LATVIAN JOURNAL OF PHYSICS AND TECHNICAL SCIENCES
2013, N 2
DOI: 10.2478/lpts-2013-0010

CROSS-SECTIONAL TEMPERATURE FIELD OF A SOLAR COLLECTOR'S ABSORBER: DEVELOPMENT OF THE MODEL

V. Barkans[1], P. Shipkovs[2], M. Vanags[2]

[1]Latvian Maritime Academy,
5B Flotes Str., Riga, LATVIA
voldemars.barkans@inbox.lv

[2]Institute of Physical Energetics,
21 Aizkraukles Str., Riga, LATVIA
shipkovs@edi.lv

In the work, the temperature field model is developed for the absorber of a round-pipe collector. As distinguished from previous models when the temperature of liquid was assumed to be constant over the entire pipe cross-section, the results obtained clearly show the temperature variations in the absorber's cross-section. In the work, optimal values are found in the work for geometrical parameters of the collector (i.e. the plate thickness and the pipe diameter) that allow the highest possible temperature of liquid to be achieved.

Key words: *solar collector, absorber, temperature field*.

1. INTRODUCTION

Solar collectors are finding ever increasing use in households for hot water preparation. From diversified solar thermal collectors those of flat-plate absorber type are simple in design and maintenance, at the same time being relatively cheap. However, while the temperature field has been calculated for different absorbers of the type (see, e.g. [1–4]), no relevant data are found for the conventional round-pipe absorber. Therefore, over many years we have concentrated attention on modelling the cross-sectional temperature field for the round-pipe absorbers of solar collectors [5–11].

In solving the cross-sectional temperature field for a round-pipe absorber [5], the periodical cross-sectional domain is divided into three sub-domains where the first sub-domain is the plate between pipes, the second – a pipe's wall, and the third – the liquid; as a result, the temperature field expressions have been obtained for all the three sub-domains. In works [6, 7] the temperature field obtained was simplified. To define its variations in time, such a field was found by solving the Laplace equation under the non-stationary time-dependent conditions [8]. The obtained results evidence that the non-stationarity might not be taken into account in long-lasting sunny weather, while it affects considerably the temperature field in a short term (e.g. in cloudy weather). The temperature field has also been found for a square-pipe absorber [9] as more technological in design.

In the present work, the temperature field model is proposed for the absorber of a round-pipe collector. As distinguished from [5], in this work the temperature of liquid is assumed to be constant over the entire pipe cross-section. Such an assumption significantly simplifies the calculation while not changing the physical essence.

2. FORMULATION OF THE PROBLEM AND ITS SOLUTION

The temperature field is sought for the absorber shape shown in Fig. 1.

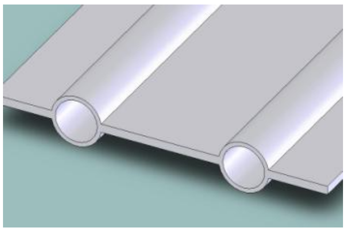

Fig. 1. A conventional absorber.

Assuming that the process is stationary, the temperature field is described by the Laplace equation [12-15]:

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=0\,. \tag{1}$$

To solve this equation, the boundary conditions are set for the collector absorber's cross-section possessing periodicity. The periodical domain is divided into two parts, D1 and D2. In part D2 the Laplace equation is written in polar coordinates.

