace, and the uneven distribution of wind speed have a strong impact on the scientific research and product performance. Thus, the even control of the wind must be taken seriously.

Considering the above factors, this paper presents a control algorithm based on the hydrodynamic theory and optimal control methods, analyzing the performance of the algorithm in order to get a satisfactory performance of the control system.

2. SYSTEM ARCHITECTURE DESIGN AND MATHEMATICAL MODELING

A. System architecture design and analysis

According to the flow field in photocatalytic air purification disinfection systems, the system architecture model has been designed, and shown as follows:

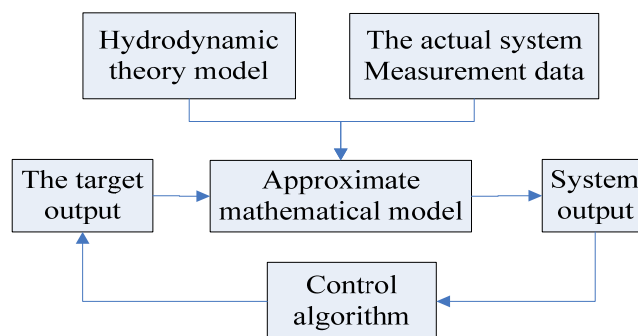


Fig.1. System architecture design diagram

After creating the theoretical model, it should be installed into the real system model by adjusting some parameters. The introduction of the model initial conditions and boundary conditions constitutes the boundary conditions of the problem [4]. The theoretical properties of these constraints and the system itself can guarantee the closed nature of the system model equations [5]. The final step is to design suitable control algorithms based on the system mode for the real-time and closed-loop control of the system. This way we were able to get a more uniform distribution of flow field and improve the overall system disinfection performance [6].

Through experimental determination, the photo-catalytic purification achieves the best results when the wind speed is around 0.8m/s [7], as is shown in Fig.2. Therefore, the wind speed should be controlled close to the optimal wind speed, which could improve the disinfection efficiency. The control algorithm is what this paper tries to figure out.

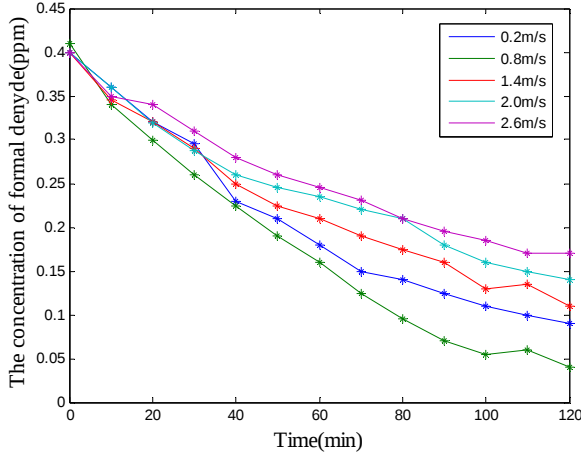


Fig.2. Relationship diagram between Photocatalytic effect and wind speed (25°C)

B. The mathematical model based on the N-S equations

As the speed of the object fluid flow is low, it can be assumed as incompressible viscous fluid. The wind speed in the chamber fits the parabolic distribution. Then take the inertial frame of reference for the study, and describe the object fluid in Euler mode. The air gravity is so low that it can be ignored. Take the fan force as the major force, and the fluid system is the laminar flow model.

According to the basic fluid mechanics equations and common model, the viscous incompressible closed equations of fluid can be taken as the model of the system. A simplified form of formula is shown in (2-1).

$$\frac{D\vec{V}}{Dt} = \frac{1}{\rho} \nabla p + \frac{\mu}{\rho} \nabla^2 \vec{V} \quad (1)$$

In equation (1), ∇ is the Hamiltonian operator; $\vec{V}$ is velocity vector; $\vec{V} = \vec{V}(x, y, z, t)$, $\frac{D}{Dt} = \frac{\partial}{\partial t} + (\vec{V} \cdot \nabla)$ is material derivative; $\vec{p} = (p_x, p_y, p_z)$ is surface force, here it mainly refers to the upward traction on the fluid from fans; density $\rho = \text{constant}$; viscosity $\mu = \text{constant}$. This is the N-S equation for the air flow in this paper.

