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EVALUATION OF THE UNIFORM FIELD DISTORTIONS PRODUCED BY A TOROIDAL DIELECTRIC BODY

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Distortions of the structure of a uniform electric field when a dielectric body with a toroidal shape is placed in it are considered in the quasi-static approximation. The rate of distortion is proposed to estimate through the effective permittivity of toroid determined by solving the corresponding boundary value problem. Some numerical estimates obtained using specially developed software in the language of Matlab are given.

Keywords: *associated Legendre functions, boundary problem, effective permittivity, Laplace's equation, toroidal coordinates.*

1. INTRODUCTION

In physics and electrodynamics, of particular interest are the field distortions caused by the introduction of the body with toroidal shape into the field (or the existence of a region of space in it). For example, ceramic washers placed in the waveguides are used as heat accumulators in the art of microwave heating. In the study of the structure of the near electromagnetic field in the first approximation a washer can be replaced by some equivalent toroid. A similar approach can be applied to the analysis of targeted disruption of the motor-car tires under microwave irradiation, and so on.

In general, the problem is formulated as follows. The body of a known configuration and the characteristic size produced by a material with absolute permittivity ε, magnetic permeability μ and conductivity σ is placed in an external field. It is necessary to assess the field distortion caused by placing the body in it.

From a physical point of view, any change in the structure of the primary field caused by an object introduced in it is provoked by secondary, or diffractional, fields. They are created by currents and charges that occur in the bulk and on the surface of the body under the influence of the incident field. The problem is reduced to the solution of Maxwell's equations with given boundary conditions for the resulting field on the surface of the body, in its origin domain and at infinity. In other words, the tangential components of the field when passing through the surface of the body must be continuous, the secondary field at infinity should disappear and take finite values at the points within the body [1].

A rigorous solution is possible, in particular, in the quasi-static approximation ($\omega \to 0$, $\lambda \to \infty$), i.e., for potential fields. Nevertheless, it gives some insight into the structure of the field in the vicinity and inside the body. As far as the body shape is concerned, such a solution can be found only in the case when the surface of the body is fully described by the equation for one of the coordinate surfaces in a certain orthogonal coordinate system. This can greatly simplify the formulation of the boundary conditions.

We will look at solutions. Thus, the dielectric body with a known configuration and the characteristic size of *a*, made from homogeneous and isotropic material, is placed in uniform field $\overline{E}_0$. Let us assume that the electrical size of the body (e.g. size in wavelengths) is small, i.e., $a / \lambda << 1$. To assess the field distortion, it is proposed introducing effective permittivity

$$\varepsilon_{\text{eff}} = \Phi/\Phi_0 , \qquad (1)$$

where $\Phi = \int\limits_S \overline{D}_1 d\overline{S}, \quad \Phi_0 = \int\limits_S \overline{D}_0 d\overline{S}$ are flows of induction vectors $\overline{D}_1 = \varepsilon_1 \overline{E}_1, \ \overline{D}_0 = \varepsilon_0 \overline{E}_0$ found in the presence of the body and without it, respectively. The ratio in (1) characterises the degree of concentration of induction vector flux through certain section *S* of the body. In contrast to (1), in some literature sources (see, e.g., [2]) the concept of the effective permittivity is used to describe the properties for a mixture of dissimilar dielectrics.

The sequence of solving the problem of effective permittivity could be as follows.

1. Select a coordinate system (α, β, φ) in such a way to simplify the boundary conditions stated below. It is necessary that one of the coordinate surface, e.g., $\alpha = \alpha_0$, could be completely superposed with body surface. If this condition is not met, for example, as the case of a cylinder of finite length, obtaining an exact solution in closed form is not possible.

2. Taking into account the type of Laplace operator Δ in the chosen coordinate system, one can compile the equation

$$\Delta F(\alpha, \beta, \varphi) = 0, \qquad (2)$$

with respect to scalar potential $F(\alpha, \beta, \varphi)$.

3. The separation of variables (or Fourier method) [3] is usually used to solve (2), representing the general solution as the superposition of particular solutions of form $F = A(\alpha)B(\beta)C(\varphi)$ and going from partial differential equations to ordinary differential equations for functions $A(\alpha)$, $B(\beta)$ and $C(\varphi)$. In total, there are 11 co-

ordinate systems that allow direct separation of variables [4]. In addition, there are changes in bispherical and toroidal systems, but with the submission

$F' = G(\alpha, \beta) f(\alpha, \beta, \varphi)$

where f is a new unknown function, $G(\alpha,\beta)$ is an additional separating factor.

