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GRID STUDY ON THE BASIS OF ALLOCATED  
POWER LOSS CALCULATION

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Studying the loss allocation, it is possible to determine the loss which appears in some grid section or line (allocated power loss) as a result of: 1) some power source sending its power to the consumers, 2) some power source sending its power to a certain power consumer, 3) power received by some consumer. Determining allocated power losses, it happens that some of them are negative. It turned out that this phenomenon is not rare and appears at a definite power distribution between suppliers. Negative allocated power loss shows that total power loss is increased. Computing the allocated loss, the possibility appears to define other grid quantities that characterize grid properties and operation, as well as to find out those suppliers and consumers who mainly affect the grid operation.

**Keywords:** *loss allocation, node voltage, power losses, voltage drop*

## 1. INTRODUCTION

Attention to power losses has been drawn at the very beginning of electricity use. Losses were considered in the grid or on specified territory without being tied to a particular source of electricity or to the consumer, for example, in [1]. Optimizing the grid in order to reduce active power losses is considered in [2] without calculating the losses attributed to a particular power plant. With the deregulated work of the power system, it became necessary to determine the losses incurred by a particular power system subject – power supplier or consumer, but neither [1] nor [2] considered allocated power losses (AL) because there were other problems to be dealt with. Let us dwell on the literature which treats the issue in the light of new requirements to the operation of power systems. Extensive literature appeared on the topic of (AL). The question is solved in two directions. Topographic direction proposed by J. Bialek [3] is based on tracing the currents. The methods of this direction [4] take a lot of work even after a tedious job is made to trace the currents in all lines is made. In analytical direction, issue is solved by matrix algebra [5]. The complicated algorithm gives a possibility to calculate the quantities (node voltages, the angles and so on) which can be given by any load flow program. To calculate the load loss allocation, it is not necessary to derive such complex expressions [6]. Both directions do not provide a simple and uniform algorithm for any grid configuration.

The procedure which allows finding the AL transmitting the electricity from a specified supplier to a certain consumer is called here a double algorithm (DA). This procedure is most fully described in [7], [8]. The DA is based on two computer programs: power flow program (more appropriate is Power World) and Matlab. The main part of the work connected with the configuration of the grid takes over power flow program. The DA has an advantage that it is apt for all grid configurations, radial grids and closed grids. Computing by the DA, the joint current (JC), AL and other extra quantities are defined. JC is current that originates in some source or supplier and, flowing in some place of a grid, flows to a certain consumer.

Making practical estimates in [8] for a closed grid, it appeared that some AL could be negative. The question arises: can negative AL be in radial grids as well; what is the reason? Moreover, computing AL by the DA, the extra (secondary) quantities and properties can be defined, which can be applied to in-depth grid characteristics. The goal of the paper is to clarify negative AL, extra quantities and properties.

Radial Q (Fig. 1 a) and closed F (Fig. 1 b) 330 kV grids are investigated. Different cases of each scheme with dissimilar loads are computed by the DA (Table 1), altogether 20 cases.

The units A, V, VA, W are used if other is not indicated.

## 2. SECONDARY QUANTITIES

AL is an initial quantity searched for by the DA. Applying the DA, some other quantities can be defined, which help to evaluate a grid in a versatile manner.

One of them is joint impedance (JI). For example, between supplier 4 and consumer 1 (Fig. 1 a) the JI  $Z_{4t1}$  can be calculated using the formula:

$$Z_{4t1} = \Delta s_{4t1} / |I_{4t1}|^2, \quad (1)$$

where  $\Delta s_{4t1}$  is AL,  $I_{4t1}$  – JC transmitting the power from supplier 4 to consumer 1.

