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PROBABILISTIC ANALYSIS FOR ESTIMATION OF THE INITIATION TIME OF CORROSION

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Abstract: In this paper, a probabilistic study on durability concrete was carried out. In such a design, initiation time of corrosion must be expressed as a mathematical model using Fick's second law and the statistical distributions properties of theirs parameters was included in this model. The scatter both in the environmental exposure conditions and structural properties was considered as random fields in the mathematical model with a probabilistic design. The main objective of this study is predicted initiation time of corrosion of concrete structures in chloride containing environments. This probabilistic study is developed using Monte Carlo simulation to determine the contribution of each input parameters and the statistical parameters of the random variables on the probability distribution functions of the initiation time of corrosion. Also, a comparison study was carried out to analyze the impact of the probability distribution on the response (the initiation time of corrosion).

Keywords: Durability, chloride effect, corrosion, spatial variability, Monte-Carlo simulation.

1. Introduction

The major cause of degradation of reinforced concrete bridge structures is chloride-induced corrosion of the reinforcing steel. This problem can impair to important serviceability and safety reductions as well as increasing repair and maintenance costs [1, 2, 3].

The initiation time of corrosion (tini) is a key factor in the service life prediction of a concrete element, because the risk of steel corrosion is highly dependent on the quality of design and construction of the concrete cover. Which concrete represents the physical barrier against any external aggressive agents. Increasing the density and impermeability of the concrete cover by reducing the water-cement materials ratio and producing properly placed, compacted, and cured concrete, reduces the apparent chloride diffusion and consequently delays the initiation of corrosion.

The analysis approach based on the probabilistic method is the most reliable way to solve uncertainty problems. The latter has attracted a lot of interest from researchers recently [1, 2, 4, 5, 6, 7, 8, 9]. As reliability concepts are better understood and more software developed, reliability-based applications move from simple, hypothetical examples using fictitious data to more complex, practical, and realistic engineering problems [10].

The present work aims to predict initiation time of corrosion (tini) of concrete structures in chloride environments using a spatial variability approach. This approach takes into account the spatial variability of the different parameters of the structure such as, the surface concentration of chloride (Cs), the concentration threshold (Cth), the diffusion coefficient (Dc) and the coating (ct) appearing in simple Model.

The Monte Carlo Simulation (MCS) methodology is used to compute the Probability Distribution Function (PDF) and the failure probability of the system response of the initiation time of corrosion. To illustrate the prediction of the service life of a reinforced concrete structure, an example of a concrete bridge element deteriorating due to chloride-initiated corrosion is analyzed [11].

2. Simplified model for predicting initiation time of corrosion

The simplified model currently used to describe chloride penetration in concrete is shown in mathematical form in Eq. (1), the concentration of free chloride ions C(x, t) at a depth x after a time t for a semi-infinite medium is:

$$C(x,t) = C_s \times \left[1 - \text{erf}\left(\frac{x}{2 \times \sqrt{D_c \times t}}\right) \right] \tag{1}$$

Based on Eq. (2), solutions for calculating have been proposed in Eq. (3). For the classical solution of Fick's law Eq. (1), is obtained when C(x, t) is equal to and x is equal to the thickness of the concrete coating as follows [12]:

$$C_{th} = C_s \times \left[1 - \text{erf}\left(\frac{c_t}{2 \times \sqrt{D_c \times t_{ini}}}\right) \right]$$
 (2)

$$t_{ini} = \frac{c_t^2 \left[erf^{-1} \left(1 - \frac{c_{th}}{c_s} \right) \right]^{-2}}{4D_c}$$
 (3)

Where:

tini: The initiation time of corrosion is calculated by comparing the chloride concentration in the concrete depth C (ct, t) with the concentration threshold (Cth).

erf-1(): is the inverse of the error function.

Where x in Eq. (1) is replaced by the concrete cover depth ct since Eq. (3) refers to the first layer of reinforcement. The Cth is defined as the chloride concentration in which the passive rust layer of the steel is destroyed and the corrosion reaction begins. Therefore, the tini is obtained when the chloride concentration at the bar coverage reaches the concentration threshold (Cth) [5, 13].

2.1. Structural discretization with expansion Karhnuen-Loeve (K-L)

In the literature, we can find several methods of discretization proposed by the researchers, which consist of breaking down the initial fields into optimal complete deterministic functions [14, 15, 16]. To do this, the discretization of a section involves a subdivision of the sample structure into small pieces of random field in the direction of the x-axis and the y-axis, as shown in Fig. 1. But, the discretization with more elements, provides more accurate results but also requires more computing time.