4. The resulting equations include the constants, called the separation constants. They are to be found taking into account the existing symmetries of the body, which determine the periodicity of solutions for certain coordinates. This simplifies the differential equations obtained and if it is possible to reduce them to the routine operations.

5. Decision of the ordinary differential equation is usually expressed in terms of some special functions with unknown coefficients to be determined. The properties of these functions are determined from selected coordinate system features.

6. Next, we have to find these coefficients. For this purpose, first, consider the limited potential in the internal region of the body and the disappearance of influence of the latter in the points of infinity outer region. This allows us to record solutions for these areas separately, thereby reducing the number of unknown coefficients.

7. These solutions should be joined due to the next reasons. They must satisfy the boundary conditions, which in this case consist in the continuity of the potential and the normal component of the induction vector on the surface of the body. If the surface is described as $\alpha = \alpha_0$, the boundary conditions take the form [3].

$$\left.\begin{aligned} &F_1 = F_2, \\ &\varepsilon \frac{\partial E_1}{\partial \alpha} = \varepsilon_0 \frac{\partial E_2}{\partial \alpha} \end{aligned}\right\} \quad \text{as } \alpha = \alpha_0 , \tag{3}$$

where F_1 и F_2 are scalar potentials into and outside the body, respectively.

Imposition of boundary conditions allows obtaining the required equations for the unknown coefficients in its final form. Their decisions complete scalar potential finding.

8. Knowing the potential, one can find the electrical field intensity

$\overline{E}_1 = -\mathrm{grad} F_1$

and induction vector $\overline{D}_1 = \varepsilon \overline{E}_1$ in the internal region in the body.

9. Determine the effective permittivity (1) in the desired section of the body under consideration

$$\varepsilon_{\text{eff}} = \Phi / \Phi_0 . \tag{4}$$

2. The Field in the Vicinity of the Toroid

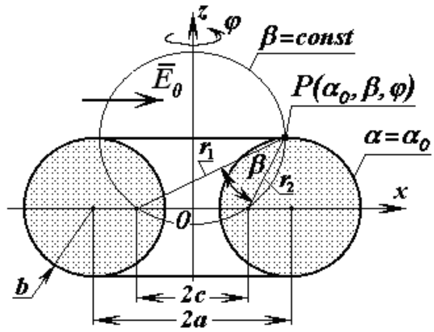

Fig. 1. Toroidal coordinates.

As already mentioned, a rigorous solution of the problem of effective permittivity is only possible for bodies whose surfaces can be completely described by the equation for one of the coordinate surfaces in some orthogonal coordinate system, which can significantly simplify the writing of the boundary conditions.

With respect to the toroidal body, system α, β, φ (Fig. 1) complies with this requirement. It is obtained by rotating around vertical axis z of two mutually orthogonal families of circles α = const, β = const. The coordinate surfaces are formed in the space as a family of tori α = const, spherical segments β = const, and the half-planes φ = const.

If the piece of straight line having length $2c$ is positioned on the x-axis, for arbitrary point P (α, β, φ) its first coordinate can be found using relationship $\alpha = \ln (r_1/r_2)$, where r_1 and r_2 are the distances from P to the ends of the piece. Coordinate β determines the angle from point P subtended by piece $2c$. The φ is the angle between the xz-plane and the plane passing through this point P and the z-axis.

Relations of toroidal α, β, φ and Cartesian x, y, z coordinates are given as follows [4]:

$$\begin{aligned} x &= c\,\mathrm{sh}\alpha \cos\varphi(\mathrm{ch}\alpha - \cos\beta)^{-1}, \\ y &= c\,\mathrm{sh}\alpha \sin\varphi(\mathrm{ch}\alpha - \cos\beta)^{-1}, \\ z &= c \sin\beta(\mathrm{ch}\alpha - \cos\beta)^{-1} \end{aligned} \tag{5}$$

where $\alpha \in [0,\infty)$, $\beta \in [-\pi,\pi]$, $\varphi \in [0,2\pi]$. Value $c = (a^2 - b^2)^{1/2}$ plays the role of the scale factor and binds radii a and b of central circle and generating one of the toroid.