Some of JI for case Qa are:  $Z_{4t1}=28+49i$ ;  $Z_{4t6}=59+191i$ ;  $Z_{5t1}=-12-120i$ ;  $Z_{5t3}=-74-238i$ ;  $Z_{7t1}=4.8+19.7i$ . For case Fa:  $Z_{4t1}=113+741i$ ;  $Z_{4t2}=21+141i$ ;  $Z_{5t2}=17+108i$ ;  $Z_{7t3}=-49-249i$ . Negative AL has negative JI. At small changes of node current values, JI can be used for quick estimation of AL.

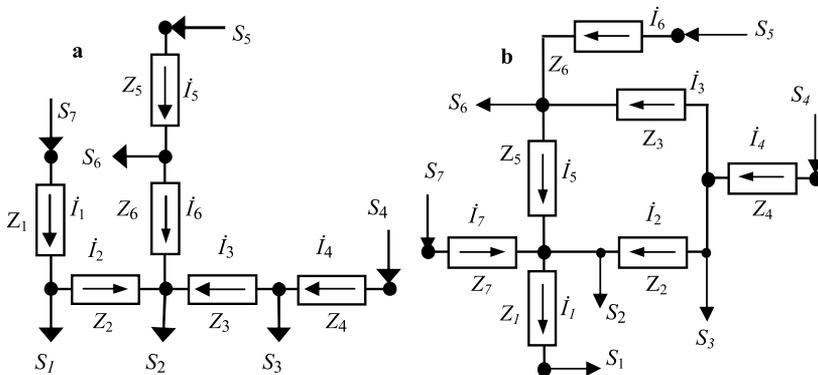


Fig. 1. 330 kV grid circuit diagrams

$S_4, S_5, S_7$  are flows to power supplying points;  $S_1, S_2, S_3, S_6$  – flows to consumers;  $I$  – branch currents;

$Z$  – transmission line impedances. Branch numbers coincide with impedance indices, node numbers – with power  $S$  indices; node currents  $J$  (not shown) correspond to node flows  $S$ . All shown quantities are complex numbers.

**a** – radial grid Q:  $Z_1=0.785+3.2i$ ,  $Z_2=5.58+28.8i$ ,  $Z_3=7.425+24i$ ,  $Z_4=1.782+5.76i$ ,  $Z_5=3.465+11.2i$ ,  $Z_6=3.366+10.88i \Omega$ ;

**b** – closed grid F:  $Z_1=11.22+70.4i$ ,  $Z_2=6.1+32i$ ,  $Z_3=6.48+25.6i$ ,  $Z_4=11.125+80i$ ,  $Z_5=1.02+6.4i$ ,  $Z_6=10.2+64i$ ,  $Z_7=0.81+3.2i \Omega$ .

Table 1

List of Computed Cases and Node Flows, MVA

Case	$S_1$	$S_2$	$S_3$	$S_4$	$S_5$	$S_6$		
MVA								
Qa	40	160+30i	20+10i	100+14i	50+5i	30+3i		
Qc				75+8.5i	75+8.5i			
Qc1				60+6.8i	90+10.2i			
Qc2				67.5+7.65i	82.5+9.35i			
Q2i				208+39i	100+14i		50+5i	
Q2i3				256+48i				
Q2r				100+100i				
Q4i3				160+50i				
Q4r				70+60i	10+1i		100+14i	40
Fa				100+14i				
Fc		56+5.6i	84+8.4i					
Fc1		42+4.2i	58+5.8i					
Fc2		25.8+2.58i	34.2+3.42i					
Fc3		18.48+1.848i	23.52+2.352i					
Fc4		12.43+1.243i	15.57+1.57i					
Fc5		15.68+1.568i	19.32+1.93i					
F2d		50+20i	100+14i	40				
F2r		40+100i						
F4w6		160+30i			0			
F4w4		50+7i						

Partial losses of any partial current in any power line can be computed. Any current out of more of them which all flow in some grid section is called here partial current. JC is partial current as well.

