

# The Effects of the Frequency and Implementation Lag of Basket Updates on the Canadian CPI

*Ning Huang<sup>1</sup>, Waruna Wimalaratne<sup>1</sup>, and Brent Pollard<sup>1</sup>*

In this article, we examine the effects of different frequencies and implementation months of basket updates on the fixed-basket price index – the Lowe index, through theoretical analysis and empirical simulation using Canadian data from 2000 to 2013. We find that both an increased frequency of basket updates and a faster implementation of these new baskets will reduce substitution bias in the CPI. However, we also find that improvements to the method of accelerating frequency has diminishing marginal returns in practice – as each subsequent increase in the frequency with which the CPI basket is updated has a less pronounced effect; and the ideal link-month when a new basket is implemented is unpredictable, since the impact of the implementation lag depends upon the consistency between short-term price movements and long-term price trends.

*Key words:* Consumer price index; Lowe formula; fixed-basket index; basket-update frequency; implementation lag; measurement bias; commodity substitution bias; superlative indexes; Fisher, Walsh, Törnqvist indexes.

## 1. Introduction

The Consumer Price Index (CPI) is the most widely used indicator of price change in Canada. It serves a variety of purposes, and is therefore of interest to governments, unions, business organizations, research institutions, and the general public. Its various uses include its function as a general indicator of inflation in Canada and as a tool for adjusting incomes, wages, and other payments to ensure that purchasing power is unaffected by any average price movement. Further details are available in a reference paper published by [Statistics Canada \(2014\)](#).

In line with the practices of most other national statistical agencies, the Consumer Prices Division (CPD) at Statistics Canada uses the Lowe index formula for aggregating its CPI at the upper level. The Lowe index formula, often described as a “Laspeyres-type” formula, is a fixed-basket formula. This means that the quantity and quality of the goods

<sup>1</sup> Statistics Canada, Consumer Prices Division, 170 Tunney’s Pasture, Ottawa, O.N., K1A 0T6, Canada. Emails: [Nings.Huang@Canada.ca](mailto:Nings.Huang@Canada.ca), [Waruna.Wimalaratne@Canada.ca](mailto:Waruna.Wimalaratne@Canada.ca), and [Brent.Pollard@Canada.ca](mailto:Brent.Pollard@Canada.ca).

**Acknowledgments:** The authors thank Ross Beck-MacNeil, Nathalie Brault, Kyle de March, Xin Ha, Mathieu Lequain, Sue Morris, Marc Prud’homme, Bradley Snider, Philip Smith, Faouzi Tarkhani, Amanda Wright, Alice Xu, Clément Yélou, two internal reviewers at Statistics Canada, and Price Measurement Advisory Committee members Jan de Haan, Erwin Diewert, Pierre Duguay, David Fenwick, John Mallon, Marshal Reinsdorf, Patrick Sabourn, Mick Silver, Kam Yu for their helpful discussions and comments. Thanks also to three anonymous reviewers for their comments. The views expressed in this article are solely those of the authors and do not necessarily reflect those of Statistics Canada.

and services included in the CPI basket must be unchanged or equivalent within the life span of a CPI basket. It is also referred to as a Cost of Goods Index (COGI).

The Lowe formula is used in practice because it offers a simple and convenient way to compile composite price indexes in a timely manner. Although the Lowe formula is a good choice for the fixed-basket concept of a CPI, its inherent limitations must be taken into consideration; for example, it cannot account for consumer's price-induced product substitution, it experiences delay in reflecting the effects of new goods and services on consumer price change, and it has difficulty in fully accounting for changes in the quality of existing consumer products. Due to these and other limitations, the official CPI, published by Statistics Canada, is not a true measure of actual changes in the cost of living.

A Cost-of-Living Index (COLI) is derived from the standpoint of an economic theory, based upon the assumption of a household's utility optimization behaviour, which assumes that a household will structure its purchases to maximize utility, or satisfaction, given a certain level of prices and a certain level of income. Since a household's utility optimization problem is dual to its cost minimization problem, a COLI then measures the change in the household's minimum cost of maintaining a fixed level of utility over two periods when faced with changes in prices. The theory of the COLI provides the conceptual framework for some countries' CPI, such as the United States (U.S.) and Sweden.