These radii can be expressed in terms of coordinate α_0 of the torus surface

$a/b = \mathrm{ch}\alpha_0$, $\quad a = c\cdot\mathrm{cth}\alpha_0$, $\quad b = c/\mathrm{sh}\alpha_0$.

Surface α_0 = Arch a/b (Fig. 1) divides the whole space into two regions: inner $\alpha_0 \le \alpha < \infty$ occupied by the toroid and external $0 \le \alpha \le \alpha_0$. Further, notation $s = \mathrm{ch}\alpha$, $s_0 = \mathrm{ch}\alpha_0$ is used for conciseness.

Let the toroid produced from a material with dielectric constant ε_1 be placed in uniform electric field $\bar{E}_0$, directed perpendicularly to the z-axis of its rotational symmetry (Fig. 1). Assuming a quasi-static problem, we turn to the Laplace's equation (2) and will seek a field in the vicinity of the toroidal shell through the scalar potential

$$F_{1,2} = F_0 + F_{1,2}^b \ , \tag{6}$$

where $F_0 = -E_0 x$ is the potential of the primary field, $F_{1,2}^b$ is the potential increment due to the influence of the body. Subscripts 1 and 2 refer to the inner and the outer regions of the toroid, respectively.

Given the symmetry of the body with respect to the coordinate axes and using the formula from (5), which relates toroidal and Cartesian coordinates, we find

$$F_0 = -E_0 x = -E_0 c(s^2 - 1)^{1/2}(s - \cos\beta)^{-1} \cos\varphi. \tag{7}$$

As it is known [4], [5], the eigenfunctions in the toroidal coordinate system are associated Legendre functions with half-integer indices. To use them for submission to x, one can multiply and divide the right-hand side of (7) to $(s - \cos\beta)^{1/2}$. Expanding the denominator of the resulting fraction in a Fourier series [5], we have

$$F_0 = \sqrt{2} M_x (s - \cos\beta)^{1/2} \sum_{n=0}^{\infty} \delta_n Q_{n-1/2}^1(s) \cos n\beta \cos\varphi, \tag{8}$$

where $M_x = 2E_0 c/\pi$, $Q^1_{n-1/2}(s)$ is the associated Legendre function of the 2nd kind of half-integer order,

$$\delta_n = \begin{cases} 1, n = 0, \\ 2, n = 1, 2\ldots \end{cases}$$

is the Kronecker delta. Potentials due to the influence of the toroid are naturally sought in the form

$$F_{1,2}^b = \sqrt{2} M_x (s - \cos\beta)^{1/2} \sum_{n=0}^{\infty} \delta_n \begin{Bmatrix} a_n P_{n-1/2}^1(s) \\ b_n Q_{n-1/2}^1(s) \end{Bmatrix} \cos n\beta \cos\varphi\,, \tag{9}$$

where the upper and lower rows refer to the inner and outer regions of the toroid, respectively. $P^1_{n-1/2}(s)$ is the associated Legendre function of the 1st kind, which meets the conditions of the disappearance of the effect of the body in the inner points of toroid where $\alpha \to \infty$. Unknown coefficients a_n and b_n are determined from the boundary conditions (3), consisting in continuity of the potential and the normal component of the induction vector passing through the surface of the toroid.

Here

$$F_1 = F_2\big|_{\alpha=\alpha_0}, \quad \varepsilon_1 \frac{\partial F_1}{\partial \alpha} = \varepsilon_2 \frac{\partial F_2}{\partial \alpha}\bigg|_{\alpha=\alpha_0}. \tag{10}$$

ε_{1} and ε_2 are the relative permittivity of the media in the inner and outer regions of the toroid, respectively. Considering the first condition (i.e., left formula) in (10) and assuming uniform convergence of the corresponding series, we find

$$a_n = \frac{Q^1_{n-1/2}(s)}{P^1_{n-1/2}(s)} b_n. \tag{11}$$

Differentiating potentials according to the second condition in (5) and using (11), after laborious transformation we obtain

$$b_n = -\frac{T_n}{g_n + T_n} \tag{12}$$

where

$$T_n = Q_{n-1/2} Q^{1'}_{n-1/2} - Q'_{n-1/2} Q^1_{n-1/2}, \tag{13}$$

$$g_n = \frac{1}{\varepsilon_2 - 1} \frac{n^2 - 1/4}{s_0^2 - 1} \frac{Q_{n-1/2}}{P^1_{n-1/2}}. \tag{14}$$

To simplify the formulas containing the associated Legendre functions, their arguments s_0 are not written in most cases. The primes denote derivatives of these functions by s_0. The numerical calculations are advantageously carried out using the integral representations [5] of the Legendre functions.