For example, in the case Qa, losses  $\Delta S_{2,4_0}$  caused by the current  $I_{2,4_0}$  ( $I_{2,4_0}$  is the part of node 4 current that flows in branch 2) in branch 2 are

$$\Delta S_{2,4_0} = (1/4)Z_2(|I_2 + I_{2,4_0}|^2 - |I_2 - I_{2,4_0}|^2), \quad (2)$$

where  $I_2$  is summary current which flows in line 2. Formula (2) is explained in Section 6.

Computed loss value is  $\Delta s_{2/40} = -16094 - 83065i$ ; although AL  $\Delta s_{4/1}$  is positive (see Table 2), supplier 4 reduces it in line 2. This can be taken into account when financing the line reconstruction.

Similarly, power losses in line 2 of JC flowing from supplier 4 to consumer 1 ( $I_{2/41}$ ) can be computed as follows:

$$\Delta s_{2/41} = 0.25 Z_2 (|I_2 + I_{2/41}|^2 - |I_2 - I_{2/41}|^2). \quad (3)$$

Loss value is  $\Delta s_{2/41} = -16094 - 83064i$ , it is equal to loss  $\Delta s_{2/40}$  since  $I_{2/40} = I_{2/41}$ . Loss  $\Delta s_{5/50} = 27263 + 88121i$ , and  $\Delta s_{5/51} = 4164.2 + 13460i$  since  $I_{5/50} \neq I_{5/51}$ .

Voltage drop across a line from partial current is a linear function of this current, e.g., voltage drop across line 2 from JC in line 2  $I_{2/41}$  (see above) is

$$\Delta u_{2/41} = Z_2 I_{2/41} \quad (4)$$

and is equal to  $-133-791i$ ; this voltage drop is negative because in Fig. 1a current  $I_{2/41}$  flows in the opposite direction. This quantity shows which partner pair causes voltage deviations in a grid.

### 3. NEGATIVE ALLOCATED POWER LOSSES

Cases Qa and Fa are used to deal with the negative losses.

Table 2 shows that radial scheme Qa has even three negative AL. Closed scheme Fa has only one – from supplier 7 to consumer 3.

To find the reason of negative AL, let us calculate AL in case Qa by transmitting electricity from supplier 5 to consumer 3, namely  $\Delta s_{5/3}$ . For clarity reason, this is done without the use of matrix algebra formulas. Current values computed by the DA are used. JC  $I_{5/3}$  flows in impedances  $Z_3, Z_5, Z_6$ :  $I_{3/5/3} = -7.0907 + 3.0637i$ ;  $I_{5/5/3} = 7.0907 - 3.0637i$ ;  $I_{6/5/3} = 7.0907 - 3.0637i$ . JC  $I_{5/3}$  or its portions do not flow in the impedances  $Z_1, Z_2, Z_4$ . Total currents in the impedances with JC  $I_{5/3}$  are  $I_3 = 140.47 - 6.09i$ ;  $I_5 = 88.16 - 9.77i$ ;  $I_6 = 35.18 - 3.63i$ . The AL  $\Delta s_{5/3}$  is the sum of partial losses in impedances  $Z_3, Z_5, Z_6$ . Computing by formulas of the type (3), we have:  $\Delta s_{3/5/3} = 0.25 Z_3 (|I_3 + I_{3/5/3}|^2 - |I_3 - I_{3/5/3}|^2)$ ;  $\Delta s_{5/5/3} = 0.25 Z_5 (|I_5 + I_{5/5/3}|^2 - |I_5 - I_{5/5/3}|^2)$ ;  $\Delta s_{6/5/3} = 0.25 Z_6 (|I_6 + I_{6/5/3}|^2 - |I_6 - I_{6/5/3}|^2)$ .

These currents in the impedances give loss values:  $\Delta s_{3/5/3} = -7534.062 - 24352.524i$ ;  $\Delta s_{5/5/3} = 2269.1769 + 7334.7132i$ ;  $\Delta s_{6/5/3} = 877.08535 + 2835.2325i$ , which being summed amount to  $\Delta s_{5/3} = -4387.8 - 14182.578i$ . This value almost does not differ from the one calculated by the DA (see Table 2). The loss in impedance  $Z_3$  is of great negative value which exceeds the sum of the positive values of the rest two impedances. It is negative because JC  $I_{3/5/3}$  is directed against total current  $I_3$ . Thus a precondition of negative AL existence is that of opposite direction of total current and JC. However, the summary result depends on other currents and impedances.

