## Jerzy Boryczka and Maria Stopa-Boryczka

# THE MULTIPERIODICAL CHANGES OF AIR TEMPERATURE IN WARSAW

The aim of the paper is to define the age anomalies of Polish climate and their causes on the basis of over 200-years' series of measurements of air temperature in Warsaw in the years 1779—1979, of Wolf's numbers in the years 1700—1978 and of frequency of atmospheric circulation in the years 1891—1967.

The periodicity of variables mentioned above was examined, assuming the sinusoidal dependence on time t

$$y = f(t) = a + b \sin\left(\frac{2\pi}{T} t + c\right) \tag{1}$$

where b — amplitude, c — phase and T — period (parameter).

The time unit is a tropical year, and the beginning (t=0) are the left-sided limits of observation sections. Changing the parameter T—sine curve period (1) — every  $\Delta T=1$  or every 0.1 years in sections: temperature  $0 < T \le 250$ , Wolf's numbers  $0 < T \le 400$ , circulation  $0 < T \le 100$ , the local  $T_r$  minima of remainder variance were found.

$$\varepsilon^{2}(T) = \frac{1}{N} \sum_{j=1}^{N} [y_{j} - f(t_{j})]^{2}$$
 (2)

in other words, the searched cycles of variables y.

In that way cycles  $T_r$  which are longer than durations of measurement sequences  $y_1, \ldots, y_N$  taken in moments  $t_1, \ldots, t_N$  were calculated.

Every local minimum  $T_r$  of remainder variance  $\varepsilon^2$  corresponds to the minimum of standard error  $\Delta=\sqrt{\frac{N}{N-3}}\,\varepsilon$  and to maxima: of correlational function  $R\left[y_j,\;f(t_j)\right],\;j=1,\ldots,N,$  of determinant coefficent  $R^2=1-\frac{\varepsilon^2}{s^2}$  (s- standard deviation of y), and of Fisher's characteristic  $F_{\rm comp}=\frac{N-3}{2}\cdot\frac{R^2}{1-R^2}$  of  $n_1=2$  and  $n_2=N-3$  of freedom degrees.

#### THE SEASONAL CHANGES IN THE SUN'S ACTIVITY

In astronomical and climatological literature the short cycles of 11, 22 years and long ones of 89—110 years of sunspots have been known. It has turned out that Wolf's numbers prove the cyclic changes of other important periods. Cycles of the Sun's activity T, their amplitudes b and phases c obtained on the grounds of empirical data, of Wolf's numbers catalogue from the years 1700-1978 (Reznikov, 1982), are compared in Table 1. Cycles of T = 11.1, 45, 56, 95, 180 > 400 years

Table 1
The Sun's Activity Cycles T, Their Amplitudes b and Phases c (1700—1978)

Tr	Years	а	ь	c	Date	Fcomp
$T_1$	5	47.33	3.682	-2.824	1821	0.656
$T_2$	11.1	47.44	28.08	-1.523	1822	51.21
$T_3$	17	47.26	4.711	0.775	1813	1.069
$T_4$	21.4	47.32	4.363	0.942	1820	0.918
$T_5$	25	47.37	3.099	-2.623	1804	0.469
$T_6$	29	47.49	5.408	2.435	1797	1.391
$\mathbf{T}_7$	33	47.42	3.248	$2.42ar{7}$	1811	0.510
$T_8$	38	47.50	4.600	-3.033	1823	1.031
$T_9$	45	47.44	9.278	2.827	1803	4.131
T10	56	47.29	13.87	-1.910	1815	9.722
$T_{11}$	95	47.22	18.15	-2.693	1812	17.94
$T_{12}$	180	46.51	9.856	-1.321	1873	4.801
$T_{13}$	400	49.05	7.423	-2.872	1783	2.508

have the biggest amplitudes b=28.08, 9.28, 13.87, 18.15, 9.86, 7.42. They are essential to  $99^{\circ}/_{\circ}$  of confidence level, because  $F_{\rm comp}$  is bigger than the critical value 4.69 at  $n_1=2$  and  $n_2=276$  of freedom degrees. The exception is the 45-years' cycle — essential to  $95^{\circ}/_{\circ}$  of confidence level. In other cases (also in the case of 21.4-years' cycle) there is no reason to reject zero hypothesis H(b=0) for  $10^{\circ}/_{\circ}$  of significance level. The strongest are 11.1-years' and 95-years' cycles with coefficients of determination of  $27^{\circ}/_{\circ}$  and  $11.5^{\circ}/_{\circ}$ .

