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ENHANCEMENT AND COMPARISON OF SIMPLE TYPES OF CLOSED NETWORKS

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As distinct from radial electric power lines, in high voltage networks in most cases consumers are fed from two sides. The advantages in this arrangement are: greater reliability of electricity supply, better power quality, and smaller active power losses. However, the losses could be reduced more fully only when the network is a uniform loop or a uniform network fed at two sides by equal EMFs. If these conditions are not complied with, circulating current (or equalizing current when EMFs at two sides are not equal) arises, and the active losses increase proportionally to its square, whereas voltage losses – proportionally to its first degree. To reduce losses in such cases, these currents should be eliminated. Irrespective of their cause, a booster transformer is mostly used for this purpose. If a non-uniform loop includes voltage of one standard rating, the circulating current can be eliminated by cheaper means than with a booster transformer: by inserting an induction coil into the loop. When the loop contains two standard voltage ratings, two transformers or autotransformers are needed. In this case, to eliminate circulating current, not only an induction coil should be inserted into the loop but also the transformer turn ratio is to be appropriately adjusted. When during a day the loads change approximately proportionally, the reactance of the induction coil may remain constant; however the turn ratio of a transformer should be changed in either case – whether the loads vary proportionally or not. When the network is not a loop but has independent EMFs at two sides, two quantities must be calculated: the magnitude of EMF difference at the ends of a network, and the inductance of the coil.

Key words: *circulating current, closed grids, network loop, non-uniform networks, quadrature booster transformer, power autotransformers, power losses, turn ratio.*

1. INTRODUCTION

For high-voltage (110 kV) energy distribution closed grids are used [1, 2], which form closed loops consisting of branches of the same standard voltage. The loops can include branches of other voltages connected via transformers or autotransformers. More specifically, a loop consisting mainly of 110 kV branches may include transmission lines of extra-high voltage (330 kV). The main reason for such an arrangement is higher reliability of the energy delivery; besides, the quality of energy is also higher. A disadvantage of closed loops including high-voltage and extra-high voltage power lines is, however, a loop's non-uniformity. This property is due to different reactance-to-resistance ratio (*X/R*) for power lines of different voltages; this is aggravated by the high reactance of autotransformers, which is the more expressed the greater the difference is between the rated powers of autotransformers included in the loop. Another disadvantage is the inequality of step-down (turn) ratios of these autotransformers. The difference of turn ratios of two autotransformers included in a loop manifests itself as an additional EMF inserted into the loop. Both factors cause a loop (circulating) current resulting in additional losses of the network. The third disadvantage of closed loops is that they need a more sophisticated relay protection. A network fed from two sides by independent EMFs is also the element of closed grids. In such a network (especially in a non-uniform one) the equalizing current arises if the EMFs are not equal. In turn, the equalizing current, similar to circulating current, also creates additional power losses. Closed grids can be met also in medium-voltage or even in low-voltage networks, although in the former mostly radial grids are used. The radial grids are free of the mentioned above adverse factors. However, in view of climate changes that cause frequent storms, on the one hand, and great farm complexes (for example, milk farms) which admit no interruption of electricity supply, on the other, a broader use of closed grids of medium- and low-voltage cannot be excluded. To prevent additional energy losses, appropriate measures should be taken to eliminate the circulating or equalizing current. One such measure is a quadrature booster whose voltage is connected in series with some branch [3, 4]. The inductance or capacitance can help as a measure in non-uniform grids [5, 6]. The corresponding issues regarding non-uniform grids are treated in more detail in [7]. In this paper, we consider possibilities to raise the accuracy of measures for eliminating the equalizing current in a double-terminal line and the circulating current in a loop possessing both the first and the second above mentioned disadvantages.

2. APPLICATION OF THE SECOND KIRCHHOFF LAW TO RINGED NETWORKS

For a non-uniform as well as for a uniform ringed network the following is valid (Fig. 1*a*):

$$\dot{Z}_1\dot{I}_1 + \dot{Z}_2\dot{I}_2 + ... + \dot{Z}_i\dot{I}_i + ... + \dot{Z}_{n+1}\dot{I}_{n+1} = 0 \ ; \ \dot{Z}_i = R_i + jX_i \, , \tag{1}$$

(for Z_i...I_i… see Fig. 1*a*).

For the uniform ringed network we can write (Fig. 1*b*):

$$\dot{Z}_{u,1}\dot{I}_{u,1} + \dot{Z}_{u,2}\dot{I}_{u,2} + ... + \dot{Z}_{u,i}\dot{I}_{u,i} + ... + \dot{Z}_{u,n+1}\dot{I}_{u,n+1} = 0 \, , \tag{2}$$

where $\dot{Z}_{u,i}$ are the impedances with the same ratio $X_i/R_i=k$ for all branches;
$\dot{I}_{u,i}$ is the branch current in a uniform loop.

Equation (2) can be rewritten as

$$\begin{aligned} &(1+jk)[R_1(I_{ua,1} + jI_{ur,1}) + ... + R_i(I_{ua,i} + jI_{ur,i}) + ... + , \\ &\quad + R_{n+1}(I_{ua,n+1} + jI_{ur,n+1})] = 0 \, , \end{aligned} \tag{3}$$

where $I_{ua,i}$; $I_{ur,i}$ are the active and reactive components, respectively, of branch current $\dot{I}_{u,i}$.

