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On Some Properties of Fuzzy Soft almost Soft Continuous Mappings

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Abstract. In this paper, we initiate and explore the interesting characterizations and properties of fuzzy soft almost soft continuous mappings in fuzzy soft classes. We also study and discuss the notions of fuzzy soft almost soft open(closed) mappings. Moreover the characterizations of composition of two fuzzy soft almost soft mappings are also studied. We hope that the findings in this paper will be useful for the researchers working in the fields such as fuzzy control systems, fuzzy automata, fuzzy logic, information systems and decision making problems.

Key words and phrases. Fuzzy soft sets, Fuzzy soft topology, Fuzzy soft classes, Fuzzy soft almost soft continuous, fuzzy soft almost soft open(closed).

1. Introduction

Fuzzy set theory initiated and studied by Zadeh[31] proved to be an important mathematical tool to solve different types of complicated problems having uncertainties in real life problems such as sociology, economics, engineering, computer and medical sciences etc. with ambiguous environment. Now a days, both mathematician and computer scientists are applying this theory, not limited to such as fuzzy control systems, fuzzy automats, fuzzy logic, fuzzy topology etc.

Molodtsov [24] introduced the concept of soft sets, which is new approach for modelling the complicated problems with uncertainties. Molodtsov et. al[24] applied the technique of soft sets into several directions. Maji et. al[22] studied the several concepts of soft sets and applied soft sets in decision making problems. Many researchers like [7-10], [27] improved the work of Maji et. al. In [27], it is observed that there exists some compact connection between soft sets and information systems. Moreover, it is pointed out that the research work on soft sets and information systems can be unified. Maji and Biswas[23] generalized the soft sets and introduced fuzzy soft sets. After that many researchers such as Z. Kong et. al[21] studied the concept of fuzzy soft sets and presented the fuzzy

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soft set theoretic approach to decision making problems.
Ahmad and Kharral[1] initiated the concept of mappings on fuzzy soft classes and introduce fuzzy soft images and fuzzy soft inverse images. The study of algebraic structures of soft sets and fuzzy soft sets have been increasing rapidly in recent years. Shabbir and Naz[28] defined and explored the concept of soft topological spaces. Further results, structures and improvements in concepts of soft topological spaces have been studied in [2], [7-9], [12-15], [25].
Hussain[16] continued to study the algebraic structures of soft semi-open(closed) sets in soft topological spaces initiated by Chen[4-5]. Chang[3] introduced and discussed the basic properties of fuzzy topological spaces. Tanay and Kandemir[29] studied the topological structures of fuzzy soft sets. Varol and Aygun[30] initiated fuzzy soft topology. Further structures of fuzzy soft topology are explored in [11], [20].
Recently, Hussain[17] initiated the concept of fuzzy soft semi-open sets as a generalization of soft semi-open sets and discussed its basic properties. In [18-19], Hussain studied and explored the weak and strong forms of fuzzy soft open sets as well as fuzzy soft semi-pre-open sets and fuzzy soft semi-pre-continuous mappings in fuzzy soft topological spaces. Hussain also developed the relationship between these newly defined concepts in [18-19].

2. Preliminaries

Definition 2.1[31]. A fuzzy set f on X is a mapping $f : X \to I = [0, 1]$. The value $f(x)$ represents the degree of membership of $x \in X$ in the fuzzy set f, for $x \in X$.
Definition 2.2[24]. Let X be an initial universe and E be a set of parameters. Let $P(X)$ denotes the power set of X and A be a non-empty subset of E. A pair (F, A) is called a soft set over X, where F is a mapping given by $F : A \to P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X. For $e \in A$, $F(e)$ may be considered as the set of e-approximate elements of the soft set (F, A).
Definition. 2.3[23]. Let I^X denotes the set of all fuzzy sets on X and $A \subseteq X$. A pair (f, A) is called a fuzzy soft set over X, where $f : X \to I^X$ is a function. That is, for each $a \in A$, $f(a) = f_a : X \to I$, is a fuzzy set on X .
Definition 2.4[23]. For two fuzzy soft sets (f, A) and (g, B) over a common universe X, we say that (f, A) is a fuzzy soft subset of (g, B) if
(1) $A \subseteq B$ and
(2) for all $a \in A$ and $b \in B$, $f_a \leq g_b$; implies f_a is a fuzzy subset of g_b.
We denote it by $(f, A)\tilde{\leq}(g, B)$. (f, A) is said to be a fuzzy soft super set of (g, B), if (g, B) is a fuzzy soft subset of (f, A). We denote it by $(f, A)\tilde{\geq}(g, B)$.
Definition 2.5[23]. Two fuzzy soft sets (f, A) and (g, B) over a common universe X are said to be fuzzy soft equal, if (f, A) is a fuzzy soft subset of (g, B) and (g, B) is a fuzzy soft subset of (f, A).
Definition 2.6[23]. The union of two fuzzy soft sets of (f, A) and (g, B) over the common universe X is the fuzzy soft set (h, C), where $C = A \cup B$ and for all $c \in C$,

$$h_c = \begin{cases} f_c, & \text{if } c \in A - B \\ g_c, & \text{if } c \in B - A \\ f_c \vee g_c, & \text{if } c \in A \cap B \end{cases}$$

We write $(f, A)\tilde{\vee}(g, B) = (h, C)$.
Definition 2.7[23]. The intersection (h, C) of two fuzzy soft sets (f, A) and (g, B) over a common universe X, denoted $(f, A)\tilde{\wedge}(g, B)$, is defined as $C = A \cap B$, and $h_c = f_c \wedge g_c$, for all $c \in C$.
Definition 2.8[23]. The difference (h, C) of two fuzzy soft sets (f, A) and (g, B) over X, denoted by $(f, A)\tilde{\setminus}(g, B)$, is defined as $(f, A)\tilde{\setminus}(g, B) = (f, A)\tilde{\wedge}(f, B)^c$.

