# NOMINAL VOLTAGES OF THE NETWORKS 

J. Survilo<br>Riga Technical University, 1 Kronvalda Blvd., Riga, LV-1010, LATVIA


#### Abstract

A great diversity of the nominal voltages in power networks can be explained based on the sizes and shape of the fed zone. To find the reason of such diversity, some assumptions should be made: the variation in parameters is continuous, the voltage is constant, the electricity consumers are spread uniformly, etc. To pass from continuous load to its discrete location, a discreteness factor is calculated for various shapes of the fed zone. The expression obtained for specific costs of the network shows that they decrease in the following cases: with voltage increasing as far as the costs of line construction are not rising too rapidly; with the load density and specific conductance of phase conductor increasing; when the consumers are located more sparsely. To reduce specific costs, an appropriate current density in phase conductors should be chosen. In the work it is shown that the choice of optimal sizes for the fed zone should be made with due consideration for the step-down transformer.


Key words: electrical network, power losses, nominal voltage, specific costs, supply zone.

## 1. INTRODUCTION

In power systems, the low-, medium-, high-, and extra-high-voltage networks are met. Nominal voltages of a transmission line (TL) are usually 110 kV and above; at a sub-transmission level they could be 66 or 33 kV , distribution network voltages - below 33 kV , while extra high are $230-765$ and up to 1200 kV [1]. Worldwide, the power systems use diversified nominal voltages. For example, in Russia the voltages of $220 ; 380 ; 660 \mathrm{~V} ; 3 ; 6 ; 10 ; 20 ; 35 ; 110 ; 220 ; 330 ; 500$; 750; 1150 kV are adopted [2-4]; in European countries: Italy - 150; 132; 20; 15; 0.4 kV ; Germany - 380; 220; 11; 20; 16; 10; $0.4 \mathrm{kV} ; \mathrm{UK}-33 ; 22 ; 11 ; 6.6 ; 0.4$; 0.23 kV ; Spain - 400; 220; 132; 66; 45; 30; 20; 15; 13; <1 kV; Poland - 400; 220; 110; 20; 15; 0.4 kV [5]; and Latvia - 380; 110; 20; 10; 6; 0.4 kV [6]. These nominal voltage values are seen as self-evident - it is implied that with them the delivery of electricity is the most profitable. Probably, there are sound reasons behind this situation; however, it allows posing some questions, e.g.: how can the most appropriate nominal voltage be chosen in particular conditions? What parameters of the network affect stronger the nominal voltage? Why are there so diversified voltages in use?

In practice of the network exploitation, a compromise is usually found between the number of nominal voltages and the most advantageous nominal voltage in every specific case. This problem is given attention also in the theoretical plane [7].

To create an appropriate mathematical model, it is necessary to make some presumptions: the capacity of transformers and the cross-section of conductors can
vary gradually; the load density is continuously distributed over the entire fed zone and is constant within it; the voltage values do not change within the zone; the simultaneity factor is equal to unity (i.e. the loads of the entire zone are changing simultaneously); the transformer capacity is equal to the maximum load.

The effect of other factors can be considered separately for every real case.
The paper is essentially a framework within which, based on the results of study, it would be possible to make improvements and useful changes in each specific case.

## 2. STATEMENT OF THE PROBLEM

For electrical network calculations the consumers' initial data are required: the power, the maximum load time, $\cos \varphi$, and some others - for example, their location. This means that for such calculations a large body of information is needed; besides, the problem can be complicated by frequent variations in the data. Whereas this last drawback is corrigible at the modern level of computing technique, such a situation does not allow the mutual relations of influencing factors to be found. To change it, some averaged indices pertaining to the consumers and their location should be taken into consideration. These indices are: the load density $\sigma$ and the consumers' spacing. The load density and ratings of current equipment are theoretically considered continuous; the spacing of consumers is also assumed to be continuous. This makes it possible to use analytical dependences. In order to take into account a discrete location of consumers, the quantity $A_{c}$ obtained for their continuous location should be multiplied by discreteness factor $k_{d}$ :

$$
\begin{equation*}
A_{d}=k_{d} A_{c} \tag{1}
\end{equation*}
$$

where $A_{d}$ is the sought-for quantity at an idealized discrete location of consumers.
The performance of a network depends on its configuration and on how the phase conductor is used (with a constant current density or a constant crosssection). We shall consider the following parameters: line length $l, \mathrm{~m}$; product $l F$, $\mathrm{m}^{3}$ ( $F$ being the cross-section); power loss $\Delta P$, W in the phase conductors of a power line; the real $U_{\Delta}$ and imaginary $U_{\delta}$ components of the voltage drop across the line, $V$ (in the modelled specifically shaped supply zones). These parameters are considered for continuous load location. To pass to the discrete load case (denoted by index ${ }_{d}$ ) according to Eq. (1), the discreteness factor $k_{d}$ is determined for each quantity specified above. In the consideration, power loss $\Delta P$ is calculated for one phase. In the tables below the power loss $\Delta P$ is given for a three-phase network (i.e. three times $\Delta P_{c}$ ).

## 3. SPATIAL MODELS OF CONSUMER LOCATION

Consideration of the kind is based on the shape of the fed zone. While the most convenient fed zone shape is hexagon [8], in the present paper a square is taken in order to simplify mathematical description. To pass to a hexagon the similarity factors can be employed. Besides, owing to territorial constraints which may arise in practice, some other shapes are considered.


Fig. 1. A strip-shaped fed zone (half area).