The transition from the representation of the continuous random field to a limited number of random variables is necessary to introduce the uncertainty of the properties of the materials into a computation model. To do this, we must choose a discretization method. In this study, the Krahunen-Loève expansion method (K-L) was adopted for the discretization of the region studied in a concrete beam of bridge in the Netherlands [11].

The corrosion phenomenon can be modeled by several methods (eg. FORM, FOSM ...) but MCS is considered the most rebade method between them [17, 18]. The Monte Carlo Method can be used to perform the reliability analysis with respect to the occurrence of each possible failure mode. In this paper, it is carried out in the space of independent standard normal variables.

In this study, the choice of the normal probability distribution is based on literature [11, 19, 20]. Also, several researchers have used this distribution to study the effect of the random variables of the response system [11,19, 20, 21, 22]. It is possible to model some phenomena with a lognormal probability distribution. For this purpose, a comparative study between the normal and lognormal distribution on the PDFs of the system response (tini) was studied in section 3.2.

Table 1

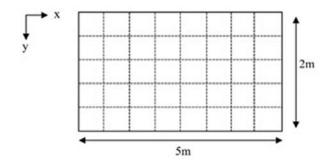


Fig. 1- 2-dimensional discretization model.

2.2. Statistical quantization of parameters in the simplified model

The parameters of a model can be quantified using real structural measurements or, if this isn't possible, the measurements should be made in the laboratory. It is therefore understandable that the statistical characteristics of the parameters vary according to the source of the measurements [23]. In this case, the quantification was based on measurements in a concrete beam of bridge in the Netherlands [11]. This region of exposed structures is located in a very aggressive marine environment.

Table 1 gives input parameters of the quantification of variables in the following simplified model. The mean and the standard deviation of the parameters are provided by authors [19, 20].

Input data for the initiation time of corrosion [19, 20].

| Variables | Description | Distribution | Mean | Standard deviation | Coefficient of variation (%) |
|-----------|----------------------------------------------|--------------|-------------------------------------|-------------------------------------|------------------------------|
| ct | Concrete cover | Normal | 41.1(mm) | 41.1(mm) 1.4(mm) | |
| Cs | Chloride ion content at the concrete surface | Normal | 5.3%(by mass of cement) | 1.47%(by mass of cement) | 28 |
| Cth | The chloride threshold concentration | Normal | 0.5%(by mass of cement) | 0.1%(by mass of cement) | 20 |
| Dc | Effective diffusion of chloride | Normal | 8.83×10 ⁻⁶ (m²/years) | 3.69×10 ⁻⁶ (m²/years) | 42 |

In table 1, the COVs of the random variables presented in the table are calculate by Eq. (4) in paragraph 2.3. We note that the concrete cover has a very small standard deviation. Rooij and Polder [20] considerate the concrete quality for the bridge elements under is very good. On the other hand in the literature ct is always to study with important values by eg. in Duracrete [21] the mean and the standard deviation are respectively equal to 40 mm, 8mm.

2.3. Design study

To study civil engineering works using the probabilistic approach, the limit state function is defined by the safety domain or in the failure domain. In this study, the failure probability are less than 10⁻³ are considered stable [24].

Also, the input parameters (Cs, Dc, Cth and ct) of the simplified model Eq. (1), were taken into account as random fields. The failure probability is calculated using Monte Carlo simulation (MCS). This has the advantage of giving accurate results, but with a very important calculation time. Also, Number of MCS should be sufficient for a rigorous calculation of the failure probability. For this purpose, it is important to perform a study on the simulation number that corresponds to a low coefficient of variation of failure probability COV(Pf) with a reasonable computation time.

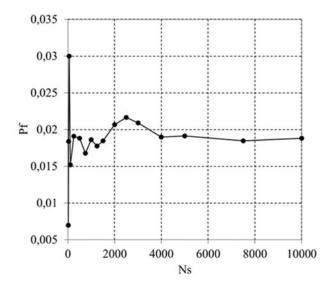


Fig. 2- Effect of the simulation number on the Pf.

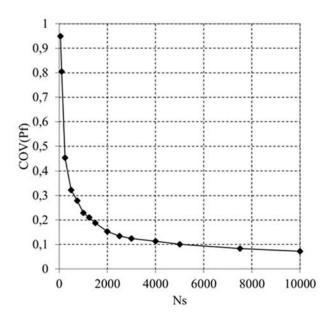


Fig. 3. Effect of the simulation number on the COV(Pf).