For our convenience, we will use the notation f_A for fuzzy soft set instead of (f, A).
Definition 2.9[29]. Let τ be the collection of fuzzy soft sets over X, then τ is said to be a fuzzy soft topology on X, if
(1) $\tilde{0}_A$, $\tilde{1}_A$ belong to τ.
(2) If $(f_A)_i \in \tau$, for all $i \in I$, then $\tilde{\bigvee}_{i\in I}(f_A)_i \in \tau$.

(3) $f_a, g_b \in \tau$ implies that $f_a \tilde{\bigwedge} g_b \tilde{\in} \tau$.
The triplet (X, τ, A) is called a fuzzy soft topological space over X. Every member of τ is called fuzzy soft open set. A fuzzy soft set is called fuzzy soft closed if and only if its complement is fuzzy soft open.

Definition 2.10[30]. Let (X, τ, A) be a fuzzy soft topological space over X and f_A be a fuzzy soft set over X. Then

(1) fuzzy soft interior of fuzzy soft set f_A over X is denoted by $(f_A)^\circ$ and is defined as the union of all fuzzy soft open sets contained in f_A. Thus $(f_A)^\circ$ is the largest fuzzy soft open set contained in f_A.

(2) fuzzy soft closure of f_A, denoted by $\overline{f_A}$ is the intersection of all fuzzy soft closed super sets of f_A. Clearly $\overline{f_A}$ is the smallest fuzzy soft closed set over X which contains f_A.

Definition 2.11[17]. Let (X, τ, A) be a fuzzy soft topological space over X. A fuzzy soft set f_A is called fuzzy soft semi-open, if there exists a fuzzy soft open set g_A such that $g_A \tilde{\leq} f_A \tilde{\leq} \overline{g_A}$. The class of all fuzzy soft semi-open sets in X is denoted by FSSO(X). Note that every fuzzy soft open set is fuzzy soft semi-open but the converse is not true in general.

Definition 2.12[17]. A fuzzy soft set f_A in fuzzy soft topological space (X, τ, A) is fuzzy soft semi-closed if and only if its complement $(f_A)^c$ is fuzzy soft semi-open. The class of fuzzy soft semi-closed sets is denoted by FSSC(X). Note that every fuzzy soft closed set is fuzzy soft semi-closed in fuzzy soft topological space (X, τ, A).

Proposition 2.13[17]. Let f_A be a fuzzy soft set in fuzzy soft topological space (X, τ, A). Then f_A is fuzzy soft semi-closed if and only if there exists a fuzzy soft closed set h_A such that $(h_A)^0 \tilde{\leq} f_A \tilde{\leq} h_A$.

Definition 2.14[17]. Let f_A be a fuzzy soft set in fuzzy soft topological space (X, τ, A). The fuzzy soft semi-closure of f_A, denoted by $scl^{fs}(f_A)$ and is defined as the intersection of all fuzzy soft closed supersets of f_A.
It is clear from the definition that $scl^{fs}(f_A)$ is the smallest fuzzy soft semi-closed set over X which contains f_A.

Definition 2.15[17]. Let f_A be a fuzzy soft set in fuzzy soft topological space (X, τ, A). The fuzzy soft semi-interior of f_A, denoted by $sint^{fs}(f_A)$ and is defined as the union of all fuzzy soft open subsets of f_A.
It is clear from the definition that $sint^{fs}(f_A)$ is the largest fuzzy soft semi-open set over X contained in f_A.

Definition 2.16[18]. Let (X, τ, A) be a fuzzy soft topological space over X. Then a fuzzy soft set f_A over X is said to be a fuzzy soft pre-open(respt. closed), if $f_A \tilde{\leq} (\overline{f_A}\ )^\circ$(respt. $\overline{(f_A)^\circ} \tilde{\leq} f_A$).

Definition 2.17[1]. Let $F(X, A)$ and $F(Y, B)$ be families of fuzzy soft sets. $u : X \to Y$ and $p : A \to B$ are mappings. Then a function $f_{pu} : F(X, A) \to F(Y, B)$ defined as :

(1) Let f_A be a fuzzy soft set in $F(X, A)$. The image of f_A under f_{pu} , written as $f_{pu}(f_A)$, is a fuzzy soft set in $F(Y, B)$ such that for $\beta \in p(A) \subseteq B$ and $y \in Y$,

$$f_{pu}(f_A)(\beta)(y) = \begin{cases} \bigvee_{x \in u^{-1}(y)} (\bigvee_{\alpha \in p^{-1}(\beta) \cap A} (f_A(\alpha)), & u^{-1}(y) \neq \phi, p^{-1}(\beta) \cap A \neq \phi \\ 0, & \text{otherwise} \end{cases},$$

for all $y \in B$. $f_{pu}(f_A)$ is known as a fuzzy soft image of a fuzzy soft set f_A.