Despite the amplitude b and phase c, the dates of minima reckoned cycles of the Sun's astivity; sine curves periods (1); were presented

Date = 
$$1700 + \frac{T}{2\pi} \left( c - \frac{3}{2} \pi \right) + kT$$
,  $k = 0, \pm 1, \dots$  (3)

Thus, the minimum of 95-years' cycle — 1812 is the lower limit of the weakest 11-years' cycle of the Sun's activity in the years 1811— —1823 — the absolute minimum of Wolf's numbers. The following minima in the 11.1-years' cycle in the years 1811, 1822 nearly agree with the dates of the minimal Sun's activity. There are the dates of other multiperiodical minima of the Sun's activity in the 95-years' cycle: 1622, 1717, 1812, 1907, 2002. On the other hand, the minimum of the Sun's activity in multiperiodical cycle of 180 years occurs in the years 1693, 1873, 2063. The forecasting minima of the Sun's activity in 45, 56, 95, 180 and over 400-years' cycles occur in the years 1884, 1983, 2002, 2063 and 2182. In the years 1700—1978 the average value of Wolf's numbers was  $\bar{y}=47,33$ , and the standard deviation was s=38.18.

## THE PERIODICAL CHANGES OF ATMOSPHERIC CIRCULATION

Analogically counted cycles of an east, west and meridional circulation (according to Vangenheim classification) on the basis of the USRR Hydrological Meteorological Service data, taken from Stachy publications (1970), have been compiled in Table 2. The number of days in the year of east air circulation, shows cycles of 16, 27, 46 and 100 years of 20, 25, 18, 26 days amplitudes — essential to  $95^{\circ}/_{\circ}$  of confidence level ( $F_{\rm comp} > 3.11$ ). The cycle of 46 years of an east circulation is probably conditioned by the cycle of the same length (45 years) of the Sun's activity. The extrema of those cycles are synchronous in the years 1891-1967. The minima of an east circulation in 46 years appear in the years: 1898, 1944, 1990 and the minima of Wolf's numbers appear in the 45-years' cycle—in the years: 1893, 1938, 1983.

Then, the west circulation undergoes the cyclic changes of 8, 28, 100-years' periods and of 13, 17, 27 days amplitudes — essential to 90% of confidence level ( $F_{comp} > 2.38$ ).

Interesting are the cycles of meridional days number (the North and the South) of air circulation T=3, 8, 16, 24, 55, > 100 years of amplitudes b=10, 9, 14, 13, 20, 16 days. The cycles of 16, 24, 55 and over 100 years are essential to  $99^{0}/_{0}$  of level. It should be stressed that the strongest, the 55-years' cycle of meridional circulation of nearly 20-days' amplitude, is more or less equal to 56-years' cycle of the Sun's activity. The phase harmony of these cycles exists, too. The minima of meridional circulation of the 55-years' cycle appear in the years 1868, 1923, 1978 and in 2033, and the minima of Wolf's numbers in the 56-years' cycle appear in the years 1871, 1927, 1983, and 2039. This indicates

that the smaller or greater air flux from the North and the South to Central Europe every 55 years depends on the Sun's activity.

## THE PERIODICAL CHANGES OF AIR TEMPERATURE

On the basis of the results of air temperature measurements in Warsaw in the years 1779—1979 (taken from the Institute of Meteorology and Water Management) the cycles  $T_r$ , their amplitudes b and phases c were established for each month (I, ..., XII) and year (I—XII). The results of zero hypothesis verification  $H(b_r=0)$ , i.e. the comparison of counted values of Fisher's characterization  $F_{\rm comp}$  with critical numbers on 0.10, 0.05, 0.01 of the significance level, at  $n_1=2$  and  $n_2-198$  degrees of freedom are very interesting.

The temperature cycles of 9-14-years' length: January-12, February—14, March—11, May—10, June—13, October—12, November—9 and December—10 years of amplitudes b=0.77, 0.88, 0.67, 0.41, 0.21, 0.38, 0.39, 0.68°C are essential to  $90^{0}/_{0}$  of confidence level. It is known from another source that the range of 9—14 years reduces the variability of the 11-years' cycle of the Sun's activity. The average annual air temperature in the years 1779—1979 shows the 13-years' cycle, essential to the same confidence level.

Similarly, the cycles of 20—24 years: January—23, March—20, April—24, May—21, June—21, August—22, September—20, November—22, and December—21 of amplitudes b = 0.76, 0.58, 0.66, 0.43, 0.37, 0.35, 0.47, 0.48, 0.71°C, are essential to 90% of confidence level. Their lengths are similar to the 22-years' cycle of sunspots. There also exists a certain convergence of about 30, 40, 60-years' periods of temperature, the Sun's activity and atmospheric circulation. For example, Wolf's numbers show the 56-years' periodicity while an average annual air temperature shows the 60-years' periodicity.