The COV(Pf) can be calculated by the following Eq. (4) [18]:

$$COV(P_f) = \sqrt{\frac{1 - P_f}{N \times P_f}} \tag{4}$$

The Fig. 3 shows the effect of the simulation number on the COV(Pf). It was found that the COV(Pf) decreases with the increase in the simulation number. It reaches a value of less than 10% from 5000 simulations. The Fig. 3 indicates that the breaking threshold decreases with successive levels. This means that the proposed procedure is valid because the realizations generated by this procedure successfully progress towards the boundary of the surface state.

Similarly, it was found that when the simulation number is equal to 2000 for Pf = 2×10^{-2} which corresponds to a COV(Pf)=15%. On the other hand, for the number of simulation is equal to 10000 the Pf= 1.9×10^{-2} in Fig. 2. However, the MCS method becomes more expensive in terms of computation time to determine the low failure probabilities Pf ($\leq 10^{-3}$) [25]. This is due to the large number of realizations needed in such a case. To this effect, to reduce the computation time, the number Ns = 2000 realizations were used to compute the failure probabilities for a considerable gain of time (8 hours).

2.4. Effect of the autocorrelation distance on the failure probability

In order, to determinate the horizontal and vertical autocorrelation distance Lx and Ly respectively, their impact on failure probability of the initiation time of corrosion Pf, has been studied.

Fig. 4 shows the effect of autocorrelation distances on the failure probability corresponding to an isotropic random field (Lx = Ly). In this case, the failure probability has been calculated and has the same average values as the random variables.

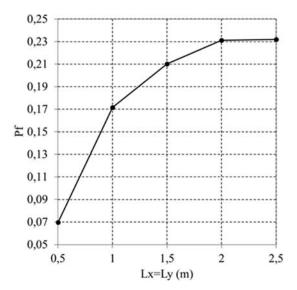


Fig. 4- Effect of the autocorrelation distance on the failure probability in the case of a random field Lx= Ly.

Fig. 4 indicates that increasing the autocorrelation distance increases the failure of probability. However, decreases the mark-up rate for large autocorrelation distances. This is because the random field tends to the case of a homogeneous material for large values of autocorrelation distances (Lx=Ly>2m). The increase in the failure of probability due to the increase of the autocorrelation distance can be explained as follows: when the autocorrelation distance is very large, the material tends to be homogeneous. In this case, the initiation time of corrosion was considered too close to that obtained during the study of a homogeneous material. For smaller autocorrelation distances, a heterogeneity of the material is obtained which results in a variability of the input parameters.

In other to study the impact of anisotropic random filed of inputs parameters, the failure probability was plotted against the vertical and horizontal autocorrelation distance in Fig. 5 and 6, respectively.

Fig. 5 and 6 show that the autocorrelation distance increase with the increase in the failure probability. This observation can be explained, when the autocorrelation distance is very large, the material tends to be homogeneous.

The low value of the autocorrelation distance (Lx or Ly) induces a great heterogeneity which results in a great variability of the random variables. This variability leads to a smaller failure probability.

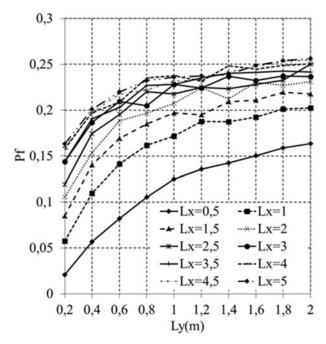


Fig. 5- Effect of the vertical autocorrelation distance Ly on the failure probability for different values of Lx.

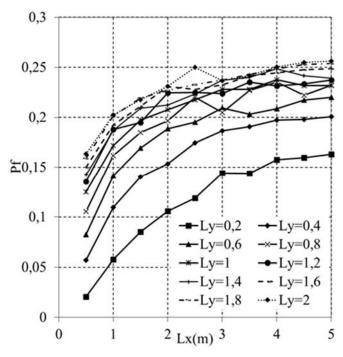


Fig. 6. Effect of the horizontal autocorrelation distance Lx on the failure probability for different values of Ly.

On the other hand, the increase in the autocorrelation distance increases the initiation time of corrosion "tini" and consequently the failure probability increases.

