

# Financial impact analysis of going public at the Warsaw Stock Exchange: Using Fuzzy Set Theory to understand behaviours of mature companies

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**Abstract.** *In this paper, we intend to contribute evidence in regard to going public financial impact and thus motivation on a sample of mature companies that launched an IPO at the Warsaw Stock Exchange between 2005 and 2015. First, we review recent literature focusing on financial and non-financial consequences of an IPO. Next, we use fuzzy sets and fuzzy reasoning to define a “mature” company and assess financial consequences of going public. Our main conclusion is that the majority of Polish mature companies use an IPO as a channel how to raise capital and accelerate their investments and growth. This contradicts many previous empirical studies highlighting that the main motivation of mature companies to go public is to use the capital raised within an IPO to rebalance their capital structure.*

**Keywords:** IPO, Going Public, Warsaw Stock Exchange, Motivation, Poland, Fuzzy Sets, Fuzzy Reasoning.

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## Introduction

It is generally agreed that companies go public when they are sufficiently mature to withstand the scrutiny of investors, institutional rating agencies, business magazines, or high-tech information intermediaries (Ritter, 1991; Chemmanur and Fulghieri, 1999; Pastor and Veronesi, 2003). Although the term “mature company” is widely used in the literature, it is not unambiguously defined. There is no fixed definition of how the “mature” company

should be old and what growth rate or what size it is supposed to have. As a rule, the “mature” company is defined only verbally and is characterised by qualitative features (e.g. Dickinson, 2011; Reiners, 2004).

This paper is an attempt to contribute to the discussion on financial consequences and thus motivation of going public. We join several recent studies and examine the impact of going public on a sample of 96 mature Polish companies, which conducted an initial public offering (IPO) between 2005 and 2015 in the Main Market of the Warsaw Stock Exchange (WSE). First, we use fuzzy sets and fuzzy reasoning to define a “mature” company. Next, we deal with the issue what the investment and financial policy of a sample of mature Polish companies within the post-IPO stage is. Are the financial funds rather used for fresh investments to strengthen the earning potential of the company or do the CFOs tend to use the raised capital for re-structuring the existing capital structure? Even in this case we struggle with the vagueness in researchers’ judgements and therefore we believe the methodology based on fuzzy set theory is appropriate to assess financial consequences of going public.

Pagano et al. (1998), Chun and Smith (2002), Jargot (2006) and many others analysed motives and financial consequences of an IPO while applying logit and probit methodologies which are two most common techniques for estimation of models with a dichotomous dependent variable (Greene, 2003). Both techniques are used to create predictive models that an event will occur and both models are estimated by maximum likelihood; goodness of fit and inferential statistics is based on the log likelihood and chi-square test statistics (Jargot, 2006). However, these techniques have significant limitations consisting in working with precise and certain information. For example, logistic regression could not be used to determine how old a “mature” company should be or what growth rate or size it is supposed to have. Researchers have to attempt to convert the measurements of age, growth and size into discrete categories characterised by qualitative features.

Therefore, in this study we deliver empirical evidence based on fuzzy set theory. While using this concept we first identify a group of entities defined as “mature companies” and next deal with financial consequences of going public. The issue of the membership of a company to a particular category has to be decided by the fuzzy logic working with the level of affiliation of a surveyed entity to the interest category. The fuzzy research approach overcomes the shortcomings of the ambiguous definition of the boundaries delimiting the category of e.g. a “mature” company or “high” growth rate of the company. The fuzzy set theory, introduced by Zadeh (1965), is commonly applied to deal with imprecise and uncertain information in the real-world problems; it is more suitable for qualitative assessment than classical evaluation methods working with precise values (Lin and Chen, 2004; Wang and Chuu, 2004).

This study extends the IPO in the way that to our knowledge we are the first who attempt to uncover the possible motivations to go public by applying the fuzzy set approach. We use data on the ex-ante and ex-post financial characteristics of surveyed sample of companies. We find support for the theory that mature Polish companies use an IPO as a channel how to raise capital and accelerate their investments and growth. This contradicts many previous empirical studies highlighting that the main motivation of mature companies to go public is to use the capital raised within an IPO to rebalance their capital structure.

## Literature review

Substantial academic literature interprets the decision of a company to conduct an initial public offering (IPO) as essential (e.g. Myers, 1984; Pagano et al., 1998; Brau and Fawcett, 2006; Jargot, 2006; Bancel and Mittoo, 2009; Peterle and Berk, 2016; Meluzín et al., 2016, 2018a,b,c). Nevertheless, the pecking order theory emphasises that this form of financing does not come into consideration until the possibilities of internal funding have been exhausted, as well as external resources, e.g. in the form of bank loans (Myers, Majluf, 1984; Michalak, 2016; Włodarczyk et al., 2018; Manuylenko et al., 2018). This is due to the costs of the share capital perceived to be higher by the financial theory compared to the costs of debt financing. Also for this reason, IPO implementation may be expected to be the last option considered when deciding on the manner of financing corporate investments, which is more likely to be used in the case of large and mature companies where an adverse selection problem is less accentuated (e. g. Ritter, 1991; Chemmanur and Fulghieri, 1999; Pastor and Veronesi, 2003; Ivanová, 2017; Ślusarczyk, 2018).

There is a broad range of motives why companies implement the IPO, being of both financial and non-financial character (Sosnowski, 2017; 2018). Ritter and Welch (2002) emphasise that the main motive is to raise capital for growth. Kim and Weisbach (2008), who examined the IPO in 38 countries, nevertheless add that most companies obtain a large volume of capital through the IPO, yet the scale of its use ranges from funding the development to rebalancing leverage. Pagano et al. (1998) also sees the main reason in rebalancing the company's capital structure after a period of high investment and dynamic growth, and further emphasises that going public improves the bargaining position of issuers towards banks and reduces debt costs and the share of bank financing in the capital structure. Auret and Britten (2008) report that IPO yields are often used to repay existing debts. Helwege and Liang (2004) add that the IPO may also represent a form of total or partial exits of the existing shareholders, whether they are in the role of company's founders, business angels, or venture capitalists.

A number of other studies indicate that the motives for the IPO implementation may consist in the efforts to build public relations; this may include, for example the visibility of the company in the media, forming the desired perception of the enterprise in the eyes of the public, or influencing the perception of the business sector in relation to its viability and risk (Brau et al., 2003, Maksimovic, Pichler, 2001; Kiseľáková et al., 2015; Bikas, Saponaitė, 2018; Wawryszuk-Misztal, 2017; Valaskova et al., 2018).

The possible IPO motive can also include optimising the assets portfolio of the existing shareholders who retain control over the company, yet selling part of the shares at the same time frees up their capital fixed in the shares of the issuing company, which they then use for other investments, repayment of debts, payment of minority shareholders etc. (Paeglis, Veeren, 2013; Wrońska-Bukalska, & Kaźmierska-Jóźwiak, 2017). Pratt and Grabowski (2008) also state that the IPO is the last option considered in deciding on how to finance development business investments due to the high cost of equity.

Brau and Fawcett (2006) conclude that the key motive of the IPO implementation in the US market conditions consists in the possibility of using the proceeds of the issue for future mergers and acquisitions, expanding the possibilities of obtaining additional funding in the future, or increasing the attractiveness of the company in recruiting managerial talent through share options. Kim and Weisbach (2008) focus on the perspectives of existing

shareholders for whom it is important to create a market for the shares of the issuing company, thus substantially increasing their liquidity.