(2) Let g_B be a fuzzy soft set in $F(Y, B)$. Then the inverse image of g_B under f_{pu}, written as $f_{pu}^{-1}(g_B)$, is a fuzzy soft set in $F(X, A)$ such that

$$f_{pu}^{-1}(g_B)(\alpha)(x) = \begin{cases} g(p(\alpha))(u(x)), & p(\alpha) \in B \\ 0, & \text{otherwise} \end{cases},$$

for all $x \in A$. $f_{pu}^{-1}(g_B)$ is known as a fuzzy soft inverse image of a fuzzy soft set g_B.
The fuzzy soft function f_{pu} is called fuzzy soft surjective, if p and u are surjective. The fuzzy soft function f_{pu} is called fuzzy soft injective, if p and u are injective.

Definition 2.18[19]. Let (X, τ, A) be a fuzzy soft topological space over X, where X is a nonempty set and τ is a family of fuzzy soft sets. Then a fuzzy soft set f_A is said to be a fuzzy soft semi-pre-open, if there exists a fuzzy soft pre-open set g_A such that $g_A \tilde{\leq} f_A \tilde{\leq} \overline{(g_A)}$.

Definition 2.19[19]. Let (X, τ, A) be a fuzzy soft topological space over X, where X is a nonempty set and τ is a family of fuzzy soft sets. Then a fuzzy soft set f_A is said to be a fuzzy soft semi-pre-closed, if there exists a fuzzy soft pre-closed set g_A such that $(g_A)^\circ \tilde{\leq} f_A \tilde{\leq} g_A$.
Note that the fuzzy soft set f_A is fuzzy soft semi-pre-open if and only if f_A^c is fuzzy soft semi-pre-closed.

Remark 2.20[19]. It is clear that any fuzzy soft semi-open as well as fuzzy soft pre-open set is a fuzzy soft

semi-pre-open set.
Definition 2.21[19]. Let f_A be a fuzzy soft set in fuzzy soft topological space (X,τ,A) over X. Then
(1) fuzzy soft semi-pre-interior of fuzzy soft set f_A denoted by F^spint^s and is defined as $F^spint^s(f_A)\tilde{=}Sup\{g_A : g_A$ is fuzzy soft semi-pre-open and $g_A\tilde{\leq}f_A\}$.
(2) fuzzy soft semi-pre-closure of fuzzy soft set f_A denoted by F^spcl^s and is defined as $F^spcl^s(f_A)\tilde{=}Inf\{g_A : g_A$ is fuzzy soft semi-pre-closed and $f_A\tilde{\leq}g_A\}$.
Definition 2.22[18]. A fuzzy soft set f_A is said to be a fuzzy soft point in (X,τ,A) denoted by $e(f_A)$, if for the element $e\tilde{\in}A$, $f(e)\tilde{\neq}\tilde{0}$ and $f(e^c)\tilde{=}\tilde{0}$, for all $e^c\in A\backslash\{e\}$.
Definition 2.23[30]. Let (X,τ_1,A) and (Y,τ_2,B) are fuzzy soft topological spaces and $f_{pu} : F(X,A)\to F(Y,B)$ be a fuzzy soft mapping. Then fuzzy soft function $f_{pu} : F(X,A)\to F(Y,B)$ is fuzzy soft continuous, if for any fuzzy soft open set g_A in (Y,τ_2,B), $f^{-1}_{pu}(g_A)$ is fuzzy soft open in (X,τ_1,A).
Definition 2.24[30]. Let (X,τ_1,A) and (Y,τ_2,B) are fuzzy soft topological spaces and $f_{pu} : F(X,A)\to F(Y,B)$ be a fuzzy soft function. Then f_{pu} is said to be fuzzy soft open(resp. fuzzy soft closed), if for any fuzzy soft open (resp. fuzzy soft closed) set h_A in (X,τ_1,A), $f_{pu}(h_A)$ is fuzzy soft open(resp. fuzzy soft closed) in (Y,τ_2,B).

3. Fuzzy soft almost soft continuous function

Definition 3.1[18]. Let f_A be a fuzzy soft set in fuzzy soft topological space (X,τ,A) over X. Then f_A is said to be fuzzy soft regular open set over X, if $(\overline{f_A})^\circ\tilde{=}f_A$.
Definition 3.2[18]. Let f_A be a fuzzy soft set in fuzzy soft topological space (X,τ,A) over X. Then f_A is said to be fuzzy soft regular closed set over X, if $\overline{(f_A)^\circ}\tilde{=}f_A$.
Theorem 3.3[18] (1) f_A is said to be fuzzy soft regular open set over X if and only if $(f_A)^c$ is said to be fuzzy soft regular closed set over X.
(2) Every fuzzy soft regular open (closed) set is a fuzzy soft open(closed) set.