In closed grids F, there is only one negative AL. Here the path between two partners (between a supplier and a consumer) ramifies, opposite directed currents are smaller and they are less. Total grid power losses are always greater when at some

partner pair or in any grid branch negative losses appear. An absolute value of negative AL is less than the increment of positive losses at other partner pair. In a closed grid, this phenomenon is less pronounced.

To present negative losses as a result of equalizing currents, the appropriate mathematical expressions are necessary.

Table 2

**RL on the Path from Supplier 4,5,7 to Consumer 1, 2, 3, 6**

Case	Consumer	supplier 4 $\Delta s_{4 1,2,3,6}$	supplier 5 $\Delta s_{5 1,2,3,6}$	supplier 7 $\Delta s_{7 1,2,3,6}$
Qa	1	20934+36622i	- 2195-22479i	4086+16658i
	2	154210+498440i	24360+78740i	93347+463380i
	3	4851+15681i	- 4387-14181i	- 1868+15691i
	6	26081+84302i	3270+10570i	14449+76158i
Fa	1	92800+609230i	18155+114630i	28654+169110i
	2	286120+1905100i	37304+237040i	18750+74074i
	3	13868+99726i	758+7274i	- 868-14510i
	6	51199+340240i	6186+38814i	1585+442i

#### 4. SUPPLIER OPTIMAL USE

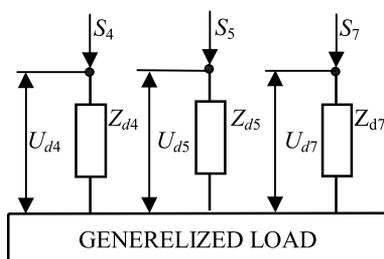


Fig. 2. Representation of the grid with the view of optimization.

Total grid power losses depend on distribution of power (which is necessary for consumers) between suppliers. If electricity price of all suppliers is the same, the farther a supplier is located from consumers the less electricity should be purchased from him due to increased losses. Optimal power distribution comes on when active components of the distinctive voltages  $U_d$  across the path from a supplier to a generalized load (Fig. 2) are equal for all suppliers.

Distinctive voltages can be defined by the following formulas:

$$U_{d4} = \Delta s_4 / |J_4|; U_{d5} = \Delta s_5 / |J_5|; U_{d7} = \Delta s_7 / |J_7|, \quad (5)$$

where  $\Delta s_4, \dots, \Delta s_7$  are power losses caused by suppliers 4; 5; 7. Those are supplier losses:

$$\Delta s_4 = \Delta s_{4t1} + \Delta s_{4t2} + \Delta s_{4t3} + \Delta s_{4t6}; \Delta s_5 = \Delta s_{5t1} + \dots; \Delta s_7 = \Delta s_{7t1} + \dots \quad (6)$$

The less active components of distinctive voltages vary, the smaller active total power losses are. This is illustrated by the following. Distinctive voltages computed for case Qa are  $U_{d4}=1163+3583i$ ,  $U_{d5}=237+594i$ ,  $U_{d7}=600+3120i$ , grid power loss is  $\Delta p=337130$ . For case Fa, the values are  $U_{d4}=2562+17048i$ ,  $U_{d5}=894+5696i$ ,  $U_{d7}=248+1233i$ ,  $\Delta p=552510$ . To find out a better option with smaller grid losses, the cases Qc, Qc1, Qc2, Fc, Fc1, Fc2, Fc3, Fc4, Fc5 were computed until a better load distribution option was found. Grid Q has a better case Qc2:  $U_{d4}=650+1929i$ ,  $U_{d5}=643+1906i$ ,  $U_{d7}=616+3179i$ , where  $\Delta p=285161$ ; closed grid – case Fc5:  $U_{d4}=463+3128i$ ,  $U_{d5}=464+2921i$ ,  $U_{d7}=427+1993i$ ,  $\Delta p=183942$ . Nevertheless, negative AL quickly disappear when active components of distinctive voltages drop.

