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RATED POWER DETERMINATION FOR MEDIUM/LOW VOLTAGE TRANSFORMERS

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The power estimation is performed for the medium/low voltage transformer of a low voltage network. The factors to be accounted for in the estimation are the load density and the fed zone shape. From variously shaped fed zones that of hexagonal shape compactly covers a greater territory and has better indices as compared with rectangles and triangles. The study is framed on an apt canonical model − a circular fed zone with constant continuous load density. The model can be used for comparison of indices at the discrete load dislocation in hexagonal or squared fed zones, extending the results obtained for a circular zone using relative ratios. The transformer power is determined by the fed zone radius, which has a natural limit, since the voltage deviation for the farthest consumer should not exceed the allowable value. Under these conditions, optimization can be performed by a given load density, changing the radius of the fed zone and the density of current in phase conductors.

Key words: *electricity consumers, electricity supply efficiency, energy loss, fed zone, medium/low voltage transformer, power loss, voltage loss.*

1. INTRODUCTION

The last conversion stage of industrial frequency voltage is its medium/ low transformation for the final consumer. The conditions of electricity supply differ for urban and rural areas; therefore, to achieve the maximum economic efficiency the analysis is needed that would take into account the factors influencing the economic indices. The ultimate aim is to determine the power of a step-down medium/low voltage transformer so that the maximum efficiency is achieved, with the least annual costs on the transformer and the corresponding low-voltage network of a fed zone, and, respectively, the least capital investments and costs of electricity losses, preserving at the same time the quality of electricity.

To carry out research of the kind, it is necessary first to choose the shape of the fed zone. Next, an appropriate mathematical model involving all the influential quantities should be worked out.

The medium/low voltage transformers have been in use from the very beginning of the electricity era; no special attention was then paid to the efficiency of network operation, and unacceptable was only deviation from the normal voltage at consumer. Nowadays, this aspect of the problem is given proper attention − not only abroad (e.g. [1]) but also in our country. In [2], the economic issues are considered based on which appropriate dependences are found as to the optimum size of a zone supplied from a higher voltage substation. Paper [3] pays more attention to load forecasting.

The low-voltage network is the lowest stage in the power system's hierarchy, which also deserves attention. When optimization has been done for a lower stage, solving the problems of a higher stage can be initiated.

Search for possible dependences should be done based on some presumptions, e.g.: the capacity of transformers and the cross-section area of conductors can vary gradually; the load density is continuously distributed throughout the entire fed zone; the low voltage is constant and equal to the nominal (400 V); the simultaneity factor is equal to unity (the loads in the entire zone change simultaneously); the transformer capacity corresponds to the maximum load. This is an ideal case considered in the framework of the study. The influence of other factors can be examined separately for every real case.

2. THE SHAPES OF FED ZONES

The conventional shape of a fed zone is hexagon. The zone should meet two requirements: it should tightly fit other ones for covering all relevant territory; it should have the best length ratio. Among diversified shapes there are many that meet the first requirement. The simplest among them are: the equilateral triangle, the square, and the hexagon. As concerns the second requirement, it is the circle that has the best length ratio k_{leci} (Fig. 1*a*) since

$$k_{leci} = \frac{R}{\pi R^2} = \frac{0.318}{R}, \tag{1}$$

but it does not meet the first criterion.

The length ratio shows the distance of a supplied zone's farthest point from the centre of this zone as compared with its area. The less the length ratio, the less the voltage drop is from the centre to the farthest point and the less power losses are. This index for a square and an equilateral triangle is (Fig.1*b*,*c*):

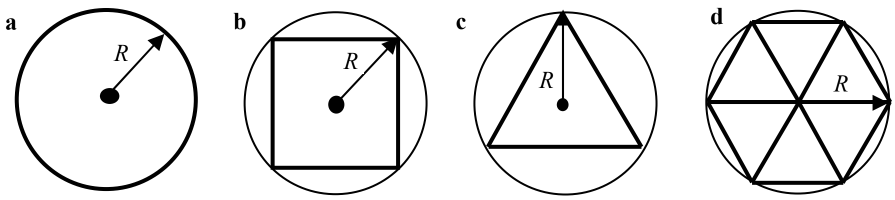

Fig. 1. Shapes of the fed zone: *a* – circle; *b* – square; *c* – equilateral triangle; *d* – hexagon.

$$k_{lesq} = \frac{R}{2R^2} = \frac{0.5}{R}; \quad k_{letr} = \frac{R}{1{,}5\sqrt{3}R^2/2} = \frac{0.77}{R};$$

$$k_{lehe} = \frac{R}{6\sqrt{3}R^2/4} = 0.385. \tag{2}$$

From the three, the second criterion is met best by a circle, and worst − by a triangle. Six such triangles form a hexagon (see Fig. 1*c,d* and Eq. (2)). To compare these length ratios with the best (for a circle) we can introduce relative length ratios as

$$\kappa_{leci} = 1\,; \quad \kappa_{lesq} = \frac{0.5}{0.318} = 1.57\,; \quad \kappa_{letr} = 2.42\,; \quad \kappa_{lehe} = 1.21\,. \tag{3}$$

Of significance are also the area ratios:

$$k_{aci} = 1\,; \quad k_{asq} = \frac{2R^2}{\pi R^2} = 0.637\,; \quad k_{atr} = \frac{1.5\sqrt{3}R^2/2}{\pi R^2} = 0.414\,;$$

$$k_{ahe} = \frac{6\sqrt{3}R^2/4}{\pi R^2} = 0{,}827\,. \tag{4}$$

If the fed territory is small enough, any other shape fitting this territory is relevant. The length ratios and the area ratios can be helpful for analyzing the parameters of a fed zone (circular as the most suitable for analysis). Using an appropriate length ratio we can model the low voltage line length of a square or a hexagon approximating them by a circle, while employing the area ratio we can do this for the transformer capacity. Any parameter of a fed zone can be ideally re-presented by that of a circle if its relative ratio is independent of the zone radius *R*.

3. APPROACH TO THE PROBLEM

To estimate the efficiency of a low-voltage network, the annual costs should be evaluated. In accordance with [4], the annual costs for a line are:

$$C_l = (i + p_\Sigma)(a + bF)l + \Delta P_{\max}(\beta'\tau + \beta'')\,, \tag{5}$$

where i is a bank's loan interest, %;
p_Σ are the costs of depreciation, maintenance and servicing, %;
a are the fitted costs of a line's construction, LVL/m;
b is the cost depending on the cross-section area F of a phase wire, LVL/(m·m^2);
l is the line length, m;
$\Delta P_{\max}$ is the maximum power loss for the entire line length, W;
β' is the cost of a line's power losses, LVL/Wh;
β'' is the peak power cost, LVL/W;
τ is the time of maximum power losses, h.

Denoting the elements of formula (5) as

$$(i + p_\Sigma)a/100 = C_a\,; \quad (i + p_\Sigma)b/100 = C_b\,; \quad \beta'\tau + \beta'' = C_w\,, \tag{6}$$

we obtain:

$$C_l = C_a l + C_b F l + C_w \Delta P_{\max}\,. \tag{7}$$

All the lines of a low-voltage network can be divided into trunk lines and branch lines (Fig. 2). A trunk one with straight branch lines (Fig. 2*a*) is better fit for urban operation with cables laid along the streets. Among other models met in practice is the leaf model (Fig. 2*b*); in the canonical model with circular branches (Fig. 2*c*) the circular shape is employed to simplify mathematical expressions. Other models can be reduced to the canonical model by appropriate relative ratios.