Expand material derivative and Hamiltonian operator, and then get the partial derivative of time and the velocity vector on the three-dimensional coordinates. As the fluid in this paper is represented by the Z-axis of laminar flow, the velocity vector in both directions in the x, y component is zero. A simplified form of formula is shown in equation (2)

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial z} = \frac{1}{\rho} \left(\frac{\partial p}{\partial x} + \frac{\partial p}{\partial y} + \frac{\partial p}{\partial z} \right) + \frac{\mu}{\rho} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \quad (2)$$

According to the law of conservation of mass, fluid continuity equation could be in this form: $\nabla \cdot \vec{V} = 0$, [4].

Meanwhile, the component of the velocity vector in the direction of x, y is zero; then $\frac{\partial u}{\partial z} = 0$. In addition, under the

assumption that in this system, forces from the fan on the fluid surface only have an effect in the z direction, the mathematical expression could be further simplified. With ideal wind speed distribution estimating the surface force, it could take the surface forces for the parabolic equation [4]. Beginning with the one-dimensional wind speed distribution,

Take $\frac{1}{\rho} \frac{\partial p}{\partial z} = p(x, t)$, and $a^2 = \frac{\mu}{\rho}$. Then the N-S

equation of the system can be expressed as equation (3).

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2} + p(x, t) \quad (3)$$

C. The application of the Galerkin method

On the basis of establishing the system's theoretical model, the methods of mathematical physics should be used in order to get a result. The system model with the controlled quality should be transformed into a form that the control theory can solve. Concerning the boundary conditions that the component of the gas flow rate on the chassis wall must be zero, the problem can be described as equation (4).

$$\begin{cases} u_t - a^2 u_{xx} = p(x, t) \\ u|_{x=0} = 0, u|_{x=l} = 0 \end{cases} \quad (4)$$

This is, in fact, the typical heat conduction problem on finite pole in the mathematical physics method [8]. In this system, l refers to the maximum length in x axis, which measures $l = 0.5$ (m).

Take $p(x, t) = q(x)f(t)$, and $q(x) = b(x - \frac{l}{2})^2 + c$,

in which b and c can be figured out through the system fitting method. Then consider the Galerkin method to solve the Boundary Value Problems

1). Select a relatively complete set of functions, like

$$\{x(l-x), x^2(l-x), \dots, x^i(l-x), \dots\}.$$

2). Take a linear combination as the approximate solution of the equation. $u_k(x, t)$, it can be expressed as follows:

$$u_k(x, t) = \sum_{i=1}^k \lambda_i(t) \cdot v_i(x) = \sum_{i=1}^k \lambda_i(t) \cdot x^i(l-x) \quad (5)$$

in which $\lambda_i(t)$ is to be determined. Substitute $u_k(x, t)$ into (2-4), and then get equation (6)

$$\sum_{i=1}^k \frac{d(\lambda_i(t))}{dt} v_i(x) = a^2 \sum_{i=1}^k \lambda_i(t) \frac{d^2(v_i(x))}{dx^2} + q(x) f(t) \quad (6)$$

3). Use the components of $u_k(x, t)$ to multiply the above equation on both sides. Do the integral in the region.