The difference between the official CPI and an underlying COLI, which can be approximated by a class of superlative indexes, is called measurement bias. According to the Consumer Price Index (CPI) Manual (ILO et al. 2004), a group of "superlative" price indexes, such as the Fisher, Walsh and Törnqvist indexes, is expected to provide "fairly close" approximations to the underlying COLI. Thus, they are recommended in the manual as the "target indexes" for the upper-level index. The main types of measurement bias include commodity-substitution bias, outlet-substitution bias, quality-change bias and new-goods bias. In this study, only the measurement bias associated with the upper-level aggregation is discussed. Apart from this bias, there could also be sampling and other non-sampling bias in the estimated elementary indexes and estimated basket weights. Note that measurement bias can be measured in terms of index level and index growth rate. In this article, commodity-substitution bias is analyzed and reported in both ways depending on the context of the article.

These measurement biases arise from the fact that any basket weights, held constant over more than one period, do not necessarily reflect the types of purchases that consumers actually make to attain the same level of welfare when relative prices change. A fixed-basket index, therefore, normally fails to account for the changes in consumers' purchasing patterns or preferences in a timely manner, and measures only the average price movement based on a specifically defined basket, resulting in measurement bias.

A COLI, on the other hand, allows for changes in the basket over time and, therefore, accounts for changes in consumer purchasing patterns when measuring average price movements over two periods. While numerous national statistical offices do not construct their CPIs as a COLI, including Statistics Canada, many of them still want to have knowledge about the measurement bias in their official CPI because of its important role as a major economic indicator and as a wage or salary indexation factor.

Since the CPI is the most commonly used indicator for tracking overall price change in Canada, measurement bias in the CPI is an important issue for both its users and compilers.

As Sabourin (2012) pointed out, “since the CPI departs from a true COLI, it is subject to measurement bias and does not necessarily reflect changes in the wellbeing of consumers, which could be problematic for monetary policy and when making cost-of-living adjustments to wages and salaries.”

Given the varying uses of the CPI, research on the measurement bias in the Canadian CPI is conducted regularly by some of its users, such as continuous research conducted by the Bank of Canada, including Crawford (1998), Rossiter (2005) and Sabourin (2012). According to Sabourin (2012), for the years from 2005 to 2011, the mean total bias in the Canadian CPI was 0.45 percentage points per year from 2005 to 2011, among which commodity-substitution bias was 0.22, outlet substitution bias was 0.04, new-good bias was 0.20 and quality adjustment bias was  $-0.01$ . Similar studies quantifying the bias in the CPI have been conducted in other countries, such as the paper by Boskin et al. (1996), also known as the Boskin Commission Report, for the U.S., which stated that “the Commission’s best estimate of the size of the upward bias looking forward is 1.1 percentage points per year. The range of plausible value is 0.8 to 1.6 percentage points per year.” The estimates of CPI bias can also be found in Shiratsuka (2006) for Japan and in Wynne and Rodriguez-Palenzuela (2002) for European countries.

This article focuses on the investigation of commodity-substitution bias, which is caused by the inability of a fixed-basket index to capture consumers’ price-induced substitution. Generally speaking, without changing the formula for compiling the CPI, this type of bias could be reduced by updating the CPI basket more frequently and by implementing the basket in a more timely fashion. Both of these methods allow a more accurate reflection of the changes in purchasing patterns due to consumers’ substitution between different combinations of goods and services. In the existing literature associated with commodity-substitution bias, there are only a limited number of studies examining the impact on the CPI of the frequency and delay of implementing new basket weights. This is likely due to the difficulties associated with acquiring such data. In the Canadian context, the annual household expenditure survey facilitated this study.

It is widely recognized that more frequent basket updates and faster implementation will lead to an index that more closely approximates a superlative measure. For instance, Japan publishes two series of CPI: the official CPI, with weights updated every five years; and a chained Laspeyres CPI, with weights updated annually.