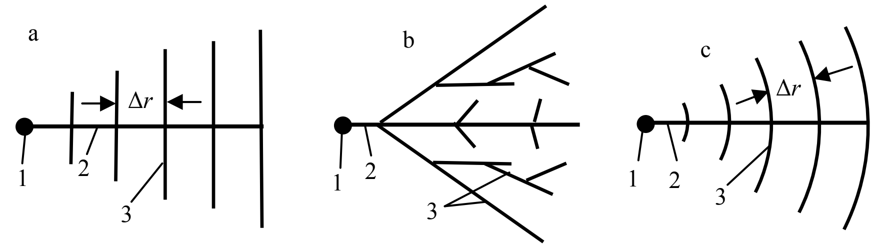

Fig. 2. Sectors of the trunk line in a fed zone: *a* – model with straight branch lines; *b* – leaf model; *c* – canonical model with circular branch lines; *1* – medium/low voltage transformer; *2* – trunk line; *3* – branch line.

A fed zone has the total length l_t of trunk lines with the total losses ΔP_{t}, and the total length l_b of branch lines with the total losses ΔP_b. The trunk lines are composed of thicker wires, since their current is greater than that of branch lines. The cross-section of a trunk line wire is constant and should be calculated corresponding to the current at its beginning. The same holds for all branch lines, with cross-section calculated corresponding to the maximum current in the branch lines. With such presumptions formula (7) will look as

$$C_{nw} = C_a l_t + C_a l_b + C_b F_t l_t + C_b F_b l_b + C_w \Delta P_t + C_w \Delta P_b \,, \tag{8}$$

where C_{nw} is the annual costs of the network.

To apply (8) for a square or a hexagon, the line lengths should be multiplied by the accordingly calculated length ratio. By the area ratio the maximum current can be modelled, with radius R remaining the same for all shapes. Expression (8) cannot characterize the efficiency of the network; this parameter will be shown when we relate the annual costs to the delivered energy (in compliance with [5]). For the simplicity sake, the per unit quantities will be sought-for in the form of per ampere (pA) notation, that is, by dividing (8) and its components by the maximum current $I_{\max}$ of the fed zone; the components with the *pA* notation will have index $_{pA}$:

$$C_{ntpA} = \frac{C_{nw}}{I_{\max}} = C_a l_{tpA} + C_a l_{bpA} + C_b (F_t l_t)_{pA} + C_b (F_b l_b)_{pA} + .$$

$$+ C_w \Delta P_{tpA} + C_w \Delta P_{bpA} \,. \tag{9}$$

In the ultimate estimation, the quantity of interest is the cost of delivery of one kWA energy, which is given by the expression:

$$C_{nwpu} = \frac{C_{nwpA}}{\sqrt{3} U T_m \cos\varphi} \,, \tag{10}$$

where U is the low phase-to-phase voltage;
T_m is the time of maximum load.

As follows from (8), to consider a network we should know rather a large number of quantities.

When considering the efficiency of a low-voltage network we should take into account a medium/low voltage transformer unit (a transformer substation). The annual costs for a transformer unit consist of the annual charges for the investments and servicing as well as of the costs of its losses:

$$C_{tu} = (i + p_{\Sigma})K_{tu} / 100 + C_w \Delta P_{ld} + C_{wnl} \Delta P_{nl}, \tag{11}$$

where K_{tu} is the price of a transformer unit;
ΔP_{ld} is the load loss;
ΔP_{nl} is the no-load loss, and

$$C_{wnl} = \beta' T + \beta'', \tag{12}$$

with T being the number of hours in a year.

The summary costs (C_{Σ}) of the transformer unit and the network according to (8) in pA and per unit notations are:

$$C_{\Sigma} = C_{tu} + C_{nw}; \quad C_{\Sigma pA} = C_{tupA} + C_{nwpA}; \quad C_{\Sigma pu} = C_{tupu} + C_{nwpu}. \tag{13}$$

The C_{tupA} and C_{tupu} values are determined in the same way as C_{nwpA} and C_{nwpu}.

The most favourable power of a medium/low voltage transformer will be when the summary cost C_{Σ} in (13) reaches minimum.

4. TRUNK LINES

In a trunk line the power loss (ΔP_t) of a fed zone is n times that of a zone sector (ΔP_{ts}), n being the number of equal sectors in this zone. The radial elementary area dA of such a sector is (see Fig. 3):

$$dA = \lambda dR = r\alpha_s dr. \tag{14}$$

The current taken from a unit area dA is:

$$di = \sigma dA = \sigma\alpha_s r dr, \tag{15}$$

where α_s is the angle of the sector;
σ is the current density of the consumer's load.

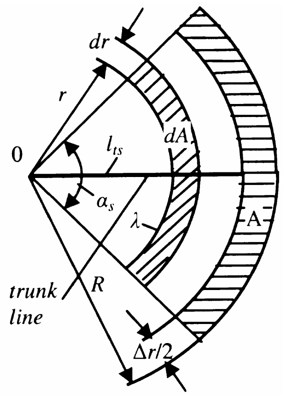

Fig. 3. Losses in a trunk line.

The current in the trunk line at distance r from the transformer unit at the point 0 is:

$$i_r = \sigma\alpha_s \int_r^R r dr = \frac{\sigma\alpha_s}{2}(R^2 - r^2). \quad (16)$$

The elementary power loss in this trunk line at the same distance from the transformer unit is:

$$d\Delta P_{ts} = i_r^{\,2} R_{0t} dr = \frac{\sigma^2 \alpha_s^{\,2}}{4}(R^2 - r^2)^2 dr, \quad (17)$$

where R_{0t} is the specific active resistance of a trunk line.

Integrating from 0 to R we obtain:

$$\Delta P_{ts} = \frac{2\sigma^2 \alpha_s^{\,2} R_{0t} R^5}{15}. \quad (18)$$

Applying the dependences:

$$\alpha_s = \frac{2\pi}{n}; \quad I_{\max} = \sigma\pi R^2; \quad R_{0t} = \frac{1}{\gamma_t F_t}; \quad F_t = \frac{I_{\max}}{n j_t}, \quad (19)$$

where $I_{\max}$ is the maximum current of a transformer unit;
γ_t, F_t are the specific conductance and cross-section area of the trunk line wire, respectively;
j_t is the current density in the trunk line wire,
we obtain a concise mathematical formula for the trunk line loss of one phase in the sector:

$$\Delta P_{ts} = \frac{8 I_{\max} j_t R}{15 n \gamma_t}. \quad (20)$$

The losses in three phases in the entire fed zone will be $3n$ times greater, i.e.:

$$\Delta P_{tci} = \frac{24 I_{\max} j_t R}{15\gamma_t} = \frac{24\sigma j_t \pi R^3}{15\gamma_t} = 5{,}0272\frac{j_t \sigma R^3}{\gamma_t}. \quad (21)$$

The trunk line extends from 0 to $R - \Delta r/2$. Of the losses, according to formula (18), the loss of the part not reached by the trunk line is to be subtracted. The sector current at a distance $R - \Delta r/2$ taken from area A (i.e. peripheral current irper) is:

$$i_{rper} = \sigma A \approx \sigma\alpha_s (R - \Delta r/4)\frac{\Delta r}{2}, \quad (22)$$

The loss of a sector's peripheral area A is:

$$\Delta P_{tsper} = \frac{1}{3} i_{rper}^{\,2} R_{0t} \frac{\Delta r}{2} = \frac{\sigma^2 \alpha_s^{\,2} (R - \Delta r/4)^2 R_{0t} \Delta r^3}{24}. \quad (23)$$

Comparative calculations by formulas (21) and (23) have shown that the peripheral losses are insignificant. Hence, the trunk loss should be calculated by formula (21).