Now we define:
Definition 3.4. Let (X,τ_1,A) and (Y,τ_2,B) be two fuzzy soft topological spaces and $f_{pu} : F(X,A)\to F(Y,B)$ be a fuzzy soft mapping. Then f_{pu} is said to be fuzzy soft almost soft continuous, if for any fuzzy soft regular-open set g_B in (Y,τ_2,B), $f^{-1}_{pu}(g_B)$ is fuzzy soft open in (X,τ_1,A).
Example 3.5. Let $X=\{h_1,h_2,h_3\}$, $Y=\{x_1,x_2,x_3\}$, $A=\{e_1,e_2\}$, $B=\{e_1^{'},e_2^{'}\}$ and $\tau=\{\tilde{0},\tilde{1},(f_A)_1,(f_A)_2,(f_A)_3,(f_A)_4\}$, $\tau^*=\{\tilde{0},\tilde{1},(g_B)_1,(g_B)_2\}$. Where $(f_A)_1$, $(f_A)_2$, $(f_A)_3$, $(f_A)_4$ are fuzzy soft sets over X and $(g_B)_1,(g_B)_2$ are fuzzy soft sets over Y, defined as follows
$f_1(e_1)(h_1)=0.5,\ f_1(e_1)(h_2)=0.3,\ f_1(e_1)(h_3)=0.2,$
$f_1(e_2)(h_1)=0.3,\ f_1(e_2)(h_2)=0.5,\ f_1(e_2)(h_3)=0.2,$
$f_2(e_1)(h_1)=1,\ f_2(e_1)(h_2)=0,\ f_2(e_1)(h_3)=0.5,$
$f_2(e_2)(h_1)=0.5,\ f_2(e_2)(h_2)=0.3,\ f_2(e_2)(h_3)=1,$
$f_3(e_1)(h_1)=0.5,\ f_3(e_1)(h_2)=0,\ f_3(e_1)(h_3)=0.2,$
$f_3(e_2)(h_1)=0.3,\ f_3(e_2)(h_2)=0.3,\ f_3(e_2)(h_3)=0.2,$
$f_4(e_1)(h_1)=1,\ f_4(e_1)(h_2)=0.3,\ f_4(e_1)(h_3)=0.5,$
$f_4(e_2)(h_1)=0.5,\ f_4(e_2)(h_2)=0.5,\ f_4(e_2)(h_3)=1.$
and
$g_1(e_1^{'})(x_1)=0.2,\ g_1(e_1^{'})(x_2)=0.3,\ g_1(e_1^{'})(x_3)=0.5,$
$g_1(e_2^{'})(x_1)=0.2,\ g_1(e_2^{'})(x_2)=0.5,\ g_1(e_2^{'})(x_3)=0.3,$
$g_2(e_1^{'})(x_1)=0.8,\ g_2(e_1^{'})(x_2)=0.7,\ g_2(e_1^{'})(x_3)=0.5,$
$g_2(e_2^{'})(x_1)=0.8,\ g_2(e_2^{'})(x_2)=0.5,\ g_2(e_2^{'})(x_3)=0.7.$
Then τ and τ^* are fuzzy soft topologies on X and Y respectively. Therefore, (X,τ,A) and (Y,τ^*,B) are fuzzy soft topological spaces over X and Y respectively. Note that the fuzzy soft closed sets in (Y,τ^*,B) are
$\{\{x_{0.8},x_{0.7},x_{0.5}\},\{x_{0.8},x_{0.5},x_{0.7}\}\}$, $\{\{x_{0.2},x_{0.3},x_{0.5}\},\{x_{0.2},x_{0.5},x_{0.3}\}\}$, $\tilde{1}$ and $\tilde{0}$.
Let us take fuzzy soft set g_B over Y defined by

$g(e_1^{'})(x_1) = 0.2$, $g(e_1^{'})(x_2) = 0.3$, $g(e_1^{'})(x_3) = 0.5$,
$g(e_2^{'})(x_1) = 0.8$, $g(e_2^{'})(x_2) = 0.7$, $g(e_2^{'})(x_3) = 0.5$.
That is, $g_B = \{\{x_{0.2}, x_{0.3}, x_{0.5}\}, \{x_{0.8}, x_{0.5}, x_{0.1}\}\}$. Calculation shows that $(\overline{g_B})^\circ \tilde{=} g_B$, which implies that g_B is fuzzy soft regular-open set.
Let us define the fuzzy soft mapping $f_{pu} : F(X,A) \to F(Y,B)$ by
$u(h_1){=}x_3, u(h_2){=}x_2$, $u(h_3){=}x_1$ and $p(e_1){=}e_1^{'}, p(e_2){=}e_2^{'}$.
Now $f_{pu}^{-1}(g_B)(e_1)h_1 = g(p(e_1))(u(h_1) = g(e_1^{'})x_3 = \{x_{0.2}, x_{0.3}, x_{0.5}\}x_3 = x_{0.5}$. Similarly, $f_{pu}^{-1}(g_B)(e_1)h_2 = x_{0.3}$ and $f_{pu}^{-1}(g_B)(e_1)h_3 = x_{0.2}$. Also
$f_{pu}^{-1}(g_B)(e_2)h_1 = g(p(e_2)(u(h_1) = g(e_2^{'})x_3 = \{x_{0.2}, x_{0.5}, x_{0.3}\}x_3 = x_{0.3}$.
Similarly, $f_{pu}^{-1}(g_B)(e_2)h_2 = x_{0.5}$ and $f_{pu}^{-1}(g_B)(e_2)h_3 = x_{0.2}$. Thus
$f_{pu}^{-1}(g_B)\tilde{=}\{e_1 = \{\{x_{0.5}, x_{0.3}, x_{0.2}\}\}, e_2 = \{\{x_{0.3}, x_{0.5}, x_{0.2}\}\}\}$, which is fuzzy soft open in $F(X,A)$. Thus fuzzy soft mapping f_{pu} is fuzzy soft almost soft continuous, because fuzzy soft set g_B is fuzzy soft regular-open in $F(Y,B)$ and $f_{pu}^{-1}(g_B)$ is fuzzy soft open in $F(X,A)$.

