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Determination of critical stress triaxiality along yield locus of isotropic ductile materials under plane strain condition

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It is widely accepted that failure due to plastic deformation in metals greatly depends on the stress triaxiality factor (TF). This article investigates the variation of stress triaxiality along the yield locus of ductile materials. Von Mises yield criteria and triaxiality factor have been used to determine the critical limits of stress triaxiality for the materials under plane strain condition. A generalized mathematical model for triaxiality factor has been formulated and a constrained optimization has been carried out using genetic algorithm. Finite element analysis of a two dimensional square plate has been carried out to verify the results obtained by the mathematical model. It is found that the set of values of the first and the second principal stresses on the yield locus, which results in maximum stress triaxiality, can be used to determine the location at which crack initiation may occur. Thus, the results indicate that while designing a certain component, such combination of stresses which leads the stress triaxiality to its critical value, should be avoided.

Keywords: *triaxiality, von Mises stress, Poisson's ratio, genetic algorithm, yield stress, FEA*

1. Introduction

The triaxiality of the stress state is known to greatly influence the amount of plastic strain which a material may undergo before ductile failure. It is defined as the ratio of hydrostatic pressure or mean stress to the equivalent von Mises stress. In order to evaluate these failure criteria and to verify their application for industry, one has to know the effect of stress triaxiality on fracture and failure of materials. The literature survey showing the effect of stress triaxiality on the fracture of ductile material is carried out in the present study. The main parameters that influence initial void nucleation and growth, and hence ductile fracture, are the triaxiality factor and the plastic strain [1]. The failure of ductile materials is often related to coalescence of microscopic voids. The stress triaxiality is one of the primary factors that influences the coalescence [2]. For three different notched specimen geometries, stress triaxiality starts to increase almost linearly as a function of the increasing plastic strain, with a rate that is peculiar to the notch radius [3]. Both the yield and failure criterion depend on the stress triaxiality [4]. The equivalent plastic strain at fracture for austenitic steel decreases exponentially with increasing stress triaxiality ratio in the range from $0 \leq \mathrm{TF} \leq 2.7$ [5]. Triaxiality ratio shows how remarkably stress state parameter varies during the post-necking plastic flow [6]. When the necking starts, the stress triaxiality effect leads to a sharp increase of the cavity growth rate [7]. The specimens with the same stress triaxiality have the same equivalent strain to crack formation, while the specimens with different stress triaxialities have different values of equivalent strain to crack formation. It has been shown that the equivalent strain and stress triaxiality are two the most important factors governing crack formation, while the stress and strain ratios cause secondary effects, and that one can make a good prediction of ductile crack formation with equivalent strain and stress triaxiality alone [8]. In most forming

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processes, the stress triaxiality is so low that the void height does not have any influence on the value of the damage variable. The damage model may be applied to stress triaxiality larger than 0.4, using any adequate void growth model [9]. Triaxiality-dependent modeling allows capturing the correct crack initiation in a notched and unnotched bar [10]. Stress triaxiality in the smooth and notched specimens varies significantly with plastic strain [11]. The macroscopic stress–strain response and the void growth and coalescence behavior of the voided cell were obtained from detailed finite element analyses and the results showed strong dependencies on the stress triaxiality [12]. For the annealed material, the ductility is slightly overestimated at the lowest and at the largest triaxiality. It was also observed that the stress triaxiality increased at very slow rate for larger values of strain hardening exponent, n [13].

The accurate and meaningful modeling of elastic and plastic behavior of ductile materials is essential for the solution of numerous problems occurring in various engineering fields. In the first part, mathematical model is formulated for stress triaxiality under plane strain condition and non-linear optimization is carried out using genetic algorithm in order to determine critical stress triaxiality. The results obtained are verified by the FE analysis of a two dimensional plate.

2. Genetic algorithm

Genetic algorithms are computerized search and optimization algorithms based on the mechanics of natural genetics and natural selection [14]. The operation of GA's begins with a population of random strings or decision variables. Thereafter, each string is evaluated to the fitness value. Three main operators viz. reproduction, crossover, and mutation are used to create a new population of points to operate the population. The population is further evaluated and tested for termination. If the termination criterion is not met, the population is iteratively operated by the above three operators and evaluated. This procedure is continued until the termination criterion is met. One cycle of these operations and the subsequent evaluation procedure is known as a generation in the GA's terminology.