Distinctive impedances which are defined as:

$$Z_{d4} = \Delta s_4 / |J_4|^2; Z_{d5} = \Delta s_5 / |J_5|^2; Z_{d7} = \Delta s_7 / |J_7|^2, \quad (7)$$

vary relatively little when the load changes. For example, in a radial grid, the loads of cases Qa and Qc vary (see Table 1), but distinctive impedances for case Qa are:  $Z_{d4}=6.56+20.21i$ ;  $Z_{d5}=2.68+6.69i$ ;  $Z_{d7}=3.37+17.02i$ ; for case Qc2:  $Z_{d4}=5.43+16.1i$ ;  $Z_{d5}=4.4+13.04i$ ;  $Z_{d7}=3.35+17.3i$ . In a closed grid, they are still closer.

It seems that supplier losses are approximately proportional to supplier current square.

## 5. FAST DETERMINATION OF NODE VOLTAGES

It goes without saying that if the supplier or consumer power changes, voltage in all grid nodes changes as well. Node sensitivity to power change depends on how far electrically one node is located from the other.

To determine this sensitivity, two modes (cases) should be computed: the first most common (base) mode where node phase voltages  $U_{vib}$  are found. Cases Qa and Fa are taken as base ones. Computing the second (auxiliary) mode, node phase voltages  $U_{via}$  are obtained, where  $i$  is node number,  $b$  and  $a$  denote base and auxiliary mode. Using these voltages, conjoint sensitivity of the nodes can be found. On the basis of these calculations, it can be fast determined what awaits each node by load or supply power change at any other node. Node voltages can be computed by any appropriate program as well, but the proposed method makes it possible to survey the situation at all nodes at once without using the computer program.

The circuit diagram in Fig. 3a illustrates the said. A grid can be deemed as an assemblage of voltage sources [9]. It can be represented by its no-load voltages  $U_{vib}$  in base mode ( $i$  stands for 1... 6,  $b$  – for basic) and by putative impedances. Slack bus voltage  $U_{v7}$  here is not topical. Figure 3a shows putative impedances  $Z_{4p1}$ , ...,  $Z_{4p3}$ ,  $Z_{4p5}$ ,  $Z_{4p6}$ ; they are internal impedances of voltage source for related influenced node and are fit only when current changes at influencing node of supplier 4.  $Z_{4s}$  is putative impedance of influencing node 4.

If power at influencing node 4 does not change ( $\Delta J_4=0$ ), eventual voltages  $U_{v1e}, \dots, U_{v6e}$  at all 1, ..., 6 nodes do not change and they are  $U_{v1b}, \dots, U_{v6b}$ . When power at influencing node changes ( $\Delta J_4 \neq 0$ ), to determine eventual voltage, e.g.,  $U_{v3e}$  at node 3 (or  $U_{v4e}$  at node 4), it is necessary to move the contact of switch S to impedance  $Z_{4p3}$  (or  $Z_{4s}$ ). Then eventual voltage at node 3 (or 4) is

$$U_{v3e} = U_{v3b} + Z_{4p3} \Delta J_4; \quad U_{v4e} = U_{v4b} + Z_{4s} \Delta J_4. \quad (8)$$

Eventual voltages at other influencing nodes are calculated in the same way. In Fig. 3b, the influencing node is consumer node 2.