The length of one trunk line is $l_{ts} = R - \Delta r/2$, while that of all trunk lines of a zone is:

$$l_t = n(R - \Delta r/2) = nRk_t\,, \tag{24}$$

where k_t is a trunk coefficient:

$$k_t = \frac{R - \Delta r/2}{R}\,. \tag{25}$$

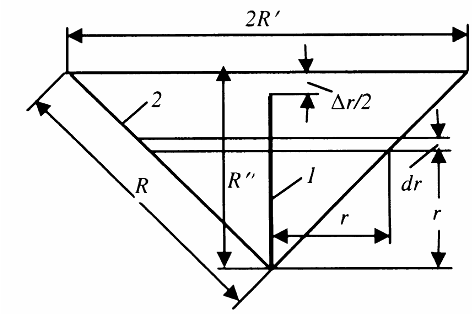

Fig. 4. Trunk losses in a sector of 1/4 square; *1* – trunk line; *2* – sector.

In the case of a square it is natural to have four sectors, i.e. n = 4. The elementary current of this sector (see Fig. 4) is:

$$i_r = \sigma \int_r^{R'} 2r dr = \sigma(R'^2 - r^2)\,. \tag{26}$$

The elementary loss is:

$$d\Delta P_{ts} = i_r^{\,2} R_{0t} dr = \sigma^2 (R'^2 - r^2)^2 R_{0t} dr \tag{27}$$

The sector loss will be:

$$\Delta P_{ts} = \int_0^{R'} d\Delta P_{ts} = \frac{8}{15}\sigma^2 R_{0t} R'^5\,. \tag{28}$$

The square zone consists of four such sectors (n = 4), hence:

$$R' = \frac{R}{\sqrt{2}}\,;\, I_{\max} = \sigma A_{sq} = 2\sigma R^2\,; \qquad \frac{I_{\max}}{n} = \frac{I_{\max}}{4} = \frac{1}{2}\sigma R^2\,;$$

$$F_t = \frac{I_{\max}}{4 j_t} = \frac{\sigma R^2}{2 j_t}\,; \qquad R_{0t} = \frac{1}{\gamma_t F_t} = \frac{2 j_t}{\gamma_t \sigma R^2}\,. \tag{29}$$

Therefore for three phases of the entire zone (multiplier 3×4 = 12) we obtain:

$$\Delta P_{tsq} = 2.2627 \frac{j_t \sigma R^3}{\gamma_t}\,. \tag{30}$$

The line lengths are found in a similar manner.

5. BRANCH LINES

Figure 5 shows the area under consideration with continuously distributed load. Current Δi_s at the beginning of cross-hatched strip $y\Delta x$ at distance x from the zone centre O is:

$$\Delta i_s = \sigma y(x)\Delta x\,. \qquad (31)$$

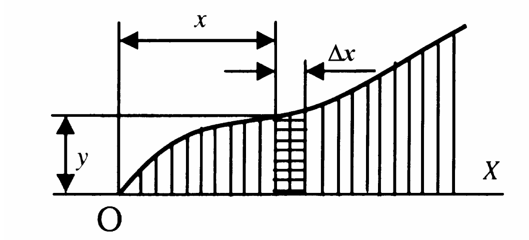

Fig. 5. Definition of branch losses at continuously distributed load.

The active resistance of strip $y\Delta x$ is:

$$R_{\Omega s} = \frac{y(x)}{\gamma_b F_s} = \frac{y(x)}{\gamma_b a\Delta x}\,, \qquad (32)$$

where γ_b is the specific conductance of branch line wire;
$a\Delta x$ is the cross-section area of the strip; a is an as yet unknown coefficient.

The losses ΔP_s of the strip are:

$$\Delta P_s = \frac{1}{3}\Delta i_s{}^2 R_{\Omega s} = \frac{1}{3}\sigma^2 y^2(x)\Delta x^2 \frac{y(x)}{\gamma_b a\Delta x} = \frac{1}{3}\frac{\sigma^2 y^3(x)\Delta x}{\gamma_b a}\,. \qquad (33)$$

If such strips cover the entire area, the total losses will be:

$$\Delta P \approx \sum \Delta P_s = \frac{1}{3}\sum_{i=1}^{m}\frac{\sigma^2 y_i{}^3 \Delta x_i}{\gamma_b a}\,. \qquad (34)$$

Consequently, we can write the exact expression for losses as

$$\Delta P_b = \frac{1}{3}\frac{\sigma^2}{\gamma_b a}\int_0^{x_{\max}} y^3(x)dx\,. \qquad (35)$$

At the maximum length $y_{\max}$, current $i_{s\max}$ through the strip (adx) at its beginning is:

$$i_{s\max} = \sigma y_{\max}(x)dx\,. \qquad (36)$$

The strip cross-section area is:

$$adx = \frac{\sigma y_{\max} dx}{j_b}\,, \qquad (37)$$

where j_b is the adopted maximum current density in the wires of branch lines.

Hence, coefficient a will be:

$$a = \frac{\sigma y_{\max}}{j_b} \tag{38}$$

and the losses of the entire shaded area:

$$\Delta P = \frac{1}{3} \frac{\sigma j_b}{\gamma_b y_{\max}} \int_0^{x_{\max}} y^3(x) dx \,. \tag{39}$$

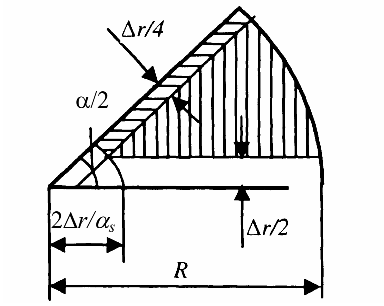

Fig. 6. Losses in branch lines.

