LATVIAN JOURNAL OF PHYSICS AND TECHNICAL SCIENCES
2013, N 2
DOI: 10.2478/lpts-2013-0013

ATOMIC AND NUCLEAR PHYSICS

SOLITON MODEL OF THE PHOTON

I. Bersons

Institute of Atomic Physics and Spectroscopy, University of Latvia,
19 Raina Blvd., Riga, LV-1586, LATVIA
e-mail: bersons@latnet.lv

A three-dimensional soliton model of photon with corpuscular and wave properties is proposed. We consider the Maxwell equations and assume that light induces the polarization and magnetization of vacuum only along the direction of its propagation. The nonlinear equation constructed for the vector potential is similar to the generalized nonlinear Schrödinger equation and comprises a dimensionless constant μ that determines the size-scale of soliton and is expected to be small. The obtained one-soliton solution of the proposed nonlinear equation describes a three-dimensional object identified as photon.

Key words: *photon, soliton, nonlinear equations, Maxwell equations*.

1. INTRODUCTION

Since ancient times people have been interested in the nature of light, long before the corpuscular and wave interpretations were proposed in the 17th and 18th centuries. Strangely that Newton having observed a typical wave phenomenon now known as Newton's rings assumed the light to be a stream of particles. Many diffraction and interference phenomena of light observed after Newton established the dominance of the wave theory. Maxwell summed up all the previous experience with electricity and magnetism in a system of equations whereof it followed that light is an electromagnetic wave the electric and magnetic fields of which are perpendicular to the direction of wave propagation. In 1900 Max Planck introduced the concept of the light quantum to explain the distribution of energy in the black-body radiation. A new constant $\hbar$ – the Planck constant – became known. At a given frequency ω the smallest portion of energy transferred by light to matter is equal to $\hbar\omega$. In 1905 Einstein explained the photoelectric effect using the hypothesis of light quanta or photons. The Compton effect interpreted as collision between two particles, photon and electron, exchanging energy and momentum is a particular manifestation of the corpuscular nature of light.

New equations of a new mechanics – the Schrödinger and Dirac equations – were proposed and used successfully to describe the motion of the electron and the structure of atoms and molecules. Only in 1927 the quantization procedure for Maxwell equations by Paul Dirac [1] provided a logical foundation to the concept of photon. Employing a Fourier series for electromagnetic field in a box with periodic boundary conditions Dirac presented the energy of the field as a collection of energies of an infinite number of linear harmonic oscillators. After quantization of the harmonic oscillators the electromagnetic field becomes an operator acting on the occupation of the states of photons. The quantization procedure allowed all the processes of emission and absorption of photons to be explained, and a new branch of physics – the quantum optics – was born.

Nevertheless, many physicists are not satisfied with the present understanding of the nature of light. Indeed, the quantization procedure is very formal, and it is hard to connect infinite number of the harmonic oscillators with the properties and finite space of vacuum. The traditional questions are: what is a photon and where it is? To quest the nature and the location of a photon the international conferences [2, 3] have been organized.

In the present paper we seek answers to the questions on the basis of two physical statements. First, since the energy of a photon is finite and equal to $\hbar\omega$, the photon must occupy a finite space. Second, the photon propagates in vacuum without getting dispersed and remaining unchanged while traveling for millions of years. Thus, the equation describing the photon, if exists, must be a nonlinear one. The quantum mechanics is a linear theory the superposition principle for which is fulfilled. By introducing a nonlinear equation we are out of the traditional quantum mechanics, the superposition principle does not work.

A nonlinear system of the relativistic invariant Maxwell and Dirac equations might serve as a starting point of obtaining a nonlinear equation for the photon. This system is the basis of quantum electrodynamics, one of the most accurate physical theories. However, attempts to derive a nonlinear equation describing the propagation of photon from the Maxwell and Dirac equations have not been successful, maybe for a very simple reason: the Dirac equation depends on the mass m of the electron, or the Compton length $\hbar/(mc)$, which looks physically unrealistic – why should the nonlinear equation describing the propagation of photon depend on the mass of electron?

We consider the vacuum as a medium without electric charges and currents and focus on the Maxwell equations. Our essential proposal is that the vacuum is a medium where light induces the polarization and magnetization along the direction of the momentum of light. The procedure of making the Maxwell equations nonlinear and including the quantum properties of light looks rather artificial. The obtained nonlinear equation is similar to the generalized nonlinear Schrödinger equation derived earlier and has the one-soliton solution with physically reasonable properties.