Putative impedances of supplying nodes are positive, but those of consumer nodes are negative. They are defined by formulas:

$$Z_{ipk} = \frac{U_{vka} - U_{vkb}}{J_{ia} - J_{ib}}; \quad Z_{is} = \frac{U_{via} - U_{vib}}{J_{ia} - J_{ib}}, \quad (9)$$

where  $U_{vka}$  and  $U_{vkb}$  are voltage of influenced node  $k$  in auxiliary and base modes;  $J_{ia}$  and  $J_{ib}$  is the current of influencing node  $i$  in auxiliary and base modes, which are obtained by the DA;  $U_{via}$  and  $U_{vib}$  – respective voltages of influencing node (Table 3). The computed putative impedances for auxiliary case Q2i (base mode being Qa) are:  $Z_{2p1} = -0.924-3.257i$ ;  $Z_{2s} = -5.667-31.652i$ ;  $Z_{2p3} = -5.096-31.569i$ ;  $Z_{2p4} = -4.875-31.505i$ ;  $Z_{2p5} = -5.466-31.648i$ ;  $Z_{2p6} = -5.641-31.653i$ . The closer the influenced node is to the influencing one, the greater is putative impedance (its absolute value), it is the greatest one for influencing node  $Z_{is}$  itself.

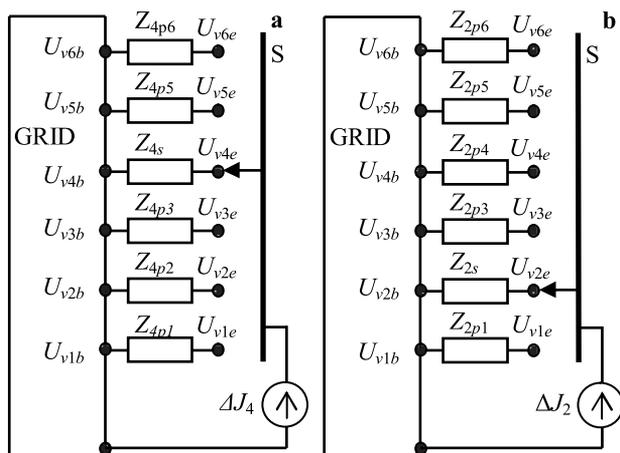


Fig.3. Updating the voltage in a grid

a – change  $\Delta J_4$  (at node 4) of supplier current,

b – change  $\Delta J_2$  (at node 2) of consumer current.

Table 3

Main Mode Data of Cases Qa and Q2i

Node	Base case Qa		Auxiliary case Q2i	
	$J$	$U_v$	$J$	$U_v$
1	70.09-0.2i	190234.4-521.7i	70.14-0.29i	190080 - 780i
2	282.29-58.10i	188245.9-3322.9i	368.15-81.14i	187030 - 5910i
3	35.19-17.60i	189435+3.1i	35.16-18.19i	188270 - 2590i
4	175.66-23.69i	189884.5+972.7i	176.39-26.24i	188740 - 1620i
5	88.16-9.77i	188818.6-1998.7i	88.56-11.05i	187620 - 4590i
6	52.98-6.14i	188403.7-2952.3i	53.22-6.92i	187190 - 5540i
7	176.73-48.58i	190528.6+5.7i	261.71-69.21i	190510

In eventual (updated) mode, voltages  $U_{vke}$  of influenced and  $U_{vie}$  of influencing modes are

$$U_{vke} = U_{vkb} + Z_{ipk}(J_{ie} - J_{ib}); \quad U_{vie} = U_{vib} + Z_{is}(J_{ie} - J_{ib}). \quad (10)$$