Now, a half of the sector (Fig. 6) of a circular fed zone will be considered. The consumers of the shade-free horizontal fragment are fed from a trunk line, while those of shaded areas are fed from a branch line. However, the branch line collects the load from the shade-free horizontal fragment to the sector's radial boundary. Hence for the shaded areas:

$$y = \frac{\alpha_s}{2} r - \frac{\Delta r}{2} = \frac{1}{2}(\alpha_s r - \Delta r)\,; \tag{40}$$

$$y_{\max} = y(x_{\max}) = \frac{\alpha_s}{2}(R - \frac{\Delta r}{2}) = \frac{\pi}{n} R k_R \,. \tag{41}$$

The integration should be done from $2\Delta r/\alpha_s$ to R. For this sector shape, observing (39)–(41), we obtain the branch losses of shaded areas per phase of the entire sector as

$$\Delta P_{bsh} = \frac{\sigma j_b}{6 \gamma_b \alpha_s R k_t} \int_{\frac{2\Delta R}{\alpha_s}}^{R} (\alpha_s r - \Delta r)^2 (\alpha_s r - \Delta r) dr \,. \tag{42}$$

The losses of the shade-free fragment are equal to the square of current at the beginning of shaded area multiplied by the resistance over $\Delta r/2$ length:

$$\Delta P_{bus} = \frac{1}{6} \frac{\sigma j_b}{\gamma_b \alpha_s R k_t} \int_{2\Delta r/\alpha_s}^{R} (\alpha_s r - \Delta r)^2 3 \Delta r dr \,. \tag{43}$$

Factor 3 before Δr means that at $\Delta r/2$ of the shade-free area the current is constant, since here the distributed load is connected not to a branch line but to the trunk one.

The branch line extends to $(\alpha_s/2)r - \Delta r/4$. Hence, losses ΔP_{bhs} of the horizontally shaded area of the sector are to be subtracted from losses $(\Delta P_{bsh} + \Delta P_{bus})$, i.e.:

$$\Delta P_{bhs} = 2(\frac{1}{3}\frac{\sigma j_b}{\gamma_b \frac{\alpha_s}{2} Rk_t}\int\limits_{2\Delta r/\alpha_s}^{R}(\frac{\Delta r}{4})^3 dr = \frac{1}{6}\frac{\sigma j_b}{\gamma_b \alpha_s Rk_t}\frac{1}{8}\int\limits_{2\Delta r/\alpha_s}^{R}\Delta r^3 dr\,. \tag{44}$$

Summing up and observing (19) for α_s, we will have the sector branch losses as

$$\Delta P_{bs} = \frac{\sigma j_b}{6\gamma_b Rk_t}(\frac{\pi^2 R^4}{n^2} - \frac{3\Delta r^2 R^2}{2} + \frac{1{,}875\Delta r^3 Rn}{2\pi} - \frac{1{,}875\Delta r^4 n^3}{8\pi^3})\,. \tag{45}$$

A zone has 3 phases and n sectors, therefore:

$$\Delta P_b = \frac{\sigma j_b}{2\gamma_b k_t}(\frac{\pi^2 R^3}{n} - \frac{3\Delta r^2 Rn}{2} + \frac{1.875\Delta r^3 n^2}{2\pi} - \frac{1.875\Delta r^4 n^4}{8\pi^3 R}) =$$
$$= \frac{j_b}{2\gamma_b k_t}(\frac{I_{\max}\pi R}{n} - \frac{3I_{\max}\Delta r^2 n}{2\pi R} + \frac{1.875 I_{\max}\Delta r^3 n^2}{2\pi^2 R^2} -$$
$$- \frac{1.875 I_{\max}\Delta r^4 n^4}{8\pi^4 R^3})\,. \tag{46}$$

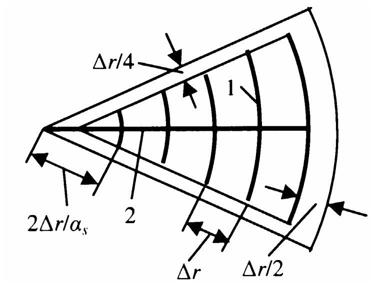

Fig. 7. View of a fed zone sector: *1* – branch lines; *2* – trunk line.

Branch lines are arranged beginning from $2\Delta r/\alpha_s$ up to $R - \Delta r/2$, and do not reach the radial boundary by the value of $\Delta r/4$ (Fig. 7). The number of branch lines in a sector is:

$$m = \frac{R - \Delta r/\alpha_s - \Delta r/2}{\Delta r} = \frac{R - (\Delta r(n+\pi))/2\pi}{\Delta r} = \frac{Rk_b}{\Delta r}\,, \tag{47}$$

where k_b is a branch coefficient:

$$k_b = \frac{R - (\Delta r(n+\pi))/2\pi}{R}\,. \tag{48}$$

If these lines were extending to the sector's radial boundary, on the entire zone scale there would be m concentric circles with the radii determined by an arithmetic series with base Δr and number m. However, beginning with the first all the consecutive circles have a radial increase of approx. Δr. Then the total length l_b' of branch lines will be:

$$l' = 2\pi\Delta r[\frac{m(1+m)}{2} + m] = 3\pi Rk_b + \pi\frac{R^2 k_b{}^2}{\Delta r}. \qquad (49)$$

Each line does not reach the sector radial boundary by $\Delta r/4$. Then the total peripheral shortage l_{bper} in the zone will be:

$$l_{bper} = \frac{\Delta r}{2}mn = \frac{Rk_b n}{2}. \qquad (50)$$

Hence, the total branch line length of a fed zone is:

$$l_b = l_b{}' - l_{bper} = \frac{\pi R^2 k_b{}^2}{\Delta r} + Rk_b(3\pi - \frac{n}{2}) = Rk_b(\frac{\pi Rk_b}{\Delta r} + 3\pi - \frac{n}{2}). \qquad (51)$$

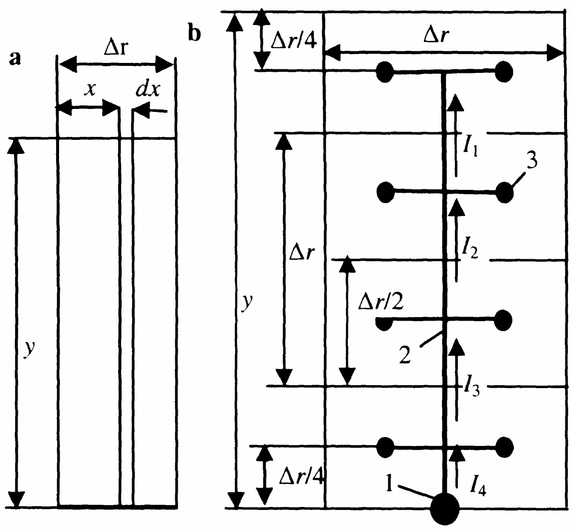

Fig. 8. A striped fed zone: *a* – analytical version; *b* – real version;
1 – transformer unit; *2* – branch line; *3* – consumer.

In the example below, the losses of branch strip (Fig. 8*a*) are calculated:

$$di_s = \sigma y dx; \qquad R_{\Omega s} = \frac{y}{\gamma_b a dx}; \qquad a = \frac{\sigma y}{j_b};$$

$$d\Delta P_s = \frac{1}{3}di_s{}^2 R_{\Omega s} = \frac{1}{3}\frac{\sigma j_b y^2 dx}{\gamma_b}. \qquad (52)$$

Three-phase losses of the strip in Fig. 8*a* are:

$$\Delta P_s{}' = \frac{\sigma j_b y^2}{\gamma_b}\int_0^{\Delta r} dx = \frac{\sigma j_b y^2 \Delta r}{j_b}. \qquad (53)$$

From quantity $\Delta P_s'$ the peripheral loss ΔP_{sper} is to be subtracted.