2. MAXWELL EQUATIONS

The Maxwell equations [4] are:

$$\operatorname{rot}\mathbf{H} - \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} = 0, \tag{1}$$

$$\operatorname{rot}\mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = 0, \tag{2}$$

$$\operatorname{div}\mathbf{D} = 0, \tag{3}$$

$$\operatorname{div}\mathbf{B} = 0. \tag{4}$$

The following respective relations between electric displacement **D** and electric vector **E** and between magnetic induction **B** and magnetic vector **H** are usually used:

$$\mathbf{D}=\mathbf{E}+\mathbf{P}, \tag{5}$$

$$\mathbf{B}=\mathbf{H}+\mathbf{M}, \tag{6}$$

where $\mathbf{P}$ is the polarization vector,
$\mathbf{M}$ is the magnetization vector.

For simplicity, we omit factor 4π often used before **P** and **M** in Eqs. (5) and (6). The energy density w and momentum density **q** of the electromagnetic field are derived from Eqs. (1) and (2) [4]:

$$w=\frac{1}{8\pi}\left(\mathbf{E}\cdot\mathbf{D}+\mathbf{H}\cdot\mathbf{B}\right), \tag{7}$$

$$\mathbf{q}=\frac{1}{4\pi c}\left(\mathbf{E}\times\mathbf{H}\right). \tag{8}$$

Further, Eqs. (1)-(4) should be transformed with account for the properties of light observed experimentally. If light is propagating in the direction **n**, momentum **q** is oriented in the same direction. Choosing this direction along the *z*-axis implies $q_x=0$ and $q_y=0$. According to Eq. (8), this is possible under the condition that $E_z=H_z=0$, whereof it follows that light waves are transverse electromagnetic waves. Since light propagates with velocity *c*, all the vectors are functions of *x*, *y*, and $\xi=z-ct$. In a dispersive medium two velocities are distinguished: the phase and the group velocity. Assuming vacuum as a non-dispersive medium, the two velocities coincide. Then the *x*-component of Eq. (1) is:

$$\frac{\partial}{\partial\xi}\left(E_x+P_x-H_y\right)=0, \tag{9}$$

whereof $H_y=E_x+P_x$.

In a similar way, for the *y*-component of Eq. (1) and the *x*- and *y*-components of Eq. (2): $H_x=-E_y-P_y$, $M_x=P_y$, $M_y=-P_x$. These components of Eqs. (1) and (2) merely define simple relations between the *x*- and *y*-components of the vectors saying nothing about the dependence of these functions on x, y and ξ. Interesting that the *z*-component of Eq. (1) coincides with Eq. (3), and the *z*-component of Eq. (2) – with Eq. (4). As a result, the Maxwell equations are reduced to the two equations:

$$\frac{\partial}{\partial x}\left(E_x+P_x\right)+\frac{\partial}{\partial y}\left(E_y+P_y\right)+\frac{\partial P_z}{\partial\xi}=0, \tag{10}$$

$$\frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y}-\frac{\partial M_z}{\partial\xi}=0. \tag{11}$$

A charge moving in a medium induces electric polarization and magnetization, **P** being oriented along the direction of **E** while **M** – along the direction of **H**. Since there are no charges in vacuum, it seems reasonable to put P_x and P_y equal to zero. Then $D_x = E_x = H_y = B_y$ and $D_y = E_y = -H_x = -B_x$ as in the traditional description of electromagnetic field in vacuum, and only *z*-components of vectors **P** and **M** remain. In general, $\mathbf{P} = \mathbf{n}P$, $\mathbf{M} = \mathbf{n}M$, and we can consider the polarization and magnetization in vacuum as induced by the momentum of electromagnetic field. The field is depending on the transverse coordinates *x* and *y* only at nonzero P_z and M_z.