From (3) follows:

$$\sum_{i=1}^{n+1} R_i I_{ua,i} = 0 \ ; \qquad \sum_{i=1}^{n+1} R_i I_{ur,i} = 0 \, . \tag{4}$$

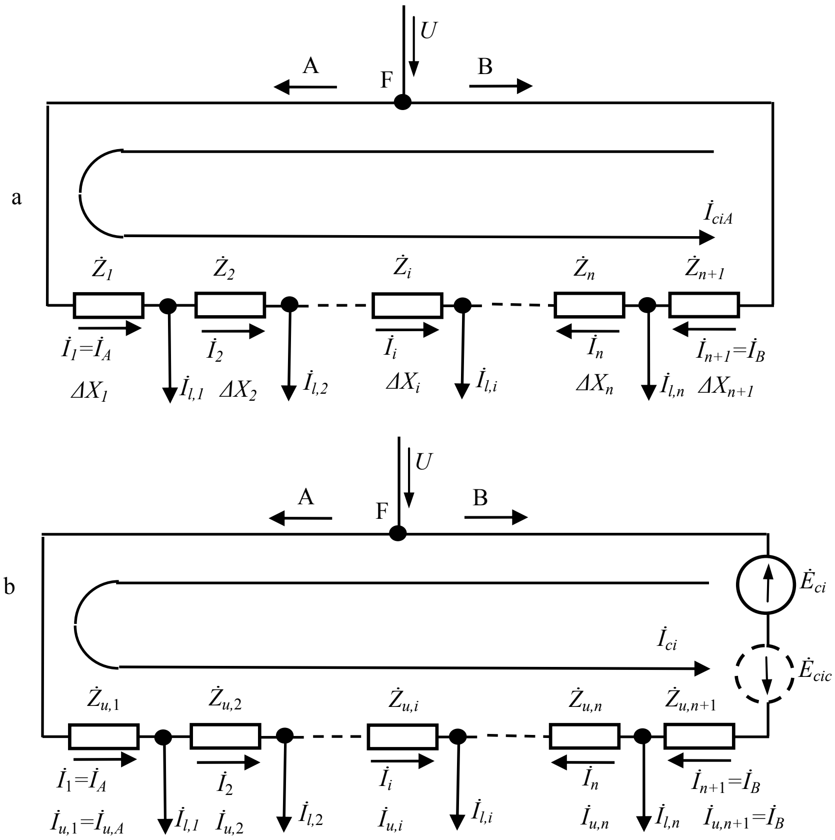

Fig. 1. Ringed network

a – non-uniform loop; *b* – substitute for non-uniform loop (uniform loop with singled out EMF $\dot{E}_{ci}$; when counter EMF $\dot{E}_{cic}$ is inserted, circulating current $\dot{I}_{ci}$ disappears.

$\dot{I}_{l,1}$ ... $\dot{I}_{l,n}$ – load currents; $\dot{I}_1$ ... $\dot{I}_{n+1}$ – branch currents; $\dot{I}_{u,1}$ ... $\dot{I}_{u,n+1}$ – branch currents of a loop with eliminated circulating current $\dot{I}_{ci}$; $\dot{Z}_1$ ... $\dot{Z}_{n+1}$ – impedances of loop branches; ΔX_1 ... ΔX_{n+1} – delta reactances; $\dot{Z}_{u,1}$ ... $\dot{Z}_{u,n+1}$ – uniform branch impedances; load current $\dot{I}_{l,1}$ flows from node 1, $\dot{I}_{l,2}$ – from node 2, and so on.

Now we obtain an important coherence:

$$R_1\dot{I}_{u,1} + ... + R_{n+1}\dot{I}_{n+1} = R_1\hat{I}_1 + ... + R_{n+1}\hat{I}_{n+1} = 0\,; \tag{5}$$

in the uniform closed loop, with the sum of products of branch resistances and branch currents being zero.

3. DETRIMENTAL EFFECT OF CIRCULATING CURRENT

Current circulating in a loop results in increased power losses in the ring and increased voltage losses along the loop.

Power losses P_{lou} in a uniform loop are:

$$P_{lou} = \sum_{m=1}^{n+1} R_m \dot{I}_{u,m}{}^2\ . \tag{6}$$

In a non-uniform loop the branch currents are the sum of branch currents $\dot{I}_u$ in a uniform ring and circulating current $\dot{I}_{ci}$:

$$\dot{I}_1 = \dot{I}_{ci} + \dot{I}_{u,1} \;\ldots\ldots\; \dot{I}_i = \dot{I}_{ci} + \dot{I}_i \quad \ldots\ldots \quad \dot{I}_{n+1} = \dot{I}_{ci} + \dot{I}_{u,n+1}\,. \tag{7}$$

The power loss in the *m*-th branch will be:

$$P_{lo,m} = R_m \dot{I}_m \widehat{I}_m = R_m (\dot{I}_{ci} + \dot{I}_{u,m})(\widehat{I}_{ci} + \widehat{I}_{u,m}) = R_m I_{ci}{}^2 + R_i \dot{I}_{u,m}{}^2 + \\ + R_m (\widehat{I}_{ci} \dot{I}_{u,m} + \dot{I}_{ci} \widehat{I}_{u,m}) = P_{loci,m} + P_{lou,m} + P_{lociu,m}, \tag{8}$$

where $P_{loci,m}$ is the power loss due to circulating current $\dot{I}_{ci}$;
$P_{lou,m}$ is the power loss due to current $\dot{I}_{u,m}$ in a uniform loop;
$P_{lociu,m}$ is the power loss due to interaction of currents $\dot{I}_{ci}$ and $\dot{I}_{u,m}$.

The total power loss in the non-uniform loop is

$$P_{los} = \sum_{m=1}^{n+1} P_{loci,m} + \sum_{m=1}^{n+1} P_{lou,m} + \sum_{m=1}^{n+1} P_{lociu,m} = P_{loci} + P_{lou} + P_{lociu}\,, \tag{9}$$

where loop losses P_{loci} due to circulating current are

$$P_{loci} = \sum_{m=1}^{n+1} R_m I_{ci}{}^2 = I_{ci}{}^2 \sum_{m=1}^{n+1} R_m\,. \tag{10}$$

Losses P_{lou} are found by Eq. (6), and losses P_{lociu} – by the expression:

$$P_{lociu} = \sum_{m=1}^{n+1} R_m (\widehat{I}_{ci} \dot{I}_{u,m} + \dot{I}_{ci} \widehat{I}_{u,m}) = \widehat{I}_{ci} \sum_{m=1}^{n+1} R_m \dot{I}_{u,m} + \dot{I}_{ci} \sum_{m+1}^{n+1} R_m \widehat{I}_{u,m} = 0\,, \tag{11}$$

and $P_{lociu} = 0$ due to Eq. (5).

We can see that the circulating current increases the active power losses P_{loci} according to Eq. (10).