According to Helwege and Liang (2004), initial public offerings are more frequent in sectors achieving rapid growth. The reason is usually the urgent need for capital to realise the planned investments (or acquisitions) in order to maintain a position in the growing market and at the same time to reduce the risk on the side of the existing shareholders, as the rapid development of the company commonly reduces the capacity to predict future developments with sufficient precision.

Bancel and Mittoo (2009), who examined the IPO motives in 12 European countries, conclude that companies most often implement IPOs to raise capital for their development investments, yet strategic considerations are also affected by efforts to improve reputation, financial credibility and financial flexibility.

In the CEE region, research into IPO motives has been limited to a few studies. Meluzín and Zinecker (2014), as well as Meluzín et al. (2016, 2018a,b,c), who performed survey-based research using the primary data from the environment of the Czech and Polish capital markets, emphasise the financial motive, both among companies that have already implemented the IPO and those which may be labelled as IPO candidates. Non-financial motives stemming from the efforts to strengthen the company's publicity and value the company's shares are also identified as essential.

Very frequently, relevant literature mentions the fact that IPO motives are different for companies which are in the growth phase of their life cycle and may therefore be described as "young", and for businesses which have already undergone a swift phase of initial development, being labelled as "stabilized" or "mature". For example, Brau and Fawcett (2006) point out the fact that when deciding on the IPO, "young" US firms, compared to their "mature" peers, are statistically more significantly motivated by the possibility to use public shares for future mergers and acquisitions, the possibility to determine the market value of their shares, and the status of a publicly traded company in order to enhance image and publicity. For the purposes of dividing the examined companies into "mature" and "young", the authors used the median age of all entities included in the research sample.

The results of the Bancel and Mittoo (2009) study suggest that "young" European businesses, in addition to strengthening publicity, place greater emphasis on the possibility of raising capital in the IPO process, which distinguishes them from "mature" companies, which more often refer to the possibility of "reducing the firm leverage", i.e. using IPO yields for the purposes of financial restructuring. Meluzín et al. (2016) have reached similar results. In addition to reducing indebtedness and strengthening the bargaining power towards external capital providers, there are the motives with a statistically significant support among the "mature" Polish companies surveyed. For the purposes of categorising companies (young vs. "mature"), both European studies repeatedly use the median age of the companies included in the research sample.

In addition to the above-mentioned empirical studies, the differences in the motivation to implement the IPO among "mature" and "young" companies are addressed in a number of theoretical models (e. g. Chemmanur and Fulghieri, 1999, Pagano et al., 1998, or Rajan, 1992). In general, it is assumed that companies do not initiate an IPO until sufficient information about them has been accumulated on the capital market to mitigate information asymmetries. Therefore, the more "mature" the enterprise is, the higher the probability of

the IPO. For more mature businesses, it is also assumed that the capital raised will be used primarily to restructure the capital structure instead of implementing development investment projects. In the case of mature companies, initial public offerings are frequently implemented only after a large-scale investment phase which has been funded, for example, by bank loans, in order to make use of the growth already achieved to raise the attractiveness of the issue to potential investors. This may lead to a greater effort concerning the IPO timing, although the growth potential of issuing mature companies tends to be significantly exhausted. The assumption of lower attractiveness of the initial public offering of mature businesses intending to raise capital is also due to easier access to alternative sources of funding compared to early-stage enterprises. The above facts support the thesis that mature businesses preferentially use the capital raised for financial restructuring. On the contrary, the motivation of young businesses is to raise capital to finance further business development.

When defining life cycle phases, i.e. their number and characteristics, the current theory is inconsistent. The main issue in identifying the life stage of a particular company consists in the fact that the existing models often work with quality characteristics which are very difficult to quantify (Konečný and Zinecker, 2017). For example, models based on quantitative variables include the one defined by Dickinson (2011). The life cycle phases of an enterprise are determined by a combination of positive and negative cash flow values from operating, investment and financial activities. On the other hand, in his model, Reiners (2004) proposes the use of relative performance indicators reflecting the year-on-year change in the growth of assets, sales and cash flow. This model also defines interval values for identifying the individual stages of the life cycle. Konečný and Zinecker (2015) recommend using the modified form of the Boston Matrix to identify the life cycle of a company. In the model, the life cycle phases are identified based on relative revenue growth where the growth rate of corporate sales is compared to the market growth and depending on the relative market share where the market share of an enterprise is compared to the volume of corporate assets. The model is limited by the assumption that the life cycle of an enterprise is identical to the life cycle of all its products.

During the life cycle, both the company's capital structure and its capital costs are transformed. Reiners (2004) states that the cost of equity has a declining trend over the life cycle, as shareholders' investment risk decreases. On the other hand, Hasan et al. (2016) believes that the course of equity cost over the company's life cycle has the form of a U-shape and thus corresponds to the trend of the debt capital costs, or the weighted average cost of capital. Following the Dickinson's model for determining the life cycle stage of the company, Faff et al. (2016) indicates that the volume of investments and issues of equity decreases in the course of the company's life cycle. The largest share in the capital structure is reached by the debt in the stages of market entry and growth, so that the value of this indicator falls in the phase of stabilization (maturity) and decline.

The rest of this paper proceeds as follows. First, we provide an overview of the methodological approach. Next, we present and interpret research findings. The final sections discuss and summarise the main conclusions.

## Methodology

In this paper, we empirically investigate the financial consequences of going public on a sample of 96 mature Polish companies which conducted an initial public offering between 2005 and 2015 in the Main Market of the Warsaw Stock Exchange. The source of financial data is the Amadeus Database provided by Bureau van Dijk. To deliver evidence on mature companies and financial consequences of going public, we formulate the following research questions: What would be an appropriate definition of a mature company within the defined subsample? What is the investment and financial policy of mature companies within the post-IPO stage? Are the financial funds rather used for fresh investments to strengthen the earning potential of the company or do the CFOs tend to use the raised capital for re-structuring the existing capital structure?

We used fuzzy sets and fuzzy reasoning to define a “mature” company and to assess financial consequences of going public. This research approach deals with indefinite quantification of a set of variables. The fuzzification of variables allowed us to query the knowledge database and to work with results, which are further used as inputs to search for parameters of typical units. In parallel with this approach, ordinary least squares (OLS) method was applied. The results, however, are not relevant as the overwhelming majority of input coefficients were statistically not significant. The validity of using OLS is reduced by the data characteristics because the time series is characterized by a small number of observations. The inappropriateness of the OLS approach is also enhanced by the fact that within the survey sample just a minority of companies can be defined as mature companies in terms of all characteristics, as defined by the review. Therefore we believe that our data set is suitable for applying the methodology based on fuzzy set theory.

## Fuzzy Set Theory

A fuzzy set theory is based on the premise that the key elements in human thinking are not numbers but words. The most important features of human thinking is the not yet well-known ability how to extract from a collection of masses of data only such items of knowledge which are relevant to the task at hand, see e.g. (Zimmermann, 1987; Skvarciany et al., 2018).

Let us take as an example of a verbal variable a size of an Industrial Production Growth (IPG). To quantify expert knowledge a set of verbal values, i.e. a dictionary, is needed. This verbal IPG dictionary could include the following set of verbal quantifiers:

*Very Low, Low, Medium, High, Very High, Extremely High*, etc. (0)

A linguistic value is transformed into a fuzzy set by the specification of a grade of membership. For example, a verbal value medium of the IPG is transformed into a fuzzy set by the grade of membership function shown in Fig. 1. A typical medium size of the IPG is:

$$\mu_{AS}(x) = 1 \text{ if and only if } b < IPG < c \quad (1)$$

Where the values of the IPG size (1) belong to the fuzzy set medium size IPG with the grade of membership equal to 1. Therefore they are a typical medium IPG size (see Fig. 1).