This time the current  $J_{ie}$  of influencing node in eventual mode is not computed by the DA or by any other program because it is embarrassing; it is calculated using known voltage  $U_{vib}$  and changed (eventual) power  $S_{ie}$  of node  $i$ :

$$J_{ie} = \frac{\hat{S}_{ie}}{3\hat{U}_{vib}}. \quad (11)$$

If there is doubt about accuracy, the calculation of eventual current  $J_{ie}$  of node  $i$  can be repeated, using voltage  $U_{vie}$  calculated by formula (10):

$$J_{ie}^{(1)} = \frac{\hat{S}_{ie}}{3\hat{U}_{vie}}. \quad (12)$$

In the paper, eventual mode is computed by the DA as well. It is done in order to check the accuracy of the method, voltages  $U_{vkeDA}$  and  $U_{vieDA}$  are obtained. The accuracy  $d_k$ ;  $d_i$  is defined as voltage difference between node voltages obtained by fast method ( $U_{vke}$ ;  $U_{vie}$ ) and by the DA ( $U_{vkeDA}$ ;  $U_{vieDA}$ ); this difference is related to the true voltage difference (the difference between eventual obtained by the DA ( $U_{vkeDA}$ ;  $U_{vieDA}$ ) and base voltages ( $U_{vkb}$ ;  $U_{vib}$ ):

$$d_k = \frac{|U_{vke} - U_{vkeDA}|}{|U_{vkeDA} - U_{vkb}|}; \quad d_i = \frac{|U_{vie} - U_{vieDA}|}{|U_{vieDA} - U_{vib}|}. \quad (13)$$

Accuracy values for some cases are shown in Table 4. The accuracy is better when node power angle of all three modes is closer (see Table 1).

Table 4

## Accuracy of Node Voltages

Case			$d_{1e}$	$d_{2e}$	$d_{3e}$	$d_{4e}$	$d_{5e}$	$d_{6e}$
basic	auxil.	event.	$d_{1e}^{(1)}$	$d_{2e}^{(1)}$	$d_{3e}^{(1)}$	$d_{4e}^{(1)}$	$d_{5e}^{(1)}$	$d_{6e}^{(1)}$
Qa	Q2i	Q2i3	0.0460	0.0771	0.0765	0.0780	0.0756	0.0757
			0.0577	0.0053	0.0018	0.0021	0.0040	0.0052
Qa	Q2i	Q2r	0.0981	0.0188	0.0375	0.0472	0.0226	0.0197
			0.0576	0.0472	0.0759	0.0864	0.0578	0.0496
Qa	Q4i3	Q4r	0.0337	0.0602	0.0261	0.0215	0.0488	0.0744
			0.0856	0.0112	0.0732	0.0661	0.0960	0.0439
Fa	F2d	F2r	0.0332	0.0102	0.0193	0.0969	0.0331	0.0115
			0.0299	0.0098	0.0214	0.0995	0.0357	0.0120
Fa	F4w6	Fw4	0.0459	0.0461	0.0454	0.0453	0.0467	0.0460
			0.0009	0.0031	0.0021	0.0021	0.0047	0.0071

$d_{1e}^{(1)}, d_{2e}^{(1)}, \dots$  is accuracy, obtained using node currents  $J_{1e}^{(1)}, J_{2e}^{(1)}, \dots$ .

## 6. CLARIFICATION OF TWO FORMULAS

1) In referred publications, using the DA algorithm, node currents  $\dot{J}$  are defined as

$$J_i = \frac{\hat{S}_i}{\sqrt{3}\hat{U}_i}, \quad (14)$$

where  $\hat{S}_i$  and  $\hat{U}_i$  are conjugate values of node power and line voltage  $\dot{U}_{ab}$ . Since phase voltage  $\dot{U}_{\rho h}$  is shifted by  $30^\circ$  [11], it should be

$$J_i = \frac{\hat{S}_i}{\sqrt{3} \cdot \text{conj}(\dot{U}_i e^{-j30^\circ})}. \quad (15)$$

However in expression (14), there is no error because PowerWorld program gives phase-to-phase voltage with phase voltage angle.