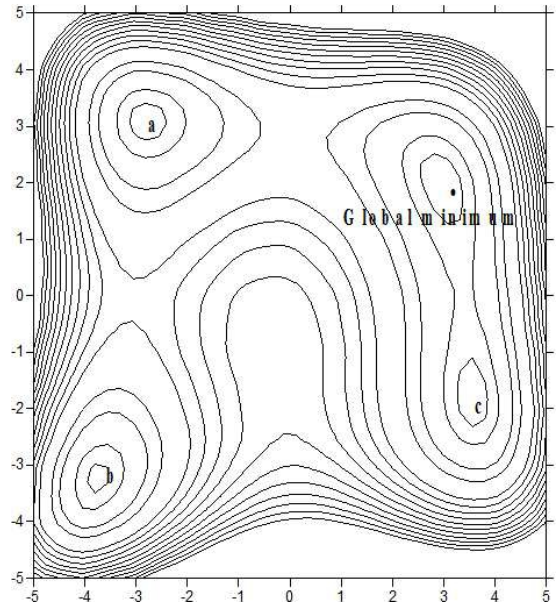

Fig. 1. Local and global optima.

The basic difference of GA's in comparison to the traditional optimization methods is that GA uses a coding of variables instead of variables directly, a population of points instead of a single point, and stochastic operators instead of deterministic operators. All these features make GA search robust, allowing it to be applied to a wide variety of problems. The advantage of using GA over other gradient based methods is that the latter can be mapped on local optimum whereas GA predicts global optima. In real-world problems, the objective function usually contains a number of optima of which one or more is a global optimum. Other optima have worse function values compared to the one at the global optimum. Therefore a designer or a decision-maker, may be interested in finding the global optimum point which corresponds to the best function value. Fig. 1 gives the schematic representation of the global and local optima.

3. Mathematical formulation

According to von Mises' yield criterion, yielding is not dependent on any particular normal

stress or shear stress, but instead, yielding depends on a function of all three values of principal stresses. Since the yield criterion is based on the differences of principal stresses, the criterion is independent of the component of hydrostatic stress. The von Mises' yield criterion involves squared terms; the result is independent of the sign of the individual stresses. This is an important advantage since it is not necessary to know which are the largest and smallest principal stresses in order to use this yield criterion. Von Mises equation of yielding is given by:

$$(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2=2\sigma_y^2 \quad (1)$$

where σ_1, σ_2 and σ_3 are the first second and third principal stresses and σ_y is the yield stress.

In plane strain condition σ_3 is given by:

$$\sigma_3=\nu\left(\sigma_1+\sigma_1\right) \quad (2)$$

where ν is Poisson's ratio.

Stress triaxiality factor (TF), which is defined by the ratio of hydrostatic stress to the equivalent von Mises stress, is given by the relation,

$$TF=\frac{\sigma_h}{\sigma_{eqv}}=\frac{1/3\left(\sigma_1+\sigma_2+\sigma_3\right)}{\frac{1}{\sqrt{2}}\sqrt{\left(\sigma_1-\sigma_2\right)^2+\left(\sigma_2-\sigma_3\right)^2+\left(\sigma_3-\sigma_1\right)^2}} \quad (3)$$

Substitution of Eq. 2 in Eq. 3 gives

$$TF=\frac{\left(\sigma_1+\sigma_2\right)+\nu\left(\sigma_1+\sigma_2\right)}{\frac{3}{\sqrt{2}}\sqrt{\left[\sigma_1-\sigma_2\right]^2+\left[\sigma_2-\nu\left(\sigma_1+\sigma_2\right)\right]^2+\left[\nu\left(\sigma_1+\sigma_2\right)-\sigma_1\right]^2}} \quad (4)$$

Stress triaxiality should reach its critical or maximum value for the crack initiation to occur in ductile materials [15, 16]. In order to get the maximum value of stress triaxiality, it is formulated as a constrained non-linear programming problem as follows:

$$\text{Objective Function}=\frac{\left(\sigma_1+\sigma_2\right)+\nu\left(\sigma_1+\sigma_2\right)}{\frac{3}{\sqrt{2}}\sqrt{\left[\sigma_1-\sigma_2\right]^2+\left[\sigma_2-\nu\left(\sigma_1+\sigma_2\right)\right]^2+\left[\nu\left(\sigma_1+\sigma_2\right)-\sigma_1\right]^2}} \quad (5)$$