First, we will study the fed zone shaped as a strip (further denoted S) (Fig. 1). The loads are located at distinct points connected by a branch line; the supply source is located at the connecting point. In the fed zone, one consumer is located at the connecting point and other $n$ consumers are above and below the centre. They are supplied through the branch line. The central consumer is not taken into account since it is not fed through the line. Hence, the number of consumers $n$ and the distance between consumers $\Delta y$ in the strip zone are:

$$
\begin{equation*}
n=\frac{y_{m}-\Delta y / 2}{\Delta y} ; \quad \Delta y=\frac{2 y_{m}}{2 n+1} . \tag{2}
\end{equation*}
$$

We will now determine the basic parameters of $\mathbf{S}$ model.

## Determination of the branch line length

At continuous load distribution the branch line length is taken from the supplying centre to the upper and lower boundaries of a zone:

$$
\begin{equation*}
l_{c}=2 y_{m} . \tag{3}
\end{equation*}
$$

In turn, at discrete load distribution this length is calculated from the supplying centre to the first (at the top and bottom) electricity consumers:

$$
\begin{equation*}
l_{d}=2\left(y_{m}-\frac{\Delta y}{2}\right)=2 y_{m} \frac{2 n}{2 n+1} . \tag{4}
\end{equation*}
$$

The discreteness factor is:

$$
\begin{equation*}
k_{d}=l_{d}: l_{c}=\frac{2 n}{2 n+1} . \tag{5}
\end{equation*}
$$

## The IF calculations

In calculations of the line length and cross-section product two cases can be met: the case $\mathbf{S j}$, when a phase conductor with a constant current density $j$ is employed, i.e. its cross-section changes with load (denoted $l F_{j}$ ); and the case $\mathbf{S F}$, when the cross-section of the phase conductor is constant and calculated according to the maximum current at the beginning of the line (denoted $l F_{F}$ ).

Case $\mathbf{S j}$. The current $i(y)$ and the cross-section with constant current density $j$ in the phase conductor at distance $y$ from the supply centre is:

$$
\begin{equation*}
i(y)=\sigma \Delta x\left(y_{m}-y\right) ; \quad F(y)=\frac{i(y)}{j}=\frac{\sigma \Delta x\left(y_{m}-y\right)}{j}, \tag{6}
\end{equation*}
$$

where $\sigma$ is the load density, $\mathrm{A} / \mathrm{m}^{2}$.
Then at continuous load distribution we will have:

$$
\begin{equation*}
\left(l F_{j}\right)_{c}=2 \int_{0}^{y_{m}} \frac{\sigma \Delta x\left(y_{m}-y\right)}{j} d y=\frac{\sigma \Delta x y_{m}^{2}}{j} . \tag{7}
\end{equation*}
$$

Case $\mathbf{S} \mathbf{j}_{\mathrm{d}}$. At the discrete load distribution with a constant current density in phase conductors we have line segments between two adjacent consumers from a set $l_{1}, l_{2}, \ldots, l_{n}$ with currents $i_{1}, i_{2}, \ldots, i_{n}$ :

$$
\begin{align*}
l_{1}=l_{2} & =\ldots=l_{n}=\Delta y ; \quad i_{1}=\sigma \Delta x 1 \Delta y ; \quad i_{2}=\sigma \Delta x 2 \Delta y ; \ldots F_{1}=\frac{i_{1}}{j}=\frac{\sigma \Delta x 1 \Delta y}{j} \\
\left(l_{1} F_{1}\right)_{d} & =\frac{\sigma \Delta x 1 \Delta y^{2}}{j} ; \quad\left(l_{2} F_{2}\right)_{d}=\frac{\sigma \Delta x 2 \Delta y^{2}}{j} ; \quad\left(l_{n} F_{n}\right)_{d}=\frac{\sigma \Delta x n \Delta y^{2}}{j} ;  \tag{8}\\
\left(l F_{j}\right)_{d} & =2 \sum_{i=1}^{n} l_{i} F_{i}=\frac{2 \sigma \Delta x \Delta y^{2}}{j}(1+2+\ldots+n)=\frac{2 \sigma \Delta x\left(2 y_{m}\right)^{2}}{j(2 n+1)^{2}} \frac{n(n+1)}{2}= \\
& =\frac{\sigma \Delta x y_{m}^{2}}{j} \frac{4 n(n+1)}{(2 n+1)^{2}}
\end{align*}
$$

In (8), expression (2) and a formula for arithmetical progression sum from [9] are employed.

The discreteness factor (similar to (5)) is:

$$
\begin{equation*}
k_{d}=\frac{4 n(n+1)}{(2 n+1)^{2}} \tag{9}
\end{equation*}
$$

Case $\mathbf{S F}_{\mathbf{c}}$. At constant cross-section and continuous load dislocation, we have:

$$
\begin{equation*}
F_{F}=\frac{i(0)}{j}=\frac{\sigma \Delta x y_{m}}{j} ; \quad\left(l F_{F}\right)_{c}=2 \int_{0}^{y_{m}} F_{F} d y=\frac{2 \sigma \Delta x y_{m}^{2}}{j} . \tag{10}
\end{equation*}
$$

Case $\mathbf{S F}_{\mathbf{d}}$. At the discrete load dislocation the following is valid:

$$
\begin{equation*}
\left(l F_{F}\right)_{d}=\frac{2 \sigma \Delta x y_{m} \Delta y n}{j}=\frac{2 \sigma \Delta x y_{m} 2 y_{m} n}{j(2 n+1)}=\frac{2 \sigma \Delta x y_{m}^{2}}{j} \frac{2 n}{2 n+1} \tag{11}
\end{equation*}
$$

The discreteness factor, similar to (5), is:

$$
\begin{equation*}
k_{d}=\frac{2 n}{2 n+1} . \tag{12}
\end{equation*}
$$

## Power loss in the phase conductor

The losses in one phase of a line and the maximum consumer load are considered.