When performing calculations, it is necessary to estimate the minimum number of members in the respective sets that must be summed to obtain a satisfactory accuracy. To this end, Figs. 2 and 3 illustrate the behaviour of the terms in (7) marked by the circles. They describe the unperturbed external field potential, depending on the number of terms n and torus parameter s_0. Dotted lines are spline approximations and are plotted for illustration purposes only.

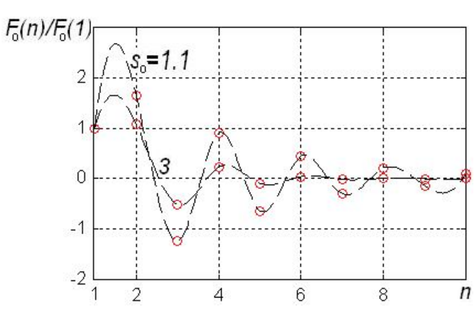

Fig. 2. Illustration of convergence of the series (7).

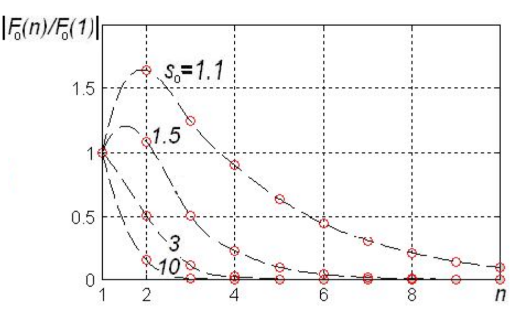

Fig. 3. Dependence of the modulus of members in (7) on s_0.

The resulting graphs confirm that the solutions, which contain the associated Legendre functions with half-integer index as the eigenfunctions, converge fast enough for all practically interesting variations of toroid geometrical dimensions. Due to this, in most cases, series (8) and (9) can be limited to 4–5 members.

The exceptions are the toroids with a relatively small diameter of the centre hole when s_0 tends to 1. In these cases, the series does not converge quickly enough, and it requires a special study. If $s_0 \to 1$, the Legendre functions in (13) and (14) can be calculated from the asymptotic formula [5]. This gives

$$g_n\Big|_{s_0\to 1}=\frac{\sqrt{2}}{(\varepsilon_2-1)(s_0+1)(s_0-1)^{3/2}}\ln\frac{(s_0-1)^{-1/2}}{\sqrt{2}\pi}, \tag{15}$$

$$T_n\Big|_{s_0\to 1}=\frac{1}{2\sqrt{2}(s_0-1)^{3/2}}[\ln\frac{(s_0-1)^{-1/2}}{\sqrt{2}\pi}-1]. \tag{16}$$

It is shown that functions g_n and T_n no longer depend on number n as $s_0 \to 1$. Therefore, the expression for the potential inside the toroid, or the top line in (9), can be greatly simplified, since it is now a series that is easily summed. After appropriate substitutions that potential takes the form:

$$F_1\Big|_{s_0\to 1}=-E_0 x\frac{1}{1+\frac{1}{4}(\varepsilon_2-1)(s_0+1)\{1+1/\ln[\sqrt{2}\pi(s_0-1)^{1/2}]\}}. \tag{17}$$

It follows from (17) that for toroids with a vanishingly small diameter of central hole, the field strength inside the body coincides with the direction of the applied field and is a constant that does not depend on the coordinates. Therefore, in this case there is a uniform polarisation of the toroid.

If the external field acts along the z-axis, a similar procedure can be applied to find the potentials. As in (7), the first step should be to search for the expansion of the coordinate, the unit vector of which is parallel to the applied field. Details of relevant computing are provided in Appendix.