The Objective Function is subjected to constraints given as:

$$\text{Subjected to } (\sigma_1-\sigma_2)^2+(\sigma_2-\nu\sigma_1-\nu\sigma_2)^2+(\nu\sigma_1+\nu\sigma_2-\sigma_1)^2=2\sigma_y^2 \quad (6)$$

$$\sigma_{1L}\le\sigma_1\le\sigma_{1U} \quad (7)$$

$$\sigma_{2L}\le\sigma_2\le\sigma_{2U} \quad (8)$$

where,

σ_{1L}= Lower limit of first principal stress

σ_{1U}= Upper limit of first principal stress

σ_{2L}= Lower limit of second principal stress

σ_{2U}= Upper limit of second principal stress

The upper and lower bounds for the first and second principal stresses considered for optimization are set as:

$$0\le\sigma_1\le1000 \quad (9)$$

$$0\le\sigma_2\le1000 \quad (10)$$

Table 1. GA Parameters.

GA Parameters	Numerical Value
Population	30
Generations	100
Reproduction Type	2 point crossover
Selection type	Sigma scaling
Mutation probability	0.005
Reproduction probability	0.85
Selection probability	0.85

Solution of the constrained programming problem, given by Eqs. 5 to 8, is carried out using GA. Specifications of GA parameters, considered for the present analysis are shown in Table 1. Fifteen sets were considered to make the optimization strategy. The sets consisting of five values of yield stress, from 200 MPa to 400 MPa with an increment of 50 MPa, have been included with every single value of Poisson's ratio, which was also varied from 0.2 to 0.4. For every set of yield stress and Poisson's ratio, the optimized values of principal stresses have been obtained and given in Table 2. The corresponding values of stress triaxiality associated with each set of principal stresses are given in Table 3. Variation of stress triaxiality with respect to yield stress and Poisson's ratio is shown in Fig. 2.

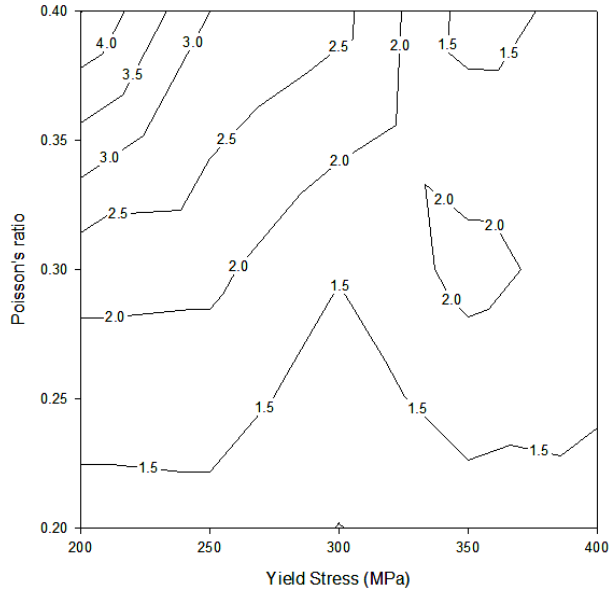

Fig. 2. Critical triaxiality map.