Finally, the monthly value of air temperature undergoes fluctuations of long periods of 64—102 and 149—250 years. They are similar to multiperiodical cycles of the Sun's activity: 95 and 180 years. It is characteristic that, a long-term cycle of temperature of 64—102 years is stronger in summer than in winter while the 149—250-years' cycle behaves in the opposite direction. The 64—102-years' cycle, in summer months: June—90, July—75, and August—79 years of amplitudes 2b = 1.06, 0.90, 0.80°C is essential to at least 95% of confidence level. While, 149—250-years' cycle in a cool half-year: November—250, December—227, January—205, March—205, and April—232 years of amplitudes 2b = 1.94, 2.24, 2.74, 1.48, 3.30, 1.18°C is essential to over 99%

Table 2

		Atmo	spheri	Atmospheric Circulation Cycles T, their Amplitudes b and Phases c (1891-1967)	ion Cyc	les T, th	eir Ampl	itudes	b and F	hases	c (1891—	1967)		
	East	East circulation				Wes	West circulation	u			Merid	Meridional ctrculation	ation	
T	q	၁	Years	Fcomp	H	Q	ပ	Years	Fcomp	H	.p.	υ	Years	Fcomp
00	13.5	-2.459	1900	2.372	ű	6.277	-2.516	1897	0.586	က	9.808	1.630	1892	2.698
10	9.615	2.272	1895	1.113	<b>0</b> 0	12.59	1.386	1895	2.497	8	9.295	-0.484		2.407
13	7.527	3.110	1894		11	9.258	669.0—	1900	1.303	11	4.305	1.867	1896	0.498
16	19.67	2.567	1836		13	8.371	-2.427	1906	1.071	16	14.10	-1.106	1906	5.897
27	24.38	-2.454	1922	9.175	16	10.88	0.178	1903	1.881	24	13.02	-2.593	1919	5.079
46	17.74	-2.551	1944		20	9.397	0.903	1903	1.361	55		1.048	1923	13.54
100	26.23	1.992	1098		28	16.50	0.827	1908	4.760	>100		2.456	1927	5.344
					45	6.937	1.682	1913	0.695					
			_		>100	27.26	0.556	1957	20.14				-	

of confidence level. Because the amplitude of that cycle is rather big (over 3°C) it causes the multiperiodical temperature anomalies. The average annual values of air temperature in Warsaw in the years 1779—1979 reveal the cycles of 89 and 217 years, which only slightly differ from the cycles of Wolf's numbers of 95 and 180 years.

The minima of air temperature in the 89-years' cycle occur in the following years: 1635, 1724, 1813, 1902, 1991, 2080 and the multiperiodical minima of the Sun's activity in the 95-years' cycle occur in the years: 1622, 1717, 1812, 1907, 2002, 2097. Therefore, they are synchronous — exactly to one 11-years' cycle. The annual minima of air temperature in the 217-years' cycle in the years 1610, 1827, 2044 are well-known dates of climate "maunder minima": 1619 and 1819 (Landsberg, 1980).

### THE AIR TEMPERATURE ANOMALIES IN THE 17TH—21ST CENTURIES

The multiperiodical trend of air temperature was received by joining together the since curves (Boryczka, 1984)

$$y = f(t) = a_o + at + \sum_{r=1}^{R} b_r \sin \left(\frac{2\pi}{T_r} t + c_r\right)$$
 (4)

of relevant  $T_r$  periods, amplitudes  $b_r$  and phases  $c_r$ .

The constant components of trend  $\frac{\partial y}{\partial t} = a + \dots$  show the growth of temperature in time limit which was caused, for instance, by the growth of dust and  $CO_2$  contents in the Earth's atmosphere. The linear part of trend  $a_0 + at$  can also approximate the arc of sine curve of very long and unknown period.