The voltage drop across the circuit between feeding node F of the non-uniform loop and some other its node P increases as we are moving around the loop, up to the current dividing node D (Fig. 2). Observing (7), at node P the voltage drop is:

$$\Delta\dot{U}_p = \dot{Z}_1(\dot{I}_{u,1} + \dot{I}_{ci}) + \dot{Z}_2(\dot{I}_{u,2} + \dot{I}_{ci}) + \ldots + \dot{Z}_p(\dot{I}_{u,p} + \dot{I}_{ci})\,, \tag{12}$$

which means that circulating current increases the voltage drop at node P or any other node between nodes 1 and D along the loop by the value:

$$\Delta\Delta\dot{U}_p = \dot{I}_{ci} \sum_{1}^{p} Z_i\,. \tag{13}$$

In branch 1 current $\dot{I}_A$ flows, which, according to [7], is:

$$\dot{I}_1 = \dot{I}_A = \frac{\sum_{k=1}^{n} \dot{I}_{l,k} \sum_{i=k+1}^{n+1} \dot{Z}_i}{\sum_{m=1}^{n+1} Z_m}\,. \tag{14}$$

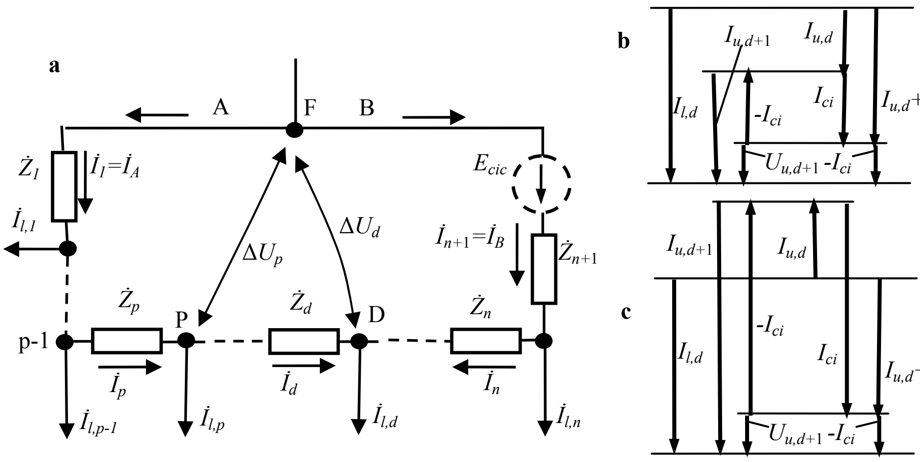

Fig. 2. Scheme of voltage drop determination
a – ringed network; *b* – node D does not move at $I_{ci} < I_{l,d}$; *c* – node D moves to the left at $I_{ci} > I_{l,d}$; ΔU_p; ΔU_d – voltage drops between nodes P and F, D and F; the rest of designations as in Fig. 1; the node number is that of load current.

In the following branches (2, ... *p*) the currents are:

$$\dot{I}_2 = \dot{I}_A - \dot{I}_{l,1}\,; \;...\;\; \dot{I}_p = \dot{I}_A - \sum_{i=1}^{p} \dot{I}_{l,i}\,. \tag{15}$$

Observing (14) and (15), the current in branch P is:

$$\dot{I}_p = \frac{\sum\limits_{k=p}^{n} \dot{I}_{l,k} \sum\limits_{i=p+1}^{n+1} \dot{Z}_i - \sum\limits_{k=1}^{p-1} \dot{I}_{l,k} \sum\limits_{i=1}^{k} \dot{Z}_i}{\sum\limits_{m=1}^{n+1} \dot{Z}_m}\,. \tag{16}$$

The maximum voltage drop will be at the current dividing node D. Branch currents before and after node D have opposite signs. Hence, for node D we will have:

$$\sum_{k=d}^{n} \dot{I}_{l,k} \sum_{i=k+1}^{n+1} \dot{Z}_i > \sum_{k=1}^{d-1} \dot{I}_{l,k} \sum_{i=1}^{k} \dot{Z}_i\,; \quad \sum_{k=d+1}^{n} \dot{I}_{l,k} \sum_{i=k+1}^{n+1} \dot{Z}_i < \sum_{k=1}^{d} \dot{I}_{l,k} \sum_{i=1}^{k} \dot{Z}_i\,. \tag{17}$$

4. THE SUBSTITUTION FOR NON-UNIFORM RINGED NETWORK

One circuit diagram substitutes for another when the functional dependence between the input and output quantities is the same. In our case, the input quantities are load currents and the output quantity is circulating current. In this specific case, active losses in the initial circuit and its substitute should remain the same.

As the first step, the circulating current of a non-uniform loop should be determined. As is shown in [7], circulating current $\dot{I}_{ciA}$ (Fig. 1) and circulating voltage $\dot{U}_{ciA}$ (both quantities calculated in direction A) can be determined by the formulas:

$$\dot{I}_{ciA} = j\frac{\sum_{k=1}^{n}\dot{I}_{l,k}\sum_{i=k+1}^{n+1}\Delta X_{\Sigma x,i}}{\dot{b}_{u\Sigma x}\sum_{m=1}^{n+1}R_m}; \quad \dot{U}_{ciA} = j\sum_{k=1}^{n}\dot{I}_{l,k}\sum_{i=k+1}^{n+1}\Delta X_i\,, \tag{18}$$

where $\dot{I}_{l,k}$ is the current of the k-th load;

R_m is the resistance of the m-th branch.

Impedance $\dot{Z}$ of a branch consists of resistance R and reactance X (see Eq. (1)).

Indices i, k, m are counted from feeding node F in the A-direction for $\dot{I}_{ciA}$ as shown in Fig. 1.

When i, k, m are counted in the B-direction, formula (18) gives:

$$\dot{I}_{ciB} = -\dot{I}_{ciA}; \qquad \dot{U}_{ciB} = -\dot{U}_{ciA}. \tag{19}$$

These two quantities have the same magnitude but opposite signs for clockwise and counter-clockwise directions in the loop. Their signs differ, because at calculations the ring was rounded along in the opposite directions. Hence, we can speak about voltage $\dot{U}_{ci}$ and current $\dot{I}_{ci}$.