There are two fuzzy intervals  $a < IPG < b$ ,  $c < IPG < d$ , see Fig. 1. It means that the relevant grade of membership is in the interval  $<0, 1>$ :

$$\mu_{medium}(x) \in <0, 1> \text{ if and only if } a < IPG < b, c < IPG < d \quad (2)$$

is the grade of membership.

These intervals represent such numerical values, which belong partially to the fuzzy set medium size IPG.

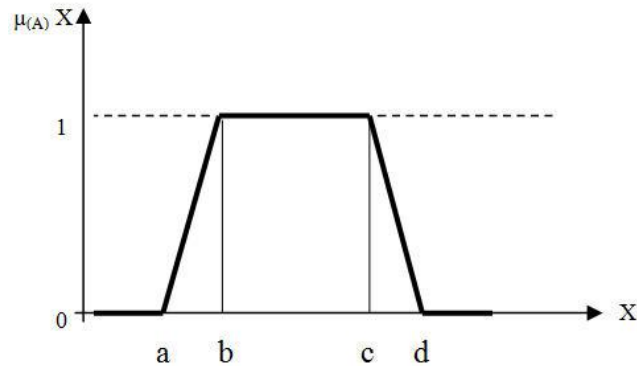


Figure 1. Trapezoidal fuzzy number  
Source: Dohnal and Dohnalova (1992)

A fuzzy model is a set of conditional statements:

If  $A_{11}$  and .....and  $A_{1n}$  then  $B_1$  or

If  $A_{21}$  and .....and  $A_{2n}$  then  $B_2$  or

(3)

If  $A_{m1}$  and .....and  $A_{mn}$  then  $B_m$

Where the fuzzy sets

$A_{ij}, B_i; i=1,2,.. m, j=1,2,...n$

(4)

Are one-dimensional fuzzy sets and can be easily specified or/and modified using points  $a, b, c$ , and  $d$  (see Fig. 1) for each set.

The model (3) describes a  $n$ -dimensional unknown function

$Y = f(X_1 \dots X_n)$

(5)

Where the fuzzy sets  $A_i$  represent independent variables  $X_i$  (5) and fuzzy sets  $B$  represent the dependent variable  $Y$ .

A given (chosen)  $n$ -dimensional fuzzy set  $Q$ :

$Q = Q_1$  and  $Q_2$  and...and  $Q_n$

(6)

is a fuzzy query. This query can be answered by the fuzzy model (4).

A similarity  $s$  of two  $n$ -dimensional fuzzy sets  $V, W$  is:

$$s(n, V, W) = \min \max_{1 \leq j \leq n} \min_{X_j} (\mu_{V_j}(X_j), \mu_{W_j}(X_j)) \quad (7)$$

Where  $\max \{ \min [\mu_{V_j}(X_j), \mu_{W_j}(X_j)] \}$  represents the fuzzy intersection of two one-dimensional fuzzy sets  $V_j \cap W_j$ .

The similarity  $s$  of the fuzzy set  $R_i$ ,

$R_i = A_{i,1}$  and  $A_{i,2} \dots$  and  $A_{i,n}$ , (see (4))

(8)

And the query  $Q$  (6) is:

$s(n, Q, R_i)$

(9)

Set  $w(Q)$  of those statements (4) which are similar to the query  $Q$  is:

$w(Q) = \{i \mid s(n, R_i, Q) > 0\}$

(10)

The answer of the fuzzy model  $R$  (4) to query  $Q$  (6) is a modification of the following union of  $B$  sets (see (4)):

$$R = \bigcup_{i \in w(Q)} B_i \quad (11)$$

Let us suppose that only two statements out of  $m$  statements (4) are similar to query  $Q$  (6). To be specific, let the first and the second statements be similar. Therefore, see (10).

$$w(Q) = \{1, 2\}$$

Let the similarities  $s$  (9) be,

$$s_1 = s(n, R_1, Q) = 0,6; \quad s_2 = s(n, R_2, Q) = 0,3 \quad (12)$$

Let the variable Industrial Production Growth (IPG) be the dependent variable and  $B_1(M)$  be EFGH and  $B_2(M)$  be IJKL, see Fig. 2 and (4). The fuzzy "answer"  $R(11)$  of the model (4) is represented by the fuzzy set  $EW_1W_2HIW_3W_4L$  (Fig. 2).

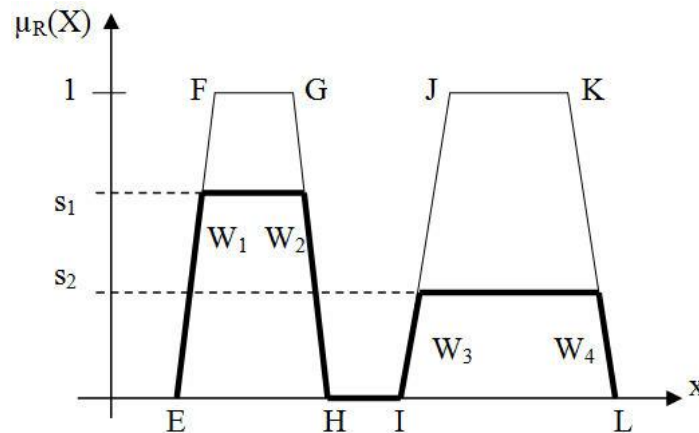


Figure 2. Example of fuzzy answer  $R$  to the query  $Q$   
Source: Dohnal and Dohnalova (1992)

Quite frequently, a fuzzy model is only a part of a more complex and perhaps conventional model. It means that the fuzzy answer  $R$  is an information item that is used for further conventional numerical calculations. Any conventional calculation is based on numbers, therefore *fuzzy*  $\rightarrow$  *numerical value* (13)

The interface is needed to generate a numerical representation of fuzzy answer  $R_{(11)}$ . There are a number of different interfaces described in literature, see e.g. (Dohnal and Dohnalová, 1992). However, this paper requires simplicity. Therefore a centre of gravity CG of the corresponding area (see Fig. 2).

$$C_{GQ}(EW_1W_2HIW_3W_4L) \quad (14)$$

is chosen as the numerical representation.

### **Fuzzy Knowledge Base**

A FES (Fuzzy Expert System) is a computer programme which can perform the above described fuzzy reasoning for any set of statements (3) and corresponding dictionaries (4). The set of statements (3) and the dictionaries (4) represent fuzzy knowledge base (FKB):

$$FKB = [\text{Statements (3), Dictionaries (4)}] \quad (15)$$

It is obvious that FES is universal as it is based on universally applicable fuzzy reasoning algorithm (11). The FKB is specific, as it represents a specific fuzzy model (3, 4).



FKB is inserted into a FES. It must be a simple software procedure. Such FES can be used to answer fuzzy query  $Q(6)$   
 $Q(6) \rightarrow FES\{FKB\} \rightarrow R(11)$  (16)  
 FKB must be developed.

### ***IPO Fuzzy Model***

It is obvious that each FKB is specific. Its specificity is due to the set of variables and the set of variables is predetermined by a studied IPO task. Moreover, the available set of IPO knowledge items must be interpreted as a set of fuzzy conditional statements. There are two basic approaches to fuzzy interpretations of the IPO knowledge: 1) expert interpretation and 2) fuzzy interpretation of non-modified IPO records.