The optimum trends f(t) of monthly temperature value in January and July and of the average annual value in the years 1779—1979 are illustrated by formulae (5), (6) and (7). January ( $F_{\rm comp}=2.66$ )

$$y = -0.084 + 0.6726 \sin\left(\frac{2\pi}{7}t + 2.865\right) + 0.793 \sin\left(\frac{2\pi}{10}t + 2.767\right) + \\ + 0.7143 \sin\left(\frac{2\pi}{13}t + 0.7362\right) + 0.5667 \sin\left(\frac{2\pi}{15}t + 0.5677\right) + \\ + 0.6587 \sin\left(\frac{2\pi}{23}t - 2.001\right) + 0.4041 \sin\left(\frac{2\pi}{27}t - 1.513\right) + \\ + 0.5378 \sin\left(\frac{2\pi}{44}t + 0.8872\right) + 0.5619 \sin\left(\frac{2\pi}{64}t + 0.3034\right) +$$

1.647 -2.883 8.522

0.604 2.710 2.499

0.493 2.384 1.736

0.449 0.013 1.431

0.284 1.546 0.574

<del>0</del>.246 3.078 0.426

0.220 1.904 0.342

0.580 2.043 2.421

0.314 1.920 0.694

1.6 0.453 0.076 0.323

0.574 0.687 1.037

0.666 -0.433 3.220

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0.593 2.823 5.544

0.277 3.059 0.983

0.262 2.292 0.886

0.328 -0.456 1.447

0.437 -2.472 2.634

1.321 19 0.196 0.514

0.868 17 0.435 2.569

-1.092 15 0.391 2.045

0.882 13 0.329 1.448

0.922 10 0,256 0.877

9.656 1.084 3.114 5 0,320 1.367 3 0.254 0.792 0.940 4 4 0.255 1.216

0.630 -1.960 2.869 0.463 7 0,458 2.867

Ν

 $\begin{array}{c} 24 \\ 0.658 \\ 2.364 \\ 5.996 \\ \cdot \\ 47 \\ 0.415 \\ -1.429 \\ 2.449 \end{array}$ 

0.085 -2.711

0.527 2.069

0.344 0.618 2.669

0.359 0.748

0.253 -0.609

0.294 -2.715

0.369 --2.330

0.349 1.515 2.594

0.395 -1.319

0.212 2.253 0.998

0.256 0.718 1.448

M

0.595 2.013 4.704

0.224 1.715 0.722

0.344 2.112 1.708

0.428 0.433 2.673

0.311 1.367 1.416

0.287 0.126 1.206

 $10 \\
0.407 \\
0.594 \\
2.415$ 

0.333 1.278 1.618

	ture in Warsaw in the years 1779—1979, — monthy I,, XII,	00000
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	n Warsaw in the years 1779—1979, — monthy I,, XII,	I-XII and their amplitudes b, phases c
	n A	Ţ