The remaining (not mentioned) quantities in formula (18) are:

$$\Delta X_{\Sigma x,i} = X_i - R_i(\sum_{m=1}^{n+1}X_m / \sum_{m=1}^{n+1}R_m); \quad \dot{b}_{u\Sigma x} = 1 + j\sum_{m=1}^{n+1}X_m / \sum_{m=1}^{n+1}R_m\,. \tag{20}$$

According to [7]:

$$\dot{b}_{u\Sigma x}\sum_{m=1}^{n+1}R_m = \sum_{m=1}^{n+1}\dot{Z}_m\,, \tag{21}$$

which means that the sum of branch impedances of a uniform loop is equal to the sum of branch impedances of a non-uniform loop; hence the denominator in Eq. 18) is the sum of impedances. Therefore, the numerator ($\dot{U}_{ci}$) can be considered as electromotive force (EMF) $\dot{E}_{ci}$ which induces circulating current $\dot{I}_{ci}$ in the uniform loop:

$$\dot{E}_{ci} = j\sum_{k=1}^{n}\dot{I}_{l,k}\sum_{i=k+1}^{n+1}\Delta X_{\Sigma x,i}\,. \tag{22}$$

This is the proof that a non-uniform ringed network can be replaced by uniform one and EMF $\dot{E}_{ci}$. This EMF, being inserted in a uniform loop, induces the same circulating current that exists in a non-uniform loop, provided that condition (21) is fulfilled, i.e. the sums of impedances of the non-uniform ring and uniform one are equal. As far as additional active losses are determined by (10) and resistances of uniform and non-uniform rings do not change, the active losses of these rings remain the same.

Thus, by inserting a counter EMF into a non-uniform ring, the circulating current and the related additional active losses can be eliminated.

5. INFLUENTIAL FACTORS IN A RINGED NETWORK

The non-uniformity of a grid is the first factor which induces circulating current in a network loop. This non-uniformity strongly manifests itself when the loop includes two standard voltage ratings, which implies two autotransformers installed in this loop. However, these autotransformers may have different power ratings due to their different maximum loads; this factor aggravates still more the non-uniformity in the loop. In the preceding section it was shown that a non-uniform loop can be replaced by a uniform one by insertion of EMF $\dot{E}_{ci}$:

$$\dot{E}_{ci} = j\sum_{k=1}^{n} \dot{I}_{l,k} \sum_{i=k+1}^{n+1} \Delta X_i = E_{ci}(-\sin\varphi_{\Delta X} + j\cos\varphi_{\Delta X}), \tag{23}$$

where the module of circulating EMF E_{ci} and the angle $\varphi_{\Delta X}$ between $\dot{E}_{ci}$ and voltage U (Fig. 1) are:

$$E_{ci} = \left| \sum_{k=1}^{n} \dot{I}_{l,k} \sum_{i=k+1}^{n+1} \Delta X_i \right|; \quad \varphi_{\Delta X} = \arg \frac{\sum_{k=1}^{n} \dot{I}_{l,k} \sum_{i=k+1}^{n+1} \Delta X_i}{U}, \tag{24}$$

with U being the voltage at point F (Fig. 1).

In this substitution we can eliminate circulating current by inserting into the ring a counter EMF: $\dot{E}_{cic} = -\dot{E}_{ci}$, so that the summary EMF is zero (Fig. 1*b*).

The same result (elimination of circulating current) will be achieved if we insert counter EMF into the initial non-uniform loop, since the total impedances of initial ring and its substitute are equal (see (21)) and the counter EMF will induce in the initial loop the same counter circulating current as in its substitute (uniform loop). We can make one more corollary: if several EMFs act in the loop, they can be summed up with appropriate one or more counter EMFs inserted to counterpoise the sum. This procedure is valid because the circuit is considered linear.

The second cause for existence of circulating current is the turn ratio inequality of transformers or autotransformers included in the network ring (Fig. 3), which looks like as additional EMF ΔE_T inserted in the loop. We shall determine the turn ratio as that of higher voltage to lower voltage of the transformer at no-load condition. For transformers at points V and W the turn ratios will be:

$$k_{TA} = \frac{U_h}{U_V}; \quad k_{TB} = \frac{U_h}{U_W}. \tag{25}$$

These turn ratios are determined at no current in the entire network, since in a linear network each factor should be found in the absence of all others in order to estimate the impact of all factors.

It is convenient to reduce the parameters of the loop elements to lower voltage side since most of them operate at lower voltage. Therefore at no-load condition we have:

$$U_V = \frac{U_h}{k_{TA}}; \quad U_W = \frac{U_h}{k_{TB}}. \tag{26}$$

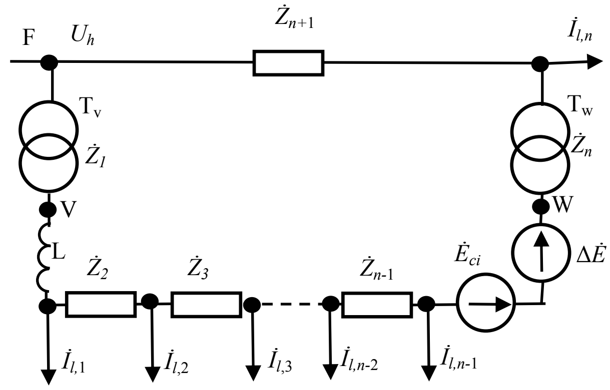

Fig. 3. Influential factors in a ringed network.
L – induction coil; E_{ci} – singled out circulating EMF of non-uniform loop; $\Delta\dot{E}$ – EMF resulting from unequal transformer turn ratios; U_h – primary (higher) voltage; T_v; T_W – transformers (autotransformers) with secondary windings connected to points V, W; remaining designations as previously.