A study by Greenlees and Williams (2009) showed that quarterly weight updates generated an index that more closely resembled a target index when compared to less frequent updates. In their study, a chained Törnqvist index was calculated as a superlative target. They simulated various weight updating periods: quarterly, semi-annual, annual and biennial. The index derived from quarterly weights approximated most closely to the superlative index. They also found that the Lowe index updated annually, which could be realistically compiled under the operational constraints, increased less than the rolling, two-year index of current methodology in four out of the six years studied (2002 to 2007). In addition, the advantage of using more timely weights was not offset by any increase in index volatility or instability.

Ho et al. (2011) examined, using data from 2002 to 2008, the impact on the New Zealand CPI of reweighting at different frequencies and at different levels of the index structure. They showed that frequent weight updates at the sub-item level and above generated CPI

series that tracked the Fisher series most closely among those generated by using other weight-update frequencies and other aggregation levels. Their current methodology with weight updates in June 2002, 2006, and 2008 quarters yielded a Laspeyres index of 117.0, while their methodology without updates produced an index of 117.9; these can be compared to a Fisher of 115.8, for the June 2008 quarter.

In addition to the frequency of basket updates, national statistical offices also need to determine when to introduce a new basket. The delay in the implementation of a new basket affects the size of commodity-substitution bias. Limited research supports this: [Généreux \(1983\)](#), using Canadian data, compared a chained Laspeyres series with eight basket updates against a chained Laspeyres series with only one basket update over the period from 1957 to 1978. He concluded “what appears to be desirable is not necessarily a more frequent updating of the CPI baskets but a more timely one.” For example, implementing the new weights in the years they refer to could considerably reduce the commodity-substitution bias. Using Canadian data, [Bérubé \(1996\)](#) also showed that introducing a basket two years after the basket reference period would reduce the annual substitution bias from 0.20 percentage points to 0.18 percentage points over the period from 1962 to 1994, compared with introducing a basket three years after the reference period.

A study from Australia Bureau Statistics ([ABS 2016](#)) showed a significant decrease in substitution bias by having shorter weight implementation lag for the period between September 2005 and September 2011. The bias declined from 0.24% per year for the CPI to 0.09%, 0.15%, and 0.16% with weight implementation lags of one, two, and three years, respectively. The Australian CPI weights are updated every six years using a household survey. In their study they utilized household final consumption expenditure from National Accounts to calculate the Lowe Index.

In 2010, Statistics Canada implemented the *CPI Enhancement Initiative*, a multi-stage program to advance the quality of the CPI. As part of this initiative, effort was directed at identifying and reducing the commodity-substitution bias. In 2013, a more frequent basket update schedule was implemented – from once every four years to once every two years. Additionally, the 2011 basket was introduced more quickly than past baskets – the time lag went from 16 months to 13 months. Interest and focus subsequently shifted to investigating the effect that changes such as these have on the quality of the CPI. The results would help inform the decision of whether to further accelerate the frequency of basket updates and further reduce the implementation lag.

The Canadian economy, similar to those of other major economies, is a knowledge-based economy, associated with dynamic technological change. With the rapid applications of new technology and emergence of new products and new market structure, consumers’ lifestyles and merchants’ pricing strategies have also experienced significant change. As a result, it is expected that a CPI basket becomes outdated more rapidly.

This in turn, raises questions for compilers of CPIs attempting to improve index quality and accuracy: does the comparison by [Généreux \(1983\)](#) between “a more frequent updating of the CPI baskets” and “a more timely one” still hold? Are empirical results from other countries, such as those revealed by [Greenlees and Williams \(2009\)](#) also valid for Canada? And, how can national statistical offices reduce the commodity substitution bias further?

Updating the basket weights of a price index such as the CPI is accomplished in various stages. How each of these are implemented will likely have some effect on the overall

index. This article will focus on the performance of the index under different scenarios for two of these stages, the weight-updating cycle, and the timeliness of the introduction of the new weights. The principal source of the data for the study is the Canadian Survey of Household Spending, which is used to reflect changes in consumers' spending patterns over time, for the period from 2002 to 2013. Price indexes from the Canadian CPI are also used.