When combining the k items, it can be integrated to the coefficient matrix of the system's equations. Evaluate integration with each element of the $k \times k$ dimensional coefficient matrix, and then the partial differential equations can be turned into ordinary differential equations. Choose $k = 3$, and substitute it into the expression of $v_i(x)$ and $q(x)$, it gets the form:

$$\begin{aligned} & \int_0^1 \begin{pmatrix} x^2(1-x)^2 & x^3(1-x)^2 & x^4(1-x)^2 \\ x^3(1-x)^2 & x^4(1-x)^2 & x^5(1-x)^2 \\ x^4(1-x)^2 & x^5(1-x)^2 & x^6(1-x)^2 \end{pmatrix} dx \begin{pmatrix} \lambda_1(t) \\ \lambda_2(t) \\ \lambda_3(t) \end{pmatrix} \\ &= a^2 \int_0^1 \begin{pmatrix} -2x(1-x) & 2x(1-3x)(1-x) & 6x^2(1-2x)(1-x) \\ -2x^2(1-x) & 2x^2(1-3x)(1-x) & 6x^3(1-2x)(1-x) \\ -2x^3(1-x) & 2x^3(1-3x)(1-x) & 6x^4(1-2x)(1-x) \end{pmatrix} dx \begin{pmatrix} \lambda_1(t) \\ \lambda_2(t) \\ \lambda_3(t) \end{pmatrix} \\ &+ \int_0^1 \begin{pmatrix} x(1-x)(b(x-1/2)^2+c) \\ x^2(1-x)(b(x-1/2)^2+c) \\ x^3(1-x)(b(x-1/2)^2+c) \end{pmatrix} dx \cdot f(t) \end{aligned} \quad (7)$$

Take $l = 0.5$, $b = 9.923$ and $c = 0.595$. In fluid mechanics, when the temperature is 300 K, the air dynamic viscosity is $\mu = 1.846 \times 10^{-5}$, and the density is $\rho = 1.161$. Then it can be concluded that $a^2 = 1.590 \times 10^{-5}$. Take the inverse of the derivative coefficient matrix of λ , and multiply on both sides of the equation (7). By doing all these, the system model can be expressed in the flowing form equation (8)

$$\dot{\lambda} = A\lambda + Bf \quad (8)$$

Where,

$$A = \begin{pmatrix} -1.7808 \times 10^{-3} & 2.2260 \times 10^{-4} & -1.4310 \times 10^{-4} \\ 1.0685 \times 10^{-2} & 0 & 2.6712 \times 10^{-3} \\ -2.1370 \times 10^{-2} & -5.3424 \times 10^{-3} & -8.0136 \times 10^{-3} \end{pmatrix}$$

$$B = (24.8072 \quad -208.3836 \quad 416.7672)^T$$

It is obvious that by using the Galerkin method to simplify the system theory model, it is easy to get the state space equation of modern control theory.

3. DESIGN OF CONTROL ALGORITHM

A. Methods of modern control theory based on state feedback

State feedback can provide richer state information and more available degrees of freedom, which make the system achieve better performance in control, so the state feedback is adopted for closed-loop control. However, normally the system state variables cannot be directly detected, which requires some specific means of estimating in order to complete the feedback. Here the wind speed distribution is considered to be expanded in the spatial domain, and the solution region is meshed with the use of the finite element method. Take the four regions which are meshed by three measurement points and border demarcation as the grid, denoted as $e_i (i = 1, 2, 3, 4)$.

Here, $y_i (i = 1, 2, 3)$ is the data from three measurement points, two segments at the measurement point are a set of basis functions, denoted as $X_i (i = 1, 2, 3)$. The wind speed distribution in the spatial domain can be expanded as shown in the form of equation (9):

$$u(x, t) = \sum_{i=1}^3 X_i(x) y_i(t) \quad (9)$$

Construct wind speed interpolation function on each interval e_i , using linear interpolation, gets:

$$\begin{cases} u_j = a_1 + a_2 x_j \\ u_m = a_1 + a_2 x_m \end{cases} \quad (10)$$

Here, j and m are interval endpoints. Calculate a_1 and a_2 from Fig.3. Get wind speed interpolation function on each region as follows:

$$\begin{cases} u_1 = 10y_1 \cdot x \\ u_2 = (2y_1 - y_2) + (-10y_1 + 10y_2)x \\ u_3 = (2y_2 - y_3) + (-5y_2 + 5y_3)x \\ u_4 = 5y_3 - 10y_3 \cdot x \end{cases} \quad (11)$$