If we assume that additional EMF ΔE_T at lower voltage is directed from A to B, then:

$$\Delta E_T = U_V - U_W = U_V(1 - \frac{k_{TA}}{k_{TB}}) = U_V(1 - K_T)\,, \tag{27}$$

where K_T is the correlation of turn ratios:

$$K_T = \frac{k_{TA}}{k_{TB}}\,. \tag{28}$$

The third factor inducing (or reducing) circulating current in the network loop is induction coil L inserted into the loop. We shall consider the case when the induction coil is connected in branch A (the first branch) and acts in the absence of circulating current. Then, according to [7], current $\dot{I}_A$ is (Fig. 1):

$$\dot{I}_A = \frac{\sum\limits_{k=1}^{n} \dot{I}_{l,k} \sum\limits_{i=k+1}^{n+1} R_i}{\sum\limits_{m=1}^{n+1} R_m}\,. \tag{29}$$

The EMF induced by induction coil L (complimentary reactance) is:

$$E_L = -\frac{\sum\limits_{k=1}^{n} \dot{I}_{l,k} \sum\limits_{i=k+1}^{n+1} R_i}{\sum\limits_{m=1}^{n+1} R_m} \omega L(-\sin\varphi_R + j\cos\varphi_R)\,; \quad \varphi_R = \arg\frac{\sum\limits_{k=1}^{n} \dot{I}_{l,k} \sum\limits_{i=k+1}^{n+1} R_i}{U_V}\,, \tag{30}$$

where U_V is to be found from (26).

To eliminate the circulating current in such a loop, the sum of active and reactive components of EMF according to (23), (27) and (30) should be zero:

$$-(\sum_{k=1}^{n} \dot{I}_{l,k} \sum_{i=k+1}^{n+1} \Delta X_i)\sin\varphi_{\Delta X} + U_V(1-K_T) + \omega L \frac{\sum_{k=1}^{n} \dot{I}_{l,k} \sum_{i=k+1}^{n+1} R_i}{\sum_{m=1}^{n+1} R_m} \sin\varphi_R = 0\,; \quad (31)$$

$$(\sum_{k=1}^{n} \dot{I}_{l,k} \sum_{i=k+1}^{n+1} \Delta X_i)\cos\varphi_{\Delta X} - \omega L \frac{\sum_{k=1}^{n} \dot{I}_{l,k} \sum_{i=k+1}^{n+1} R_i}{\sum_{m=1}^{n+1} R_m} \cos\varphi_R = 0\,. \quad (32)$$

From the last equation, the value of complementary induction L is:

$$L = \frac{\cos\varphi_{\Delta X}}{\omega \cos\varphi_R} (\sum_{m=1}^{n+1} R_m) \operatorname{tg}\alpha_{\Sigma\Delta}\,, \quad (33)$$

where $\operatorname{tg}\alpha_{\Sigma\Delta}$ according to [7] is:

$$\operatorname{tg}\alpha_{\Sigma\Delta} = \frac{\sum_{k=1}^{n} \dot{I}_{l,k} \sum_{i=k+1}^{n+1} \Delta X_i}{\sum_{k=1}^{n} \dot{I}_{l,k} \sum_{i=k+1}^{n+1} R_i}\,. \quad (34)$$

In such a way the complementary inductance can be found, regardless of the actual transformer ratios k_{TA} and k_{TB}. If a negative value of reactance is obtained by formula (30), this would mean that, instead of induction, capacitance with the same module of reactance should be inserted into the ring; however, this may not be advisable; in this case calculations by (23)–(34) should be made from B direction in order to obtain positive value of reactance.

The induction L determined, the ratio K of transformer ratios can be derived from (31) as

$$K_T = 1 + \frac{(\omega L / \sum_{m=1}^{n+1} R_m)(\sum_{k=1}^{n} \dot{I}_{l,k} \sum_{i=k+1}^{n+1} R_i)\sin\varphi_R - (\sum_{k=1}^{n} \dot{I}_{l,k} \sum_{i=k+1}^{n+1} \Delta X_i)\sin\varphi_{\Delta x}}{U_V}\,. \quad (35)$$

By these measures, additional active power losses can be eliminated without booster transformers also in the network rings which contain transmission lines of two voltage standard ratings. A booster transformer can be replaced by the induction coil (current reactor) of specified inductance with corresponding transformation ratios of transformers in the ring. Such measures will have the stronger effect the more accurate are the initial data required for calculations by (23)–(34). With loads changing, the calculated quantities will also change, though to a different degree: induction L will not change with proportionally changing loads; however turn ratios should be changed with load changing. To a high degree of probability, it can be said that if the induction coil is chosen adequately, it is necessary to change turn ratios only.

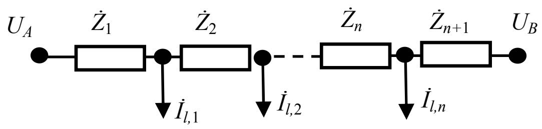

Fig. 4. Two-terminal circuit with loads $\dot{I}_l$
U_A; U_B – voltages at both terminals of the circuit

We shall now consider a non-uniform two-terminal network (Fig. 4). When voltages $\dot{U}_A$ and $\dot{U}_B$ are equal, the network is equivalent to an ordinary loop with one feeding source. However, in usual practice it is not so. In the general case, instead of ΔE_T by (27) for a two-transformer non-uniform ring, we will have:

$$\Delta \dot{E} = \dot{U}_A - \dot{U}_B = \Delta E(\cos\varphi_{\Delta E} + j\cos\varphi_{\Delta E})\,; \qquad \varphi_{\Delta E} = \arg(\frac{\dot{U}_A - \dot{U}_B}{\dot{U}_A})\,. \tag{36}$$

Instead of (31) and (32) we have:

$$-(\sum_{k=1}^{n} \dot{I}_{l,k} \sum_{i=k+1}^{n+1} \Delta X_i)\sin\varphi_{\Delta X} + \Delta E\cos\varphi_{\Delta E} + \omega L \frac{\sum\limits_{k=1}^{n} \dot{I}_{l,k} \sum\limits_{i=k+1}^{n+1} R_i}{\sum\limits_{m=1}^{n+1} R_m} \sin\varphi_R = 0\,; \tag{37}$$

$$(\sum_{k=1}^{n} \dot{I}_{l,k} \sum_{i=k+1}^{n+1} \Delta X_i)\cos\varphi_{\Delta X} + \Delta E\sin\varphi_{\Delta E} - \omega L \frac{\sum\limits_{k=1}^{n} \dot{I}_{l,k} \sum\limits_{i=k+1}^{n+1} R_i}{\sum\limits_{m=1}^{n+1} R_m} \cos\varphi_R = 0\,. \tag{38}$$