To estimate the substitution bias, this article compares the results of the Lowe price index with those of the Fisher price index. This approach differs from the more common method of estimation, which compares the results between the Laspeyres price index and the Fisher price index. Another difference lies in the focus of the analysis: instead of only reporting the empirical results derived from Canadian data, the divergence in the resulting indexes obtained under various scenarios (different weight-updating schedules, and different implementation lags of the introduction of new weights) is analyzed in detail using a mathematical approach. Consequently, this article will shed new light on how to mitigate the well-known and pervasive substitution bias which characterizes a fixed-basket CPI for national statistical offices in countries facing similar situations as Canada. For these countries, the findings will play an important role in determining the desired frequency of weight updates and time of implementing new weights.

The remainder of this article is organized as follows: Section 2 discusses the data sources and data construction methods; Section 3 defines the target price index, which belongs to a group of superlative series that closely approximate a COLI, used in this study; Section 4 addresses the effects of the frequency and implementation lag of weight updates on the Canadian CPI in detail; and Section 5 concludes the article.

## 2. Data Construction

The two main elements required for the calculation of a price index series are prices and quantities. To this end, this study makes use of two main sources of data – the Consumer Price Index (CPI) and the Survey of Household Spending (SHS). The CPI provides data on the price indexes for each of its measured goods and services at the basic class level of aggregation. Basic classes are the lowest-level aggregates of products, chosen by Statistics Canada, for which a set of weights is fixed for the duration of the CPI basket. The SHS data are used in constructing fixed-basket weights for twelve years going from 2000 to 2011 based on the 2005 CPI classification structure. In this way, the estimated substitution bias would not be affected by the impact of changes in the specification and the appearance of new products.

The “price” component of the index calculation comes from the CPI over the period from January 2000 to December 2013. The original price indexes are unlinked price indexes for each of the corresponding published CPI basket. To facilitate the index reconstruction, the indexes were linked together based on the classification of the 2005 basket and rebased to January 2000 = 100. The reconstructed indexes, therefore, represent the price movement from the price reference period of January 2000 to a given price observation month.

The “quantity” component of the index comes from the SHS, which contains detailed information about consumer spending during a given reference year. The SHS sample has

a cross-sectional design, and is selected from the Labour Force Survey sampling frame and carried out in private households. The SHS is the main source of the expenditure weights data for the CPI.

In the first stage of the data construction, we derived expenditure weights for the years without official CPI weights – 2000, 2002 to 2004, 2006 to 2008, and 2010, using data from the SHS. Official CPI weights data were used whenever they were available; specifically the 2001, 2005, 2009, and 2011 baskets. However, some adjustments were made in order to align them with the 2005 classification of the CPI at the basic class level of aggregation. The 2005 classification structure that was in use in the official CPI from May 2007 to April 2011 was maintained across time to preserve uniformity and avoid complications arising from the introduction of new items. For non-official basket update years, some expenditure values were unavailable from the SHS; for example, the low level details for some basic classes under the food classification. To estimate these expenditure values, we used a modified price-updating method which used a weighted average of expenditures for those years with detailed SHS information. With this method, relatively greater importance is assigned to expenditures in baskets from periods closer to the imputed period. For example, the unknown expenditure for item  $i$  in 2003 can be imputed from the formula:

$$p_i^{2003} q_i^{2003} \equiv \underbrace{(6/8) p_i^{2001} q_i^{2001} \frac{p_i^{2003}}{p_i^{2001}}}_{\text{upward price update}} + \underbrace{(2/8) p_i^{2009} q_i^{2009} \frac{p_i^{2003}}{p_i^{2009}}}_{\text{backward price update}} \quad (1)$$

Finally, similar imputation strategies were employed for calculating the weights for the mortgage interest cost basic class as well as some components of the clothing classification. In the case of the mortgage interest cost index, where Statistics Canada has a special treatment, data were available only for the official basket reference years. As a remedy, weights for the remaining years were calculated using the same method as employed for food classification. For the replacement cost basic class, the SHS lacked detailed housing data for non-official basket update years, and so a combination of internal and external data was used to calculate its value.