The elementary peripheral loss is:

$$d\Delta P_{sper} = \frac{1}{3} di_{sper}{}^2 R_{\Omega sper} = \frac{\sigma j_b \Delta r^3 dx}{192\gamma_b y}. \tag{54}$$

Since peripheral loss can be obtained as

$$\Delta P_{sper} = \frac{\sigma j_b \Delta r^3}{192\gamma_b y} \int_0^{\Delta r} dx = \frac{\sigma j_b \Delta r^4}{192\gamma_b y}, \tag{55}$$

the strip losses ΔP_s are:

$$\Delta P_s = \Delta P_s' - \Delta P_{sper} = \frac{\sigma j_b y^2 \Delta r}{j_b} - \frac{\sigma j_b \Delta r^4}{192\gamma_b y}. \tag{56}$$

6. COMPARISON WITH THE DISCRETE LOAD MODEL

So far the consideration has concerned a continuously distributed load. We shall calculate the same quantities (losses and line lengths) in the discrete-load model with a hexagon (Fig. 9) and a square (Fig. 10). Both the models with discrete loads have the same radius (see Fig. 1) of 297 m and the load density $\sigma = 0.00118$ A/m^2. Each discrete load is situated in the centre of a 30×30 m square,

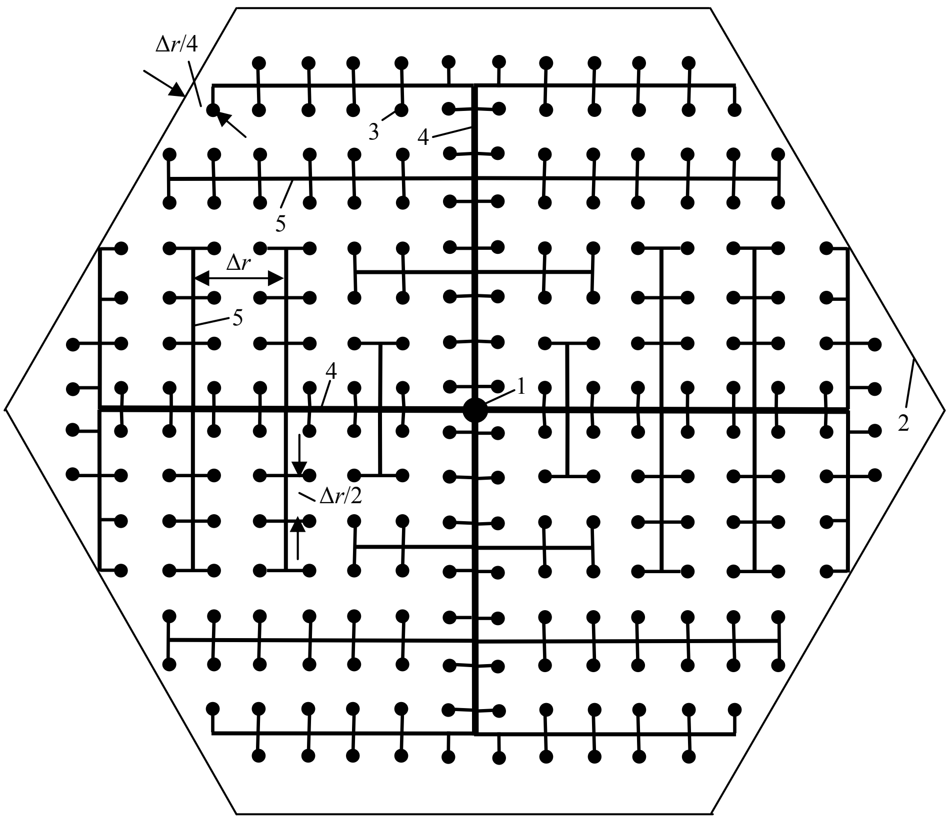

Fig. 9. Example of a hexagonal fed zone; *1* – medium/low voltage transformer; *2* – zone boundary; *3* – electricity consumer; *4* – trunk line; *5* – branch line.

hence the load current (that of a single consumer) is i_{dl} = 0.00118×30×30 = 1.062 A, while Δr = 60 m, $\Delta r/2$ = 30 m, $\Delta r/4$ = 15 m. The current density of aluminium (specific conductance γ = 32 A/mm^2) phase wires in all models is $j_t = j_b$ = 1 m/(Ω·mm^2).

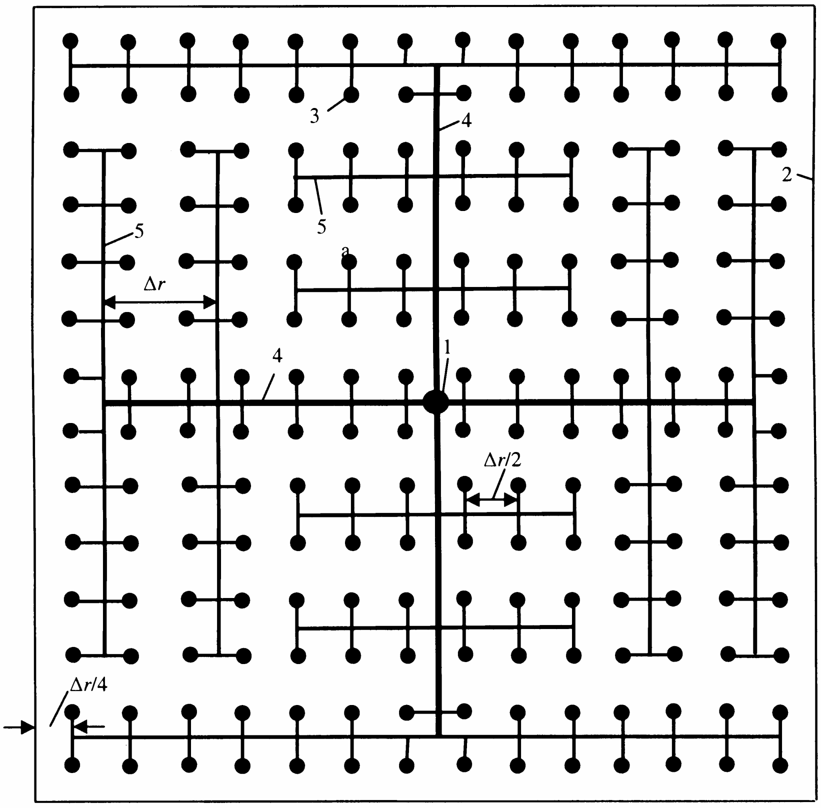

Fig. 10. Example of a square fed zone; *1* – medium/low voltage transformer; *2* – zone boundary; *3* – electricity consumer; *4* – trunk line; *5* – branch line.

To show the way in which losses in hexagonal and squared zones with discrete loads are handled, we shall calculate the losses in the strip model according to Fig. 8, assuming the length to be y = 120 m. From the beginning, the cross-section area of the strip model phase wires was taken F = 10 mm^2. The current density in the phase wires is $j_{ob} = I_{\max}/F$ = 0.00118·120·60/10=0.85 A/mm^2.