Solving (37) and (38) together, we obtain:

$$L = \frac{\cos(\varphi_{\Delta X} - \varphi_{\Delta E})}{\omega\cos(\varphi_R - \varphi_{\Delta E})}(\sum_{m=1}^{n+1} R_m) tg\alpha_{\Sigma\Delta}\,; \tag{39}$$

$$\Delta E = \frac{(\sum\limits_{k=1}^{n} \dot{I}_{l,k} \sum\limits_{I=k+1}^{n+1} \Delta X_i)\sin\varphi_{\Delta X} - \omega L \dfrac{\sum\limits_{k=1}^{n} \dot{I}_{l,k} \sum\limits_{i=k+1}^{n+1} R_i}{\sum\limits_{m+1}^{n+1} R_m} \sin\varphi_R}{\cos\varphi_{\Delta E}}\,. \tag{40}$$

Hence, in the general case, when voltages at the terminals of a network do not coincide, we have a similar procedure: the inductance of complementary reactance and the magnitude of voltage difference should acquire definite values to remove the equalizing current. The additional quantity – the direction of voltage difference – should be taken into account.

Next, we shall consider the situation with the maximum voltage drop at current dividing point D (Fig. 2). When counter EMF is not inserted, the voltage drops calculated to the left (ΔU_{dl}) and right (ΔU_{dr}) of point D are:

$$\Delta \dot{U}_{dl} = \sum_{i=1}^{d} \dot{Z}_i(\dot{I}_{u,i} + \dot{I}_{ci})\,; \; \Delta \dot{U}_{dr} = \sum_{i=d+1}^{n+1} \dot{Z}_i(\dot{I}_{u,i} - \dot{I}_{ci})\,; \tag{41}$$

$$\left|\Delta\dot{U}_{dl}\right| = \left|\Delta\dot{U}_{dr}\right|. \tag{42}$$

When the counter EMF is inserted, the circulating (equalizing) current disappears, and at node D we have a smaller voltage drop:

$$\Delta\dot{U}_{dl} = \sum_{i=1}^{d}\dot{Z}_i\dot{I}_{u,i}\ ;\ \Delta\dot{U}_{dr} = \sum_{i=d+1}^{n+1}\dot{Z}_i\dot{I}_{u,i} - E_{cic}\,. \tag{43}$$

From Fig. 2*a* it is obvious that (42) holds. The maximum voltage drop is achieved at node D:

$$\Delta\Delta\dot{U}_{dl} = \dot{I}_{ci}\sum_{1}^{d}Z_i\ , \tag{44}$$

provided that node D remains at the same place of the loop. Node D will not move when

$$\dot{I}_{ci} < \dot{I}_{l,d}\,, \tag{45}$$

since after eliminating current $\dot{I}_{ci}$, the current $\dot{I}_{u,d}$ still remains positive (Fig. 2*b*) and flowing to node D, whereas at

$$\dot{I}_{ci} > \dot{I}_{l,d}\ , \tag{46}$$

current $\dot{I}_{u,d}$ becomes negative (Fig. 2*c*) and flows to the left of the former node D. This means that actual node D shifts to the left, and the maximum voltage drop according to (41) and (44) is still smaller (the gain ΔU_{dl} increases), since in (41) the $\Sigma\dot{Z}_i$ value is greater than in (43).

We thus can see that by eliminating the circulating current the voltage drop could be reduced at least according to (44), with node D determined by (17) in the initial non-upgraded loop.

6. COMPARISON OF SIMPLE NETWORKS

Simple networks can be structured as: 1) radial networks [2] (according to [8] – single line service), Fig. 5*a,c,d*; 2) a closed loop [2] (according to [8] – ring main unit or a closed interconnector-distributer (ID)), Fig.5e; 3) a divided loop or a divided interconnector-distributer, Fig. 5*f,g,h*. At the loop configuration, buses A and B are both connected to one source; at the ID configuration, buses A and B are connected to different sources. To the closed loop and closed ID the consumers are connected through automatically operated switching devices (Fig. 5*e*). To the divided loop and divided ID the consumers can be connected by tap (Fig. 5*f*), through manually operated switching devices from one side (Fig. 5*g*) or from two sides (Fig. 5*h*). The divided structures (Fig. 5*d,g,h*) can be operated using auto-reclosing automatics and load switches. Two halves of a closed divided loop and a closed divided ID at normal operation are connected by a switching device at the current dividing node D, which in emergency situation automatically opens and, when the damaged line span is isolated, it is reclosed. Two halves of the open divided loop or the open divided ID function separately, but in the emergency situation, when damaged line span is isolated, the switching device D is on.