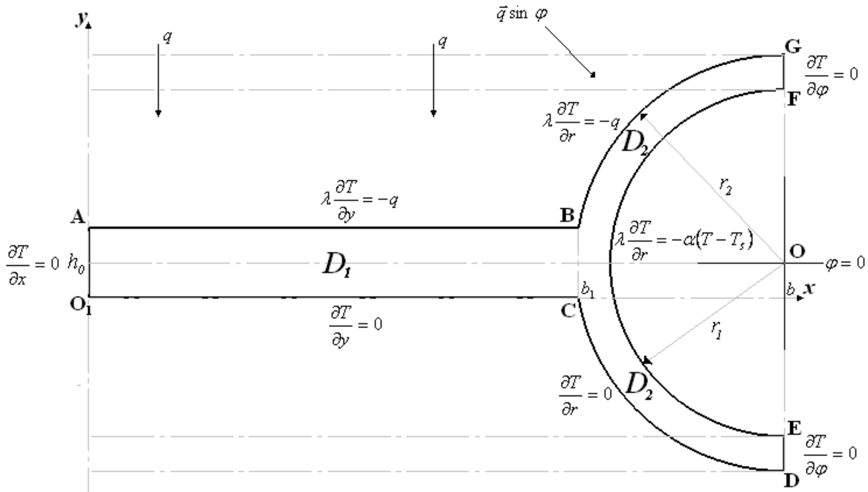

Fig. 2. Absorber cross-sections with the set boundary conditions.

On the Sun-oriented surface the incidence of solar radiation is perpendicular to all its points except the pipe surface, for which the perpendicular radiation component is calculated. The area of periodical cross-section and the boundary conditions are shown in Fig. 2. The bottom part is isolated, therefore, the heat flow perpendicular to the isolated surface is zero. The same situation is for the symmetry axes AO_1, GF and DE. On the Sun-facing surface the heat flow is proportional to the solar radiation density. In turn, on the surface where the cooling liquid meets the internal wall of a pipe the heat flow is described by a conventional heat flow equation where T_s is the liquid temperature and T is the temperature of a pipe's inside wall.

To simplify the analytical solution of Laplace's equation the dimensionless parameters are used [8-11]:

$$\xi = \frac{x}{b}; \quad \xi_1 = \frac{b_1}{b}; \quad \xi_2 = \frac{b_2}{b}; \quad \begin{pmatrix} 0 \le x \le b; \\ 0 \le \xi \le 1; \end{pmatrix} \tag{2}$$

$$\eta = \frac{y}{b}; \quad \eta_0 = \frac{h_0}{b}; \quad \begin{pmatrix} -r_2 \le y \le r_2; \\ -\rho_2 \le \eta \le \rho_2; \end{pmatrix} \tag{3}$$

$$\rho = \frac{r}{b}; \quad \rho_1 = \frac{r_1}{b}; \quad \rho_2 = \frac{r_2}{b}; \quad \begin{pmatrix} r_1 \le y \le r_2; \\ \rho_1 \le \rho \le \rho_2; \end{pmatrix} \tag{4}$$

The Q and Bi parameters are defined by the formulas:

$$Q = \frac{q}{\lambda}\frac{b}{T_0}; \quad Bi = \frac{\alpha}{\lambda} b. \tag{5}$$

The dimensionless temperatures are written as

$$\Theta_1 = \frac{T_1}{T_0}; \quad \Theta_2 = \frac{\Delta T_2}{T_0}, \tag{6}$$

where

$$\Delta T_2 = T_2 - T_s. \tag{7}$$

The designations in the formulas and figures describing the temperature field of the absorber are as follows.

$2b$ is the distance between the pipe axes, m;
$2h_0$ is the thickness of the absorber's plate, m;
r_1 is the internal radius of the pipe, m;
r_2 is the external radius of the pipe, m;
q is the solar radiation density, W/m^2;
λ is the thermal conductivity of the absorber, W/m K;
α is the coefficient of convective heat transfer from the surface, W/m^{2}K;
T_s is the temperature of liquid, K;
T_0 is the initial temperature of liquid, K.

Having written the boundary conditions in dimensionless parameters and solving the Laplace equation, we obtain the following expression for the temperature field in domain D1:

$$\Theta_1(\xi,\eta)=\frac{Q}{4\eta_0}\left(\eta^2-\xi^2\right)+F\ , \tag{8}$$

where

$$F=1-Q\eta_0 . \tag{9}$$

The temperature field in domain D2 is sought-for in polar coordinates, with its dimensionless form written as

$$\Theta_2(\rho,\phi)=\frac{Q}{\pi}\left(\frac{1}{Bi\rho_1}+\ln\frac{\rho}{\rho_1}\right)-\frac{Q}{\pi\eta_0}\sum_{k=1}^{\infty}\frac{\rho}{k}\left(\frac{\rho}{\rho_2}\right)^{k-1}\cdot$$