Case $\mathbf{S j} \mathbf{j}$. A strip-shaped network is treated, with constant current density $j$ in the line phase conductor and continuous load distribution.

The specific linear phase conductor resistance $(\Omega / \mathrm{m})$ at a distance $y$ from the supply point will be:

$$
\begin{equation*}
R_{0}(y)=\frac{1}{\gamma F(y)} \tag{13}
\end{equation*}
$$

where $\gamma, 1 /(\Omega \cdot \mathrm{m})$ is the specific conductance of the phase wire material;
$F(y), \mathrm{m}^{2}$ is the cross-section of the phase conductor in accordance with (6). Then the $R_{0}(y), d R_{\Omega}$ and $d \Delta P(y)$ values - the elementary increments of phase conductor resistance and of power losses at a distance $y$, respectively - will be:

$$
\begin{align*}
& R_{0}(y)=\frac{j}{\gamma i(y)} ; \quad d R_{\Omega}(y)=\frac{j}{\gamma i(y)} d y \\
& d \Delta P(y)=i^{2}(y) d R_{\Omega}(y)=\frac{j}{\gamma} i(y) d y \tag{14}
\end{align*}
$$

The power loss in a phase conductor with constant current density is:

$$
\begin{equation*}
\Delta P_{c}=2 \int_{0}^{y_{m}} d \Delta P(y) d y=\frac{j \sigma \Delta x y_{m}^{2}}{\gamma} \tag{15}
\end{equation*}
$$

Case $\mathbf{S j}_{\mathbf{d}}$. It is similar to the preceding case (Case $\mathbf{S} \mathbf{j}_{\mathbf{c}}$ ) but at discrete load distribution (see Fig. 1). The considered strip zone may be part of a greater territory with uniform distribution of consumers. Then each consumer is located in the centre of a square, i.e. $\Delta x=\Delta y$. The current $\Delta i$ collected from each consumer's plot area is:

$$
\begin{equation*}
\Delta i=\sigma \Delta x \Delta y \tag{16}
\end{equation*}
$$

consequently, currents $i_{1}, i_{2}, \ldots, i_{n}$ in the line at distance $\Delta y$ between the neighbouring consumers are:

$$
\begin{equation*}
i_{1}=1 \Delta i ; \quad i_{2}=2 \Delta i ; \ldots ; i_{n}=n \Delta i \tag{17}
\end{equation*}
$$

where $n$ is determined from (2).
Cross-section $F_{1}$, specific resistance $R_{01}$, resistance $R_{1}$ and power loss $\Delta P_{1}$ at the first distance of the phase conductor are:

$$
\begin{align*}
& F_{1}=\frac{i_{1}}{j} ; \quad R_{01}=\frac{j}{\dot{i}_{1}} ; \quad \Delta R_{1}=R_{01} \Delta y \\
& \Delta P_{1}=\Delta R_{1} i_{1}^{2}=\frac{j \Delta y i_{1}}{\gamma}=\frac{j \sigma \Delta x \cdot 1 \Delta y^{2}}{\gamma} \tag{18}
\end{align*}
$$

similarly:

$$
\begin{equation*}
\Delta P_{2}=\frac{j \sigma \Delta x \cdot 2 \Delta y^{2}}{\gamma} ; \quad \Delta P_{n}=\frac{j \sigma \Delta x \cdot n \Delta y^{2}}{\gamma} \tag{19}
\end{equation*}
$$

The power loss in the phase conductor of the entire strip, observing (2), is:

$$
\begin{equation*}
\Delta P_{d}=2 \sum_{i=1}^{n} \Delta P_{i}=2 \frac{j \sigma \Delta x \Delta y^{2}}{\gamma}(1+2+\ldots+n)=\frac{j \sigma \Delta x y_{m}{ }^{2}}{\gamma} \frac{4 n(n+1)}{(2 n+1)^{2}} \tag{20}
\end{equation*}
$$

Discreteness factor, similar to (5), is:

$$
\begin{equation*}
k_{d}=\Delta P_{d}: \Delta P_{c}=\frac{4 n(n+1)}{(2 n+1)^{2}} . \tag{21}
\end{equation*}
$$

Cases $\mathbf{S F}_{\mathbf{c}}$ and $\mathbf{S F}_{\mathbf{d}}$ are considered in a similar way. For three phases the power loss $\Delta P$ is three times that according to (15) and (20).

## Real component of the voltage drop

The real component $U_{\Delta}$ of voltage drop at distance $y_{m}$ (its maximum value) from the supply point at the maximum consumer load is calculated as follows.

Case $\mathbf{S j}_{\mathbf{c}}$.

$$
\begin{equation*}
U_{\Delta c}=\int_{0}^{y_{m}} i(y) d R_{\Omega}=\frac{j y_{m}}{\gamma} . \tag{22}
\end{equation*}
$$

Case $\mathbf{S j}_{\mathbf{d}}$. Observing (2), (17), (18) we have:
$U_{\Delta 1}=R_{01} \Delta y i_{1}=\frac{j \Delta y}{\gamma} ; U_{\Delta 2}=R_{02} \Delta y i_{2}=\frac{j \Delta y}{\gamma} ; \ldots ;$
$U_{\Delta n}=R_{0 n} \Delta y i_{n}=\frac{j \Delta y}{\gamma}$.
$U_{\Delta d}=\frac{j \Delta y}{\gamma} n=\frac{j y_{m}}{\gamma} \frac{2 n}{2 n+1}$.
The discreteness factor:
$k_{d}=U_{\Delta d}: U_{\Delta c}=\frac{2 n}{2 n+1}$.
Cases $\mathbf{S F}_{\mathbf{c}}$ and $\mathbf{S F}_{\mathbf{d}}$ are considered in a similar way.