0.538 0.510 1.210

0.468 0.754 0.905

0.282 : -1.307 t 0.319

0.585 -1.444 1.404

 $\begin{array}{c} 23 \\ 0.756 \\ -2.121 \\ 2.418 \end{array}$ 

0.443 1.864 0.808

0.544 0.577 1.274

0.682 0.710 1.934

0.771 2.861 2.475

0.648 2.833 1.744

 $\begin{array}{c} 5 \\ \textbf{0.422} \\ -2.100 \\ \textbf{0.731} \end{array}$ 

Months

0.367 -1.597 0.552 6 0.774 -3.136 3.064

0.737 3.084 2.462

0.366 1.181 0.660

0.094 0.861 0.045

0.256 0.905 0.306

0.387 2.807 0.716

0.224 2.496 0,234

0.689 2.499 2.265

0.797 1.038 3.085

0.525 0.699 1.322

156 0.373 2.150 2.819	250 0.320 1.656 1.845					
75 0.453 0.460 4.917	149 0.219 0.929 1.055	187 0.241 2.954 1.278		250 0.967 —3.079 13.24		
52 0.192 0.345 0.894	79 0.397 0.929 3.448	87 0.324 2.233 2.313	250 0.261 -2.071 1.717	100 0.297 2.902 1.051		
31 0.089 2.255 0.223	$50 \\ 0.320 \\ -0.652 \\ 2.225$	56 0.461 1.969 5.041	99 0.394 —2.849 2.656	66 0.273 3.133 0.896	227 1.119 2.895 7.632	217 0.568 —2.969
26 0.121 0.359 0.376	$45 \\ 0.295 \\ -2.671 \\ 1.888$	42 0.212 0.640 1.097	60 0.286 1.106 1.401	50 0.212 1.811 0.570	$102 \\ 0.363 \\ -2.676 \\ 0.684$	89 0.243 2.336 3.067
23 0.207 0.090 1.003	32 0.279 1.308 1.699	27 0.363 0.896 3.084	$34 \\ 0.423 \\ -1.162 \\ 3.069$	39 0.293 1.811 1.027	65 0.576 2.235 1.695	60 0.178 1.320 1.724
$\begin{array}{c} 20 \\ 0.185 \\ -2.498 \\ 0.806 \end{array}$	27 0.193 0.669 0.861	23 0.1.77 0.22.7 0.764	27 0.388 3.103 2.614	31 0.353 0.977 1.487	35 0.491 2.475 1.205	41 0.119 0.700 0.745
18 0.164 2.380 0.649	22 0.351 0.189 2.678	20 0.369 	$23 \\ 0.235 \\ 2.100 \\ 0.924$	22 0.483 1.607 2.844	29 0.361 1.964 0.661	22 0.153 1.494 1.310
15 0.240 1.292 1.321	19 0,342 0.055 2.523	18 0.416 -1.561 4.100	19 0.246 1.671 1.040	17 0.293 1.821 1.051	$21 \\ 0.708 \\ -0.747 \\ 2.590$	19 0.132 0.417 0.968
12 0.202 2.293 0.943	16 0.417 1.120 3.823	16 0.358 3.061 2.997	17 0.310 2.760 1.646	15 0.382 1.074 1.752	18 0.687 -3.004 2.459	17 0.148 1.399 1.204
7 0.208 3.076 1.010	12 $0.240$ $-0.973$ $1.320$	14 0.237 0.040 1.338	12 0.383 2.959 2.539	$13 \\ 0.386 \\ -0.909 \\ 1.788$	15 0.819 1.368 3.504	13 0.211 0.228 2.476
5 0.096 2.580 0.251	10 0.256 0.165 1.435	8 0.284 1.513 1.897	$7 \\ 0.169 \\ -1.083 \\ 0.485$	9 $0.531$ $-0.330$ $3.447$	$10 \\ 0.683 \\ -1.874 \\ 2.437$	$\begin{array}{c} 9\\ 0.111\\ -0.380\\ 0.683 \end{array}$
3 0.205 2.685 0.985	4 0.385 0.498 3.241	5 0.272 0.635 1.739	5 0.402 2.109 2.789	4 0.244 2.378 0.713	6 0.537 0.795 1.485	4 0.156 1.119 1.337
T Q o	F Q o	T Q O H	H V o H	H O o H	T o o s	T Q
VII	VIII	XI	×	×	XII	ROK

ROK

$$+0.1402 \sin\left(\frac{2\pi}{205}t - 2.787\right) \tag{5}$$

 $July (F_{comp} = 2.042)$ 

$$y = 18.56 + 0.2191 \sin\left(\frac{2\pi}{7}t - 3.043\right) + 0.2650 \sin\left(\frac{2\pi}{15}t + 1.269\right) +$$

$$+0.2054 \sin\left(\frac{2\pi}{23}t + 0.0090\right) + 0.4107 \sin\left(\frac{2\pi}{75}t + 0.3722\right) +$$

$$+0.2891 \sin\left(\frac{2\pi}{156}t + 2.288\right)$$

$$(6)$$

 $Year (F_{comp} = 5.115)$ 

$$y = 7.603 - 0.001504 \ t + 0.1581 \ \sin\left(\frac{2\pi}{4}t + 1.132\right) +$$

$$+ 0.2029 \ \sin\left(\frac{2\pi}{13}t + 0.2304\right) + 0.1507 \ \sin\left(\frac{2\pi}{22}t + 1.585\right) +$$

$$+ 0.1542 \ \sin\left(\frac{2\pi}{60}t + 1.769\right) + 0.1700 \ \sin\left(\frac{2\pi}{89}t + 2.453\right) +$$

$$+ 0.6367 \ \sin\left(\frac{2\pi}{217}t - 2.916\right)$$

$$(7)$$

They stimulate the most credible cycles of air temperature from among those listed in Table 3. The zero hypothesis  $H(b_1 = \ldots = b_R = 0)$  was verified by the comparison of calculated value of Fisher's characteristic ( $F_{\text{comp}}$  — of  $n_1 = 2R$ ,  $n_2 = N - 2R - 1$  of degree of freedom) with critical numbers  $F_{\text{critical}}$  on 0.05, 0.01 of significance level. Thus, the hypothesis  $H(b_1 = \ldots = b_R = 0)$  is wrong on such levels of confidence as: January—99% and July—95%. The hypothesis  $H(a = b_1 = \ldots b_R = 0)$  for the annual temperature value for 99% of confidence level is also untrue.

Extrapolating the value of aproximating functions (5), (6) and (7) within time interval  $-279 \leqslant t < 0$ , the main temperature extrema of the 16th-18th centuries were reconstructed. Extrapolating the value of trends f(t) within time  $201 < t \leqslant 521$ , the forecast of air temperature anomaly in the 21st-23rd centuries, the so-called "sounding of future" was made. Parts of function's diagrams (5), (6) and (7) in the years 1779-2130 are illustrated by the curves in Figures 1, 2 and 3. For the sake of comparison, time curves of monthly (January, July) and annual values of air temperature measured in Warsaw in the years

1779—1979, were introduced. There is a great analogy between theoretical f(t) and observational  $y_1, \ldots, y_N$  curves.