2) Based on [10]; [7]; [8], power loss in impedance  $Z$  with currents  $I = I_1 + I_2 + \dots + I_n$  (Fig. 4) from any of currents (for example, from current  $I_2$  power loss  $\Delta s_2$ ) can be defined by expression:

$$\Delta s_2 = k \frac{Z}{4} \left( \left| \dot{I} + \frac{\dot{I}_2}{k} \right|^2 - \left| \dot{I} - \frac{\dot{I}_2}{k} \right|^2 \right) = \frac{1}{\kappa} \frac{Z}{4} \left( |\dot{I} + \kappa \dot{I}_2|^2 - |\dot{I} - \kappa \dot{I}_2|^2 \right); \quad \kappa = \frac{1}{k}. \quad (16)$$

The substitution  $\kappa=1/k$  is made for convenience in further consideration.

For example, if  $k=100$ , then  $\Delta s_2 = 25Z(|i + 0.01\dot{I}_2|^2 - |i - 0.01\dot{I}_2|^2)$ , (see [6]).

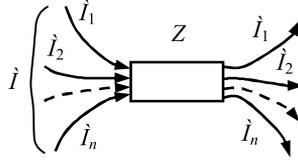


Fig. 4. Branch Z with several currents.

Complex numbers can be represented as:

$$\dot{I} = I(\cos \alpha + j \sin \alpha); \quad \dot{I}_2 = I_2(\cos \beta + j \sin \beta). \quad (17)$$

Inserting (17) in expression  $|\dot{I} + \kappa\dot{I}_2|^2 - |\dot{I} - \kappa\dot{I}_2|^2$  we obtain (18) and inserting (18) in equation (16), we obtain (19):

$$|\dot{I} + \kappa\dot{I}_2|^2 - |\dot{I} - \kappa\dot{I}_2|^2 = 4\kappa I I_2 [\cos(\alpha - \beta)] = 4\kappa I I_2 (\cos \alpha \cos \beta + \sin \alpha \sin \beta); \quad (18)$$

$$\Delta s_2 = Z I I_2 (\cos \alpha \cos \beta + \sin \alpha \sin \beta). \quad (19)$$

Whatever  $k$  (or  $\kappa=1/k$ ) we introduce into (16), we still get (19). When we assume  $k=1$ , we see a simpler formula similar to expressions (2) and (3):

$$\Delta s_2 = 0.25Z(|\dot{I} + \dot{I}_2|^2 - |\dot{I} - \dot{I}_2|^2). \quad (20)$$

The decrease in  $k$  times of partial current (it was conjoint current) was necessary in the very beginning [10] when AL was looked for.

The sum of partial losses, obtained on the basis of expressions (16) and (20), is equal to the loss of summary current which corresponds to the meaning of electrical engineering.

The author of the research does not know any formula of a different type where the sum of partial losses is equal to the summary current losses.

## 7. CONCLUSIONS

1. Partial power losses and partial voltage drops can be computed for each grid branch, this allows fixing the source of hard mode in the grid.
2. Negative AL indicate a particular market partner pair; they signify that total grid power losses are increased. Negative AL can appear when JC in some branch is directed opposite to the total current of this branch.
3. In the grid with minimal power losses, values of active components of distinctive voltages for all suppliers are equal or close.
4. Using putative impedances and node voltage levels, the grid response to power changes can be surveyed.

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## TĪKLU PĒTĪJUMI UZ SAVSTARPĒJO ZUDUMU APRĒĶINU BĀZES

J. Survilo

### K o p s a v i l k u m s

Nosakot saistītos jaudas zudumus (zudumus, kas rodas pārvadot jaudu no kāda piegādātāja kaut kādam patērētājam) tīklā, gadās, ka daži saistītie zudumi ir negatīvi. Izrādījās, ka šī parādība nav reta un rodas pie noteiktas piegādātāja jaudas sadales. Negatīvie saistītie zudumi uzrāda, ka ir palielināti kopējie jaudas zudumi tīklā. Aplēšot zudumus, ir iespēja noteikt citus tīkla lielumus, kuri raksturo tīkla īpašības un darbu, kā arī ļauj noskaidrot tos piegādātājus un patērētājus, kuri ietekmē tīkla darbu.

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