## Imaginary component of the voltage drop

The imaginary component of voltage drop $U_{\delta}$ (defined at the maximum consumer load) almost does not depend on the phase conductor's diameter, since specific reactance $X_{0}, \Omega / \mathrm{m}$ of the line depends but slightly on the wire diameter. Hence, cases $\mathbf{S j}_{\mathbf{c}}, \mathbf{S j}_{\mathbf{d}}, \mathbf{S F} \mathbf{c}, \mathbf{S F}$ merge into the two cases: $\mathbf{S}_{\mathbf{c}}$ and $\mathbf{S}_{\mathbf{d}}$.

Case $\mathbf{S}_{\mathbf{c}}$. Observing (6) we have:

$$
\begin{equation*}
U_{\delta c}=\int_{0}^{y_{m}} i(y) X_{0} d y=\frac{X_{0} \sigma \Delta x y_{m}^{2}}{2} \tag{26}
\end{equation*}
$$

Case $\mathbf{S}_{\mathrm{d}}$. Reactance $\Delta x$ between adjacent consumers and voltage drops across them are:

$$
\begin{equation*}
\Delta x=X_{0} \Delta y ; \quad U_{\delta 1}=\Delta x i_{1} ; \quad U_{\delta 2}=\Delta x i_{2} ; \ldots ; \quad U_{\delta n}=\Delta x i_{n} . \tag{27}
\end{equation*}
$$

Observing (2), (16) and (17) we have:

$$
\begin{equation*}
U_{\delta d}=\sum_{i=1}^{n} U_{\delta i}=X_{0} \sigma \Delta x \Delta y^{2}(1+2+\ldots+n)=\frac{X_{0} \sigma \Delta x y_{m}{ }^{2}}{2} \frac{4 n(n+1)}{(2 n+1)^{2}} . \tag{28}
\end{equation*}
$$

The discreteness factor:

$$
\begin{equation*}
k_{d}=\frac{4(n+1)}{(2 n+1)^{2}} . \tag{29}
\end{equation*}
$$

Calculation results are shown in Table 1 for two cases: $\mathbf{S j}$ and $\mathbf{S F}$. Tabulated are the characteristic quantities for continuous load distribution $\left(l ; l F, \Delta P, U_{\Delta}, U_{\delta}\right.$, and discreteness factor $k_{d}$ ). The quantities for discrete load distribution are defined by (1).

Table 1
Characteristic quantities for a strip-shaped fed zone

| Case | $\begin{gathered} l \\ k_{d} \end{gathered}$ | $\begin{aligned} & \hline l F \\ & k_{d} \end{aligned}$ | $\begin{aligned} & \hline \Delta P \\ & k_{d} \end{aligned}$ | $\begin{aligned} & \hline U_{\Delta} \\ & k_{d} \end{aligned}$ | $\begin{aligned} & \hline U_{\delta} \\ & k_{d} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Sj | $2 y_{m}$ | $\begin{aligned} & \frac{\sigma \Delta x y_{m}{ }^{2}}{j} \\ & \frac{4 n(n+1)}{(2 n+1)^{2}} \end{aligned}$ | $\begin{aligned} & \frac{3 j \sigma \Delta x y_{m}{ }^{2}}{\gamma} \\ & \frac{4 n(n+1)}{(2 n+1)^{2}} \end{aligned}$ | $\begin{aligned} & \frac{j y_{m}}{\gamma} \\ & \frac{2 n}{2 n+1} \end{aligned}$ | $\frac{X_{0} \sigma \Delta x y_{m}{ }^{2}}{2}$ |
| SF | $\frac{2 n}{2 n+1}$ | $\begin{gathered} \frac{2 \sigma \Delta x y_{m}{ }^{2}}{j} \\ \frac{2 n}{2 n+1} \end{gathered}$ | $\begin{gathered} \frac{2 j \sigma \Delta x y_{m}{ }^{2}}{\gamma} \\ \frac{2(n+1)}{2 n+1} \end{gathered}$ | $\begin{gathered} \frac{j y_{m}}{\gamma} \\ \frac{2(n+1)}{2 n+1} \end{gathered}$ | $\frac{4 n(n+1)}{(2 n+1)^{2}}$ |

B. Fed zone in the shape of a bar (Fig. 2).

A bar-shaped network includes branch lines and trunk lines whose phase conductors can be made of different materials, and therefore may have their own parameters $j_{b}, \gamma_{b}$ and $j_{t}, \gamma_{t}$. Whereas for a strip we had two cases: at constant $j$ and constant $F$, for a bar we have four cases: two branch line cases and the other two trunk line cases. Case Bbj holds for a branch line with constant $j$; case $\mathbf{B b F}$ - for a branch line with constant $F$; cases $\mathbf{B t j}$ and $\mathbf{B t F}$ concern a trunk line with constant $j$ and $F$, respectively. The number of branch lines in one direction from the supply centre is $N$ (the middle branch line is also accounted for in the calculations); the number of consumers supplied from one direction of the branch line is $n$. The calculation results are shown in Table 2, where the quantities are similar to those in Table 1.


Fig. 2. Fed zone in the shape of a bar (a fourth of the area).