When the autocorrelation distance is very large, the problem becomes similar to that of the random field dimension (1D), hence the failure probability is less than the two-dimensional probability. Therefore, to give our program a more realistic approach (heterogeneous medium in a reinforced concrete structure), we used small values for distances Lx = 0.5 and Ly = 0.4.

3. Parametric study

A parametric study was undertaken to investigate the effect of the random variables (the coating ct, the chloride surface concentration Cs, the chloride threshold concentration Cth and the diffusion coefficient Dc)on the probability distribution functions (PDF) of the system response (tini). Also, the probability distribution of the random variables on the PDFs responses was studied.

3.1. Impact of the coefficients of variation (COV) of the different random variables on the PDFs of the system response (tini)

To investigate the effect of COV of the random variables on the PDFs of the system response, the COV for this variable is increased or decreased by 50% from its baseline value given in Table 2.

 $Table\ 2$ Effect of coefficients of variation of random variables on corrosion initiation time.

| Random variables | Coefficient of variation COV(Random variable) % | Statistical parameters of PDF | | | | | | |
|---------------------|-------------------------------------------------|-------------------------------|----------------------------------|----------|----------|-------------------------------------------|--|--|
| | | Mean (years) | Standars deviation (years) | Skewness | Kurtosis | Coefficient of variationof PDF % | | |
| ct | 1.7 | 35.196 | 3.889 | 1.021 | 5.854 | 11.049 | | |
| | 3.4 | 35.068 | 3.709 | 1.257 | 8.626 | 10.577 | | |
| | 5.1 | 35.039 | 3.278 | 1.025 | 5.867 | 9.355 | | |
| Cs | 14 | 34.832 | 2.909 | 1.12 | 7.460 | 8.352 | | |
| | 28 | 35.068 | 3.709 | 1.26 | 8.626 | 10.577 | | |
| | 42 | 35.516 | 4.555 | 1.44 | 9.122 | 12.825 | | |
| Cth | 10 | 35.313 | 4.153 | 1.10 | 6.296 | 11.761 | | |
| | 20 | 35.068 | 3.709 | 1.26 | 8.626 | 10.577 | | |
| | 30 | 34.799 | 2.999 | 0.703 | 3.868 | 8.618 | | |
| Dc | 21 | 34.409 | 1.591 | 0.545 | 3.880 | 4.624 | | |
| | 42 | 35.068 | 3.709 | 1.257 | 8.626 | 10.577 | | |
| | 63 | 36.208 | 8.689 | 3.361 | 30.885 | 23.997 | | |

To initiate corrosion, external chlorides must be transported inside the concrete and reach the recessed steel [25, 26]. For this reason, the thickness of the coating is one of the most important parameters affecting the service life of the structures. However, Fig. 7 shows that when the coefficient of variation COV of the coating (ct) increases, the mean and the standard deviation of the initiation time of corrosion vary slightly. It is found that the COV proposed by Ying [11] is less important than those found in the literature [6, 27]. For this purpose, the coefficient of variation COV of the coating (ct) has practically no impact on the PDF. It is to be supposed that the thickness is uniform along the structure.

Although the thickness of the coating is theoretically considered a constant value, it varies from place to place and this variation is closely related to the level of quality control during construction. For example, in the Lounis and Amleh study [27] of the now-demolished Dickson Bridge in Montreal, the thickness of the concrete cover was measured directly at many locations on the bridge. The average coating depth was 36.6 mm with a coefficient of variation of COV of 45%. The specified depth of overlap was 25 mm. It is therefore proposed to use values of coefficients of variation of 10%, 20% and 30% (and beyond) for satisfactory, moderate and weak quality control of construction, respectively [6].

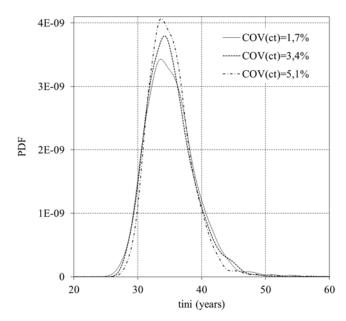


Fig. 7- Influence of coefficients of variation of coating "ct" on PDFs.

To study the effect of the coefficient of variation of surface chloride concentration on the probability of depassivation, values ranging from 14% to 42% were used to obtain the probabilities. The results are shown in Fig. 8.

In a corrosive environment, Cs has a significant impact on the initiation phase of steel reinforcement corrosion. Fig. 8 clearly shows this influence also in Table 2, the mean and standard deviation of tini increases with increasing COV(Cs). The skewness and the kurtosis of the responses is also affected by the increase of the coefficient of variation.