It seems that potentials $\mathbf{A}$ and Φ defined in [4] as

$$\mathbf{B} = \operatorname{rot}\mathbf{A}, \tag{12}$$

$$\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} - \operatorname{grad}\Phi \tag{13}$$

are more fundamental quantities in the quantum mechanics than vectors $\mathbf{E}$, $\mathbf{D}$, $\mathbf{H}$, and $\mathbf{B}$. Indeed, the interaction of particles with the electromagnetic field in Schrödinger's and Dirac's equations is defined by potentials. Describing photon as a quantum object, one may also use potentials and put $\Phi = 0$. In our coordinate system $A_z = 0$, $E_x = \partial A_x / \partial \xi$, $E_y = \partial A_y / \partial \xi$, therefore Eqs. (10) and (11) may be rewritten as

$$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + P_z = 0, \tag{14}$$

$$\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} + M_z = 0. \tag{15}$$

If $P_z = 0$, $M_z = 0$, the functions A_x and A_y do not depend on x and y and are arbitrary functions of ξ. Then, as usual, the vector potential of a monochromatic wave can be presented as

$$\mathbf{A} = \mathbf{e}\left(a e^{ik\xi} + a^* e^{-ik\xi}\right), \tag{16}$$

where **e** is the unit polarization vector,
a is the amplitude,
a^* is the complex conjugate value,
the wave vector $k = \omega / c$.

Quantization of the light field (see e.g. [5]) yields $a = Nb$, $a^* = Nb^+$, $N = \sqrt{2\pi\hbar c / kV}$, where b^+ and b are the creation and annihilation operators of photons, respectively, and *V* is the quantization volume.

3. SEARCH OF NONLINEAR EQUATION FOR THE PHOTON

Now the task is to modify Eqs. (14) and (15) to include the quantum properties of light and make them nonlinear. Actually, this means that P_z and M_z

should be presented as functions of A_x, A_y and their derivatives. Assuming that photon propagates along a straight line in the direction of *z*-axis it is natural to use cylindrical coordinates with polar radius ρ and polar angle φ defined by equations $x - x_0 = \rho\cos\phi$, $y - y_0 = \rho\sin\varphi$. Equation (16) well describes the emission and absorption of photons. Since function (16) does not depend on the polar angle φ, the functions A_x and A_y are axially symmetric depending only on variables ρ and ξ. However, this leads to a difficulty in Eqs. (14) and (15): the derivative with respect to x is proportional to $x - x_0$, and the derivative with respect to y is proportional to $y - y_0$. Thus, P_z and M_z must also contain such terms. We propose the following representation for P_z and M_z:

$$P_z = \frac{(\mathbf{r} - \mathbf{r}_0)}{\rho} \cdot T\mathbf{A} \ , \tag{17}$$

$$M_z = \frac{(\mathbf{r} - \mathbf{r}_0)}{\rho} \cdot \left(\mathbf{n} \times T\mathbf{A}\right), \tag{18}$$

where $\mathbf{n}$ is the unit vector in direction *z*,
T is a scalar operator applied to **A**.

Equations (14) and (15) can now be presented as

$$\frac{\partial A_x}{\partial \rho} + TA_x = 0 \ , \tag{19}$$

$$\frac{\partial A_y}{\partial \rho} + TA_y = 0 \, . \tag{20}$$

Further, supposing the light being linearly polarized along the *x*-axis when $A_y = 0$, we will start from Eq. (19). Since little is known about the structure and properties of vacuum, we are proceeding with some dimensional, mathematical and physical reasoning as follows. Each term of operator T is related to a dimensional or a dimensionless parameter. The purpose is to construct an equation of minimum unknown parameters. Operator T must comprise terms containing derivatives with respect to ξ, for example, $\gamma_2 \partial / \partial\xi$, $\gamma_3 \xi \partial^2 / \partial\xi^2$ or $l_1 \partial^2 / \partial\xi^2$. The dimension of operator T is the reciprocal distance, so parameters γ_2 and γ_3 are dimensionless while dimension of l_1 is the distance; therefore, it is not reasonable to include the last term in the equation. The nonlinear Schrödinger equation containing a term of $|E_x|^2$ is very popular in the nonlinear optics [6]. We choose two nonlinear terms $\xi|A_x|^2$ and $\int |A_x|^2 \mathrm{d}\xi$ having the dimension of energy. Multiplying derivative terms by $\hbar c$, we obtain the equation:

$$\hbar c \gamma_1 \frac{\partial A_x}{\partial \rho} + \hbar c \gamma_2 \frac{\partial A_x}{\partial \xi} + \hbar c \gamma_3 \xi \frac{\partial^2 A_x}{\partial \xi^2} + \gamma_4 \xi |A_x|^2 A_x + \gamma_5 A_x \int_{-\infty}^{\xi} |A_x|^2 \mathrm{d}\xi' = 0 \, , \tag{21}$$

where γ_i are dimensionless parameters.