Characteristic quantities for a bar-shaped fed zone

| Case | $\begin{gathered} l \\ k_{d} \end{gathered}$ | $\begin{gathered} l F \\ k_{d} \end{gathered}$ | $\begin{gathered} \Delta P \\ k_{d} \end{gathered}$ | $\begin{gathered} U_{\Delta} \\ k_{d} \end{gathered}$ | $\begin{gathered} U_{\delta} \\ k_{d} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Bbj |  | $\begin{aligned} & \frac{2 \sigma y_{m}{ }^{2} x_{m}}{j_{b}} \\ & \frac{4 n(n+1)}{(2 n+1)^{2}} \end{aligned}$ | $\begin{gathered} \frac{6 j_{b} \sigma y_{m}{ }^{2} x_{m}}{\gamma_{b}} \\ \frac{4 n(n+1)}{(2 n+1)^{2}} \end{gathered}$ | $\begin{aligned} & \frac{j_{b} y_{m}}{\gamma_{b}} \\ & \frac{2 n}{2 n+1} \end{aligned}$ | $\frac{X_{0} \sigma x_{m} y_{m}{ }^{2}}{2 n+1}$ |
| BbF |  | $\begin{gathered} \frac{4 \sigma y_{m}{ }^{2} x_{m}}{j_{b}} \\ \frac{2 n}{2 n+1} \end{gathered}$ | $\begin{gathered} \frac{4 j_{b} \sigma y_{m}{ }^{2} x_{m}}{\gamma_{b}} \\ \frac{2(n+1)}{2 n+1} \end{gathered}$ | $\begin{aligned} & \frac{j_{b} y_{m}}{2 \gamma_{b}} \\ & \frac{2(n+1)}{2 n+1} \end{aligned}$ | $\frac{4 n(n+1)}{(2 n+1)^{2}}$ |
| Btj | $2 x_{m}$ | $\begin{aligned} & \frac{2 \sigma y_{m} x_{m}{ }^{2}}{j_{t}} \\ & \frac{4 N(N+1)}{(2 N+1)^{2}} \end{aligned}$ | $\begin{aligned} & \frac{6 j_{t} \sigma y_{m} x_{m}{ }^{2}}{\gamma_{t}} \\ & \frac{4 N(N+1)}{(2 N+1)^{2}} \end{aligned}$ | $\begin{aligned} & \frac{j_{t} x_{m}}{\gamma_{t}} \\ & \frac{2 n}{2 n+1} \end{aligned}$ | $X_{0} \sigma y_{m} x_{m}{ }^{2}$ |
| BtF | $\frac{2 N}{2 N+1}$ | $\begin{gathered} \frac{4 \sigma y_{m} x_{m}{ }^{2}}{j_{t}} \\ \frac{2 N}{2 N+1} \end{gathered}$ | $\begin{gathered} \frac{4 j_{t} \sigma y_{m} x_{m}{ }^{2}}{\gamma_{t}} \\ \frac{2(N+1)}{2 N+1} \end{gathered}$ | $\begin{gathered} \frac{j_{t} x_{m}}{2 \gamma_{t}} \\ \frac{2(N+1)}{2 N+1} \end{gathered}$ | $\frac{4 N(N+1)}{(2 N+1)^{2}}$ |

$\mathbf{S +}$. The following consideration relates to the square network with cruciform trunk lines (Fig. 3). Lines of the kind are suitable for the cities with perpendicular arrangement of streets. In a square $n=N$, which means that the number of branch lines in one direction from the supply centre and the maximum number of consumers supplied from one direction of a branch line is $n$. The principle of case designation is the same as for Table 2. The results are shown in Table 3.


Fig. 3. A square fed zone with cruciform trunk lines (a fourth of the area). $i_{1}, i_{2}, \ldots, i_{n}$ - currents in the entire branch line (on both sides of the trunk line); $i_{\mathrm{J}} i_{\mathrm{II}}, \ldots$ - currents in a trunk line segment.


Fig. 4. A square fed zone with $\mathbf{x}$-like. trunk lines (fourth of the area).
$i_{1}, i_{2}, \ldots i_{n}$ - currents in the entire branch line (on both sides of the trunk line); $i_{\mathrm{I}}, i_{\mathrm{II}}$, ... - currents in a trunk line segment.

Characteristic quantities of a square fed zone ( $\mathbf{S}+$ )

| Case | $\begin{gathered} l \\ k_{d} \end{gathered}$ | $\begin{gathered} l F \\ k_{d} \end{gathered}$ | $\Delta P$ $U_{\Delta}$ <br> $k_{d}$ $k_{d}$ | $\begin{aligned} & U_{\delta} \\ & k_{d} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| S+bj | $4 x_{m} n$ | $\begin{gathered} \frac{4 \sigma x_{m}{ }^{3}}{3 j_{b}} \\ \frac{4 n(n+1)}{(2 n+1)^{2}} \end{gathered}$ | $\frac{4 j_{b} \sigma x_{m}{ }^{3}}{}$ $\frac{j_{b} x_{m}}{\gamma_{b}}$ <br> $\frac{4 n(n+1)}{(2 n+1)^{2}}$ $\frac{2 n}{2 n+1}$ | $\frac{X_{0} \sigma x_{m}{ }^{3}}{2 n+1}$ |
| S+bF |  | $\frac{8 \sigma x_{m}{ }^{3}}{3 j_{b}}$ | $\frac{8 j_{b} \sigma x_{m}{ }^{3}}{3 \gamma_{b}}$ $\frac{j_{b} x_{m}}{2 \gamma_{b}}$ | $4 n^{2}$ |
|  | $\frac{2 n}{2 n+1}$ | $\frac{4 n(n+1)}{(2 n+1)^{2}}-\frac{6 n(n+1)}{(2 n+1)^{3}}$ | $\frac{4 n(n+1)}{(2 n+1)^{2}}-\frac{6 n(n+1)}{(2 n+1)^{3}} \quad \frac{2 n}{2 n+1}$ | $\overline{(2 n+1)^{2}}$ |
| $\mathbf{S}+\mathbf{t j}$ | $\begin{gathered} 4 x_{m} \\ \frac{2 n}{2 n+1} \end{gathered}$ | $\frac{8 \sigma x_{m}{ }^{3}}{3 j_{t}}$ | $\frac{8 j_{t} \sigma x_{m}{ }^{3}}{\gamma_{t}}$ $\frac{j_{t} x_{m}}{\gamma_{t}}$ | $\frac{2 X_{0} \sigma x_{m}{ }^{3}}{3}$ |
|  |  | $\frac{4 n(n+1)}{(2 n+1)^{2}}$ | $\frac{4 n(n+1)}{(2 n+1)^{2}}$ $\frac{2 n}{2 n+1}$ | $\frac{4 n(n+1)}{(2 n+1)^{2}}$ |
| $\mathbf{S}+\mathbf{t F}$ | $\begin{gathered} 4 x_{m} \\ \frac{2 n}{2 n+1} \end{gathered}$ | $\begin{gathered} \frac{4 \sigma x_{m}^{3}}{j_{t}} \\ \frac{8 n^{2}(n+1)}{(2 n+1)^{3}} \end{gathered}$ | $\begin{gathered} \Delta P=\frac{32 j_{t} \sigma x_{m}{ }^{3}}{5 \gamma_{t}} ; U_{\Delta}=\frac{2 j_{t} x_{m}}{3 \gamma_{t}} ; k_{d U}=1 \\ k_{d P}=\frac{15}{2}\left(\frac{1}{3}-\frac{n(n+1)}{(2 n+1)^{2}}+\frac{3 n^{2}+3 n-1}{15(2 n+1)^{2}}\right) \end{gathered}$ | $\begin{aligned} & \frac{2 X_{0} \sigma x_{m}{ }^{3}}{3} \\ & \frac{4 n(n+1)}{(2 n+1)^{2}} \end{aligned}$ |