Once the “price” and “quantity” components were built, a data validation was performed by reconstructing the official CPI using the analytical database. Comparing the constructed CPI with the official CPI, we believe that the analytical series was a very good approximation.

### 3. Target Index Formula

To determine the magnitude of the commodity-substitution bias, first, it is necessary to select a target index with which to compare the estimates of this study. The Fisher, Walsh, and Törnqvist indexes have been widely used for this purpose, as they belong to a small class of “superlative indexes”.

An important characteristic of superlative indexes is that they include the prices and quantities in both periods being compared, they are therefore symmetrically weighted indexes. Moreover, these three index number formulas are flexible and provide second-order approximation to each other. In other words, different superlative indexes tend to

have similar properties, yield similar results and behave in very similar ways. In addition, they are expected to provide a close approximation to the underlying conditional cost-of-living index (COLI). Diewert (1976) showed that superlative indexes provide close approximations to any true cost-of-living price index if the underlying utility function is linear homogeneous. As a close approximation to the unknown COLI, superlative indexes are recommended in the CPI ILO Manual as the theoretical target indexes. The difference between the Laspeyres-type index, which does not permit the commodity-substitution induced by relative price changes, and the target indexes can be treated as a measure of commodity-substitution bias at the upper level of index aggregation when holding classification structure unchanged.

In this study, we aim at comparing chained-CPI series constructed by applying different weights. The target indexes are, therefore, estimated by using the chain-linked Fisher, Walsh, and Törnqvist index number formulas with annual weight-updating, as detailed monthly expenditure data are unavailable. The corresponding annual CPI series are derived by taking the unweighted arithmetic average of monthly price indexes of the twelve months in the calendar year. Using the Fisher index number formula  $P_{ChF}^{(2003+t)/2003}$  as an example, we show how the chain-linked index between 2003 and 2011 is constructed:

$$\begin{aligned}
 P_{ChF}^{(2003+t)/2003} &= \prod_{j=1}^t P_F^{(2003+j)/(2003+j-1)} \\
 &= \prod_{j=1}^t \left( \frac{\sum_{i=1}^N p_i^{2003+j} q_i^{2003+j-1}}{\sum_{i=1}^N p_i^{2003+j-1} q_i^{2003+j-1}} \frac{\sum_{i=1}^N p_i^{2003+j} q_i^{2003+j}}{\sum_{i=1}^N p_i^{2003+j-1} q_i^{2003+j}} \right)^{1/2} \quad (2) \\
 &\quad P_L^{2003+j/2003+j-1} \quad P_P^{2003+j/2003+j-1} \\
 t &= 1, 2, \dots, 8
 \end{aligned}$$

where  $P_{ChF}^{(2003+t)/2003}$  denotes chained-Fisher from 2003 to 2003 + t;  $P_F^{(2003+j)/(2003+j-1)}$  denotes the direct Fisher index from 2003 + j - 1 to 2003 + j;  $P_L^{(2003+j)/(2003+j-1)}$  denotes the direct Laspeyres index from 2003 + j - 1 to 2003 + j and  $P_P^{(2003+j)/(2003+j-1)}$  denotes the direct Paasche index from 2003 + j - 1 to 2003 + j. N is the total number of goods and services included in the CPI basket. The chained-Walsh index and chained-Törnqvist index can be compiled similarly. The three superlative indexes are expected to behave similarly, which is confirmed by the numerical results over the period from 2003 to 2011 reported in Table 1, where the average growth rate using the chained Fisher index as an example, is calculated as  $\sqrt[8]{\left(P_{ChF}^{2011/2003}/100\right)} - 1$ .

In the next section, the target index values in Table 1 can be compared with the CPI series compiled with different CPI weight-updating schedules to produce the estimates of the upper-level commodity-substitution bias. More specifically, the chained-Fisher index is used as an example to estimate the commodity-substitution bias in this article.

Table 1. Superlative price indexes (2003 = 100).