$$\frac{1+\left(\frac{\rho_1}{\rho}\right)^{2k}\frac{k-Bi\rho_1}{k+Bi\rho_1}}{1-\left(\frac{\rho_1}{\rho_2}\right)^{2k}\frac{k-Bi\rho_1}{k+Bi\rho_1}}\cos\frac{k\pi}{2}\cdot$$

$$\cdot\left\{\eta_0\left[\frac{\cos(k-1)\phi_2}{k-1}-\frac{\cos(k+1)\phi_2}{k+1}\right]-\left[\frac{\sin(k+1)\phi_2}{k+1}+\frac{\sin(k-1)\phi_2}{k-1}\right]+\right.$$

$$\left.+\rho_2\left[\frac{\sin(k+2)\phi_2}{k+2}+\frac{\sin(k-2)\phi_2}{k-2}\right]\right\}\cos k\left(\varphi-\frac{\pi}{2}\right), \tag{10}$$

where $\left(\rho_1\le\rho\le\rho_2;\quad \frac{\pi}{2}\le\phi\le\frac{3\pi}{2}\right)$

3. RESULTS AND DISCUSSION

Figure 3 shows the temperature field on the collector surface with the parameters given in Table 1 for different *b* (i.e. distance from the plate centre to the pipe centre). The temperature in the direction from the plate centre to the pipe is decreasing – as might be expected since the liquid flowing through the pipe has a lower temperature than in the plate, and this difference creates a temperature gradient in the direction of which the heat is flowing.

Table 1

The output data for the model

Plate thickness, h, m	Pipe external diameter, m	Pipe internal diameter, m	Solar radiation density, q, W/m^2	Plate thermal conductivity, λ, W/m K	Convective heat transfer, α, W/m^2 K	Initial temperature, K
0.001	0.012	0.01	1000	385	1000	373

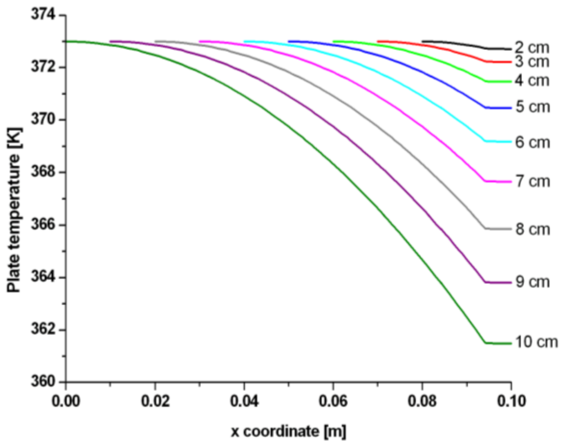

Fig. 3. Temperature field at different *b* values.

Figure 4 shows the temperature field in the pipe coating at variable *b* value. As compared with temperature variation in the plate (Fig. 3), in the coating it is much less, but in any case at increasing *b* the temperature difference between the top of pipe and the point of its contact with the plate increases.

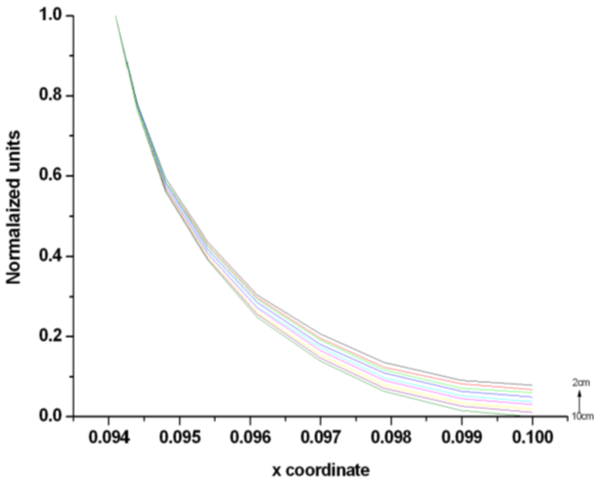

Fig. 4. Temperature variations on the coating surface at different *b* values.