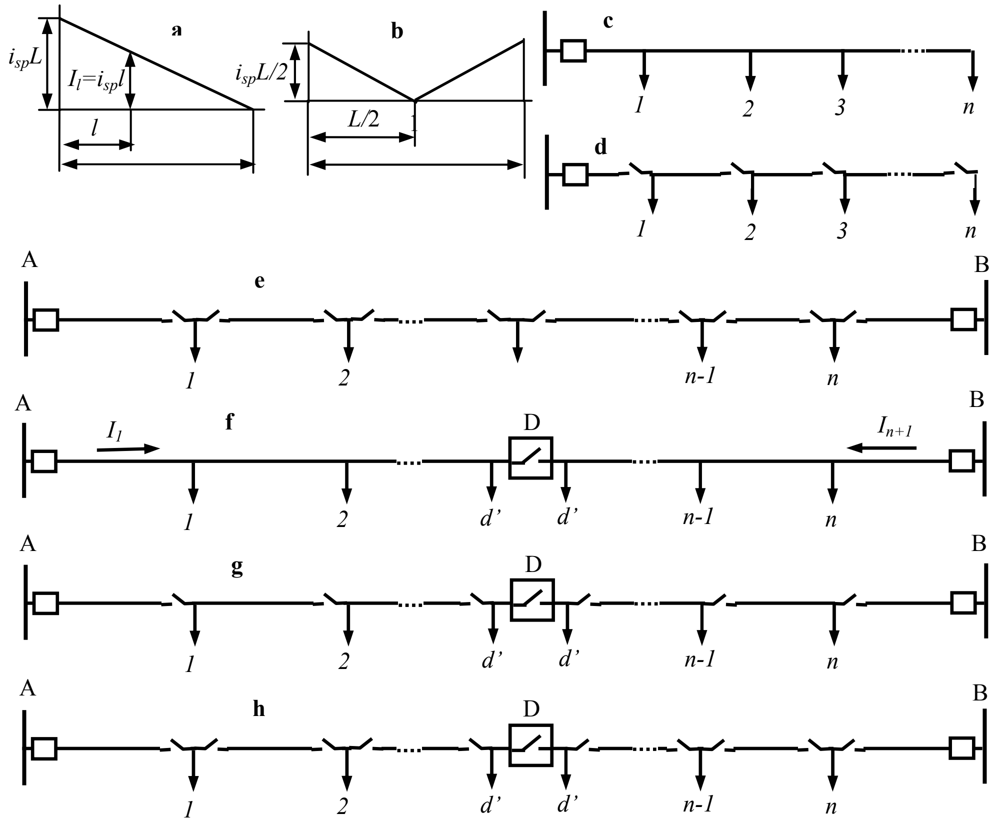

Fig. 5. Types of simple electric networks
a and *b* – linearly distributed load in radial and closed network; *c* and *d* – radial tapped and radial with switching devices networks; *e* – closed loop or ID; *f, g, h* – divided loop or ID: *f* – tapped, *g* – with single-sided, and *h* – with double-sided load connections through manually or automatically operated switching devices; D – dividing switching device.

The divided network is broken into two halves at current dividing point D (Fig. 5) of a uniform or a non-uniform network. Additional losses in closed and open divided networks are calculated as those due the circulating current, which in closed divided networks is determined as in previous sections. Current $\dot{I}_{ci}$ in open divided networks is calculated in another way: for particular loads of the network, currents $\dot{I}_{u,1}$ and $\dot{I}_{u,n+1}$ for the closed uniform version of this network should be calculated; in an open real network (either uniform or non-uniform) for the same loads these currents will be $\dot{I}_1$ and $\dot{I}_{n+1}$; the circulating current can be calculated as usual – as the difference between currents $\dot{I}_l$ and $\dot{I}_{u,1}$, i.e. $\dot{I}_{ci} = \dot{I}_1 - \dot{I}_{u,1}$ (from direction A). However, a more accurate result will be achieved if it is calculated as the half-sum calculated from both directions:

$$\dot{I}_{ci} = \frac{(\dot{I}_1 + \dot{I}_{u,n+1}) - (\dot{I}_{n+1} + \dot{I}_{u,1})}{2}. \tag{47}$$

First, the uniform networks with linearly distributed load are considered (Fig. 5*a,b*) in order to have the general case and simpler expressions. The parameters of power lines remain the same. Therefore, current $\dot{I}_l$, elementary resistance

dR and impedance dZ at distance l from the end of a radial network (Fig. 5a) or from node D of a divided one (Fig. 5b) will be:

$$I_l = i_{sp} l \, ; \quad dR = R_{sp} dl \, ; \quad dZ = Z_{sp} dl \, , \tag{48}$$

where i_{sp} is the load current per line unit; R_{sp} and Z_{sp} are the specific resistance and impedance, respectively.

Active power losses ΔP_r, ΔP_c and ΔP_{op} for radial, closed with eliminated circulating current, and open networks respectively are:

$$\Delta P_r = \int_0^L I_l^{\,2} dR = \int_0^L i_{sp}^{\,2} R_{sp} l^2 dl = i_{sp}^{\,2} R_{sp} \frac{L^3}{3} \, ; \tag{49}$$

$$\Delta P_c = P_{losc} = 2 \int_0^{L/2} I_l^{\,2} dR = 2 \int_0^{L/2} i_{sp}^{\,2} R_{sp} l^2 dl = i_{sp}^{\,2} R_{sp} \frac{L^3}{3 \bullet 4} = i_{sp}^{\,2} R_{sp} \frac{L^3}{12} \, ; \tag{50}$$

$$\Delta P_{op} = i_{sp}^{\,2} R_{sp} \frac{L^3}{12} + R_{sp} L I_{ci}^{\,2} \, , \tag{51}$$

where I_{ci} is to be found by (47).

The maximum voltage losses in the networks are:

$$\Delta \dot{U}_{r\max} = \int_0^L I_l dZ = \int_0^L i_{sp} Z_{sp} l dl = i_{sp} Z_{sp} \frac{L^2}{2} \, ; \tag{52}$$

$$\Delta U_{c\max} = \Delta U_{sc\max} = \int_0^{L/2} I_l dZ = \int_0^{L/2} i_{sp} Z_{sp} l dl = i_{sp} Z_{sp} \frac{L^2}{2 \bullet 4} = i_{sp} Z_{sp} \frac{L^2}{8} \, ; \tag{53}$$

$$\Delta U_{op\max} = i_{sp} Z_{sp} \frac{L^2}{2 \bullet 4} + Z_{sp} \frac{L}{2} \bullet I_{ci} = i_{sp} Z_{sp} \frac{L^2}{8} + I_{ci} Z_{sp} \frac{L}{2} \, . \tag{54}$$

In practical open divided networks the power and voltage losses should be calculated as shown in Sect. 3, where circulating current I_{ci} is determined by (47).

The maximum voltage jumps due to load current jolts ΔI_j are:

$$\Delta U_{jr\max} = Z_{sp} L \Delta I_j \, ; \quad \Delta U_{jc\max} = Z_{sp} \frac{L}{4} \Delta I_j \, ; \quad \Delta U_{jop\max} = Z_{sp} \frac{L}{2} \Delta I_j \, . \tag{55}$$

To bring numerical figures nearer to real situations, the calculations have been done for several numbers n of equal loads between equal branch impedances (Table 1); the total network load and the total line length for all n are 10 conventional units.