Sx. Under consideration is a square-shaped network with $\mathbf{x}$-like trunk lines (Fig. 4). Lines of the kind might be suitable for countryside. For the square $n=N$. The principle of case denomination is the same as in Table 2. Consideration results are shown in Table 4.

Table 4
Characteristic quantities of a square fed zone ( $\mathbf{S x}$ )

| Case | $\begin{gathered} l \\ k_{d} \end{gathered}$ | $\begin{aligned} & l F \\ & k_{d} \end{aligned}$ | $\begin{gathered} \Delta P \\ k_{d} \end{gathered}$ | $\begin{gathered} U_{\Delta} \\ k_{d} \end{gathered}$ | $\begin{aligned} & U_{\delta} \\ & k_{d} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Sxbj | $4 x_{m} n$ | $\begin{gathered} \frac{4 \sigma x_{m}{ }^{3}}{3 j_{b}} \\ \frac{4 n(n+1)}{(2 n+1)^{2}} \end{gathered}$ | $\begin{aligned} & \frac{4 j_{b} \sigma x_{m}{ }^{3}}{\gamma_{b}} \\ & \frac{4 n(n+1)}{(2 n+1)^{2}} \end{aligned}$ | $\begin{aligned} & \frac{j_{b} x_{m}}{\gamma_{b}} \\ & \frac{2 n}{2 n+1} \end{aligned}$ | $\frac{X_{0} \sigma x_{m}{ }^{3}}{2 n+1}$ |
| SxbF | $\frac{2 n}{2 n+1}$ | $\begin{gathered} \frac{8 \sigma x_{m}{ }^{3}}{3 j_{b}} \\ \frac{4 n(n+1)}{(2 n+1)^{2}}-\frac{6 n(n+1)}{(2 n+1)^{3}} \end{gathered}$ | $\begin{gathered} \frac{8 j_{b} \sigma x_{m}{ }^{3}}{3 \gamma_{b}} \\ \frac{4 n(n+1)}{(2 n+1)^{2}}-\frac{6 n}{(2 n+1)^{3}} \end{gathered}$ | $\begin{gathered} \frac{j_{b} x_{m}}{2 \gamma_{b}} \\ \frac{2(n+1)}{2 n+1} \end{gathered}$ | $\frac{4 n(n+1)}{(2 n+1)^{2}}$ |
| Sxtj | $4 \sqrt{2} x_{m}$ | $\begin{aligned} & \frac{4 \sqrt{2} \sigma x_{m}{ }^{3}}{3 j_{t}} \\ & \frac{4 n(n+1)}{(2 n+1)^{2}} \end{aligned}$ | $\begin{aligned} & \frac{4 \sqrt{2} j_{t} \sigma x_{m}{ }^{3}}{\gamma_{t}} \\ & \frac{4 n(n+1)}{(2 n+1)^{2}} \end{aligned}$ | $\begin{gathered} \frac{\sqrt{2} j_{t} x_{m}}{\gamma_{t}} \\ \frac{2 n}{2 n+1} \end{gathered}$ | $\frac{\sqrt{2} X_{0} \sigma_{m}{ }^{3}}{3}$ |
| SxtF |  | $\begin{gathered} \frac{4 \sqrt{2} \sigma x_{m}{ }^{3}}{j_{t}} \\ \frac{2 n}{2 n+1} \end{gathered}$ | $\begin{gathered} \frac{12 \sqrt{2} j_{t} \sigma x_{m}{ }^{3}}{5 \gamma_{t}} \\ \frac{4(n+1)\left(3 n^{2}+3 n-1\right)}{3 n(2 n+1)^{2}} \end{gathered}$ | $\begin{gathered} \frac{\sqrt{2} j_{t} x_{m}}{3 \gamma_{t}} \\ \frac{n+1}{n} \end{gathered}$ | $\frac{4 n(n+1)}{(2 n+1)^{2}}$ |

Taking the data from Tables $1-4$, we can compare the parameters of different zone shapes. Of particular interest is a square $\mathbf{S}+$ and a bar, the latter with $n=N\left(\right.$ when $\left.x_{m}=y_{m}\right)$. For example, $\mathbf{S}+\mathbf{t j}, \mathbf{S}+\mathbf{t F}: U_{\delta c}=2 X_{0} \sigma x_{m}^{3} / 3, l=4 x_{m} ; \mathbf{B t j}, \mathbf{B t F}$ : $U_{\delta c}=X_{0} \sigma x_{m}{ }^{3}, l=2 x_{m}$. It is also noteworthy to compare the losses in the lines.