In Fig. 5 the temperature of liquid is shown in dependence on *b* value for different outside diameters of the pipe and a constant thickness of its wall (1 mm). At the same pipe diameter, with *b* increasing the temperature of liquid decreases. In physical terms this is explainable with the fact that the temperature gradient should be in the whole cross-section periodical system as implied by the mathematical expressions obtained for the temperature field. If the distance to the lowest temperature point is increasing, the difference between the initial temperature and the lowest one (the temperature of liquid in the given model) also increases.

It is seen that at increasing pipe diameter the difference (as compared with that for the previous sizes) becomes smaller, which points to a critical value for the diameter after which it is of no use to increase this value.

Dependence of the liquid temperature on the pipe diameter at a constant *b* value (0.07 m) is illustrated in Fig. 6, where the curve has a saturated character. It could be seen that at the diameter value of 12 mm the liquid temperature is changing but slightly with increasing diameter. Therefore, this curve shows the way of optimizing the pipe diameter.

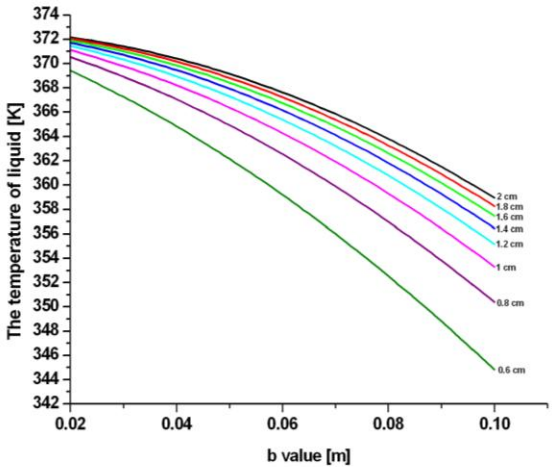

Fig. 5. The temperature of liquid *vs*. *b* value at different pipe diameters (the wall thickness is 1 mm).

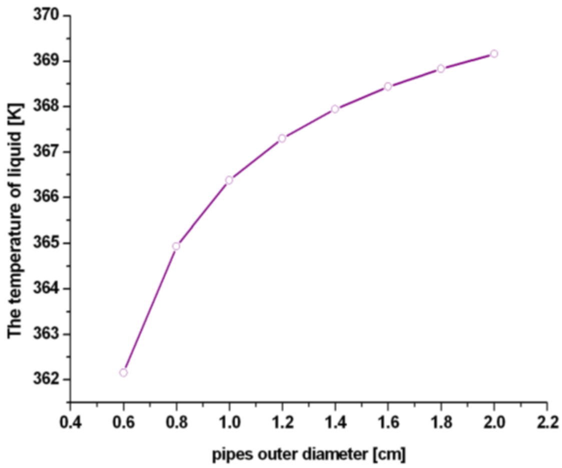

Fig. 6. The liquid temperature *vs.* pipe diameter.

The dependence on *b* value at different plate thicknesses is shown in Fig. 7 (the initial parameters are as given in Table 1, the pipe diameter is 12 mm, the pipe wall thickness is 1 mm). At increasing plate thickness the dependence of liquid temperature on the initial temperature weakens in the same manner as it is for the case with increasing diameter (see Fig. 5).

The curve shown in Fig. 8 also possesses a saturated character, which evidences that there exists a threshold value of the plate thickness after which its increasing is of no sense.

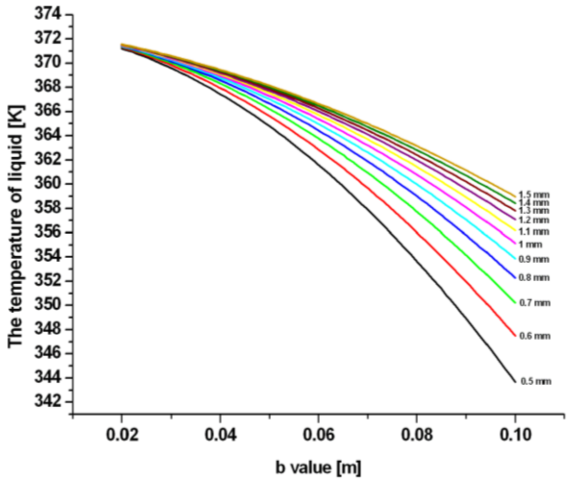

Fig. 7. The liquid temperature *vs*. *b* value for different plate thicknesses (*h*).