Year (2003 + <i>t</i> )	Fisher		Walsh		Törnqvist	
	Chained index	Annual inflation	Chained index	Annual inflation	Chained index	Annual inflation
2003	100.000		100.000		100.000	
2004	101.728	1.728	101.730	1.730	101.730	1.730
2005	103.746	1.984	103.750	1.986	103.750	1.986
2006	105.475	1.667	105.480	1.668	105.482	1.669
2007	107.401	1.826	107.409	1.829	107.410	1.828
2008	109.624	2.069	109.632	2.070	109.633	2.069
2009	109.670	0.042	109.684	0.047	109.688	0.050
2010	111.404	1.581	111.422	1.585	111.422	1.581
2011	114.389	2.679	114.408	2.680	114.405	2.677
Average growth rate (2003–2011)		1.695		1.697		1.696

4. Approaches to Reducing Commodity-Substitution Bias

In general, the commodity-substitution bias could be measured as the difference between the published CPI and the target index, both of which are estimated by keeping the items in the baskets fixed over time. The source of this substitution bias varies. Two important sources could be the frequency of CPI basket weight updates, and the time lag between the end of the basket reference year and the initial implementation time of a new CPI basket in the CPI calculation. Using the 2011 basket update as an example, we illustrate the relationship among different time periods involved in the index calculation using the following timeline.

On the timeline in Figure 1, the basket reference year (during which the SHS is conducted to collect the necessary information for the CPI basket) is 2011. The 2011 CPI basket was implemented with the February 2013 CPI, which is defined as the implementation month in this paper. The duration from January 2012 to January 2013 is the implementation lag, which in this case is 13 months. January 2013 is the link month for the implementation of the 2011 CPI basket.

In this section, how the frequency and implementation lag of the CPI weight affect the magnitude of the upper-level commodity-substitution bias will be explored in further detail.

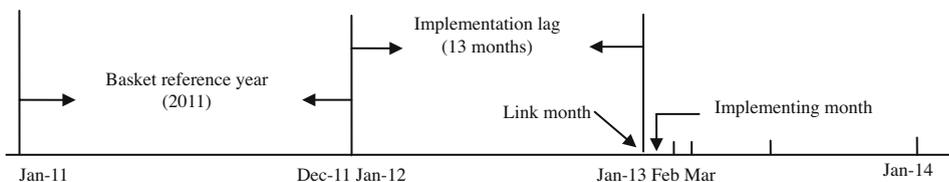


Fig. 1. Timeline of CPI basket update.

#### 4.1. Commodity-Substitution Bias and the Frequency of Basket Updates

##### 4.1.1. Conceptual Framework to Measure the Impact of the Basket Update Frequency

The CPI basket is designed to reflect consumers’ spending patterns. As a result of both relative price changes and some long-term effects on consumers’ spending behaviour, such as the impact of demographic factors and technological changes, the weights might become out-of-date and less representative of current consumption patterns. The bias in a Lowe index is likely to increase as the basket weights age. Therefore, CPI weights should be updated periodically to reflect the changes in these patterns.

To identify the pure impact of the frequency of weight updates on the magnitude of the CPI bias, we fix the implementation lag at 13 months and vary only the frequency of weight updates when calculating the All-items CPI, which measures price change of all the goods and services included in the Canadian CPI, for the period from January 2002 to December 2013.

A direct Lowe index  $P_{Lo}(p^0, p^t, q^b)$  can be defined in terms of a quantity vector  $q^b \equiv [q_1^b, \dots, q_N^b]$ , a price vector of base period  $p^0 \equiv [p_1^0, \dots, p_N^0]$  and a price vector of current period  $p^t \equiv [p_1^t, \dots, p_N^t]$ :

$$P_{Lo}(p^0, p^t, q^b) = \frac{\sum_{i=1}^N p_i^t q_i^b}{\sum_{i=1}^N p_i^0 q_i^b} \tag{3}$$

where  $N$  is the total number of goods and services included in the CPI weight structure.