## 4. CONSIDERATION OF A LOW-VOLTAGE NETWORK

The model of a supplied zone in the form of square $\mathbf{S}+$, with a given (or assumed) number $n$ of discrete loads is considered. The trunk line in the model has a constant current density ( $\mathbf{S}+\mathbf{t j}$ ) and branch lines - a constant cross-section ( $\mathbf{S}+\mathbf{b F}$ ) specific for each branch line in accordance with the maximum current at its beginning.

The annual costs according to [10] for a line are:

$$
\begin{equation*}
C_{l}=\left(i+p_{\Sigma}\right)(a+b F) l+\Delta P\left(\beta^{\prime} \tau+\beta^{\prime \prime}\right), \tag{30}
\end{equation*}
$$

where $i$ is a bank's loan interest, $\%$;
$p_{\Sigma}$ are the costs of depreciation, maintenance and servicing, $\%$;
$a$ are the fitted costs of a line's construction, $\mathrm{LVL} / \mathrm{m}$, which comprise not only the power line itself but also its ancillary equipment (distribution nodes, minor switches, fuses, etc); $b$ is the cost depending on the crosssection $F$ of a phase wire, $\mathrm{LVL} / \mathrm{m}^{3}$;
$l$ is the line length, $m$;
$\Delta P$ is the power loss at the maximum consumer load for the entire line length, W;
$\beta^{\prime}$ is the cost of a line's power losses, $\mathrm{LVL} / \mathrm{Wh}$;
$\beta^{\prime \prime}$ is the peak power cost, $\mathrm{LVL} / \mathrm{W}$;
$\tau$ is the time of maximum power losses, h .
Having performed the following substitutions:

$$
\begin{equation*}
\left(i+p_{\Sigma}\right) / 100=C_{p f} ; C_{p f} a=C_{a} ; C_{p f} b=C_{b} ; \beta^{\prime} \tau+\beta^{\prime \prime}=C_{w}, \tag{31}
\end{equation*}
$$

we obtain:

$$
\begin{equation*}
C_{l}=C_{a} l+C_{b} l F+C_{w} \Delta P . \tag{32}
\end{equation*}
$$

We will not seek the line costs separately but those of the entire fed zone:

$$
\begin{equation*}
C_{n t}=C_{a}\left(l_{t j}+l_{b F}\right)+C_{b}\left[(l F)_{t j}+(l F)_{b F}\right]+C_{w}\left(\Delta P_{t}+\Delta P_{b}\right) . \tag{33}
\end{equation*}
$$

The quantities $l_{t j}, l F_{t j}, \Delta P_{t j}$ for trunk lines and $l_{b F}, l F_{b F}, \Delta P_{b F}$ for branch lines are found in Table 3 for continuous load density (for discrete load distribution we have to use discreteness factor $k_{d}$ ). Trunk lines are taken with constant current density $j$ in the phase conductors, whereas branch lines - with constant crosssection $F$. The material of phase conductors is assumed to be the same for both types of line.

Based on Table 3, expression (33) can be rewritten as

$$
\begin{align*}
C_{n t}= & 4 C_{a} x_{m} \frac{2 n(n+1)}{2 n+1}+\frac{16 C_{b} \sigma x_{m}{ }^{3}}{3 j}\left[\frac{4 n(n+1)}{(2 n+1)^{2}}-\frac{3 n(n+1)}{(2 n+1)^{3}}\right]+  \tag{34}\\
& +\frac{32 C_{w} j \sigma x_{m}{ }^{3}}{3 \gamma} \cdot\left[\frac{4 n(n+1)}{(2 n+1)^{2}}-\frac{3 n(n+1)}{2(2 n+1)^{3}}\right] .
\end{align*}
$$

Better cost index is presented by specific costs $C_{n t p s}$, which are defined as the ratio of $C_{n t}$ and the delivered maximum power $S$ in a three-phase network.

$$
\begin{align*}
& C_{n t p S}=C_{n t} / S ; \quad S=\sqrt{3} U I \\
& C_{n t p S}=C_{n t} /(\sqrt{3} U I) ; \quad I=4 \sigma x_{m}^{2}, \tag{35}
\end{align*}
$$

where $I, \mathrm{~A}$ is the current collected from the entire zone at the maximum consumer load (conjugate is not shown, since only the absolute value is significant);
$U$ is the phase-to-phase low voltage $(400 \mathrm{~V})$.

The second multiplier of each term in (34) is a function of discreteness factor defined by number $n$, hence it is a function of $n$ :

$$
\begin{align*}
& f_{1}(n)=\frac{2 n(n+1)}{2 n+1} ; \quad f_{2}(n)=\frac{4 n(n+1)}{(2 n+1)^{2}}-\frac{3 n(n+1)}{(2 n+1)^{3}} ; \\
& f_{3}(n)=\frac{4 n(n+1)}{(2 n+1)^{2}}-\frac{3 n(n+1)}{2(2 n+1)^{3}} . \tag{36}
\end{align*}
$$

Expressing $n$ according to (2) by $x_{m}$ and $\Delta x$ and inserting it into $f_{1}(n)$, we obtain:

$$
\begin{equation*}
f_{1}(n)=\frac{x_{m}}{\Delta x}-\frac{0.25 \Delta x}{x_{m}} \approx \frac{x_{m}}{\Delta x} \tag{37}
\end{equation*}
$$

At $\Delta x / x_{m}=0.5$ and 0.25 the difference between $f_{1}(n)$ and $x_{m} / \Delta x$ is 6.3 and $1.6 \%$, respectively.