Most of the quantum mechanical equations are written in energy representation: the operators of kinetic and potential energy are applied to the wave function. The Dirac equation contains operators such as $\hbar c\partial/\partial x$. So, Eq. (21) looks like a quantum mechanical equation but nonlinear. By inserting $A_x = \sqrt{\hbar c}F$ and choosing $\gamma_1 = i$, $\gamma_2 = 2\mu$, $\gamma_3 = \mu$, $\gamma_4 = \gamma_5 = 2\mu$, Eq. (21) is reduced to a generalized nonlinear Schrödinger equation (GNLSE) [7–10]:

$$i\frac{\partial F}{\partial \rho} + 2\mu\left[\frac{\partial F}{\partial \xi} + \frac{\xi}{2}\frac{\partial^2 F}{\partial \xi^2} + \xi|F|^2 F + F\int_{-\infty}^{\xi}|F|^2 \, \mathrm{d}\xi'\right] = 0 \ . \tag{22}$$

The GNLSE is equivalent to the equation of spin evolution of the Heisenberg spin chain with inhomogeneities at the continuum limit [7–10]. The difference between Eq. (22) and the earlier derived equation for the dynamics of Heisenberg's spin system is the derivative $\partial F/\partial\rho$ in Eq. (22) instead of derivative $\partial F/\partial t$ with respect to time in the spin system. As seen from Eq. (22), parameter μ determines the scale of polar coordinate ρ. If $\mu = 0$, then F does not depend on ρ, and for a monochromatic wave $F = \exp(ik\xi)$. We suppose Eq. (22) to be suitable for describing the photon.

4. ONE-SOLITON SOLUTION

Equation (22) has been shown to be integrable [10]. Its one-soliton solution can be presented as

$$F = kb(\rho)\operatorname{sech}\left[k\xi b(\rho)\right]\exp\left[i\theta(\rho,\xi)\right] , \tag{23}$$

where

$$\xi = z - ct , \tag{24}$$

$$\rho = \sqrt{(x - x_0)^2 + (y - y_0)^2} , \tag{25}$$

$$d = \frac{s^2}{(s^2 + 1)} , \tag{26}$$

$$a(\rho) = \frac{d + \mu k\rho}{(d + \mu k\rho)^2 + d^2/s^2} , \tag{27}$$

$$b(\rho) = \frac{d/s}{(d + \mu k\rho)^2 + d^2/s^2} , \tag{28}$$

$$\theta(\rho,\xi) = k\xi a(\rho) + 2\arctan\left[s(1 + \mu k\rho/d)\right] . \tag{29}$$

Here $k = 2\pi/\lambda$ is the wave vector,
λ is the wavelength, and
s is a dimensionless parameter.

In the one-soliton solution of Eq. (22) derived in [10] the factor with arctan in the phase is absent. We suppose that one-soliton solution (23) describes a linearly polarized photon of frequency $\omega = kc$ propagating along the *z*-axis.

The electric field of the photon is $E_x = \sqrt{\hbar c}\partial F / \partial \xi$ and the energy density (7) is equal to $|E_x|^2 / 4\pi$. Full energy W of the photon is equal to the integral of this density over the whole space:

$$W = \frac{\hbar c}{4\pi} \int_0^{2\pi} \mathrm{d}\phi \int_0^{\infty} \rho \mathrm{d}\rho \int_{-\infty}^{\infty} \left| \frac{\partial F}{\partial \xi} \right|^2 \mathrm{dz} . \qquad (30)$$

Evaluating the integrals and equating W to the photon energy $\hbar\omega$, we obtain the equation:

$$\frac{4\mu^2 s}{s^2+1} = 1 + \frac{1}{3(s^2+1)} - \frac{s\pi}{2} + s \arctan s , \qquad (31)$$

which connects two dimensionless parameters s and μ. If μ is small, then s is large and approximately equal to $1/\left(6\mu^2\right)$.

As follows from Eqs. (27) and (28), $a(\rho = 0) = 1$ and $b(\rho = 0) = 1/s$. The scale of coordinates ρ and $\xi = z - ct$ is determined by wavelength $\lambda = 2\pi / k$. For small μ the transverse size of photon is by factor μ^{-1} larger than λ, the longitudinal size surpassing λ by factor μ^{-2}. Therefore, the region around the center of the photon where vector potential A_x is proportional to the classical value $\exp(ik\xi)$ is large enough. The vector potential decreases exponentially at $\xi \to \pm\infty$ and decreases as ρ^{-2} with increasing ρ. The frequency of oscillation of the field is $\omega a(\rho)$ and decreases from its maximum value ω on the symmetry axis to zero at $\rho \to \infty$.