Organize equation (11). Get the expression of X_i as shown in equation (12):

$$\begin{cases} X_1(x) = \begin{cases} 10x & 0 \leq x < 0.1 \\ 2-10x & 0.1 \leq x < 0.2 \end{cases} \\ X_2(x) = \begin{cases} 10x-1 & 0.1 \leq x < 0.2 \\ -5x+2 & 0.2 \leq x < 0.4 \end{cases} \\ X_3(x) = \begin{cases} -1+5x & 0.2 \leq x < 0.4 \\ 5-10x & 0.4 \leq x \leq 0.5 \end{cases} \end{cases} \quad (12)$$

Define the indicator function:

$$J_e = \int_D \left(\sum_{j=1}^3 X_j(x) y_j(t) - \sum_{k=1}^3 x^k (1-x) \lambda_k(t) \right) X_i(x) dx = 0 \quad (13)$$

Get a set of relationships between measuring the output and estimated state as shown below:

$$\begin{cases} \int_0^{0.5} X_1(x) \sum_{j=1}^3 X_j(x) y_j(t) dx = \int_0^{0.5} X_1(x) \sum_{k=1}^3 x^k (0.5-x) \lambda_k(t) dx \\ \int_0^{0.5} X_2(x) \sum_{j=1}^3 X_j(x) y_j(t) dx = \int_0^{0.5} X_2(x) \sum_{k=1}^3 x^k (0.5-x) \lambda_k(t) dx \\ \int_0^{0.5} X_3(x) \sum_{j=1}^3 X_j(x) y_j(t) dx = \int_0^{0.5} X_3(x) \sum_{k=1}^3 x^k (0.5-x) \lambda_k(t) dx \end{cases} \quad (14)$$

Substituting (3-4) into (3-6), get the following relations:

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 0.04114 & 0.00308 & 0.00017 \\ 0.06541 & 0.01367 & 0.00257 \\ 0.04570 & 0.01894 & 0.00756 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} \quad (15)$$

Equation (15) gives the relationship matrix of the system output and system state. This relationship is based on equation (13), while the purpose is to estimate the system state according to the output of the three measurement point. The interpolation function remains continuous, that is, satisfying the compatibility conditions, for using a linear interpolation. Therefore, the calculation of the expansion of different forms can get the estimate of system state, which solves the problem of undetectable system's state that is used as feedback.

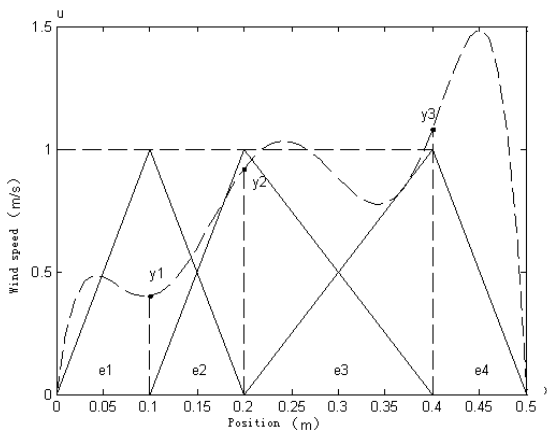


Fig.3. Schematic diagram of the wind speed distribution
Basis function sin spatial domain base

In normal circumstances, state feedback matrix K can be figured out, using the state estimates. But the control objective in this system is the uniformity of the wind surface. It is not possible to determine the Pole assignment

expectations of the matrix K . Optimal control theories should be introduced into the problem.

B. Optimal control algorithm

Use the energy and the error of the system to construct the linear quadratic, and design an infinite time state regulator to control the system. According to the estimation of system state space expression, take the performance function as follows:

$$J = \frac{1}{2} \int_0^\infty (x^T Q_1 x + u^T Q_2 u) dt \quad (16)$$

In the function of performance, error vector X is equivalent to the system state vector λ and estimates $\hat{\lambda}$. u , which denotes the control constraints in the dynamic control process, is equivalent to the control volume f in the actual system. Q_1 and Q_2 represent the intensity of the controller.