By virtue of (53) the strip losses are: $\Delta P_s'$ = 0.00118·0.85·120^2·60/32 = 27.081 W; according to (55), the peripheral losses are ΔP_{sper} = 0.00118·0.85·60^4/(192·32·120) = 0.0176 W, and according to (56) we have: ΔP_s = 27.061 – 0.0176 = 27.0.63 W.

Now, we shall calculate the losses of discrete consumers. The phase wire resistivity is: $R_{\Omega/m} = 1/(\gamma F) = 1/(32 \cdot 10) = 0.003125$ Ω/m. For three-phase losses, when the current of one phase is used for loss calculations, the resistance of a phase wire should be taken three times greater. Then the resistance of a $\Delta r/4$ long wire will be $R_{\Delta r/4}$ = 3·0.003125·15 = 0.140625 Ω; other resistances are: $R_{\Delta r/2}$ = 3·0.003125·30 = 0.28125 Ω; $R_{3\Delta r/4}$ = 3·0.003125·45 = 0.421875 Ω. The currents in the branch line of Fig. 8*b* are: I_1 = 2·1.062 = 2.124; I_2 = 4.248; I_3 = 6.372; I_4 = 8.496 A; the losses due to these currents are: $\Delta P_1 = I_1^2 \cdot R_{\Delta r/2} = 2.124^2 \cdot 0.28125$ = 1.2688; ΔP_2 = 5.0753; ΔP_3 = 11.4194; ΔP_4 = 10.1506 W. The total losses of a strip with discrete load: ΔP_{sdl} = 27.914 W, the discrepancy with ΔP_s being 3%.

If the losses should be recalculated for another current density or cross-section area, a new value of specific resistance $R_{\Omega/m}'$ is to be determined and the resistance factor k_Ω $R_{\Omega/m}'/R_{\Omega/m}$ calculated; the new value of losses is the product of this factor and the previous value of losses. For example, if j_b' = 1 A/mm^2, then $R_{\Omega/m}' = 1/(\gamma F')$; hence $F' = I_{\max}/j_b'$ = 0.00118·120·60/1 = 8.496 mm^2. This is a hypothetical value; however, in the analysis we shall adopt it in the model calculations. We will thus have: $R_{\Omega/m}'$=1/(32·8.496)=0.0036782, and the resistance factor $k_\Omega = R_{\Omega/m}'/R_{\Omega/m}$ = 0.0036782/0.003125 = 1.177. New loss values will be: ΔP_s = 27.063·1.177=31.853; ΔP_{sdl} = 27.914·1.177=32.855.

To develop the final expression for *pA* losses of the network in a circular model, we shall write, observing (19), the formulas in the *pA* notation for the corresponding quantities as

$$\Delta P_{tcipA} = \frac{24 j_t R}{15\gamma_t}; \quad l_{tpA} = \frac{l_t}{\sigma\pi R^2} = \frac{nk_t}{\sigma\pi R}; \quad (F_t l_t)_{pA} = \frac{Rk_t}{j_t};$$
$$\Delta P_{bpA} = \frac{j_b}{2\gamma_b k_R}\left(\frac{\pi R}{n} - \frac{3\Delta r^2 n}{2\pi R} + \frac{1.875\Delta r^3 n^2}{2\pi^2 R^2} - \frac{1.875\Delta r^4 n^4}{8\pi^4 R^3}\right); \tag{57}$$
$$l_{bpA} = \frac{k_b}{\sigma}\left(\frac{k_b}{\Delta r} + \frac{3\pi - n/2}{\pi R}\right); \quad (F_b l_b)_{pA} = \frac{k_b k_t \pi R \Delta r}{n j_b}\left(\frac{k_b}{\Delta r} + \frac{3\pi - n/2}{\pi R}\right).$$

Applying (57) to (9), we obtain the following expressions for the pA network costs:

$$C_{ntpA} = \frac{C_a n k_t}{\sigma\pi R} + \frac{C_a k_b}{\sigma}\left(\frac{k_b}{\Delta r} + \frac{3\pi - n/2}{\pi R}\right) + \frac{C_b R k_t}{j_t} + \frac{C_b k_b k_t \pi R \Delta r}{n j_b} \cdot$$
$$\cdot\left(\frac{k_b}{\Delta r} + \frac{3\pi - n/2}{\pi R}\right) + \frac{24 C_w j_t R}{15\gamma_t} +$$
$$+ \frac{C_w j_b}{2\gamma_b k_t}\left(\frac{\pi R}{n} - \frac{3\Delta r^2 n}{2\pi R} + \frac{1.875\Delta r^3 n^2}{2\pi^2 R^2} - \frac{1.875\Delta r^4 n^4}{8\pi^4 R^3}\right). \tag{58}$$

The results of calculations are shown in Table 1.

Table 1

Comparison of parameters for differently shaped fed zones

Fed zone	Load distribution	l_{tpA}, m/A	l_{bpA}, m/A	ΔP_{tpA}, W/A	ΔP_{bpA}, W/A
Circle	Continuous	3.266	13.578	14.850	3.711
Hexagon	Discrete	3.591	12.688	11.384	4.729
Square	Discrete	3.459	11.818	10.864	4.227

Losses in a circle are computed by formulas (57); those in a hexagon and a square are calculated in the manner shown for a strip. The greatest discrepancy is for per ampere (pA) losses in trunk lines; presumably, it is due to the maximum current irregularities in branch lines. The branch pA losses show the same tendency as for the strip. The maximum current ratios are close to the area ones, with a considerable discrepancy for a hexagon due to the free space along its boundaries (Fig. 9). On the whole, the circle model (Eq. (58)) could admittedly be used to reveal the influence of various factors on the network efficiency for other zone shapes.

To optimize the efficiency, we can vary zone radius R, while n, Δr and σ are conditioned by the territory planning. Other parameters are determined by the wire material, the type of consumers, and the economic factors. Varying radius R, we must abide by the main constraint: the maximum voltage loss that in a low-voltage network is equal to the voltage drop across the active resistance of a trunk wire ($U_{\Delta t}$) and a branch wire ($U_{\Delta b}$).