The expert interpretation is based on verbal descriptions of experts' feelings generated by the very nature of verbal quantifications. This approach is highly likely to increase the vagueness of the resulting fuzzy model, for details see e.g. Dohnal and Dohnalová (1992). This is the main reason why the non-modified alternative of fuzzy model development was chosen.

Let us suppose that a set of  $M$  relevant  $n$ -dimensional records of IPO is available. Many IPO variables are non fuzzy by their very nature. For example TAS (Total Assets Size) given in EUR. This value is known absolutely precisely and therefore its fuzzy interpretation can be accurate, i.e. non-fuzzy:

$a = b = c = d$ , see Fig. 1 (You-Shyang Chen and Ching-Hsue Cheng, 2012).

However, common sense indicates that for reasoning purposes TAS interpretation can be partially fuzzy:

$X$  – specific numerical value of TASS; e.g.  $X = 1.82$

$b = X = c$ , see Fig. 1

$a = (1 - \varepsilon)X$ ;  $d = (1 + \varepsilon)X$ ; Dincer et al. (2016).

It means that the fuzzy set ( $R_2$ ) is represented by a triangle and not by a trapezoid as in Fig. 1.

The following variables were chosen by a team of experts, see Table 1.

*Table 1. Variables, Abbreviations and Dimensions*

Variable	Abbreviation	Dimension
Total Assets $(it-1)$	B20	logarithm of assets (in '000 EUR)
Leverage $(it-1)$	D20	total debt over total debt plus equity
Shareholders Funds $(it-1)$	F20	in '000 EUR
ROA $(it-1)$	L20	EBITDA over total assets
Growth $(it/it-1)$	N24	annual growth rate in sales computed as $(it/it-1)$
Investments $(it/it-1)$	O28	growth rate of fixed assets computed as $(it/it-1)$
Leverage $(it)$	R20	total debt over total debt plus equity
Age $(it)$	U20	number of years from incorporation till $it$
Growth $(it+1/it)$	Y24	annual growth rate in sales computed as $(it+1/it)$
Investments $(it+1/it)$	Z	growth rate of fixed assets computed as $(it+1/it)$

Source: Authors' own research.

The fuzzy knowledge base has 94 statements and 10 variables. Each variable has its dictionary. Each dictionary is given by the corresponding numerical values given in Appendix

1 and by the relevant values of the fraction  $\varepsilon$  ( $R_2$ ). The following fractions are used in the text, see Table 2.

*Table 2. Relevant Values of the Fraction  $\varepsilon$*

Variable	Abbreviation	Definition of $\varepsilon$	Auxiliary defining variable $i$
Total Assets $_{(it-1)}$	B20	0.43	-
Leverage $_{(it-1)}$	D20	$= i/X$	0.05
Shareholders Funds $_{(it-1)}$	F20	0.42	-
ROA $_{(it-1)}$	L20	$= i/X$	5
Growth $_{(it/it-1)}$	N24	$= i/X$	0.5
Investments $_{(it/it-1)}$	O28	$= i/X$	0.25
Leverage $_{(it)}$	R20	$= i/X$	0.05
Age $_{(it)}$	U20	0.23	-
Growth $_{(it+1/it)}$	Y24	$= i/X$	0.5

Source: Authors' own research.

### **Fuzzy Queries**

The algorithm of answering queries (16) is relatively simple and very flexible. For example, there is no need to specify all variables, see Tab. 1. Some variables can be ignored. This flexibility can be easily used to perform a dialogue loop:

Investor  $\rightarrow$  FES{FKB}  $\rightarrow$  Investor  $\rightarrow$  FES{FKB}  $\rightarrow$  etc.

However, to take an advantage of the fuzzy flexibility just a subset of  $n$  variables (5) is used. For example, just the following variables (see Tab. 1) are used to specify the query: B20, F20, L20, U20. Just one variable  $V$  (5) represents the answer  $R(11)$ , see (16). It means that the function (5) is modified as follows:  $O28 = f(B20, F20, L20, U20)$

The expert system will then reply an answer with (O28) – based on the information incorporated in the fuzzy knowledge base. The result is calculated as the position of the centre of gravity of the variable fuzzy set measured from the origin of the axis  $x$ .

When defining the concept of the “mature company”, we follow the relevant literature (e.g. Dickinson, 2011; Reiners, 2004; Konečný and Zinecker, 2015) which indicates that the this concept has not been firmly anchored, as there is no fixed definition of how the “mature” company should be old at what rate it is to grow or what size it is supposed to have. As a rule, the “mature” company is defined only verbally and is characterised by qualitative features. Consequently, the concept itself is largely indefinite and it is not clear where the boundary is between the “young” and the “mature” company. In addition, companies operate in a variety of industries where the characteristics of “maturity” may vary. The issue of the membership of the companies surveyed to the category of mature companies has therefore been decided by the fuzzy logic working with the level of affiliation of the surveyed entity to the interest category. The approach allows us to work with the category of “mature” companies without explicitly expressing the sharp boundaries (values) of criteria which would decide whether or not the enterprise belongs to the category of “mature” companies. Fuzzy logic is an instrument which can effectively solve the absence of sharp boundaries and which also allows incorporating into the research the entities on the borderland to meet the definition characteristics.

Based on the relevant literature (Jargot, 2006; Dickinson, 2011; Reiners, 2004; Konečný and Zinecker, 2015), the following variables characterising the “mature” company have been selected: assets, leverage, annual growth rate in sales, number of years from incorporation, equity, and growth rate of fixed assets.

Using the knowledge base, we estimate the development of selected financial characteristics in the pre-IPO (it-1) and post-IPO (it+1) period. As we wish to find out if the proceeds of the issue are used in the surveyed sample of companies for the purpose of financing investments, we focus in particular on comparing the growth rate of investments (fixed assets) in the period before and after the IPO. The values of the selected fuzzy variables are available in the Amadeus database. In determining the qualitative and quantitative definition of fulfilment of the individual criteria, we followed the literature, expert estimates and distribution of the quantitative characteristics of specific criteria in the basic set of surveyed companies. We selected the definition area in order to meet the criteria either in absolute or relative terms, with respect to the examined IPO basic set.

In the first stage, we defined a mature company through the assets variable (B20), when we assumed, in line with the literature (Reiners, 2004) that the values for mature companies would tend to oscillate around the upper limit of the observed asset-size range of all companies conducting the IPO in the examined period. The amount of assets in the surveyed sample of companies (basic set) ranges from EUR 443,000 to 267,000,000. Based on the aforementioned assumption that mature companies tend to be “bigger”, we determined a fuzzy threshold for these businesses in the range of EUR 75 – 350 million, with the “center” of the interval in the form of the highest membership in group was assumed at EUR 200 million. Alternatively, we also calculated the lower limit at EUR 55 million and 100 million. The definition of the size of the assets of companies which can be considered as mature was carried out at the level of the upper quartile of the total assets of all surveyed companies (basic set) which conducted the IPO in the period 2005 – 2015. The “center” means a value which best represents the value of the specific criterion. The lower and upper limits are the boundaries of admissible values, yet not the most common ones.

In addition, we used the variable of the “age” (U20) of the company in the pre-IPO period. We assumed that it will tend to be longer among mature companies (the assumption is based on the fact that mature companies had to undergo the previous stages of the company’s life cycle, which must have taken a certain amount of time). The lower limit was set at 15 years, while the upper limit at 200 years. The centre is then 45 years. The companies in the basic set reach the values from 2 to 124 years.