Theorem 3.6. Let (X,τ_1,A) and (Y,τ_2,B) be two fuzzy soft topological spaces and $f_{pu} : F(X,A) \to F(Y,B)$ be a fuzzy soft mapping. Then the following statements are equivalent:
(1) f_{pu} is fuzzy soft almost soft continuous mapping.
(2) For any fuzzy soft regular-closed set g_B in (Y,τ_2,B), $f_{pu}^{-1}(g_B)$ is fuzzy soft closed in (X,τ_1,A).
(3) For any fuzzy soft open set h_B in (Y,τ_2,B), $f_{pu}^{-1}(h_B)\tilde{\leq}(f_{pu}^{-1}((\overline{h_B})^\circ))^\circ$.
(4) For any fuzzy soft closed set k_B in (Y,τ_2,B), $\overline{f_{pu}^{-1}\{\overline{(k_B)^\circ}\}}\tilde{\leq}f_{pu}^{-1}(k_B)$.

Proof. (1) $\Leftrightarrow$ (2) This directly follows from Theorem 3.3.
(1) $\Rightarrow$ (3) Let h_B be fuzzy soft open in (Y,τ_2,B). Then $h_B\tilde{\leq}(\overline{h_B})^\circ$. Therefore, $f_{pu}^{-1}(h_B)\tilde{\leq}f_{pu}^{-1}((\overline{h_B})^\circ)$. By Using Theorem 5.9(b)[18], we have that $(\overline{h_B})^\circ$ is a fuzzy soft regular-open set in (Y,τ_2,B). Since f_{pu} is fuzzy soft almost soft continuous mapping, then $f_{pu}^{-1}((\overline{h_B})^\circ)$ is fuzzy soft open set in (X,τ_1,A). Hence, $f_{pu}^{-1}(h_B)\tilde{\leq}f_{pu}^{-1}((\overline{h_B})^\circ)\tilde{=}(f_{pu}^{-1}((\overline{h_B})^\circ))^\circ$.
(3) $\Rightarrow$ (1) Let h_B be fuzzy soft regular-open in (Y,τ_2,B). Then
$f_{pu}^{-1}(h_B)\tilde{\leq}(f_{pu}^{-1}((\overline{h_B})^\circ))^\circ\tilde{=}(f_{pu}^{-1}(h_B))^\circ$. This implies that $f_{pu}^{-1}(h_B)\tilde{=}(f_{pu}^{-1}(h_B))^\circ$. Hence $f_{pu}^{-1}(h_B)$ is fuzzy soft open set in (X,τ_1,A).
(2) $\Leftrightarrow$ (4) This can be proved in similar way.
This completes the proof.

Remark 3.7. (1) Fuzzy soft semi-continuous mapping and fuzzy soft almost soft continuous mappings are independent notions.
(2) If f_{pu} is fuzzy soft almost soft continuous mapping then f_{pu} is fuzzy soft continuous mapping, since every fuzzy soft regular-open set is fuzzy soft open but the converse is not true in general[18].

Definition 3.8. A fuzzy soft topological space (X,τ,A) is said to be fuzzy soft semi-regular space, if the collection of all fuzzy soft regular-open sets of X forms a fuzzy soft base for fuzzy soft topological space (X,τ,A).

The proof of the following lemma follows by using Theorems 3.9 and 3.11[30].

Lemma 3.9. Let (X,τ,A) be fuzzy soft topological space and f_{A_i} be a collections of fuzzy soft sets in (X,τ,A). Then $\tilde{\bigvee}_i\overline{f_{A_i}}\tilde{\leq}\overline{\tilde{\bigvee}_i f_{A_i}}$ and $\tilde{\bigvee}_i\overline{f_{A_i}}\tilde{=}\overline{\tilde{\bigvee}_i f_{A_i}}$, if f_{A_i} is a finite set. Moreover, $\tilde{\bigvee}_i(f_{A_i})^\circ\tilde{\leq}(\tilde{\bigvee}_i f_{A_i})^\circ$.

The following theorem shows that the converse of Remark 3.7(2) is true, if (Y,τ_2,B) is fuzzy soft semi-regular space.

Theorem 3.10. Let (X,τ_1,A) be fuzzy soft topological space, (Y,τ_2,B) be a fuzzy soft semi-regular space and $f_{pu} : F(X,A) \to F(Y,B)$ be a fuzzy soft mapping. Then f_{pu} is fuzzy soft almost soft continuous if and only if f_{pu} is fuzzy soft continuous.

Proof. Using Remark 3.7(2), it is sufficient to prove that if f_{pu} is fuzzy soft almost soft continuous then it is fuzzy soft continuous. Suppose h_B be fuzzy soft open in (Y,τ_2,B). Then $h_B\tilde{=}\tilde{\bigvee}h_{B_\alpha}$, where each h_{B_α} is fuzzy soft regular-open set in (Y,τ_2,B). Now using Theorem 3.10(3)[1], Lemma 3.9 and Theorem 3.6(3), we have
$f_{pu}^{-1}(h_B)\tilde{=}\tilde{\bigvee}f_{pu}^{-1}(h_{B_\alpha})\tilde{\leq}\tilde{\bigvee}(f_{pu}^{-1}((\overline{h_{B_\alpha}})^\circ))^\circ\tilde{=}\tilde{\bigvee}(f_{pu}^{-1}(h_{B_\alpha}))^\circ\tilde{\leq}(\tilde{\bigvee}f_{pu}^{-1}(h_{B_\alpha}))^\circ\tilde{=}(f_{pu}^{-1}(h_B))^\circ$. This implies that $f_{pu}^{-1}(h_B)$ is fuzzy soft open in (X,τ_1,A). Hence the proof.