4. Verification using finite element analysis

In order to verify the results obtained by using GA in the preceding sections, finite element (FE) analysis is carried out with a two dimensional square plate having dimensions of 100 mm×100 mm. PLANE82 element type is considered for plane strain FE modeling. The element is defined by eight nodes having two degrees of freedom at each node: translations in the nodal x and y directions. An element edge length is taken as 2 mm. Young's modulus is taken as 2×10^5 N/mm^2 with Poisson's ratio varied from 0.2 to 0.4. Boundary conditions are applied on the left and bottom edges. The left edge is constrained in x direction such that $u_x = 0$ and the bottom edge is constrained in y direction such that $u_y = 0$. Because of regular and axisymmetric geometry, mapped meshing is preferred, as shown in Fig. 3. There are a total of 2500 elements connected with 7651 nodes. Uniformly distributed loads are applied on each node in +FX and +FY directions. The load is divided by the number of nodes present on the top and right edges of the plate so that the uniformly distributed loads should be provided throughout the edge. The applied load is determined by dividing the principal stresses provided in Table 2 by the area of cross section of the plate. FE results obtained after simulation give the value of equivalent von Mises stress which is found to be equal to the yield stresses at different Poisson's ratios considered for the present analysis and are provided in Table 4. This verifies the correctness of the result obtained by optimizing the mathematical model given by Eqs. 5 - 8 using GA. Fig. 4 (a to d) depicts the distribution of stress throughout the plate.

5. Results and discussion

Mathematical formulation given by Eqs. 5 - 8 is optimized by using GA. Table 1 gives the values of GA parameters considered for the present

Table 2. Maximized values of the set of principal stress obtained through GA.

Yield Stress	$\nu = 0.2$		$\nu = 0.3$		$\nu = 0.4$	
(σ_y)	σ_1	σ_1	σ_1	σ_1	σ_1	σ_1
200	352.06	290.39	500.18	499.84	996.61	938.14
250	425.63	405.72	641.27	582.85	914.54	693.52
300	487.03	254.71	652.58	407.51	996.73	712.01
350	677.33	487.86	880.10	869.27	684.03	26.03
400	671.59	661.42	947.99	697.56	932.83	501.68

Table 3. Critical value of stress triaxiality at different Poisson's ratios.

Yield Stress (σ_y)	$\nu = 0.2$	$\nu = 0.2$	$\nu = 0.2$
200	1.284	2.166	4.515
250	1.330	2.121	3.001
300	0.989	1.511	2.658
350	1.263	2.165	1.306
400	1.333	1.763	1.673

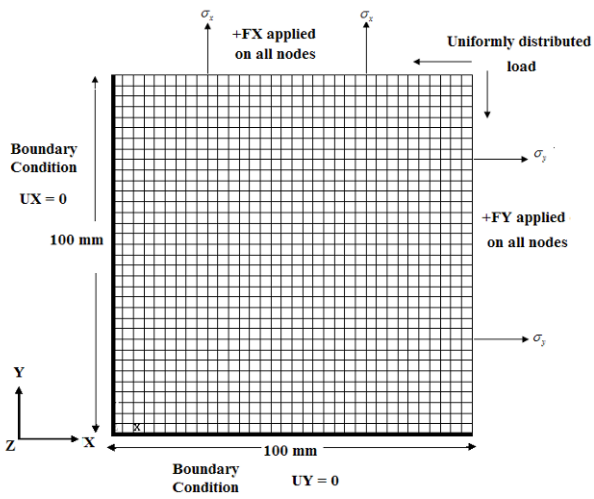

Fig. 3. Finite element mesh.

Table 4. Equivalent von Mises stress at different Poisson's ratios obtained through FEA.

Yield Stress (σ_y)	$\nu = 0.2$	$\nu = 0.2$	$\nu = 0.2$
200	200.26	200.02	200.23
250	252.57	250.11	250.61
300	300.89	300.52	300.36
350	350.07	350.43	350.02
400	400.23	400.08	400.13

optimization problem. Tables 2 and 3 give the critical values of the first and second principal stresses obtained by GA and the corresponding critical stress triaxiality. The contour map of critical triaxiality is depicted in Fig. 2, which shows that an increase in Poisson's ratio leads to an increase in stress triaxiality, which reflects that for a particular set of principal stress, higher Poisson's ratio is more prone to initiate cracks as compared to lower Poisson's ratio. On the yield locus of isotropic ductile materials many combinations of principal stresses are possible. Tables 2 and 3 yield the critical values of principal stresses and corresponding stress triaxiality at different yield stress and Poisson's ratios, which can decide about the location of crack initiation occurring on the yield locus. Thus, such combinations should be avoided and clearly mentioned while designing the components.

Results obtained by GA are verified by using finite element analysis. Fig. 3 depicts the loading and boundary conditions taken into account for carrying our FE analysis. It is found that equivalent von Mises stresses obtained for all the fifteen cases given in Table 4 are found to be in very good agreement with the actual yield stress and Poisson's ratio considered in case of mathematical formulation.