The gradual depletion of the growth potential of a company was captured by the variable “sales growth” in the pre-IPO period (N20). Such companies achieve positive growth, yet rather lower, single-digit or low two-digit. When determining the parameters of the criterion, we again rely on the assumption of the life cycle stage of the company. Mature companies will have completed a rapid growth phase. However, this growth will be moderate or even negative. In order to define the membership of the companies, we set the lower limit at 0%, the upper limit at 15%, and the “centre” at 5%. Alternatively, the values are as follows: lower limit minus 5%, upper limit 10%, and centre remains unchanged.

The financial leverage indicator prior to the IPO (D20) was also used to define a mature company. We proceed again from the assumption that if a business has reached the stage of a mature company, in the past, it had to undergo a phase allowing it to accumulate

resources for its own development. Therefore, in the range of the leverage values, we considered values rather lower, taking into account the successful history of the company. We propose the level of the parameter so that the lower limit of the indicator is 0, the center at 0.25 and the upper limit at 0.45, alternatively also at 0.75.

## Research Findings

Through the defined variables (B20, U20, N20, and D20), we performed an initial query into the knowledge base concerning the return on assets (ROA) in pre-IPO stage (variable L20). The answers provided by the knowledge base concerning the return on assets of a company prior to the IPO were most frequently in the range of 7 to 13% depending on the range of acceptable thresholds of the input variables (B20, U20, N20, and D20), while only in one case the value of the indicator was negative. The upper limit of 13% was not exceeded. The resulting values of the reruns on assets prior to the IPO (L20) can be perceived as a confirmation of the suitability of selecting the criteria defining a mature company. The values did not acquire extreme values and are positive. They correspond to the idea of a well-established, successful business, however, with already limited growth opportunities, which corresponds to the characteristics of a mature company.

The interval of the return on assets (ROA) values estimated by the knowledge base was further used as part of the knowledge base query concerning the investment growth of a mature company prior to the IPO. Other variables defining the mature company included again the amount of assets prior to the IPO (variable B20), the amount of which is expected to be at the higher limit of the achieved values, the company's age (again rather higher), and the amount of equity in the absolute values (F6). The value of equity ranges from EUR 26,719 to EUR 296,032, with the centre of the interval at the amount of EUR 49,819. In the case mature companies, the amount of equity was expected to reach the upper limit values. Based on the definition, mature companies have successfully passed through the growth phase, thus being large (variable B20) and having sufficient own resources (variable F20).

The answers of the knowledge base concerning the query on the growth rate of investment of mature companies in the pre-IPO period depending on the variability of the input parameters were represented in most cases by positive values ranging from 15% to 35%. Extreme values then reached negative values in the amount of single-digit per cent or positive values at a level slightly above 50% (in the entire surveyed set of companies, there were companies which, in the year prior to the IPO, achieved an 80% investment decrease or, on the other hand, the growth of investments by more than 500%). The recorded answers of the knowledge base are consistent with the idea of a moderate investment development for mature companies.

For further queries into the knowledge base concerning the companies' growth and the growth of their investment in the post-IPO phase, we performed expert adjustments of these variables. We eliminated extreme results from the previous queries, i.e. that the investment growth in the pre-IPO phase exceeded 50% per year, and we also excluded the possibility of a decline in investment. In the long-term, the value of investment growth over 50% is not consistent with the growth of the company in the mature phase of its development. Slightly negative values seem to be more in line with reality, yet in the long-term they will concern businesses which are already losing their positions.

We used the defined investment growth in the pre-IPO phase, together with other variables, to query the knowledge base in order to determine whether the companies in the post-IPO phase accelerate their growth (measured by sales) or not. Other variables used in this query included the assets (B20) in the pre-IPO phase and the age of the company (U20). The return on assets (ROA) variable (L20) in the pre-IPO phase is the outcome of the first query. A supplementary criterion included the leverage in the year of conducting the IPO defined by lower values. The answer using the knowledge base determined the growth (Y19) of the defined companies after conducting the IPO in the range of 29 – 37% per year. This considerably exceeds the original assumptions of the company growth in the pre-IPO phase identified in the first query by defining the N20 variable (the values up to 15% or slightly negative in some variants).

The values related to the growth of corporate sales after conducting the IPO (variable Y19) were used along with other definition features of mature companies estimated on the basis of previous queries into the knowledge base (i.e. the return on assets in the pre-IPO phase (L20) and the investment growth in the pre-IPO phase (O20)) to estimate the investment activity of mature companies after the IPO (Z). The response of the knowledge base indicates that companies after conducting the IPO substantially increase their investment. Depending on the setting of the definition parameters of this group of companies, they achieve the investment growth at the level of top tens per cent, yet the most typical situation consists in more than doubling the investment activity. In the whole set of surveyed companies included in the knowledge base, there are also companies that have experienced a decline in investments after the IPO.

Figure 3 shows the sequence of the queries to the knowledge base, including the defining variables.

## Discussion and Conclusions

The research results show that mature companies, as we defined them through fuzzy variables, significantly accelerate their growth in the post-IPO phase. While their sales grow at a rate of up to 15% per annum before the IPO, the value of this indicator more than doubles in the post-IPO phase. This means that a large number of companies after the IPO significantly increase their investment activity. The finding is especially important for potential investors at the Warsaw Stock Exchange, as academic literature and the results of previous empirical studies suggest that mature companies usually have sufficient equity and access to loan financing. The main motive for the IPO implementation usually includes the efforts to optimise the capital structure, rather than the implementation of new business projects. In the case of these companies, there may also be stronger efforts concerning the IPO timing, i.e. they are listed on the stock exchange only after completing extensive investment policies, being funded, for example, by bank loans. The past growth is supposed to serve as a signal, i.e. to make the issue more attractive in the eye of potential investors (e.g. Chemmanur and Fulghieri, 1999; Pagano et al., 1998; Rajan, 1992).

However, our initial results, which take the form of answers to the knowledge base, show that this is not the case for mature companies entering the Warsaw Stock Exchange. Through the fuzzification of the variables and the subsequent dialogue with the knowledge base, we have succeeded in identifying a group of entities which we define as mature companies and subsequently evaluating their investment activity in the post-IPO phase. The

proposed methodological approach (the fuzzification of the variables and the subsequent dialogue with the knowledge base) overcomes the shortcomings of ambiguous definition or certain indefiniteness of the boundaries defining the category of “mature” companies.