It can be also written in terms of the hybrid share form as follows:

$$\begin{aligned} P_{Lo}(p^0, p^t, s^{0:b}) &= \frac{\sum_{i=1}^N p_i^t q_i^b}{\sum_{i=1}^N p_i^0 q_i^b} = \sum_{i=1}^N \left( \frac{p_i^t}{p_i^0} \right) \frac{p_i^0 q_i^b}{\sum_{i=1}^N p_i^0 q_i^b} \\ &= \sum_{i=1}^N \left( \frac{p_i^t}{p_i^0} \right) s_i^{0:b} \end{aligned} \tag{4}$$

where the hybrid expenditure shares  $s_i^{0:b}$  corresponding to the quantity weights vector  $q^b$  measured at base period price vector  $p^0$  are defined as:

$$s_i^{0:b} = \frac{p_i^0 q_i^b}{\sum_{i=1}^N p_i^0 q_i^b}, \quad i = 1, 2, \dots, N \tag{5}$$

If more than one basket, say baskets  $b1$  and  $b2$ , are in use, it is necessary to calculate the chain-linked Lowe index, where the indexes calculated using different CPI baskets are linked together. To explain this concept, let  $p^{y,m}$  be the elementary price vector for year  $y \geq 2002$  and month  $m = 1, 2, \dots, 12$ ; the chain-linked Lowe index for year  $y$  and month  $m$ , with every  $x$  years as the frequency of weight updates, is denoted as  $P_{ChLo_x}(y, m)$ . The calculation of the chain-linked Lowe index depends on which basket is currently used and in which month it is linked to the previous basket. In general,

a chain-linked Lowe index can be defined as:

$$P_{ChLo_x}(y, m) = P_{ChLo_x}(link\_month) P_{Lo}(p^{link\_month}, p^{y,m}, q^b) \tag{6}$$

where  $P_{ChLo_x}(link\_month)$  is a chain-linked Lowe index for the link month that chains together indexes using the current basket  $q^b$  and the previous baskets.

If the CPI basket is assumed to be updated every  $x$  years, where  $x$  can be 1, 2, 3, 4, or 5, after the adoption of the 2000 basket, the Equation (6) can be applied to compile the CPI series. With the implementation lag set equal to 13 months, a new basket 2000 +  $kx$  is introduced in February of year 2002 +  $kx$  with January ( $m = 1$ ) of year 2002 +  $kx$  as the link month,  $k = 1, 2, \dots$  such that 2002 +  $kx \leq y$  ( $y$  is the year of the price index). With these assumptions, the chain-linked Lowe index can be calculated by substituting the corresponding values in Equation (6), yielding the following results:

$$P_{ChLo_x}(y, m) = P_{ChLo_x}(2002 + kx, 1) P_{Lo}(p^{2002+kx,1}, p^{y,m}, q^{2000+kx}) \tag{7}$$

The first component of the right-hand side of the Equation (7),  $P_{ChLo_x}(2002 + kx, 1)$ , is the link factor, which is also a chain-linked Lowe index for January of year 2002 +  $kx$ , which is the link month of the current basket (2000 +  $kx$ ); the second component,  $P_{Lo}(p^{2002+kx,1}, p^{y,m}, q^{2000+kx})$ , is the direct Lowe index comparing the current month ( $y, m$ ) with the link month (2002 +  $kx, 1$ ), January of year 2002 +  $kx$ .

The link factor  $P_{ChLo_x}(2002 + kx, 1)$  can be also defined as the product of several direct Lowe indexes as follows:

$$\begin{aligned} P_{ChLo_x}(2002 + kx, 1) &= P_{Lo}(p^0, p^{2002+x,1}, q^{2000}) \\ &\quad P_{Lo}(p^{2002+x,1}, p^{2002+2x,1}, q^{2000+x}) \\ &\quad \dots P_{Lo}(p^{2002+(k-1)x,1}, p^{2002+kx,1}, q^{2000+(k-1)x}) \end{aligned} \tag{8}$$

where  $k$  denotes the number of times the CPI basket is updated since the price reference period, which is assumed to be during the life span of the basket  $q^{2000}$ .