Table 1

Quality indices for various load numbers *n*

n	ΔP_r	ΔP_c	ΔP_{op}	$\Delta U_{r\max}$	$\Delta U_{c\max}$	$\Delta U_{op\max}$	ΔU_{jrmax}	ΔU_{jcmax}	ΔU_{jopmax}
2	417	167	$167+10I_{ci}^2$	50	17	$17+3.3I_{ci}$	$6.6I_j$	$2.2I_j$	$3.3I_j$
3	389	139	$139+10I_{ci}^2$	50	17	$17+5I_{ci}$	$7.5I_j$	$2.5I_j$	$5I_j$
4	375	125	$125+10I_{ci}^2$	50	15	$15+4I_{ci}$	$8I_j$	$2.4I_j$	$4I_j$
9	352	102	$102+10I_{ci}^2$	50	14	$14+4I_{ci}$	$9I_j$	$2.5I_j$	$4I_j$

In all types of networks the probability of line damage is $p_d = \lambda L$, where λ is the specific line failure intensity. However, the number n_{sht} of short-term affected consumers (while the damage is revealed and the necessary commutations are made manually) differs from number n_{lt} of long-term affected consumers (while the damage is eliminated). The numbers n_{sht} and n_{lt} of affected consumers also depend on how the consumers are connected to PL: n_{Tsht}, n_{Tlt} when they are connected by tap (Fig. 5*c,f*); n_{D1sht}, n_{D1lt} when the power line (PL) is sectioned by disconnectors according to Fig. 5*d,g*; n_{D2lt} when PL is sectioned according to Fig. 5*h*.

For a radial network the following is valid:

$$n_{Tlt} = n\,; \quad n_{D1sht} = n - n_{D1lt}\,. \tag{56}$$

For a closed network (Fig. 5*e*) $n_{sht} = n_{lt} = 0$ because all consumers are fed from both sides and the damaged PL span is isolated automatically by protection means.

For a closed divided network (Fig. 5*h*) with automatic operation of switch D:

$$n_{Tsht} = n_{D2lt} = 0\,; \quad n_{Tlt} = n_{D1sht} = n_{D2sht} = n/2\,; \quad n_{D1lt} = 1\,. \tag{57}$$

For closed divided networks with manual operation of switch D:

$$n_{Tsht} = n_{D1sht} = n_{D2sht} = n\,; \quad n_{Tlt} = n/2\,; \; n_{D1lt} = 1\,; \quad n_{D2lt} = 0\,. \tag{58}$$

For open networks:

$$n_{Tsht} = n_{D1sht} = n_{D2sht} = n/2\,; \quad n_{Tlt} = n/2\,; \quad n_{D1lt} = 1\,; \quad n_{D2lt} = 0\,. \tag{59}$$

For divided structures of Fig. 5*d*,*g* operated using auto-reclosing automatics and load switches, the number of short-term affected consumers is zero, since the faulty line detection and the necessary commutations are made automatically; whereas the structure of Fig. 5*h* can be considered as a closed network (Fig. 5*e*).

7. CONCLUSIONS

1. In a non-uniform loop the power losses increase proportionally to the squared circulating current, and the maximum voltage loss increases proportionally to the circulating current.
2. A non-uniform loop can be presented by a uniform one and by correspondingly calculated circulating EMF.
3. The loop containing two transformers or autotransformers with unequal turn ratios can be presented by a uniform loop and by summary circulating EMF.
4. A non-uniform or a uniform network – a loop or a two-terminal circuit with unequal EMFs at the ends – can be presented by a uniform loop and by summary circulating EMF. The circulating current in such a network can be eliminated by inserting a complementary reactance (inductance or capacitance) and adequate counter EMF.
5. The closed networks have substantially smaller active power losses and voltage deviations.
6. In a closed network at PL damage there are no affected consumers; in other types of network the situation is different.

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VIENKĀRŠO SLĒGTO TĪKLU VEIDU UZLABOŠANA UN SALĪDZINĀJUMS

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K o p s a v i l k u m s

Atšķirībā no radiālajām elektriskajām līnijām ir tīkli, kur patērētāji tiek baroti no divām pusēm. To priekšrocība ir tā, ka ir lielāka elektroenerģijas piegādes drošība, labāka elektroenerģijas kvalitāte, kā arī mazāki aktīvās jaudas zudumi. Bet zudumi samazinās pilnā mērā tikai tad, ja tīkls ir homogēns gredzens, vai homogēns tīkls, kas tiek barots no divām pusēm ar vienādu eds. Ja šis nosacījums nav izpildīts, tad rodas cirkulējošā vai izlīdzinošā strāva un aktīvās jaudas zudumi palielinās proporcionāli tās kvadrātam, bet sprieguma zudumi – proporcionāli tās pirmajai pakāpei. Lai samazinātu zudumus šajos gadījumos, jānovērš cirkulējošā strāva. Lai to novērstu neatkarīgi no šīs strāvas cēloņa, pielieto bustertransformatoru. Nehomogēnajā gredzenveidīgā tīklā ar vienu sprieguma nominālu, cirkulējošo strāvu var novērst, pielietojot lētāku pasākumu: ievietojot gredzenā indukcijas spoli (reaktoru). Ja gredzenveidīgs tīkls satur divus sprieguma nominālus, tad gredzenā ir nepieciešami divi transformatori vai autotransformatori. Šajā gadījumā, lai novērstu cirkulējošo strāvu, ir nepieciešama ne tikai indukcijas spole, bet arī attiecīgi jāpielāgo transformācijas koeficients. Ja diennakts laikā slodzes mainās aptuveni savstarpēji proporcionāli, indukcijas spoles induktivitāti var nemainīt, bet transformācijas koeficients jāmaina neatkarīgi no tā, kā mainās slodzes. Jā tīkls tiek barots no divām pusēm, ir jārēķina divi lielumi: divu pušu eds starpība un spoles induktivitāte, ievērojot eds starpības vektora fāzi.

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