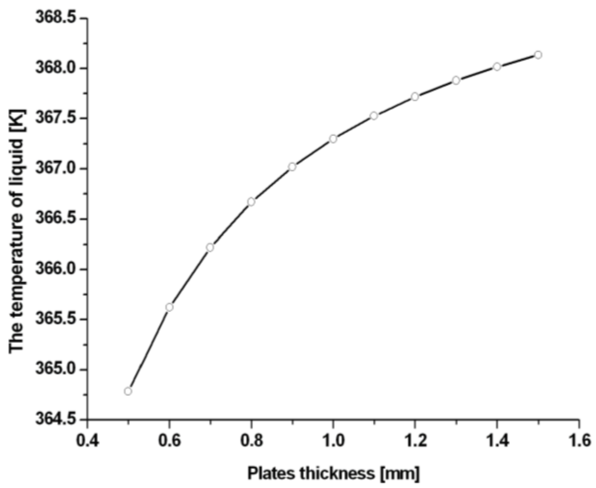

Fig. 8. Liquid temperature *vs*. plate thickness (*h*).

From the above it follows that a compromise is to be found for the difference of liquid temperature from the initial. If this difference is large (i.e. *b* value is large, with the pipe diameter and the plate thickness being small), the heat flow from the plate centre to the pipe will be large, since this flow is directly proportional to the temperature gradient. In turn, if this difference is small (i.e. *b* value is small while the pipe diameter and the plate thickness are large), the heat flow will be weaker. In any case, of importance is to achieve that the heat flow be large; at the same time, if *b* value is large and the pipe diameter is small, the amount of liquid per area unit will also be small, which would mean less per area unit power for the solar collector. Clear enough that the power decrease will be compensated at some time moment by a heat flow increase.

The proposed mathematical model shows that in all the cases there exist threshold values for the plate thickness and the pipe diameter – i.e. some optimal values that are not to be increased further.

4. CONCLUSIONS

In the work, the cross-sectional temperature field has been obtained for the solar collector's absorber under the assumption that the heat flow is stationary. The results clearly show the temperature variations in the absorber's cross-section.

The dependence of liquid temperature on different geometrical parameters points to the existence of their optimal values.

Under the conditions when solar radiation is strong and constant for a long time, it is expedient to make the collector's absorber with a small distance between the plate and pipe centres, with a large pipe diameter (up to 14 mm) and a large plate thickness (up to 1.2 mm); at the same time, when cloudiness dominates, it would be better to design the solar collector with a greater *b*, a smaller pipe diameter (from 6 to 8 mm) and a small plate thickness (0.5 mm).

ACKNOWLEDGEMENT

This work has been supported by the National Research Programme 2010–2013 "Technologies for Innovative Production and Use of Energy Resources and Provision of Low Carbon Emissions by Means of Renewable Energy Sources, Support Measure for the Mitigation of Environment and Climate Degradation – LATENERGI"