For arbitrary polarization of the photon, both Eqs. (19) and (20) must be taken into account. Defining

$$A_x = \sqrt{\hbar c}\, c_1 F \exp(i\delta_1), \quad A_y = \sqrt{\hbar c}\, c_2 F \exp(i\delta_2) , \qquad (32)$$

where c_i and δ_i are real constants and $c_1^2 + c_2^2 = 1$, we obtain that $|\mathbf{A}|^2 = \hbar c |F|^2$, where function F is defined by Eq. (23).

If the photon propagates in an arbitrary direction $\mathbf{k}(k_x, k_y, k_z)$, the vector potential of the photon is $\sqrt{\hbar c}\,\mathbf{e}F$, where the unit vector $\mathbf{e}$ is orthogonal to $\mathbf{k}$ direction: $\mathbf{e} \cdot \mathbf{k} = 0$. Function F of the one-soliton solution is given by expression (23) where $\mathbf{k} \cdot \mathbf{r} - \omega t$ stands instead of $kz - \omega t$ and $\sqrt{\mathbf{k}^2 (\mathbf{r} - \mathbf{r}_0)^2 - (\mathbf{k} \cdot \mathbf{r} - \mathbf{k} \cdot \mathbf{r}_0)^2}$ – instead of $k\rho$.

5. CONCLUSIONS

The proposed nonlinear equation for the vector potential of collinearly propagating photons is similar to the generalized nonlinear Schrodinger equation (GNLSE). The one-soliton solution of this equation describes a three-dimensional object with corpuscular and wave properties that could be identified as photon. The cubic terms of A_x in Eq. (21) are responsible for the calculated energy of the photon proportional to $\hbar\omega$. A problem remains with the unknown constant μ determining the size-scale of photon. The constant is supposed to be small; theoretical arguments or experimental possibilities to evaluate it are not easy to find. Probably, this constant is proportional to the fine-structure constant or its power.

The proposed model is only a model. It seems to be of importance here to find a two-photon solution of the generalized nonlinear Schrödinger equation (GNLSE).

ACKNOWLEDGEMENT

The reported study has been supported by the Latvian Science Council (Grant No. 10-5/116) and Project No. 2009/0210/1DP/1.1.1.2.0/09/APIA/VIA/100 financed by the European Social Fund (ESF).

REFERENCES

1. Dirac, P.A. (1927). *Proc. Roy. Soc., A 114*, 243.
2. Roychoudhuri, C., & Roy, R. (Eds.) (2003). The Nature of Light: What is a Photon? *Special issue of Optics and Photonics News.*
3. Roychoudhuri, C., Creath, K., & Kracklauer, A. (Eds.) (2005). The Nature of Light: What is a Photon? *Proceedings of SPIE, 5866.* SPIE (Bellingham, WA).
4. Born, M., & Wolf, E. (1999). *Principles of Optics.* Cambridge: Cambridge University Press.
5. Davydov, A.S. (1965). *Quantum Mechanics.* Oxford: Pergamon Press.
6. Kivshar, Y.S., & Agrawal, G.P. (2003). *Optical Solitons.* San Diego: Academic Press.
7. Calogero, F., & Degasperis, A. (1978). *Lett. Nuovo Cimento, 22,* 420.
8. Lakshmanan, M., & Bulloush, R.K. (1980). *Phys. Lett., 80A*, 287.
9. Balakrishnan, R. (1982). *J. Phys. C: Solid State Phys., 15,* L1305.
10. Porsezian, K., & Lakshmanan, M. (1991). *J. Math. Phys., 32,* 2923.

FOTONA SOLITONA MODELIS

I. Bērsons

K o p s a v i l k u m s

Fotona aprakstam tiek piedāvāts trīsdimensiju solitona modelis ar daļiņas un viļņa īpašībām. Tiek apskatīti Maksvela vienādojumi un pieņemts, ka gaisma inducē vakuuma polarizāciju un magnetizāciju tikai gaismas izplatīšanās virzienā. Konstruētais nelineārais vienādojums vektora potenciālam ir līdzīgs vispārinātajam nelineārajam Šrēdingera vienādojumam un satur bezdimensionālu konstanti μ, kura nosaka solitona izmērus un kura varētu būt maza. Atrastais vienādojuma vien-solitona atrisinājums tiek identificēts kā fotons.

28.11.2012.