3. The Effective Permittivity of the Toroid

To calculate the effective permittivity from (1), it is necessary to find the fluxes

$$\Phi=\int_S \overline{D}_1 d\overline{S},\quad \Phi_0=\int_S \overline{D}_0 d\overline{S}$$

of electrical induction vectors $\overline{D}_1=\varepsilon\overline{E}_1$, $\overline{D}_0=\varepsilon_0\overline{E}_0$ through central cross section S (it is shaded in Fig. 1) in the presence of the toroid and without it, respectively. In this case, value of E_1 is φ-component of the field strength inside the toroid.

It is easy to determine through potential F_1

$$E_{1\varphi} = -\frac{1}{h_\varphi}\frac{\partial F_1}{\partial \varphi}$$

where h_φ is the Lamé coefficient [4]. The elementary area on section S can be represented as $dA_S = h_\alpha h_\beta d\alpha d\beta$. After corresponding substitutions and elaborate calculations, one can find the flux of induction

$$\begin{aligned} \Phi &= 4\sqrt{2}\varepsilon_0\varepsilon_r cM_x \sin\varphi \sum_{n=0}^{\infty}\delta_n u_n \int_{s_0}^{\infty}\frac{Q_{n-1/2}^{1}(s_0)}{s^2-1}\int_0^{\pi}\frac{\cos n\beta}{(s-\cos\beta)^{1/2}}\mathrm{d}s\mathrm{d}\beta = \\ &= 8\varepsilon_0\varepsilon_r cM_x \sin\varphi (s_0^2-1)\sum_{n=0}^{\infty}\delta_n u_n(s_0)T_n(s_0) \end{aligned} \tag{18}$$

where $\varepsilon_r = \varepsilon_2/\varepsilon_1$ is the relative permittivity of the material, u_n=1+ b_n. Values b_n and T_n may be obtained from (12) and (13), respectively. In the central section of the toroid φ = π/2 (for the right half of this section) and φ = 3π/2 (for the left half).

The flow in the absence of the toroid is proportional to twice the area of a circle with radius b (Fig. 1):

$$\varepsilon_{\text{eff}} = \frac{\Phi}{\Phi_0} = \frac{8}{\pi^2}\varepsilon_r(s_0^2\text{-}1)\sum_{n=0}^{\infty}\delta_n u_n(s_0)T_n(s_0). \tag{19}$$

Finally, we find that the effective permittivity of the toroid is described by an infinite series

$$\varepsilon_{\text{eff}} = \frac{\Phi}{\Phi_0} = \frac{8}{\pi^2}\varepsilon_r(s_0^2\text{-}1)\sum_{n=0}^{\infty}\delta_n u_n(s_0)T_n(s_0). \tag{20}$$

Figure 4 depicts a family of graphs describing the dependence of the effective permittivity of the toroid on geometrical parameter s_0 = a/b and relative permittivity ε_r of the material. The initial region of the curves is shown for convenience on a larger scale in Fig. 4 *b* separately. The sum of the first 10 items of the series is taken in the future for the exact value of the effective permittivity.

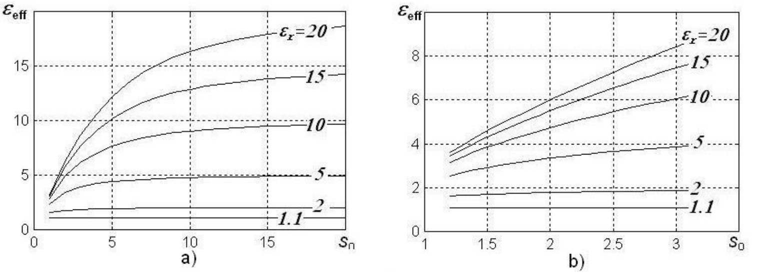

Fig. 4. Effective permittivity of toroid change depending on its geometry and material.

As the series converges rapidly enough, the number of terms taken into account can be significantly reduced by the cost of some decrease in accuracy. Thus, while limiting its only zeroth member the error does not exceed 5–10 % in almost any combination of the values of the geometric parameters s_0 and permittivity ε_r of the toroid material. It remains below a specified upper limit, but tends to increase with increasing ε_r and to decrease with increasing s_0. Error exceeds this limit only for small values of $1< s_0 \leq 1.25$ of the toroids with a small centre hole diameter. In this case, the potential (17) is recommended to be used as an outcoming point to calculate the effective permittivity.