The voltage drop across the trunk line up to the circle boundary, observing (16) and (19), will be:

$$U'_{\Delta t} = \int_0^R i_r R_{0t} dr = \frac{j_t}{\gamma_t R^2} \int_0^R (R^2 - r^2) dr = \frac{2 j_t R}{3\gamma_t}; \tag{59}$$

and the peripheral voltage drop:

$$U_{\Delta tper} = \frac{j_t}{\gamma_t R^2} \int_{Rk_t}^R (R^2 - r^2) dr = \frac{j_t R}{\gamma_t} [\frac{2}{3} - k_t (1 - \frac{k_t^2}{3})]. \tag{60}$$

In turn, the maximum voltage loss on a trunk line is:

$$U_{\Delta t} = U'_{\Delta t} - U_{\Delta tper} = \frac{j_t R}{\gamma_t} k_t (1 - \frac{k_t^2}{3}); \tag{61}$$

and that of a branch line up to the radial boundary:

$$U'_{\Delta b} = \frac{1}{2} i_{b\max} R_{0b} l'_{b\max}. \tag{62}$$

The maximum length up to the radial boundary of branch line and its cross-section will be:

$$l'_{b\max} = \frac{\pi}{n} R k_t; \qquad F_b = \frac{i_{b\max}}{j_b}. \tag{63}$$

The maximum branch current and specific branch line resistance are:

$$i_{b\max} = \sigma A_b = \sigma l'_{b\max} \Delta r = \sigma \frac{\pi}{n} R k_t \Delta r;$$

$$R_{0b} = \frac{1}{\gamma_b F_b} = \frac{j_b}{\gamma_b i_{b\max}} = \frac{\sigma \pi R k_t \Delta r}{n j_b}; \tag{64}$$

$$U'_{\Delta b} = \frac{j_b \pi R k_t}{2 n \gamma_b}. \tag{65}$$

A real branch line does not reach the radial boundary by $\Delta r/4$ (Fig. 7), hence:

$$U_{\Delta bper} = \frac{1}{2} i_{b\Delta r/4} R_{0b} \frac{\Delta r}{4} = \frac{j_b n \Delta r^2}{32\gamma_b \pi R k_t}. \tag{66}$$

The maximum voltage loss on a branch line is:

$$U_{\Delta b} = U'_{\Delta b} - U_{\Delta bper} = \frac{j_b \pi R k_t}{2n\gamma_b} - \frac{j_b n \Delta r^2}{32\gamma_b \pi R k_t}, \tag{67}$$

and the maximum voltage loss in a fed zone:

$$U_\Delta = \frac{j_t R}{\gamma_t} k_t (1 - \frac{k_t^2}{3}) + \frac{j_b \pi R k_t}{2n\gamma_b} - \frac{j_b n \Delta r^2}{32\gamma_b \pi R k_t}. \tag{68}$$

From (68), the maximum R satisfying the admissible voltage loss U_Δ is:

$$R = \frac{U_\Delta + \sqrt{U_\Delta^2 + 4[\frac{j_t(1-k_t^2/3)}{\gamma_t} + \frac{j_b \pi}{2n\gamma_b}]\frac{j_b n \Delta r^2}{32\gamma_b \pi}}}{2[\frac{j_t(1-k_t^2/3)}{\gamma_t} + \frac{j_b \pi}{2n\gamma_b}} + \frac{\Delta r}{2}. \tag{69}$$

The first value of R is calculated with assumed k_t, whereas more exact its values are calculated introducing the trunk coefficient k_t obtained from (25).

In expression (58), the variable quantities are R, j_t, j_b, since distance Δr is conditioned by the dislocation of consumers, whereas the number n of sectors − by the territory planning. Constants C_a, C_b, C_w are determined by the existing technical and economic conditions. The influence of various factors can be elucidated analyzing expression (58) − if not mathematically then by the case calculations, since the influence of some quantities is very intricate.

According to the authors of [6], building of a 1 km 0.4 kV line with insulated aluminium wires costs ~10000 LVL, since K_0 = 10 LVL/m. In [4] this quantity is given as $K_0 = a+bF$, with a and b defined according to (5). The cross-section of a phase wire could be taken 50 mm^2, which according to [7] costs ~2 LVL/m. Hence bF = 2 LVL/m, b = 2/50 = 0.04 LVL/(m·mm^2) = 40000 LVL/(m·m^2); $a = K_0 - bF$ = 10–2 = 8 LVL/m. It is supposed that i = 10–14%, $p_\Sigma \approx$ 4%; the assumed values: i = 12%, p_Σ = 4%, β' = 0.000033 LVL/Wh, β'' = 0.00365 LVL/W. By (6), C_a = 1.28 LVL/m; C_b = 0.0064 LVL/(m·mm^2) = 6400 LVL/(m·m^2); C_w = 0.10265 LVL/W. The remaining quantities are: σ = 0,00118 A/m^2; n = 4; Δr = 60 m; U = 400 V; U_Δ = 0.05·U/1.732=11.5 V; $j_t = j_b = 1\cdot 10^6$ A/m^2; $\gamma_t = \gamma_b = 32\cdot 10^6$ m/(Ω·m^2); k_t is found from (25), and k_b − from (48); τ = 3000 h; T_m = 4600 h; C_{nwpA}, C_{nwpu} are found from (58) and (10), respectively.

The end results show that pA and pu values are slightly decreasing with zone radius: at R = 361 m, C_{nwpu} = 0.01046 LVL/kW; 250 m – 0.00971; 150 m – 0.00885, all the three results being obtained for $j_t = j_b$ = 1 A/mm^2. The result is quite understandable: the smaller the fed zone radius the less are losses in phase wires. When optimized for admissible voltage loss of 11.5 V, radius R_{con} is determined from (69). Hence, we can change the radius by changing the current densities j_t and j_b. Here we can see an inverse dependence with respect to the current density: at $j_t = j_b = 1.5$ A/mm^2 we will have $R_{con} = 246.6$ m, $C_{nwpu} = 0.01$ LVL/kWh, while at $j_t = j_b = 2$ A/mm^2 − $R_{con} = 190$ m and $C_{nwpu} = 0.00889$ LVL/kWh.

The inference is: a low-voltage network should be optimized together with its transformer unit.

The transformer unit cost K_{tu} consists of the cubicle cost K_{cu} and the transformer cost K_{tr}. According to [6], $K_{tu} \approx 20000$ LVL for a unit with one transformer.

Currently, three medium/low voltage transformers are available at the Latvian Branch of International Electro-Technical Concern ABB:

	S_{tr}, kVa	Price, LVL+VAT	Load loss ΔP_{ld}, W	No-load loss ΔP_{nl}, W
1)	25	1765	790	160
2)	40	1931	1300	160
3)	63	2145	1800	240

Hence, the transformer cost (VAT included) can be modelled as

$$K_{tr} = 1840 + 0.0117 S_{tr}\,. \tag{70}$$

If the assumed power of a transformer in the transformer unit is 40 kVA (1931+VAT≈2310 LVL), the cost of a cubicle itself is: 20000-2310=17690 LVL. The transformer unit cost, irrespective of the transformer capacity sharing, is: 17690+1840=19530 LVL. Therefore, this cost (VAT included) can be modelled as

$$K_{tu} = 19530 + 0.0117 S_{tr}\,. \tag{71}$$

The load losses and no-load losses of a transformer can be determined as functions of its capacity [8]:

$$\Delta P_{ld} = \kappa_{ld} S_{tr}^{\;3/4}\;; \quad \Delta P_{nl} = \kappa_{nl} S_{tr}^{\;3/4}\,. \tag{72}$$

For the transformers under consideration the factors κ_{ld} and κ_{nl} are:

$$\kappa_{ld} = \frac{\Delta P_{ld}}{\sqrt[4]{S_{tr}^{\;3}}}\;;\; \kappa_{nl} = \frac{\Delta P_{nl}}{\sqrt[4]{S_{tr}^{\;3}}}\,. \tag{73}$$