6. Conclusion

The main focus of this paper is to develop a general methodology for determination of critical triaxiality of engineering material and its influence on Poisson's ratio and the set of the first and the second principal stresses for ductile materials.

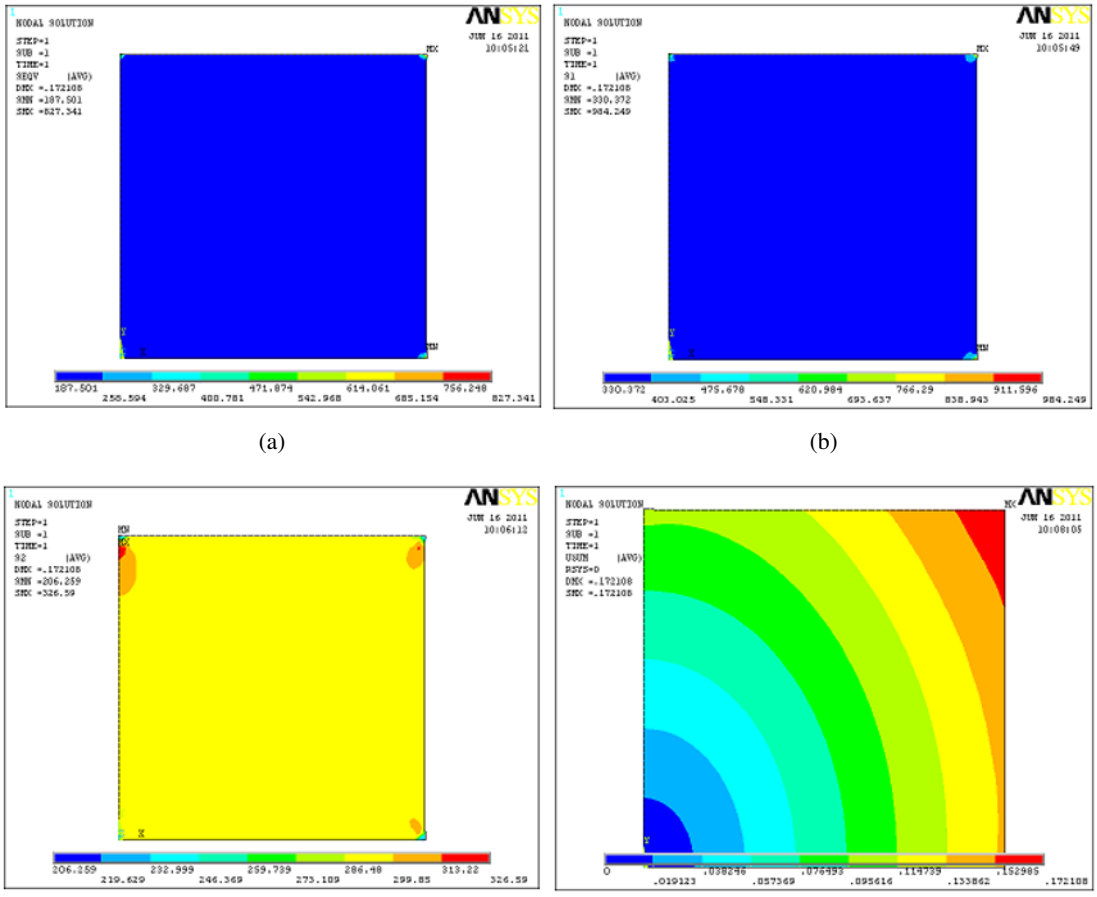

Fig. 4. Distribution of (a) von Mises stresses (b) first principal stress (c) second principal stress (d) vector sum of x and y displacements.

A mathematical model for the stress triaxiality under plain strain condition has been formulated and optimized using GA for the search of critical triaxiality. To verify the results obtained by GA, FE analysis is carried out on a two dimensional square plate. Based on the investigations of this study, it is found that there exists a critical stress triaxiality factor along yield locus of isotropic ductile materials, which decides about the location of expected crack initiation to occur. Thus, the stress triaxiality can be considered as an unavoidable parameter in product design for specific engineering applications.

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