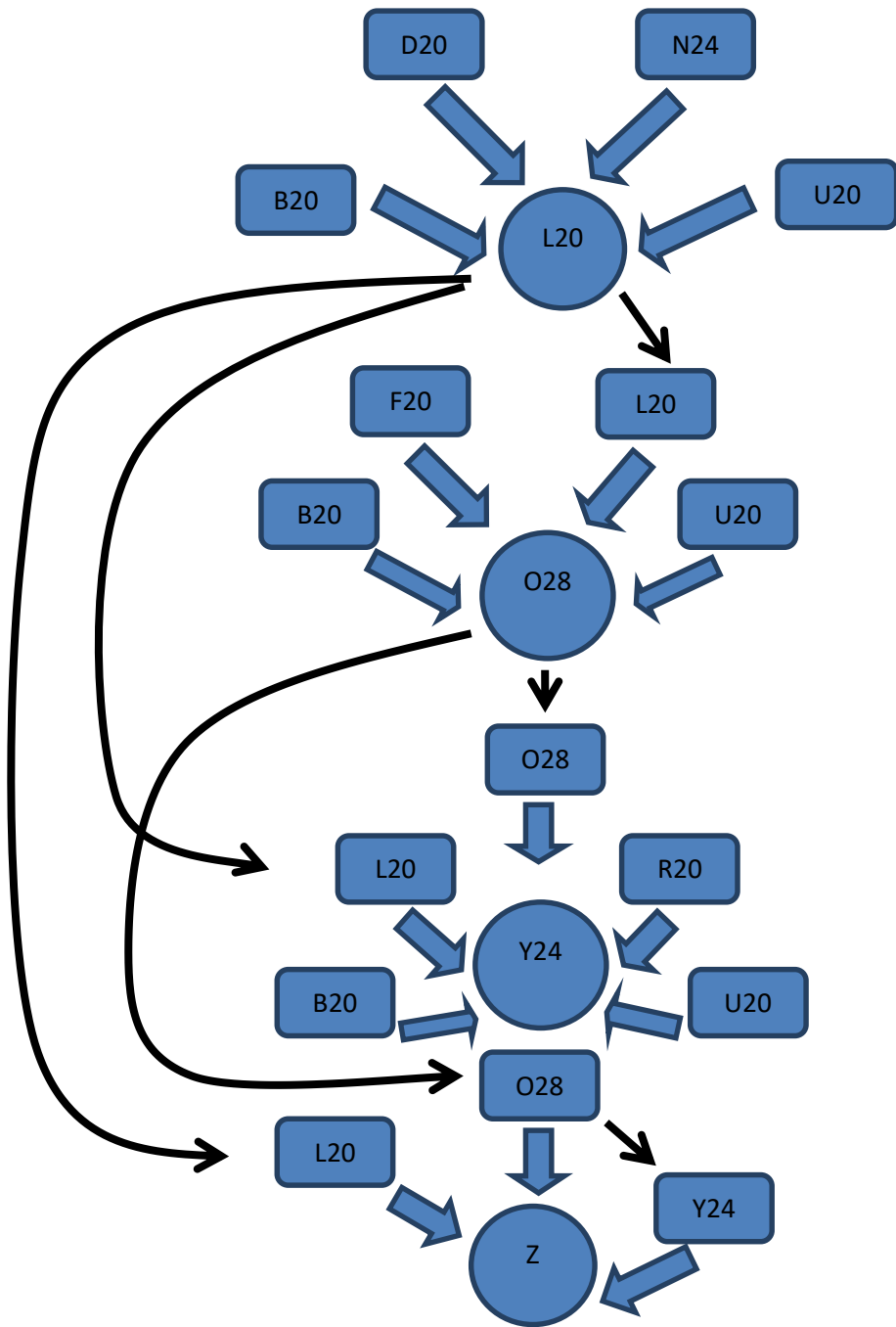


Figure 3. Sequence of queries raised by the knowledge base and using the acquired values for further querying the base  
Source: Authors' own research.

Although the survey methodology used here reveals the very likely motivation of going public, it may also be a source of some limitations. We have the following concerns: research results based on financial data do not represent direct insights from the CFOs; a bias cannot be excluded, because only survey data from decision makers could confirm our conclusions. We analysed financial data of enterprises operating from a part in unfavorable macroeconomic conditions covering financial and economic crisis between 2009 and 2011; this might have affected the financial results and decisions in companies. In follow-up research, we aim to extend the data experiment to other points in time and involve more countries in Europe with a significant IPO activity.

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## Appendix. Fuzzy description of 94 known IPO cases

A1	B1	C1	D1	E7	F1	G	H	I	J	K	L1	M1	N1	O1	P1	Q1	R1	S7	T1	U1	V6	W8	X1	Y9	Z10	0.58
A1	B1	C1	D1	E1	F1	G	H	I	J	K	L1	M8	N2	O1	P1	Q1	R1	S1	T1	U1	V3	W1	X1	Y4	Z15	0.58
A1	B7	C1	D2	E6	F1	G	H	I	J	K	L1	M1	N1	O1	P9	Q1	R5	S7	T1	U9	V1	W1	X1	Y12	Z11	0.58
A1	B7	C1	D2	E1	F2	G	H	I	J	K	L6	M1	N2	O2	P5	Q8	R1	S1	T4	U7	V6	W1	X1	Y17	Z12	0.58
A1	B1	C6	D1	E6	F1	G	H	I	J	K	L5	M5	N1	O7	P1	Q6	R1	S2	T1	U7	V2	W5	X7	Y6	Z9	0.58
A1	B1	C9	D2	E5	F4	G	H	I	J	K	L8	M1	N1	O1	P1	Q1	R1	S2	T1	U4	V6	W4	X5	Y17	Z15	0.58
A1	B2	C1	D3	E6	F2	G	H	I	J	K	L7	M5	N1	O1	P1	Q1	R3	S1	T2	U4	V1	W8	X5	Y15	Z17	0.58
A2	B1	C1	D1	E1	F1	G	H	I	J	K	L2	M2	N7	O3	P1	Q1	R1	S1	T1	U8	V5	W9	X1	Y12	Z16	0.64
A2	B1	C1	D1	E5	F1	G	H	I	J	K	L9	M1	N1	O1	P1	Q1	R1	S4	T1	U9	V1	W1	X1	Y12	Z12	0.64
A2	B1	C1	D1	E1	F1	G	H	I	J	K	L8	M7	N1	O3	P1	Q1	R9	S1	T1	U1	V1	W7	X6	Y15	Z16	0.64
A2	B1	C1	D1	E1	F1	G	H	I	J	K	L6	M7	N1	O8	P1	Q1	R1	S1	T1	U1	V8	W1	X1	Y10	Z11	0.64
A2	B7	C1	D1	E6	F8	G	H	I	J	K	L1	M8	N1	O9	P7	Q1	R9	S3	T7	U1	V1	W1	X8	Y14	Z13	0.64
A2	B8	C9	D1	E6	F5	G	H	I	J	K	L7	M1	N1	O1	P7	Q9	R1	S9	T5	U1	V7	W6	X7	Y15	Z16	0.64
A2	B9	C1	D1	E1	F1	G	H	I	J	K	L1	M1	N1	O1	P9	Q1	R1	S1	T7	U6	V4	W1	X1	Y12	Z11	0.64
A2	B1	C1	D1	E1	F1	G	H	I	J	K	L3	M2	N8	O2	P1	Q1	R1	S1	T1	U5	V9	W1	X1	Y12	Z17	0.64
A2	B1	C2	D1	E1	F1	G	H	I	J	K	L1	M1	N1	O1	P2	Q2	R8	S1	T3	U4	V1	W1	X1	Y17	Z18	0.64
A3	B1	C1	D1	E1	F1	G	H	I	J	K	L1	M1	N1	O1	P1	Q1	R1	S4	T1	U1	V6	W1	X1	Y14	Z15	0.70
A3	B1	C1	D9	E6	F1	G	H	I	J	K	L1	M8	N1	O1	P1	Q1	R9	S2	T1	U1	V1	W8	X7	Y16	Z5	0.70
A3	B8	C1	D1	E1	F4	G	H	I	J	K	L6	M1	N1	O1	P8	Q1	R1	S1	T7	U6	V8	W4	X4	Y7	Z4	0.70
A3	B1	C1	D9	E1	F1	G	H	I	J	K	L1	M1	N1	O1	P1	Q1	R8	S1	T1	U1	V1	W1	X1	Y3	Z10	0.70
A3	B1	C1	D9	E4	F1	G	H	I	J	K	L1	M1	N1	O8	P1	Q1	R5	S6	T1	U1	V1	W1	X1	Y3	Z10	0.70
A3	B1	C1	D1	E1	F1	G	H	I	J	K	L1	M1	N1	O9	P1	Q1	R8	S1	T1	U1	V1	W1	X1	Y14	Z17	0.70
A3	B8	C1	D1	E9	F9	G	H	I	J	K	L1	M1	N1	O1	P1	Q1	R6	S8	T1	U1	V1	W1	X1	Y10	Z8	0.70
A3	B3	C6	D1	E1	F2	G	H	I	J	K	L1	M1	N6	O1	P5	Q5	R5	S1	T6	U4	V1	W5	X4	Y10	Z16	0.70
A3	B1	C1	D1	E7	F1	G	H	I	J	K	L1	M1	N9	O1	P1	Q1	R7	S9	T1	U1	V1	W1	X1	Y3	Z10	0.70
A3	B1	C1	D1	E1	F8	G	H	I	J	K	L9	M1	N1	O1	P1	Q1	R1	S1	T1	U1	V1	W8	X1	Y16	Z5	0.70
A3	B1	C1	D9	E1	F1	G	H	I	J	K	L3	M2	N2	O1	P2	Q5	R9	S3	T2	U1	V1	W1	X1	Y18	Z5	0.70
A3	B1	C1	D5	E9	F1	G	H	I	J	K	L9	M6	N1	O7	P1	Q1	R7	S2	T1	U1	V1	W1	X8	Y3	Z5	0.70
A3	B1	C1	D6	E1	F1	G	H	I	J	K	L1	M8	N1	O1	P1	Q1	R6	S1	T1	U7	V1	W1	X1	Y4	Z7	0.70
A3	B5	C8	D1	E1	F6	G	H	I	J	K	L1	M1	N1	O7	P7	Q9	R6	S7	T8	U1	V1	W1	X1	Y8	Z12	0.70
A3	B6	C7	D1	E1	F5	G	H	I	J	K	L1	M1	N1	O1	P7	Q9	R9	S2	T7	U1	V1	W1	X1	Y7	Z6	0.70
A3	B5	C6	D1	E1	F4	G	H	I	J	K	L1	M1	N1	O2	P1	Q1	R9	S1	T1	U5	V1	W9	X6	Y7	Z12	0.70
A3	B1	C8	D1	E1	F9	G	H	I	J	K	L3	M4	N1	O1	P1	Q9	R1	S1	T1	U1	V9	W4	X3	Y9	Z4	0.70
A3	B4	C6	D2	E8	F7	G	H	I	J	K	L1	M1	N1	O1	P5	Q7	R2	S6	T8	U9	V1	W1	X1	Y2	Z8	0.70
A3	B9	C1	D1	E1	F8	G	H	I	J	K	L0	M9	N1	O6	P7	Q1	R1	S4	T4	U1	V3	W1	X1	Y4	Z13	0.70
A3	B2	C3	D1	E9	F2	G	H	I	J	K	L1	M1	N1	O1	P3	Q4	R7	S5	T4	U6	V1	W1	X9	Y3	Z14	0.70
A3	B6	C7	D1	E1	F5	G	H	I	J	K	L5	M5	N1	O3	P4	Q8	R1	S1	T3	U1	V2	W3	X1	Y3	Z2	0.70
A3	B2	C5	D7	E1	F3	G	H	I	J	K	L4	M3	N5	O1	P1	Q5	R1	S1	T1	U1	V9	W1	X1	Y2	Z2	0.70
A3	B3	C2	D3	E1	F4	G	H	I	J	K	L1	M1	N1	O1	P4	Q2	R1	S1	T5	U8	V1	W9	X5	Y5	Z14	0.70