REFERENCES

1. Patancar, S.V., & Spalding, D.B. (1972). A calculation procedure for heat, mass and momentum transfer in three-dimensional parabolic flows. *International journal of heat and mass transfer*, *15*, 1787.
2. Sopian, K., Syahri, M., Abdullah, S., Othman, M.Y., & Yatim, B. (2004). Performance of a non-metallic unglazed solar water heater with integrated storage system. *Renewable Energy, 29*, 1421–1430.
3. Gorla, R.S.R. (1997). Finite element analysis of a flat plate solar collector. *Finite Elements in Analysis and Design*, *24*, 283–290.
4. Turgut, O., & Onur, N. (2009). Three-dimensional numerical and experimental study of forced convection heat transfer on solar collector surface. *International Communication in Heat and Mass Transfer, 36,* 274–279.
5. Shipkovs, P., Barkans, V., Temkins, A., Shipkovs, J., Lebedeva, K., & Vanags, M. (2005). Heat conduction process on plane surfaces of a solar collector's absorber: the mathematical description. *Latv. J. Phys. Tech. Sci*., *6*, 3–15.
6. Shipkovs, P., Kashkarova, G., Lebedeva, K., Shipkovs, J., Vanags, M., & Leitans, K. (2006). The mathematical description for heat conduction process on the surface of a solar collector's absorber. *Proceedings of World Renewable Energy Congress IX*, 19-25 August Florence (Italy). CD Full Proceedings 2006, p. 5.
7. Barkans, V., Temkins, A., Shipkovs, P., Kashkarova, G., Vanags, M., Lebedeva, K., & Shipkovs, J. (2006). Mathematical modeling of plane surface solar collectors. *Proceedings of LMA 8th International Conf. „Maritime transport and infrastructure – 2006"*, 130 p.
8. Barkans, V., Temkins, A., Shipkovs, P., & Vanags, M. (2007). Nonstationary heat conduction process on a solar collector. *Proceedings of LMA 9th International Conf. „Maritime transport and infrastructure – 2007"*, 214 p.
9. Shipkovs, P., Vanags, M., Lebedeva, K., Kashkarova, G., Shipkovs, J., & Barkans, V. (2008). Modeling of a solar collector's absorber. *Proceedings of the 2nd International*

Association of Science and Technology for Development of Power and Energy Systems (African PES), 8–10 Sept., Gaborone (Bostwana).
10. Shipkovs, P., Vanags, M., Lebedeva, K., Kashkarova, G., Shipkovs, J., Barkans, V., & Jirgens, M. (2008). Optimization of the Solar Collectors Absorbers Parameters. *The 10th World Renewable Energy Congress – WREC X*, 19-25 July, Glasgow, Scotland, CD Proceedings, pp. 187–1873.
11. Shipkov, P., Barkans, V., Vanags, M., & Vasilevska, L. (2010). A mathematical model for the temperature field in the absorber of a new type solar collector. *Latv. J. Phys. Tech. Sci*., *47* (4), 44–50.
12. Riekstiņš E. (1969). *Matemātiskās fizikas metodes*. Rīga: Zvaigzne, 620 lpp.
13. Кошляков, Н.С, Глинер, Э.Б., & Смирнов, М.М.. (1962). *Основные дифференциальные уравнения математической физики*. Москва: Физматгиз, 768 стр.
14. Тихонов, Н., & Самарский, А. А. (1956). *Уравнения математической физики*. Москва: Наука, 724 стр.
15. Прудников, А.П., Бычков, Ю.А., & Маричев, О.И. (1983). *Интегралы и ряды. Специальные функции.* Москва: Наука, 750 стр.

SAULES KOLEKTORA ABSORBERA ŠĶĒRSGRIEZUMA TEMPERATŪRAS LAUKS APAĻAS CAURULĪTES GADĪJUMĀ – FINĀLA MODELIS

V. Barkans, P. Šipkovs, M. Vanags

K o p s a v i l k u m s

Darbā iegūts temperatūras lauks saules kolektora absorbera šķērsgriezumam, uzskatot, ka siltumplūsmas process ir stacionārs. Rezultāti labi parāda temperatūras izmaiņu absorbera šķēsgriezumā.

Šķidruma temperatūras atkarība no dažādiem ģeometriskajiem parametriem liecina, ka tiem eksistē optimālas vērtības, kuras nav jēgas palielināt, jo tas neatstāj ietekmi uz šķidruma temperatūru.

Apstākļos, kuros saules radiācija ir liela un konstanta ilgu laika periodu, kolektora absorbers būtu jātaisa ar mazu *b* vērtību (attālumu no plates centra līdz caurulītes centram), lielu caurulītes diametru (līdz 14 mm) un lielu plates biezumu (līdz 1,2 mm). Apstākļos, kuros dominē mākoņainība, labāk konstruēt saules kolektoru ar lielāku *b* vērtību, mazu caurulītes diametru (no 6 līdz 8 mm) un mazu plates biezumu (0,5 mm).

29.01.2013.