In January the lowest monthly value of temperature was recorded in the following years:

date: 1803, 1823, 1850, 1893, 1940  $y_i$ : -17.8, -14.1, -11.6, -13.1, -12.1°C

Nevertheless the superposition of 9 sinusoidal cycles in that month (formula 5) gives the main minima of temperature in the following years:

date: 1801, 1823, 1850, 1892, 1941  $y_{min}$ : -8.6, -7.0, -7.6, -7.1, -7.0°C

The absolute minimum of theoretical curve  $y_{\rm min}=8.6^{\circ}{\rm C}$ ) (1801) differs only about one year from the date when the absolute temperature minimum  $-17.8^{\circ}{\rm C}$  (1803) was measured. It also differs only about one 11-years' cycle from the minimum of the absolute Sun's activity. The reconstructed main minima of the 16th-18th centuries which appeared in 1592 ( $y_{\rm min}=-8.3^{\circ}{\rm C}$ ) and in 1621 ( $y_{\rm min}=-8.1^{\circ}{\rm C}$ ) are very close to the year 1619 — the date of temperature "maunder minimum". Similar minima of temperature, as they were on the turn of the 15th /16th and 17th/18th centuries could be expected in the years: 2011 ( $y_{\rm min}=-7.9^{\circ}{\rm C}$ ), 2032 ( $y_{\rm min}=-7.8^{\circ}{\rm C}$ ) and in 2243 ( $y_{\rm min}=-7.9^{\circ}{\rm C}$ ).

In July, there exists quite a big compatibility in time of computed (according to formula 6) and measured main temperature minima. The dates of the coolest Julies are listed below:

date: 1800, 1815, 1832, 1844, 1898, 1979 y<sub>i</sub>: 15.9, 15.7, 14.1, 15.3, 15.6, 15.1°C

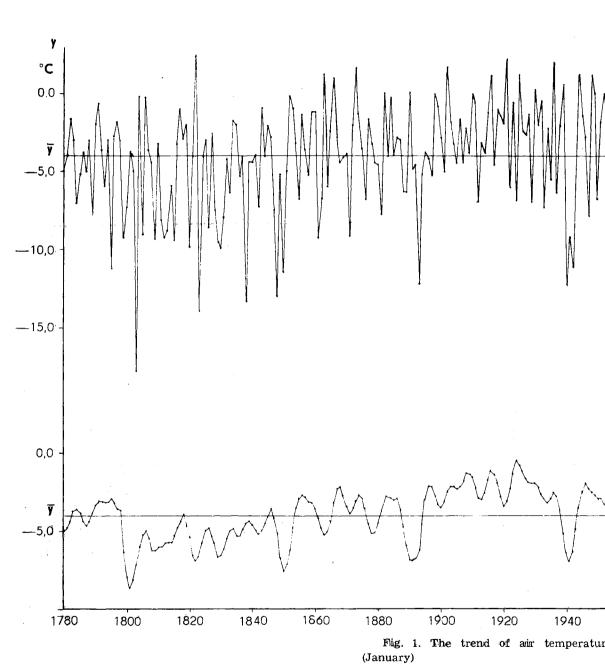
Whereas the curve's minima (6) appear in the years:

date: 1801, 1815, 1943, 1907, 1981  $y_{\min}$ : 18.4, 17.7, 17.6, 17.8, 17.3°C

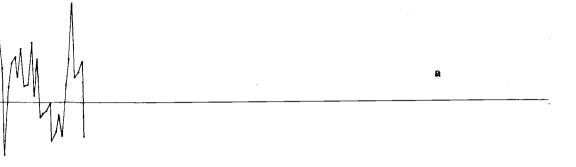
Similar minima of temperature appeared in 1512 ( $y_{\min} = 17.6^{\circ}\text{C}$ ), 1683 ( $y_{\min} = 17.8^{\circ}\text{C}$ ), and they could be expected near the dates: 2054 ( $y_{\min} = 18.0^{\circ}\text{C}$ ), 2117 ( $y_{\min} = 17.7^{\circ}\text{C}$ ), 2145 ( $y_{\min} = 17.6^{\circ}\text{C}$ ) and in 2208 ( $y_{\min} = 17.7^{\circ}\text{C}$ ).