A3	B12	C18	D16	E9	F11	G7	H7	I2	J7	K4	L7	M8	N12	O16	P13	Q17	R17	S9	T11	U8	V4	W7	X9	Y6	Z12	0.70
A3	B9	C3	D2	E3	F12	G7	H7	I2	J7	K4	L5	M3	N6	O14	P8	Q3	R3	S4	T11	U14	V17	W4	X3	Y4	Z3	0.70
A3	B4	C5	D19	E9	F3	G7	H7	I2	J7	K4	L1	M1	N3	O11	P4	Q4	R4	S7	T5	U18	V17	W2	X2	Y1	Z2	0.70
A3	B19	C16	D11	E1	F19	G7	H7	I2	J7	K4	L2	M2	N2	O12	P19	Q14	R17	S4	T18	U14	V3	W3	X3	Y6	Z8	0.70
A4	B18	C19	D20	E15	F13	G8	H5	I3	J3	K7	L17	M19	N10	O6	P17	Q19	R16	S12	T17	U11	V5	W20	X20	Y9	Z14	0.76
A4	B18	C18	D16	E11	F17	G8	H5	I3	J3	K7	L9	M11	N6	O3	P17	Q18	R14	S10	T17	U18	V6	W12	X15	Y4	Z13	0.76
A4	B16	C18	D9	E11	F17	G8	H5	I3	J3	K7	L13	M13	N3	O9	P16	Q17	R5	S13	T17	U2	V16	W6	X5	Y11	Z14	0.76
A4	B4	C3	D1	E6	F8	G8	H5	I3	J3	K7	L2	M2	N15	O18	P3	Q4	R7	S9	T4	U16	V14	W1	X2	Y17	Z18	0.76
A4	B12	C17	D20	E8	F6	G8	H5	I3	J3	K7	L12	M19	N7	O13	P13	Q17	R16	S7	T11	U18	V5	W16	X18	Y3	Z14	0.76
A4	B17	C16	D19	E7	F14	G8	H5	I3	J3	K7	L4	M5	N8	O9	P16	Q15	R20	S6	T10	U10	V1	W4	X7	Y14	Z4	0.76
A4	B5	C7	D12	E12	F5	G8	H5	I3	J3	K7	L15	M15	N17	O16	P7	Q9	R13	S9	T6	U7	V7	W15	X16	Y11	Z15	0.76
A4	B5	C4	D3	E15	F5	G8	H5	I3	J3	K7	L13	M8	N3	O6	P3	Q4	R7	S11	T4	U2	V14	W10	X6	Y11	Z9	0.76
A4	B3	C4	D12	E12	F3	G8	H5	I3	J3	K7	L6	M16	N3	O15	P3	Q3	R8	S15	T3	U9	V13	W12	X10	Y16	Z5	0.76
A4	B6	C4	D6	E13	F7	G8	H5	I3	J3	K7	L17	M16	N4	O13	P5	Q3	R2	S10	T7	U12	V18	W16	X13	Y5	Z7	0.76
A4	B1	C2	D6	E6	F1	G8	H5	I3	J3	K7	L15	M11	N5	O15	P1	Q2	R3	S10	T1	U9	V18	W5	X3	Y1	Z2	0.76
A4	B3	C4	D12	E17	F3	G8	H5	I3	J3	K7	L7	M5	N2	O5	P2	Q3	R10	S17	T2	U2	V10	W7	X7	Y5	Z6	0.76
A5	B20	C15	D15	E14	F19	G5	H2	I6	J1	K5	L6	M6	N15	O7	P19	Q19	R11	S17	T19	U1	V10	W16	X16	Y13	Z15	0.82
A5	B9	C7	D16	E15	F8	G5	H2	I6	J1	K5	L11	M14	N6	O3	P7	Q16	R13	S16	T6	U12	V8	W14	X15	Y10	Z17	0.82
A5	B8	C7	D2	E1	F11	G5	H2	I6	J1	K5	L1	M2	N8	O4	P5	Q7	R4	S13	T7	U4	V17	W2	X2	Y16	Z15	0.82
A5	B7	C5	D1	E13	F11	G5	H2	I6	J1	K5	L17	M10	N5	O4	P6	Q5	R1	S8	T9	U14	V20	W15	X8	Y14	Z12	0.82
A5	B12	C8	D16	E15	F10	G5	H2	I6	J1	K5	L7	M8	N1	O4	P10	Q5	R18	S1	T8	U15	V3	W3	X3	Y16	Z5	0.82
A6	B18	C20	D17	E17	F16	G11	H7	I8	J3	K3	L9	M13	N9	O12	P18	Q20	R19	S17	T16	U2	V2	W8	X15	Y8	Z2	0.88
A6	B16	C19	D16	E17	F15	G11	H7	I8	J3	K3	L9	M11	N7	O9	P16	Q19	R14	S17	T16	U6	V6	W8	X8	Y8	Z7	0.88
A6	B12	C15	D15	E13	F11	G11	H7	I8	J3	K3	L8	M9	N18	O9	P16	Q16	R16	S18	T15	U15	V5	W7	X9	Y10	Z3	0.88
A6	B20	C10	D16	E15	F19	G11	H7	I8	J3	K3	L3	M3	N19	O5	P19	Q15	R11	S18	T19	U2	V10	W6	X6	Y1	Z3	0.88
A6	B10	C13	D1	E17	F0	G11	H7	I8	J3	K3	L14	M13	N8	O7	P9	Q12	R11	S7	T9	U7	V9	W13	X14	Y14	Z18	0.88
A6	B13	C14	D20	E8	F7	G11	H7	I8	J3	K3	L14	M19	N9	O2	P10	Q13	R20	S15	T3	U16	V1	W14	X20	Y6	Z3	0.88
A6	B17	C10	D16	E8	F15	G11	H7	I8	J3	K3	L2	M2	N8	O14	P17	Q10	R18	S7	T16	U12	V3	W4	X5	Y18	Z2	0.88
A6	B6	C7	D3	E18	F9	G11	H7	I8	J3	K3	L17	M15	N10	O8	P6	Q7	R3	S18	T8	U7	V18	W17	X14	Y6	Z10	0.88
A6	B5	C7	D13	E15	F5	G11	H7	I8	J3	K3	L8	M7	N5	O16	P5	Q6	R18	S12	T3	U16	V3	W6	X11	Y5	Z4	0.88
A6	B18	C13	D15	E3	F18	G11	H7	I8	J3	K3	L15	M16	N1	O11	P18	Q6	R15	S3	T18	U10	V6	W13	X18	Y17	Z3	0.88
A6	B3	C2	D3	E19	F5	G11	H7	I8	J3	K3	L7	M13	N11	O11	P4	Q2	R2	S19	T5	U6	V18	W17	X13	Y13	Z9	0.88
A6	B6	C3	D6	E1	F8	G11	H7	I8	J3	K3	L16	M16	N19	O18	P9	Q6	R13	S1	T8	U11	V7	W10	X16	Y2	Z1	0.88
A6	B20	C7	D2	E16	F20	G11	H7	I8	J3	K3	L14	M8	N2	O3	P18	Q5	R2	S15	T20	U2	V18	W3	X8	Y3	Z2	0.88
A7	B14	C13	D4	E5	F16	G3	H6	I4	J4	K6	L6	M5	N16	O8	P13	Q14	R11	S4	T13	U15	V9	W12	X12	Y16	Z7	0.94
A7	B6	C14	D7	E17	F7	G3	H6	I4	J4	K6	L20	M19	N13	O19	P8	Q13	R6	S15	T9	U9	V15	W19	X19	Y15	Z16	0.94
A7	B14	C14	D7	E12	F16	G3	H6	I4	J4	K6	L5	M4	N13	O3	P12	Q13	R8	S10	T13	U10	V13	W14	X12	Y9	Z7	0.94
A7	B14	C15	D13	E6	F14	G3	H6	I4	J4	K6	L13	M14	N5	O1	P10	Q13	R10	S8	T10	U10	V10	W18	X18	Y7	Z11	0.94
A7	B9	C13	D17	E15	F6	G3	H6	I4	J4	K6	L4	M4	N7	O13	P8	Q11	R13	S12	T7	U15	V7	W7	X6	Y10	Z7	0.94
A7	B14	C9	D8	E17	F15	G3	H6	I4	J4	K6	L4	M3	N17	O2	P11	Q11	R8	S17	T12	U2	V12	W17	X16	Y9	Z5	0.94