To evaluate these factors, a 63 kVA transformer was taken:

$$\kappa_{ld} = 1800 / \sqrt[4]{63000^3} = 0.4525 \text{ W/(VA)}^{3/4};$$

$$\kappa_{nl} = 240 / \sqrt[4]{63000^3} = 0.0603\,. \tag{74}$$

Now, observing (71)–(74), we can rewrite (11) as

$$C_{tu} = (i + p_\Sigma)195.3 + 0.000117(i + p_\Sigma) S_{tr} + \\ + 0.4525 C_w S_{tr}^{\;3/4} + 0.0603 C_{wnl} S_{tr}^{\;3/4}\,. \tag{75}$$

Assuming the transformer capacity to correspond to the maximum load current ($S_{tr} = \sqrt{3}UI_{\max}$) we can write:

$$C_{tu} = 195.3(i + p_\Sigma) + 0.08104 I_{\max} + 61.106 C_w \sqrt[4]{I_{\max}^3} +$$
$$+ 8.143 C_{wnl} \sqrt[4]{I_{\max}^3};$$
$$C_{tupA} = \frac{62.168(i + p_\Sigma)}{\sigma R^2} + 0.08104(i + p_\Sigma) + \frac{45.899 C_w}{\sqrt[4]{\sigma}\sqrt{R}} + \frac{6.116 C_{wnl}}{\sqrt[4]{\sigma}\sqrt{R}}; \quad (76)$$
$$C_{tupu} = \frac{1}{T_m \cos\varphi}[\frac{0.08973(i + p_\Sigma)}{\sigma R^2} + 0.0001169(i + p_\Sigma) +$$
$$+ \frac{0.06625 C_w}{\sqrt[4]{\sigma}\sqrt{R}} + \frac{0.008829 C_{wnl}}{\sqrt[4]{\sigma}\sqrt{R}}].$$

Using (58) and (76), we can search for the minimum costs C_Σ by (13), varying radius R and current density j, with due regard for limitations by (69). It would be simpler to reach the target by a case study, i.e. substituting in (13) proper values of current density j and radius R. Studying (13) shows that per unit costs C_Σ strongly depend on the load density σ (grow with σ decreasing). More detailed dependences can be seen from Table 2, where R_{con}, R_{opt}, R_{ado} are, respectively: the radius in view of (69), the optimized (by the least costs) radius, and that adopted observing (69); the current densities being $j_t = j_b = j$.

Table 2

The case study results

σ, MVA/km^2	Δr, m	j, A/mm^2	R_{con}, m	R_{opt}, m	R_{ado}, m	S_{tr}, MVA	$C_{\Sigma pu}$, LVL/kWh
15	60	1.2	303	190	190	1.7	0.00275
5	60	1.2	304	270	270	1.145	0.00433
1	60	0.8	448	455	448	0.63	0.011
0.1	60	0.5	707	900	707	0.157	0.069
0.1	200	0.6	633	950	633	0.126	0.0305

7. CONCLUSIONS

1. Although other than hexagonal fed zone shapes can be met in practice, for the analysis more convenient is a circular fed zone.
2. The line lengths, load losses, and the maximum current of a hexagonal or a squared fed zone can be calculated as approximately proportional to those of a circular fed zone.
3. The radius of a fed zone, the capacity of a medium/low voltage transformer and, to a lesser degree, the current density in line conductors depend on several factors − first of all on the load density. They should be adapted to local conditions by the least annual costs criterion.
4. The analysis has revealed the interdependence of the main parameters of a low-voltage network.

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REFERENCES

1. Szodenyi, A. (2004). Optical filter type influence on transparent WDM network's size. *Hiradastechnika,* (12), 55–58.
2. Venghaus, H. (2006). *Wavelength Filters in Fibre Optics.* Berlin: Springer, 454.
3. Azadeh, M. (2009). *Fiber Optics Engineering*. New York: Springer, 374.
4. Agrawal, G. (2001). *Nonlinear Fiber Optics (3rd ed-n).* London: Academic Press, 466.
5. Ozoliņš, O., & Ivanovs, Ģ.. (2010). Evaluation of Band-Pass Filters Influence on NRZ Signal in HDWDM Systems. *Lithuanian Journal of Electronics and Electrical Engineering.* **4(100)** 65–68.
6. Cisco Systems (2008). Cisco Visual Networking Index – Forecast and Methodology 2007–2012. *White paper,* (1), 1–15.
7. Voges, E., & Peteramann, K. (2002) *Optische Kommunikationstechnik – Handbuch für Wissenschaft un Industrie.* Berlin: Springer-Verlag, 1072.
8. Lyubomirsky, I. (2007). Advanced modulation Formats for Ultra-Dense Wavelength Division Multiplexing. *White paper,* (1), 1–14.
9. Agrawal, G. P. (2002) *Fiber Optic Communication Systems (3rd ed-n).* New York: John Wiley and Sons, 546.
10. *OptSim 5.0 User Guide* http://www.rsoftdesign.com/
11. Bobrovs, V., & Ivanovs, G. (2008). Investigation of Mixed Data Rate and Format Transmission in WDM Networks. *Lithuanian Journal of Electronics and Electrical Engineering.* **4(84)** 63–66.
12. Ozoliņš, O., & Ivanovs, Ģ. (2009). Realization of Optimal FBG Band–Pass Filters for High Speed HDWDM. *Lithuanian Journal of Electronics and Electrical Engineering.* **4(92)** 41–44.

VIDSPRIEGUMA/ZEMSPRIEGUMA TRANSFORMATORU JAUDAS NOTEIKŠANA

J. Survilo

K o p s a v i l k u m s

Zemsprieguma tīklam kopā ar vidsprieguma/zemsprieguma transformatoru jāatbilst efektivitātes prasībām, piegādājot nepieciešamās kvalitātes elektroenerģiju patērētājiem. Izmaksās galvenokārt ietilpst ikgadējās maksas par transformatoru, zemsprieguma tīklu un par zudumiem tīklā un transformatorā. Šīs izmaksas un citi parametri ir atkarīgi no vairākiem faktoriem. Galvenais no tiem ir slodzes blīvums. Sešstūra apgādājamā zona pārklāj blīvi visu lielāku teritoriju un tai ir labākie citi rādītāji, salīdzinot ar taisnstūri un trīsstūri. Izskatīšana tika veikta uz ērtā šim nolūkam kanoniskā modeļa, kas ir apaļa apgādājamā zona ar nepārtrauktu slodzes blīvumu. Uz šo modeli var paļauties, kad izskata sešstūra vai kvadrātisku zonu ar diskrēto slodzi. Rezultātus, kas iegūti uz apļa zonas modeļa, var pārnest uz kvadrāta vai sešstūra zonu ar relatīvo koeficientu palīdzību. Transformatora jauda ir noteikta ar apgādājamās zonas rādiusa vērtību. Rādiusam ir nepārvarams ierobežojums – sprieguma novirze pie visattālākā patērētāja nedrīkst pārsniegt pieļaujamo vērtību. Šajos apstākļos nav daudz iespēju, lai optimizētu tīkla efektivitāti. Optimizāciju pie uzdotā slodzes blīvuma var panākt, mainot apgādājamās zonas rādiusu un strāvas blīvumu fāžu vados.

10.06.2010.