Theorem 3.11. Let f_A be a fuzzy soft set in fuzzy soft topological space (X, τ, A) over X. If f_A is fuzzy soft semi-pre-open, then $\overline{f_A}$ is fuzzy soft regular-closed.

Proof. Suppose that f_A is fuzzy soft semi-pre-open. Then using Theorem 3.8(1)[19], we get $f_A\tilde{\leq}\overline{((\overline{f_A})^\circ)}$. This implies that $\overline{f_A}\tilde{\leq}\overline{((\overline{f_A})^\circ)}$. Also $\overline{((\overline{f_A})^\circ)}\tilde{\leq}\overline{f_A}$. Therefore, $\overline{f_A}\tilde{=}\overline{((\overline{f_A})^\circ)}$. Hence $\overline{f_A}$ is fuzzy soft regular-closed. This completes the proof.

Theorem 3.12. Let (X, τ_1, A) and (Y, τ_2, B) are fuzzy soft topological spaces and $f_{pu} : F(X, A) \to F(Y, B)$ be a fuzzy soft mapping. Then the following statements are equivalent to each other.

(1)f_{pu} is fuzzy soft almost soft continuous.

(2) For every fuzzy soft semi-pre-open set g_B in (Y, τ_2, B), $\overline{f_{pu}^{-1}(g_B)}\tilde{\leq}f_{pu}^{-1}(\overline{g_B})$.

(3) For every fuzzy soft semi-open set g_B in (Y, τ_2, B), $\overline{f_{pu}^{-1}(g_B)}\tilde{\leq}f_{pu}^{-1}(\overline{g_B})$.

(4) For every fuzzy soft pre-open set g_B in (Y, τ_2, B), $f_{pu}^{-1}(g_B)\tilde{\leq}(f_{pu}^{-1}((\overline{g_B})^\circ))^\circ$.

Proof. (1) $\Rightarrow$ (2) Suppose that g_B be fuzzy soft semi-pre-open set in (Y, τ_2, B). Then by Theorem 3.11, we have $\overline{g_B}$ is fuzzy soft regular-closed in (Y, τ_2, B). Since f_{pu} is fuzzy soft almost soft continuous, then Theorem 3.6 implies that $f_{pu}^{-1}(\overline{g_B})$ is fuzzy soft closed in (X, τ_1, A). Therefore, $\overline{f_{pu}^{-1}(g_B)}\tilde{\leq}f_{pu}^{-1}(\overline{g_B})$.

(2) $\Rightarrow$ (3) This directly follows using Remark 2.20 that every fuzzy soft semi-open set is fuzzy soft semi-pre-open.

(3) $\Rightarrow$ (1) Suppose g_B be a fuzzy soft regular closed set in (Y, τ_2, B). Then $g_B\tilde{=}\overline{((g_B)^\circ)}$. Theorem 3.3(2) implies that g_B is fuzzy soft semi-open set in (Y, τ_2, B). This follows that $\overline{f_{pu}^{-1}(g_B)}\tilde{\leq}f_{pu}^{-1}(\overline{g_B})\tilde{=}f_{pu}^{-1}(g_B)$. Therefore, $f_{pu}^{-1}(g_B)$ is fuzzy soft closed. Hence by Theorem 3.6, f_{pu} is fuzzy soft almost soft continuous.

(1) $\Rightarrow$ (4) Suppose that g_B be fuzzy soft pre-open set in (Y, τ_2, B). Then $g_B\tilde{\leq}(\overline{g_B})^\circ$ and $(\overline{g_B})^\circ$ is fuzzy soft regular-open. Since f_{pu} is fuzzy soft almost soft continuous, then by Theorem 3.6, $f_{pu}^{-1}((\overline{g_B})^\circ)$ is fuzzy soft open set in (X, τ_1, A). Therefore, $f_{pu}^{-1}(g_B)\tilde{\leq}f_{pu}^{-1}((\overline{g_B})^\circ)\tilde{=}(f_{pu}^{-1}(\overline{g_B})^\circ)^\circ$.

(4) $\Rightarrow$ (1) Suppose that g_B be fuzzy soft regular-open set in (Y, τ_2, B). Then g_B is fuzzy soft pre-open set in (Y, τ_2, B). Therefore, $f_{pu}^{-1}(g_B)\tilde{\leq}(f_{pu}^{-1}(\overline{g_B})^\circ)^\circ\tilde{=}(f_{pu}^{-1}(g_B))^\circ$. Hence $f_{pu}^{-1}(g_B)$ is fuzz soft open set in (X, τ_1, A). This follows that f_{pu} is fuzzy soft almost soft continuous function. Hence the proof.

The following lemma directly follows from definition of fuzzy soft pre-closure $F^s pcl^s$ of the fuzzy soft set f_A.

Lemma 3.13. Let $e(f_A)$ be a fuzzy soft point in a fuzzy soft topological space (X, τ, A). Then $e(f_A)\tilde{\in}F^s pcl^s(h_A)$ if and only if for any fuzzy soft pre-open set k_A in (X, τ, A) with $e(f_A)\tilde{\in}k_A$ implies $h_A\tilde{\bigwedge}k_A\tilde{\neq}\tilde{\phi}$.

Theorem 3.14. Let (X, τ, A) be fuzzy soft topological space and f_A is fuzzy soft semi-open set in (X, τ, A). Then $F^s pcl^s(f_A)\tilde{=}\overline{f_A}$.