4. The Toroid Shape Permittivity

Eliminating in (20) dependence on εr by taking limit $\varepsilon_r \to \infty$, we obtain an expression for the toroid shape permittivity in the form:

$$\varepsilon_{\mathrm{f}} = \varepsilon_{\mathrm{eff}}\Big|_{\varepsilon_r \to \infty} = \frac{8}{\pi^2}(s_0^2 - 1)\sum_{\mathrm{n}=0}^{\infty} \delta_n (\mathrm{n}^2 - \frac{1}{4}) \frac{Q_{n-1/2}(s_0)}{P^1_{n-1/2}(s_0)} \tag{21}$$

The family of curves constructed in accordance with (20) is shown in Fig. 5. It depicts how the limit of effective permittivity $\varepsilon_{\mathrm{eff}}$ has been reached at different values of s_0. These curves allow characterising the gain in the value of the flow of electric induction through the central section of the toroid due to the properties of the dielectric, implemented for a given body size. On the other hand, it is possible to judge on the degree of utilisation of these properties.

They occur most fully at low permittivity material when $\varepsilon_{\mathrm{eff}}$ practically equals ε_{f} even at relatively low values of s_0. With an increase in ε_r, the relation $\varepsilon_{\mathrm{eff}}(\varepsilon_r)$ is becoming weaker. It is easy to establish that the spread of values ε_r that inevitably arises due to technological reasons affects the change in the value of ε_{eff}: the stronger it is, the more geometrical parameter s_0. Graph of function $\varepsilon_{\mathrm{eff}}(s_0)$, i.e., the maximum attainable value of the gain, is shown in Fig. 5 *b*. It allows for the specified toroid size finding such permittivity of the dielectric material, which having a further increase will not lead to an increase in the value of ε_{eff}.

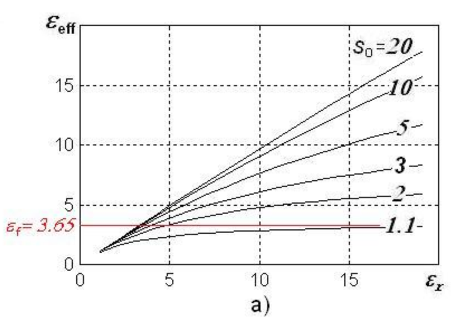

Fig. 5 a. The behaviour of function $\varepsilon_{\mathrm{eff}}(\varepsilon_r)$.

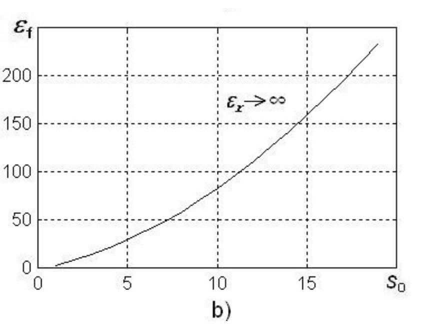

Fig. 5 b. The toroid shape permittivity.

From a physical point of view, limiting transition $\varepsilon_r \to \infty$ turns any dielectric material of toroid into a perfect metal, on the surface of which the tangential field components, i.e., E_φ and E_β, become zero. Thus, the structure of the field in the outer region will be determined by single component E_α = - gradF_2, which is normal to the surface of the body. Illustrations of the field structure near a perfectly conducting toroid for some particular cases can be found in [6].

5. CONCLUSIONS

The problem of the incidence of a plane electromagnetic wave to a toroidal dielectric body in many cases can be considered quasi-stationary. The corresponding scalar potentials obtained by solving the Laplace's equation are represented as the series containing the associated Legendre functions with half-integer indices. Assessment of convergence of the series shows that for almost all possible combinations of the geometric dimensions of the toroid and the dielectric constant of the material the series damps sufficiently quickly. With this, it is permissible in practical calculation to retain in the sum 4–5 members only. Certain exceptions are the toroids with small ($s_0 \rightarrow 1$) diameter of the central hole.

Distortion of a uniform external field caused by a body being placed in it is proposed to be estimated by the value of the effective permittivity of this body. It is equal to the gain in the value of the flow of electric induction vector through a certain section of the body that arises due to its dielectric properties. Necessary relations and appropriate ratio calculations for the central section of the toroid are deduced. The toroid shape permittivity is found. All calculations have been performed in the computing environment Matlab using specially designed programs.