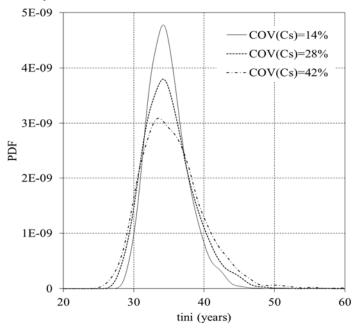


Fig. 8- Influence of the coefficients of variation of chloride concentration on the "Cs" surface on PDFs.

In addition to the surface concentration (Cs) of the structure studied, another concentration of chloride called chloride threshold (Cth) was taken into account in this study. The chloride threshold level is the concentration of chlorides required to break down the protective passive film on the reinforcing surface and initiate corrosion. The chloride thresholds proposed in the literature cover a wide range of values. Glass and Buenfeld [28] discussed the different factors affecting the threshold value and summarized the reference values.

Fig. 9 shows the effect of the COV coefficient of variation of the chloride threshold con-centration (Cth) on the tini. These results were obtained using various COVs of Cth.

In this study, the mean chloride threshold value (Cth) is 0.5% located just between the value proposed by the ACI committee [29], which is at the conservative end of the range, and the value of 0.4% used. In Europe, which seems to be a more appropriate value. In this effect, Fig. 9 and Table 2 show that the variation of the COV(Cth) has an impact on the tini. In other words, the tini decreases with the increase of the COV(Cth).

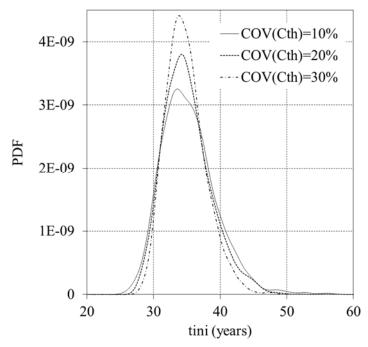


Fig. 9- Influence of the coefficients of variation of the "Cth" chloride threshold concentration on PDFs.

Figure 10 shows the effect of the COV of the diffusion coefficient (Dc) on the PDFs. From this figure, it was observed that the coefficient of diffusion has a great impact on the PDF of system response.

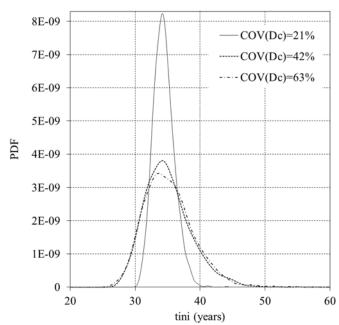


Fig. 10- Influence of coefficients of variation of effective diffusion coefficient of chloride "Dc" on PDF.

Also, we can see from Table 2 the COV(Dc) increases systemically with the random field (From 34.5 years for COV(Dc)=21% to 36 years for COV(Dc)=63%). This means that random field has important impact in the variability of the system response (tini).

Kurtosis is a descriptor of the shape of a probability distribution. Table 2 shows that with increasing standard deviation, kurtosis increases for Cs and Dc. The measurement made of the dispersion given

by the standard deviation shows that the distribution of the probability masses is around their center. In other words, from Figs. 8 and 10, each time the COV increases, the flattening of the curves is noted. On the other hand, for the Figs 7 and 9 the flattening of the curves are negligible.

The mean of tini increases in a weak way with the increase of the COV(Dc), unlike the other statistical moments (standard deviation, skewness, kurtusis) which increase of a very important way.

3.2. Impact of probability distributions on PDFs of system response (tini)

The impact of the probability distribution (Normal "DN" or Lognormal "DLGN") of the random field parameters on the PDFs of the studied system response was studied. In the Fig. 11, 12, 13 and 14 the probability density function of the initiation time of corrosion (tini) is computed on the basis of the input parameters present in Table 1 using normal and lognormal distributions.

It is noted in Fig. 11 and 12 that the probability distribution change for the random fields Cth and ct has no effect on their PDFs responses unlike Dc and Cs.

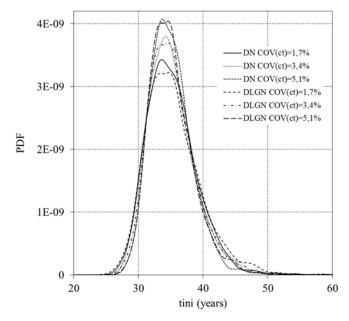


Fig. 11- Influence of the variation of the probability distribution of effective diffusion of chloride "ct" on PDF.