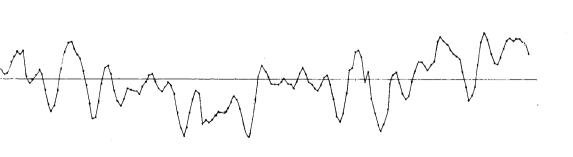
Of special cognitive importance in climatology are the implications resulting from formula (17) which introduces the multiperiodical changes of average annual value of air temperature in Warsaw over the past centuries. The function, approximating empirical data from the years 1779—1979 has "the deep" minima in the years:

date: 1633, 1812, 2049, 2257  $y_{\min}$ : 6,6, 6.2, 6.1, 5.8°C



a) observation data in the years 1779-1979 b) the diagram of approximating function (5)





b

1960 1980 2000 2020 2040 2060 2080 2100 2120 Years in Warsaw in the years 1779—2130

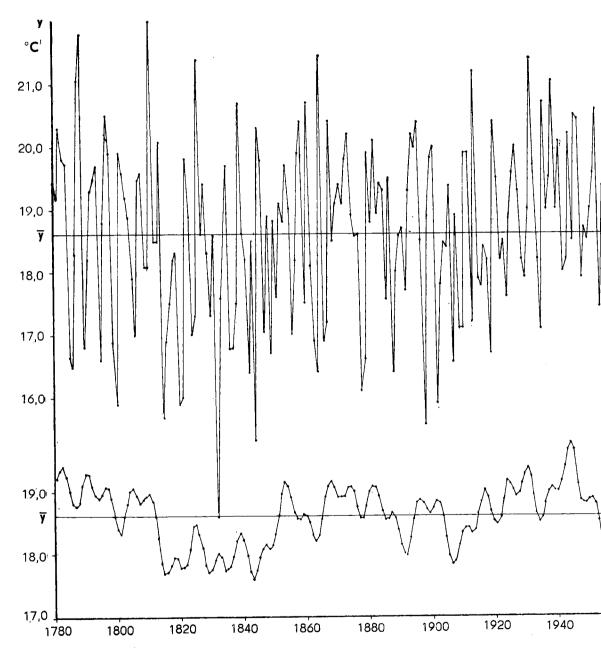
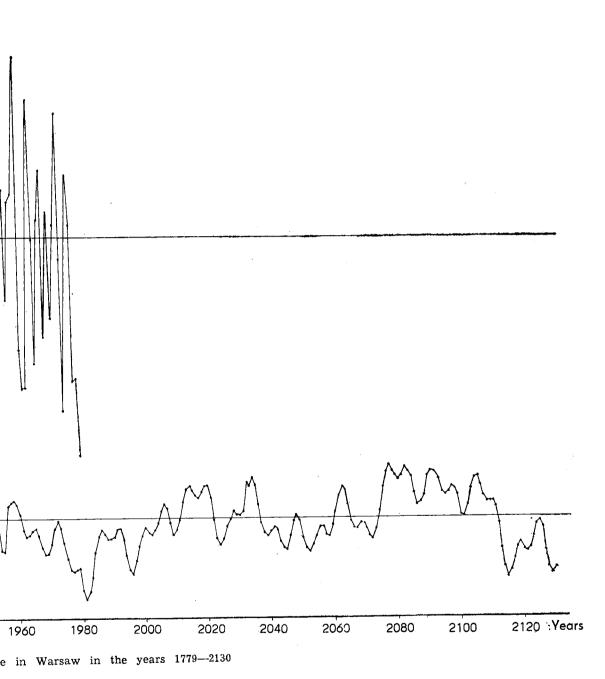
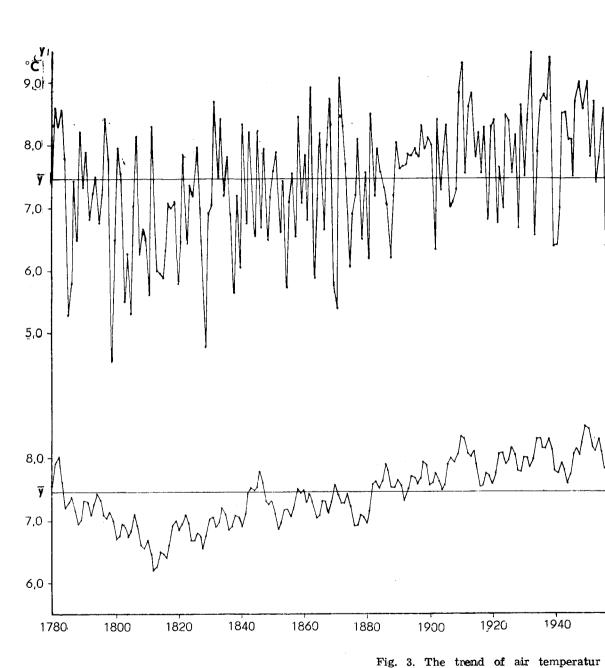