$Q_1 = \begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix}$, $Q_2 = 1$, then:

$$J = \frac{1}{2} \int_0^\infty (3\lambda_1^2 + 4\lambda_2^2 + f^2) dt \quad (17)$$

Infinite time state regulator requires that the system is completely controllable, so the controllability of the system needs to be verified. The rank of controllability matrix is $rank(M) = 2$, so this system is not completely controllable. The necessary and sufficient condition for linear time-invariant system, which can be stabilized when state feedback is used for controlling, is that the uncontrollable subsystem is asymptotically stable. Therefore, decompose the system in accordance with the controllability. The result is shown in equation (18). The relationship between the output of the system and the system state is shown in equation (19). As the feature of one-dimensional control subsystem is negative, this system is stabilized.

$$\dot{\hat{\lambda}} = \begin{pmatrix} 0 & 3.4615 \times 10^{-4} & 0 \\ 1 & 0.0564 & 0.0115 \\ -5.4629 & -0.3258 & -0.0661 \end{pmatrix} \hat{\lambda} + \begin{pmatrix} 0 \\ 1 \\ 85.2 \end{pmatrix} f \quad (18)$$

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 0.45081 & -0.00241 & 0.00017 \\ -0.15689 & 0.00195 & 0.00257 \\ 0.33687 & -0.00159 & 0.00756 \end{pmatrix} \begin{pmatrix} \hat{\lambda}_1 \\ \hat{\lambda}_2 \\ \hat{\lambda}_3 \end{pmatrix} \quad (19)$$

Consider the question of the applicability of the optimal control method. The control exerted within a limited time can make the controllable components return to the zero state from the non-zero status, when it comes to the controllable subsystem. And the uncontrollable subsystem is asymptotically stable. So it is feasible to exert the state

feedback control on the system. Set the system matrix and control matrix, A' and B' , respectively, and the controllable subsystem expression as follows:

$$\dot{\hat{\lambda}}' = \begin{pmatrix} 0 & 3.4615 \times 10^{-4} \\ 1 & 0.0564 \end{pmatrix} \hat{\lambda}' + \begin{pmatrix} 0 \\ 1 \end{pmatrix} f \quad (20)$$

Q_1 and Q_2 are both positive definite, so the optimal control could be :

$$\begin{aligned} u^*(t) &= -Q_2^{-1} B'^T P \hat{\lambda}'(t) = -1 \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{pmatrix} \begin{pmatrix} \hat{\lambda}_1(t) \\ \hat{\lambda}_2(t) \end{pmatrix} \\ &= -p_{12} \hat{\lambda}_1(t) - p_{22} \hat{\lambda}_2(t) \end{aligned} \quad (21)$$

Here the matrix P is the positive definite solution of the Riccati algebraic equation $-PA' - A'^T P + PB'Q_2^{-1}B'^T P - Q_1 = 0$. Under the condition that P is ensured to be positive definite,

the solution is $P = \begin{pmatrix} 1.1400 \times 10^4 & 3 \\ 3 & 2.0577 \end{pmatrix}$. Hence, the

optimal control is $u^*(t) = -3\hat{\lambda}_1(t) - 2.0577\hat{\lambda}_2(t)$, and the optimal feedback gain matrix K is $K = -Q_2^{-1}B'^T P = \begin{pmatrix} -3 & -2.0577 \end{pmatrix}$. According to the optimal control theory and previous discussion, it is known that this closed-loop system is stable. The structure of the completed closed-loop system is shown in Fig.4, with the closed-loop system state equation as

$$\dot{\hat{\lambda}}' = \begin{pmatrix} 0 & 3.4615 \times 10^{-4} \\ -2 & -2.0013 \end{pmatrix} \hat{\lambda}'$$