It should be noted that the overall nature of behaviour of toroid permittivity on the size and properties of the material remains the same as for the bodies of other shapes, e.g., such as sphere and ellipsoid [7], [8]. This testifies to the proximity of the physical processes that occur under the influence of homogeneous external fields on bodies with different geometry. Common features are generated by the similarity of the charge and polarisation current distributions inside the body.

APPENDIX: ON DECOMPOSITION CONTAINING THE ASSOCIATED LEGENDRE FUNCTIONS

In many boundary value problems whose solution requires the use of toroidal coordinates, it is necessary to present the z-coordinates of points belonging to the boundary of the area under consideration as a series whose coefficients are expressed in terms of the associated Legendre functions.

Let us find this expansion. Using coupling equations (5) between Cartesian and toroidal coordinates one can write

$$z = \frac{c\sin\beta}{s-\cos\beta} = z'c(s-\cos\beta)^{1/2}, \tag{A1}$$

where

$$z' = \frac{\sin\beta}{(s-\cos\beta)^{3/2}}. \tag{A2}$$

Expand function ($A2$) in a Fourier along sine β in the interval [-π, π]:

$$z' = \sum_{n=0}^{\infty} k_n \sin n\beta, \tag{A3}$$

where

$$k_n = \frac{1}{\pi}\int_{-\pi}^{\pi} z' \sin n\beta d\beta$$

are coefficients to be determined. Thus, it is necessary to calculate the integral

$$I = \int_{-\pi}^{\pi} \frac{\sin n\beta \sin \beta d\beta}{\left(s-\cos\beta\right)^{3/2}} \tag{A4}$$

Writing the numerator of the integrand in the form

$$\sin n\beta \sin \beta = \frac{1}{2}[\cos(n-1)\beta - \cos(n+1)\beta]$$

and using the integral representation [5] to the associated Legendre function of the 2nd kind of the first order, we find

$$\int_{0}^{\pi} \frac{\cos m\beta d\beta}{\left(s-\cos\beta\right)^{3/2}} = -2\sqrt{2}\frac{Q^1_{m-1/2}}{\left(s^2-1\right)^{1/2}},$$

then

$$I = -\frac{2\sqrt{2}}{\left(s^2-1\right)^{1/2}}\left[Q^1_{n-3/2}(s) - Q^1_{n+1/2}(s)\right].$$

Using the recurrence formula [5]

$$Q^1_{\nu-1}(s) - Q^1_{\nu+1}(s) = -(2\nu+1)\left(s^2-1\right)^{1/2} Q_\nu(s)$$

in case of $\nu = n$-1/2, one can obtain instead of (A4)

$$I = 4\pi\sqrt{2} n Q_{n-1/2}(s). \tag{A5}$$

hence

$$k_n = \frac{4\sqrt{2}}{\pi} n Q_{n-1/2}(s).$$

Substituting this value of the coefficient in ($A3$) and back to ($A1$), we have the desired expansion

$$z = \frac{2\sqrt{2}}{\pi} c(s-\cos\beta)^{1/2} \sum_{n=0}^{\infty} n\delta_n Q_{n-1/2}(s) \sin n\beta. \tag{A6}$$

In the well-known monograph [4, v.2, ch. 10, example 10.37], formula (*A*6) is shown in the wrong way.

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NOVĒRTĒJUMS VIENDABĪGA ELEKTRISKĀ LAUKA IZKROPĻOJUMIEM, KURUS RADA TOROĪDA FORMAS DIELEKTRISKS ĶERMENIS

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K o p s a v i l k u m s

Darbā apskatīts kvazi-statisks tuvinājums viendabīga elektriskā lauka izkropļojumiem gadījumos, kad tajā tiek ievietots dielektrisks toroīda formas ķermenis. Izkropļojumu apmēru tiek piedāvāts novērtēt ar toroīda efektīvo caurlaidību, kas tiek noteikta, atrisinot atbilstošo robežvērtību uzdevumu. Tiek doti skaitliski novērtējumi, kas iegūti, lietojot speciāli valodā Matlab izstrādātu programmatūru.

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