Proof. For any fuzzy soft set f_A, $F^s pcl^s(f_A)\tilde{\leq}\overline{f_A}$ is obviously true. Thus it is enough to show that $\overline{f_A}\tilde{\leq}F^s pcl^s(f_A)$, for fuzzy soft semi-open set f_A in (X, τ, A). For this, let $e(g_A)\tilde{\in}\overline{f_A}$ and $e(g_A)\tilde{\in}k_A$, where k_A is fuzzy soft pre-open set in (X, τ, A). Then $e(g_A)\tilde{\in}k_A\tilde{\in}(\overline{k_A})^\circ$ and hence $f_A\tilde{\bigwedge}(\overline{k_A})^\circ\tilde{\neq}\tilde{\phi}$. Now f_A is fuzzy soft semi-open follows that $f_A\tilde{\bigwedge}(\overline{k_A})^\circ\tilde{\leq}\overline{((f_A)^\circ)}\tilde{\bigwedge}(\overline{k_A})^\circ\tilde{\leq}\overline{((f_A)^\circ\tilde{\bigwedge}\overline{k_A})}\tilde{\leq}\overline{(f_A\tilde{\bigwedge}k_A)}$. Hence, we get $\overline{(f_A\tilde{\bigwedge}k_A)}\tilde{\neq}\tilde{\phi}$. This implies that $f_A\tilde{\bigwedge}k_A\tilde{\neq}\tilde{\phi}$. Using Lemma 3.13, we get $e(g_A)\tilde{\in}F^s pcl^s(f_A)$. Therefore, $\overline{f_A}\tilde{\leq}F^s pcl^s(f_A)$. This completes the proof.

The proof of the following theorem follows in similar lines as in Theorem 3.12 and is therefore omitted.

Theorem 3.15. Let (X, τ_1, A) and (Y, τ_2, B) are fuzzy soft topological spaces and $f_{pu} : F(X, A) \to F(Y, B)$ be a fuzzy soft mapping. Then the following statements are equivalent to each other.

(1)f_{pu} is fuzzy soft almost soft continuous.

(2) For every fuzzy soft semi-pre-open set g_B in (Y, τ_2, B), $\overline{f_{pu}^{-1}(g_B)}\tilde{\leq}f_{pu}^{-1}(F^s cl^\alpha(g_B))$.

(3) For every fuzzy soft semi-open set g_B in (Y, τ_2, B), $\overline{f_{pu}^{-1}(g_B)}\tilde{\leq}f_{pu}^{-1}(F^s pcl^s(g_B))$.

(4) For every fuzzy soft pre-open set g_B in (Y, τ_2, B), $f_{pu}^{-1}(g_B)\tilde{\leq}(f_{pu}^{-1}(scl^{fs}(g_B)))^\circ$.

Definition 3.16. Let (X, τ_1, A) and (Y, τ_2, B) are fuzzy soft topological spaces and $f_{pu} : F(X, A) \to F(Y, B)$ be a fuzzy soft mapping. Then f_{pu} is said to be fuzzy soft almost soft open(resp. closed), if for any fuzzy soft regular-open (resp. closed) set h_A in (X, τ_1, A), $f_{pu}(h_A)$ is fuzzy soft open(resp. closed) in (Y, τ_2, B).

Definition 3.17. Let (X, τ_1, A) and (Y, τ_2, B) are fuzzy soft topological spaces and $f_{pu} : F(X, A) \to F(Y, B)$ be a fuzzy soft mapping. Then fuzzy soft surjective fuzzy soft mapping $f_{pu} : F(X, A) \to F(Y, B)$ is fuzzy soft almost soft quasi-compact if and only if $f_{pu}^{-1}(g_A)$ is fuzzy soft regular-open in (X, τ_1, A) follows g_A is fuzzy soft open in (Y, τ_2, B).

The proof of the following theorem directly follows from the above Definition 3.17.

Theorem 3.18. Let (X, τ_1, A) and (Y, τ_2, B) are fuzzy soft topological spaces and $f_{pu} : F(X, A) \to F(Y, B)$ be a fuzzy soft mapping. Then the following statements are equivalent:
(1) f_{pu} is fuzzy soft bijective and fuzzy soft almost soft quasi-compact.
(2) The fuzzy soft inverse image of every fuzzy soft regular-open(resp. closed) set is fuzzy soft open(resp. closed).

Now we establish the following characterizations of fuzzy soft almost soft open functions as:

Theorem 3.19. Let (X, τ_1, A) and (Y, τ_2, B) are fuzzy soft topological spaces and $f_{pu} : F(X, A) \to F(Y, B)$ be a fuzzy soft bijective mapping. Then the following statements are equivalent:
(1) f_{pu} is fuzzy soft almost soft open.
(2) f_{pu} is fuzzy soft almost soft closed.
(3) f_{pu} is fuzzy soft almost soft quasi-compact.
(4) f_{pu}^{-1} is fuzzy soft almost soft continuous.