Observing (33)-(37), the expression for specific per unit of the network's annual costs (without transformer) is:

$$
\begin{equation*}
C_{n t p S}=\frac{C_{a}}{\sqrt{3} \sigma \Delta x U}+\frac{4 C_{b} x_{m}}{3 \sqrt{3} j U} f_{2}(n)+\frac{8 C_{w} j x_{m}}{3 \sqrt{3} \gamma U} f_{3}(n) \tag{38}
\end{equation*}
$$

Functions $f_{1}(n) \ldots f_{3}(n)$ are displayed in Fig. 5. Expression (38) shows the influence of the parameters on the specific costs of the network.


Fig. 5. Functions $f_{1}(\mathrm{n}) \ldots f_{3}(n)$.
To optimize a network's operation it is necessary to include a transformer into consideration.

Some remaining questions are to be considered in the next publications.

## 5. CONCLUSIONS

1. In order to investigate the electrical network it is more convenient to use a model with continuously changing parameters and calculate the discreteness factor for supplying zone.
2. Specific costs of a network decrease with voltage increasing as far as the fitted costs of line construction are not increasing too rapidly with voltage.
3. Specific costs of a network decrease with the load density and specific conductance of phase conductor increasing when the consumers are spread more sparsely. To reduce specific costs, an appropriate current density should be chosen.
4. To choose the optimum dimensions of the supplying zone a step-down transformer should be included into consideration.

## REFERENCES

1. http://en.wikipedia.org/wki/Electric_power_transmission.
2. Веников, В.А., \& Строева, В.А., (1998). Электрические системы. Электрические сети (гл. 1). Москва: Высшая школа.
3. Блок, В.М. (1986). Электрические сети и системыя (гл. 1). Москва: Высшая школа.
4. Идельчик, В.И., (1989). Электрические сети и системыи (гл. 6). Москва: Энергоатомиздат.
5. ENKS - ST 2001 - 00522. DISPOWER. Distributed generation with high penetration of renewable energy sources. Structure and data concerning electrical grids for Italy, Germany, Spain, UK, Poland. www.iset. uni-kassel. de/dispower.
6. Vanags, A. (2007). Elektriskie tīkli un sistēmas. Rīga: RTU (in Latvian).
7. Krišāns, Z., \& Oḷeiņikova, I. (2007). Elektroenergètisko uzņēmumu vadības pamati. Riga: RTU (in Latvian).
8. Guseva, S., Skobel̦eva, N., Brenners, N., \& Borščevskis, O. (2009). Modelling of service zones of urban transformer substations. Scientific proceedings of Riga Technical University; Power and Electrical Eng-g., 24 (4), 24-31.
9. Бронштейн, И.Н., \& Семендяев, К.А. (1954). Справочник по математике для инженеров и учащихся втузов. Москва: Гос. изд-во технико-теоретической литры (с. 159).
10. Vanags, A., \& Krišāns, Z. (2005). Elektriskie tīkli un sistēmas, II daļa. Rīga: RTU (in Latvian).

## TĪKLU NOMINĀLIE SPRIEGUMI

## J. Survilo

## Kopsavilkums

Elektriskajās sistēmās izmanto dažādus nominālos spriegumus. Kāds pamats ir tādai gradācijas pārpilnībai? Atbilde tiek meklēta uz apgādājamās zonas un tās izmēru bāzes. Novērtējums tiek veikts, izvēloties dažus pieņēmumus: parametru izmaiņa ir nepārtraukta, spriegums ir konstants, elektrības patērētāji ir izvietoti nepārtraukti ar konstantu blīvumu utt. Lai rezultātus atgrieztu reālajā situācijā, tiek aprēk̦ināts un lietots atsevišks koeficients dažādas formas apgādājamām zonām. Analizējot tīkla apgādājamās zonas izdevumu formulu, redzam, ka īpatnējie izdevumi samazinās: palielinot tīkla spriegumu (ja līnija pārlieku nesadārdzinās), kad patērētāji ir izvietoti retāk, kad ir lielāks slodzes blīvums un fāzes vada materiāla vadāmība, pie optimālā strāvas blīvuma fāzes vadā. Raksta nosaukumā ietvertās problēmas pētīšana turpināsies.
11.04.2011.

## MISTAKE CORRECTION*

## REFERENCES

1. Веников, В.А., \& Строева, В.А. (1998). Электрические системы. Электрические сети. Москва: „Высшая школа" (гл. 10, 11).
2. Krišāns, Z. \& Oḷeiņikova, I. (2007). Elektroenerg̀ētisko uzņēmumu vadības pamati. Riga: RTU (11.nod.).
3. Guseva, S., Borščevskis, O., Skobeļeva, N., \& Brenners, N. (2009). Load Forecasting till 2020 of Existing and Perspective Transformer Substations in Riga. Scientific Proceedings of Riga Technical Uiversity; Power and Electrical Engineering, 25 (4), 77 80.
4. Vanags, A., \& Krišāns, Z. (2005). Elektriskie tīkli un sistēmas, II daļa. Rīga: RTU (2. nod.).
5. Блок, В.М., (1986). Электрические сети и системы. Москва: Высшая школа (гл. 9).
6. www.Latvenergo.lv.
7. www.jauda.com.
8. Survilo, J. (2008). Starpsprieguma 1 kV pielietošanas lietderīgums sadales tīklos. Latv. J. Phys. Tec. Sci., 2, 17-30.
[^0]
[^0]:    * Latv. J. Phys.Tech. Sci., 2010, N 6, p. 29. REFERENCES.