Fig. 2. The trend of air temperatu (July)

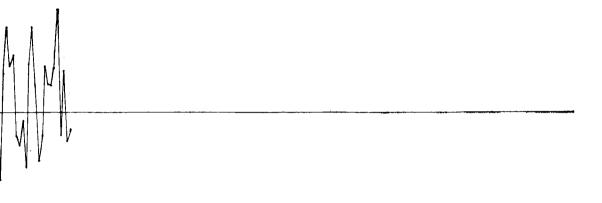
a) observation data in the years 1779—1979 b) the diagram of approximating function (6)

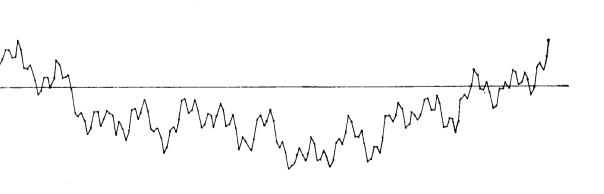




(Year)

a) observation data in the years 1779—1979 b) the diagram of aproximating function (7)





2060

2040

2080

21′

b

1960 1980 2000 2020 in Warsaw in the years 1779—2130

Two first dates specify the time of existence of known "maunder minima" of air temperature on the Earth in the years: 1619 and 1819 (Landsberg, 1980). The absolute minimum of annual average temperature in Warsaw within the observation interval (1812) is the lower limit of the weakest 11-years' cycle of the Sun's activity in the years 1811—1823. Multiperiodical maxima of annual temperature occur in the years:

date: 1495, 1715, 1950, 2131  $y_{\text{max}}$ : 9.3, 8.7, 8.5, 8.2°C

The absolute maximum of annual average temperature in the years 1779—1979 (1950) is slightly ahead of the date of the strongest 11-years' cycle of sunspots in the years 1955—1964 and is ahead of the absolute maximum date of Wolf's numbers (1957).

The probability of crossing the trend's line f(t) (in plus or in minus) can be defined on the basis of the size of remainder  $\varepsilon_j = y_j - f(t_j)$  (Table 4). The remainder deviations  $\varepsilon = \sqrt{\varepsilon^2}$  are: January  $\varepsilon = 3.100$ ;

Table 4 The Distribution of Remainder Size  $\varepsilon_j=y_j-f(tj)$  in Months: January, July and within a Year

ε <sub>j</sub> /ε	January	July	Year
< -3.0	1	0	0
-3.02.0	6	3	6
-2.01.0	26	. 33	31
-1.0 — $0.0$	56	59	54
0.0 — 0.1	86	73	84
0.0 — 2.0	25	29	24
2.0 — 3.0	1	4	2
> 3.0	0	0	0

July  $\varepsilon=1.424$ ; a year  $\varepsilon=0.829^{\circ}\mathrm{C}$ . For example, the probability of annual temperature occurrence in an interval  $f(t)\pm\varepsilon$  is  $68,7^{\circ}/_{\circ}$  and the occurrence of higher temperature than  $f(t)+2\varepsilon$  is less than  $1^{\circ}/_{\circ}$ .

In conclusion, we can note that at present "we are" in the declining part of the curve of average annual temperature values, below the "standard" i.e. the average 201 years equal to 7.46°C, It refers both to the whole year and to extreme months:

January — below the average  $\bar{y} = -4.02$ °C and July  $-\bar{y} = 18.58$ °C. In the last two decades of the 20th century the declining tendency of monthly average temperature in January and the growth of average temperature in July should be observed. Naturally, temperature of those months will undergo short-term fluctuation of great amplitudes.

Formula (7) also implicates another important conclusion. In Warsaw, in the years 1779—1979 there existed a constant decreasing tendency of annual air temperature values  $a=-0.15^{\circ}\text{C}/100$  years. It is probably the total effect of growth of dust and  $\text{CO}_2$  concentration in the Earth's atmosphere in recent years.

Multiperiodical changes of climatological elements in Poland, with special reference to Warsaw, have been the fundamental research problems of the Chair of Climatology of the Faculty of Geography and Regional Studies of Warsaw University (Kaczorowska, 1962; Okołowicz, 1964; Stopa-Boryczka, 1973; Kossowska 1973; Kopacz-Lembowicz, 1978).

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