Proof. (1) $\Rightarrow$ (2) Suppose f_A be fuzzy soft regular-closed in (X, τ_1, A). Then $(f_A)^c$ be fuzzy soft regular-open. Since f_{pu} is fuzzy soft almost soft open, then $f_{pu}((f_A)^c)$ is fuzzy soft open. This implies that $(f_{pu}(f_A))^c$ is fuzzy soft open. Hence $f_{pu}(f_A)$ is fuzzy soft closed. Therefore, f_{pu} is fuzzy soft almost soft closed.
(2) $\Rightarrow$ (3) Suppose that $f_{pu}^{-1}(h_A)$ is fuzzy soft regular-closed. Then Theorem 3.18 implies that $f_{pu}f_{pu}^{-1}(h_A)$ is fuzzy soft closed. This follows that h_A is fuzzy soft closed. Therefore, f_{pu} is fuzzy soft almost soft quasi-compact.
(3) $\Rightarrow$ (4) Suppose that k_A be fuzzy soft regular open in (X, τ_1, A). Then $f_{pu}^{-1}f_{pu}(k_A)\tilde{=}k_A$ is fuzzy soft regular-open. Therefore, $f_{pu}(k_A)$ is fuzzy soft open. That is, $(f_{pu}^{-1})^{-1}(k_A)$ is fuzzy soft open. This follows that f_{pu}^{-1} is fuzzy soft almost soft continuous.
(4) $\Rightarrow$ (1). Suppose f_A be a fuzzy soft regular-open in (X, τ_1, A). Then by (4), $(f_{pu}^{-1})^{-1}(f_A)$ is fuzzy soft open in (Y, τ_2, B). Hence f_{pu} is fuzzy soft almost soft open. Hence the proof.

Theorem 3.20. Let (X, τ_1, A), (Y, τ_2, B) and (Z, τ_3, C) are fuzzy soft topological spaces and $f_{pu} : F(X, A) \to F(Y, B)$ be a fuzzy soft open and fuzzy soft continuous mapping, where $u : X \to Y$ and $p : A \to B$ are mappings and let $g_{qv} : F(Y, B) \to F(Z, C)$ be a fuzzy soft mapping, where $v : Y \to Z$ and $q : B \to C$ are mappings. Then $(gof)_{pv}$ is fuzzy soft almost soft continuous if and only if g_{qv} is fuzzy soft almost soft continuous.

Proof. ($\Rightarrow$) Suppose that g_{qv} is fuzzy soft almost soft continuous. Let (H, C) be fuzzy soft regular-open subset in (Z, τ_3, C). This implies that
$(gof)_{pv}^{-1}(H, C)$ is fuzzy soft open in (X, τ_1, A) and hence $f_{pu}^{-1}(g_{qv}^{-1}(H, C))$ is fuzzy soft open in (X, τ_1, A). Since f_{pu} is fuzzy soft open, then $f_{pu}(f_{pu}^{-1}(g_{qv}^{-1}(H, C)))$ is fuzzy soft open in (Y, τ_2, B). Therefore, $g_{qv}^{-1}(H, C)$ is fuzzy soft open in (Y, τ_2, B). Thus g_{qv} is fuzzy soft almost soft continuous.
($\Leftarrow$) Suppose that g_{qv} is fuzzy soft almost soft continuous and let (H, C) be fuzzy soft regular-open subset in (Z, τ_3, C). Then $g_{qv}^{-1}(H, C)$ is fuzzy soft open in (Y, τ_2, B). Since f_{pu} is fuzzy soft continuous, then $f_{pu}^{-1}(g_{qv}^{-1}(H, C))$ is fuzzy soft open in (X, τ_1, A). Therefore, $(gof)_{pv}^{-1}(H, C)$ is fuzzy soft open in (X, τ_1, A). Hence $(gof)_{pv}$ is fuzzy soft almost soft continuous. This completes the proof.

Theorem 3.21. Let (X, τ_1, A), (Y, τ_2, B) and (Z, τ_3, C) are fuzzy soft topological spaces and $f_{pu} : F(X, A) \to F(Y, B)$ be a fuzzy soft mapping, where $u : X \to Y$ and $p : A \to B$ are mappings and let $g_{qv} : F(Y, B) \to F(Z, C)$ be a fuzzy soft mapping, where $v : Y \to Z$ and $q : B \to C$ are mappings. If f_{pu} is fuzzy soft almost soft continuous and $(gof)_{pv}$ is fuzzy soft open(closed) then g_{qv} is fuzzy soft almost soft open(closed).

Proof. Suppose that f_{pu} is fuzzy soft almost soft continuous and $(gof)_{pv}$ is fuzzy soft open(closed). Let (G, B) be a fuzzy soft regular-open subset in (Y, τ_2, B). Then $f_{pu}^{-1}(G, B)$ is a fuzzy soft open(closed) set in (X, τ_1, A). Also $(gof)_{pv}$ is fuzzy soft open(closed) implies that $(gof)_{pv}(f_{pu}^{-1}(G, B))$ is fuzzy soft open(closed) set in (Z, τ_3, C). But $(gof)_{pv}(f_{pu}^{-1}(G, B))\tilde{=}g_{qv}(G, B)$. Therefore, $g_{qv}(G, B)$ is fuzzy soft open(closed) in (Z, τ_3, C). Thus g_{qv} is fuzzy soft almost soft open(closed). Hence the proof.

Conclusion. We initiated and explored the interesting characterizations and properties of fuzzy soft almost soft continuous mappings in fuzzy soft classes. It is observed that every fuzzy soft continues mapping $f_{pu} : F(X,A) \to F(Y,B)$ from fuzzy soft class $F(X,A)$ to fuzzy soft class $F(Y,B)$ is fuzzy soft almost soft continuous but the converse is not true in general. The converse is true, if the underlying fuzzy soft topological spaces (Y,τ_2,B) is fuzzy soft semi-regular space. We also studied and discussed the notions of fuzzy soft almost soft open(closed) mappings. Moreover, we also studied the characterizations of composition of two fuzzy soft almost soft mappings. We hope that the findings in this paper will be useful for the researchers working in the fields such as fuzzy control systems, fuzzy automata, fuzzy logic, information systems and decision making problems.

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