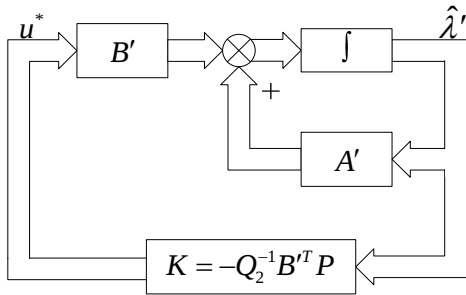


Fig.4. Structure of closed-loop system of wind surface control algorithm based on the model

4. SYSTEM SIMULATION AND PERFORMANCE ANALYSIS

In order to verify the control performance of the designed algorithm, the system is simulated with Simulink in Matlab 2007b, and a system output curve is drawn. Fig.5 compares the control performance of the new algorithm with the traditional PID algorithm.

It can be seen that the uniformity of the wind field distribution has been further improved compared with the original condition. The application of the newly designed

algorithm to photocatalytic air purification and disinfection system will increase its overall performance. In this case, the selection of wind speed value in three measurement points for control, $x_1 = 10$, $x_2 = 20$ and $x_3 = 40$, respectively, has its reasons. As is shown in the red curve of Fig.5, these three points represent the low, gentle and high points of the curve, respectively, which possess the most representative features. Therefore, in order to avoid the system calculating a large dimension, choosing these three points to control is the most representative, which gets a better performance. We have tried to change the different parameters, and the control effect has proved this idea. Using the device, its sterilization efficiency was tested for *Staphylococcus albus*. The results showed that the device could reduce the number of bacteria in the air significantly.

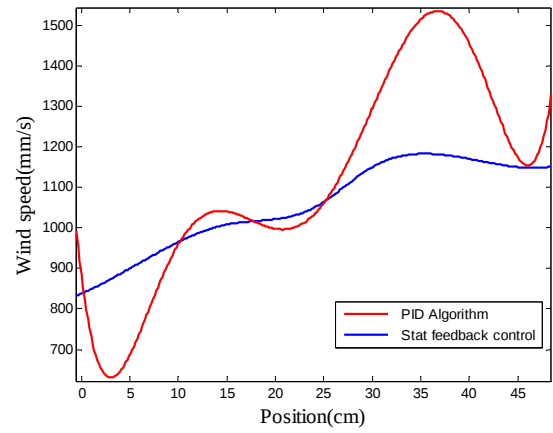


Fig.5. The comparison chart of wind surface uniformity from different algorithms

Table 1. Sterilization efficiency of the device for *Staphylococcus albus*

Time [min]	Control group		Test group	
	Number of colonies [CFU/m ³]	Natural Decline rate [%]	Number of colonies [CFU/m ³]	Purification rate [%]
0	249509.00±27209.86	0	253482.70±34935.74	0
30	189615.70±25703.85	24.15±1.97	224.00±49.68	99.88±0.01
60	149308.00±21379.87	40.31±2.13	16.33±14.57	99.99±0.01
90	105026.70±17763.23	57.88±1.77	0	100.00±0.00

5. CONCLUSION

In this paper the experimental platform is the Nano air disinfection and purification system. The control objective is derived from actual requirements, and this paper gives a certain solution in the control algorithm, which is rare in the previous researches. In order to process the N-S equations, the idea of the Galerkin method is introduced to simplify the complex system model. Expanding the wind speed

distribution in the time domain and space domain, respectively, helps solving the problem of system state estimation. Finally, the optimal control through state feedback is realized. The system simulation results show that the design of the control algorithm in this paper could improve the wind speed distribution, and make the overall system performance better.

There is still much room for the control algorithm in this paper to improve. For example, the wind speed distribution can be extended to higher dimensional space; viscous fluid laminar flow model can be more complex, etc. In addition, if the designed algorithm is applied to the system on site and achieves good effect, great progresses will be made in the theoretical research and engineering application of fluid motion control area.

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