A7	B15	C12	D12	E15	F15	G3	H6	I4	J4	K6	L11	M9	N4	O10	P14	Q9	R11	S12	T15	U16	V10	W9	X8	Y13	Z14	0.94
A7	B9	C11	D2	E10	F12	G3	H6	I4	J4	K6	L19	M17	N6	O10	P7	Q10	R4	S10	T9	U11	V17	W19	X18	Y11	Z10	0.94
A7	B13	C10	D2	E11	F17	G3	H6	I4	J4	K6	L12	M6	N5	O5	P11	Q9	R4	S11	T14	U11	V16	W14	X10	Y12	Z8	0.94
A7	B4	C6	D2	E19	F8	G3	H6	I4	J4	K6	L19	M18	N16	O15	P5	Q7	R1	S19	T8	U7	V19	W16	X11	Y13	Z6	0.94
A7	B9	C8	D17	E2	F6	G3	H6	I4	J4	K6	L8	M13	N7	O12	P8	Q8	R17	S2	T5	U9	V3	W13	X18	Y7	Z11	0.94
A7	B15	C6	D13	E1	F15	G3	H6	I4	J4	K6	L7	M8	N10	O3	P14	Q6	R13	S1	T14	U2	V8	W4	X4	Y9	Z12	0.94
A7	B4	C4	D12	E2	F4	G3	H6	I4	J4	K6	L11	M10	N7	O7	P3	Q5	R9	S4	T3	U15	V11	W19	X18	Y9	Z6	0.94
A8	B17	C19	D18	E8	F14	G4	H4	I5	J2	K5	L3	M2	N5	O8	P15	Q18	R19	S8	T10	U14	V1	W3	X2	YU N	ZU N	1.00
A8	B14	C18	D16	E19	F13	G4	H4	I5	J2	K5	L11	M14	N11	O13	P15	Q18	R18	S12	T13	U10	V3	W10	X16	YU N	ZU N	1.00
A8	B17	C15	D15	E19	F17	G4	H4	I5	J2	K5	L3	M3	N18	O6	P16	Q16	R16	S20	T15	U3	V5	W4	X3	YU N	ZU N	1.00
A8	B11	C14	D1	E12	F16	G4	H4	I5	J2	K5	L19	M16	N2	O5	P10	Q11	R1	S11	T14	U14	V20	W16	X10	YU N	ZU N	1.00
A8	B18	C9	D2	E6	F19	G4	H4	I5	J2	K5	L11	M6	N3	O11	P17	Q7	R4	S5	T19	U4	V16	W6	X5	YU N	ZU N	1.00
A8	B5	C4	D10	E16	F6	G4	H4	I5	J2	K5	L5	M4	N10	O2	P3	Q5	R18	S7	T2	U3	V3	W1	X1	YU N	ZU N	1.00
A8	B16	C3	D4	E20	F18	G4	H4	I5	J2	K5	L2	M2	N2	O5	P15	Q2	R5	S5	T17	U16	V16	W3	X3	YU N	ZU N	1.00
A8	B2	C1	D1	E20	F3	G4	H4	I5	J2	K5	L19	M15	N14	O1	P2	Q1	R2	S15	T3	U5	V18	W1	X2	YU N	ZU N	1.00

Source: Authors' own research.