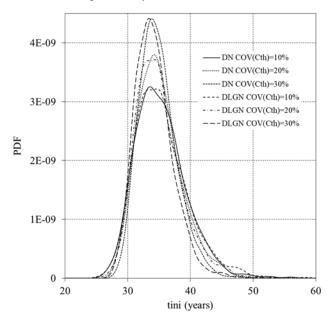


Fig. 12- Influence of the variation of the probability distribution of effective diffusion of the chloride "Cth" on PDF.

With the increase of COV(Dc), in Fig. 13, the impact of the probability distribution increases. It has been noticed that there is a slight influence of the probability distribution on the mean of tini and the PDF (COV(Dc)=63% the difference of tini between the distribution of the normal law and log normal is 7 years). In other words, it can be said that for homogeneous materials (with low COVs) the impact of probability distributions is negligible.

The use of the lognormal distribution in the input random fields gives much greater tini than in the case of taking into account a normal distribution. As a result, the tini output parameter will be overestimated, which may lead to an estimate of the wrong maintenance time.

With the increase of COV, it was noticed that there is a slight influence of the distribution law on the mean of tini (COV(Cs)=42% the difference of tini between the distribution of the normal law and log normal is 3 years), its shows in Fig. 14.

Skewness demonstrated a measure of the asymmetry of the probability distribution of random variables around their mean. If skewness is > 0 then they say that the distribution is asymmetric left, <0 asymmetric right else = 0 the distribution is symmetric [30]. In figs. 13 and 14 the distribution is left-skewed for the use of the lognormal distribution for different COVs. In figs. 11 and 12 the distribution almost symmetric.

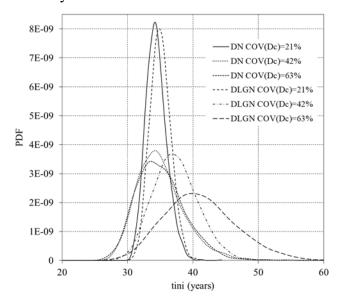


Fig. 13- Influence of the variation of the probability distribution of effective diffusion of chloride "Dc" on PDF.

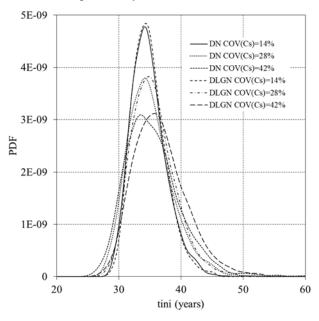


Fig. 14- Influence of the variation of the probability distribution of effective diffusion of chloride "Cs" on PDF.

4. Conclusions

This paper aims to present a probabilistic analysis of the corrosion initiation time caused by chloride ions using the Monte Carlo simulation method. The random fields were the coating, the chloride surface concentration, the chloride threshold concentration and the diffusion coefficient. The deterministic model was based on Fick's second law describing conventionally the network initiation time of corrosion implemented in the Matlab software. The main conclusions of the document can be summarized as follows:

A parametric study has shown that the initiation time of corrosion was mainly induced by diffusion coefficient (Dc) and the chloride surface concentration (Cs). the thickness of the coating and the chloride threshold concentration have practically negligible contribution and proves to be factors that does not affect the reinforced concrete structure to be studied. This last parameters have a negligible weight in the variability of response. It can be considere this parameters deterministic. So, we can reduce the computation time of the probabilistic analysisand the cost of the experimental investigation of new similar projects when detecting the most influential parameters on the variability of the system responses, then the non-influential parameters do not need a thorough experimental investigation on their variability.

The coefficient of variation of diffusion coefficient (Dc) and the chloride surface concentration (Cs) was found much greater than the random fields Cth and ct. Therefore, the input uncertain parameters that have a significant weight in the variability of this response (i.e. tini) should be thoroughly investigated in practice. The input parameters for whith the COVs have the largest contribution, are of most significant influence on the variability of a system response.

The analysis results showed that using different distribution type for problem parameters, proportion with distribution type is effective in calculation results. The initiation time of corrosion can be overestimated by the choice of type of distribution of the random input fields. This overestimation may lead to an estimate of the incorrect maintenance time.

For the low COVs of the different random fields, the impact of the probability distributions on PDFs of system response (tini) is negligible. On the other hand, the impact is important for the random fields (Dc and Cs) when there are considerable values of their COVs.

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