We now describe how the chain-linked Lowe index can be constructed if the weights are updated every two years, that is  $x = 2$ . Denote the chain-linked Lowe index for year  $y$  and month  $m$ , with an update frequency of every two years, by  $P_{ChLo_2}(y, m)$ . In this case, the direct Lowe index, which uses the 2000 basket only, is employed from February 2002 to January 2004, with January 2002 as the link month, the overlapping period that links the old and new CPI series. Applying (9), we have  $P_{ChLo_2}(2002, 1) = P_{Lo}(p^{2002,1}, p^{2002,1}, q^{2000}) = 1$ . Thus, the chain-linked Lowe index defined by (9) is, for the first 24 months running from February 2002 to January 2004, equal to the direct Lowe index:

$$P_{ChLo_2}(y, m) = P_{Lo}(p^{2002,1}, p^{y,m}, q^{2000}) \tag{9}$$

(with  $y = 2002, 2003$ ; and  $m = 1, 2, \dots, 12$  and  $y = 2004; m = 1$ )

The same direct Lowe index on the right hand side of (9) is, therefore, used to define the chain-linked Lowe index for January 2004:

$$P_{ChLo_2}(2004, 1) = P_{Lo}(p^{2002,1}, p^{2004,1}, q^{2000}) \tag{10}$$

The above chain-linked Lowe index for January 2004 corresponds to the link factor that chains together indexes using the 2000 basket and the 2002 basket. For the remaining months in 2004 and 2005, the annual quantity weights vector  $q^{2002}$  becomes available and the chain-linked Lowe index is defined as follows:

$$P_{ChLo_2}(y, m) = P_{ChLo_2}(2004, 1) P_{Lo}(p^{2004,1}, p^{y,m}, q^{2002})$$

(11)

(with  $y = 2004, 2005; m = 1, 2, \dots, 12; y = 2006; m = 1$ )

The chain-linked Lowe index for January 2006 is, therefore, defined as follows:

$$P_{ChLo_2}(2006, 1) = P_{ChLo_2}(2004, 1) P_{Lo}(p^{2004,1}, p^{2006,1}, q^{2002}) \tag{12}$$

Here again, the chain-linked Lowe index for January 2006 is the link factor that chains indexes based on 2004, 2002, and 2000 baskets respectively. From February 2006 to January 2008, the annual quantity weights vector  $q^{2004}$  becomes available and the chain-linked Lowe for this time span is defined as follows:

$$P_{ChLo_2}(y, m) = P_{ChLo_2}(2006, 1) P_{Lo}(p^{2006,1}, p^{y,m}, q^{2004})$$

(13)

(with  $y = 2006, 2007; m = 1, 2, \dots, 12; y = 2008; m = 1$ )

Once more, the link factor chaining the indexes together across baskets is the chain-linked Lowe index for January 2008 which continues to be defined by the right-hand side of (13), as follows:

$$P_{ChLo_2}(2008, 1) = P_{ChLo_2}(2006, 1) P_{Lo}(p^{2006,1}, p^{2008,1}, q^{2004}) \tag{14}$$

Continuing the above process, we can construct the chain-linked Lowe index for other months in the other years.

To show how the defined process works, here we compile a chain-linked Lowe index for a particular month, say August 2011, as an example. Assume the weight-updating frequency is two ( $x = 2$ ) and implementation lag is 13 months. The chained Lowe index is then denoted by  $P_{ChLo_2}(2011, 8)$ . Based on the described process, the current period, August 2011, is identified to be in the time span going from February 2010 to January 2012 and the associated quantity weights vector is  $q^{2008}$ , with January 2010 as the link month. The chain-linked Lowe index  $P_{ChLo_2}(2011, 8)$  can then be constructed as:

$$P_{ChLo_2}(2011, 8) = P_{ChLo_2}(2010, 1) P_{Lo}(p^{2010,1}, p^{2011,8}, q^{2008}) \tag{15}$$

where  $P_{ChLo_2}(2010, 1)$  is the link factor that chains together the price indexes using the 2008 basket and the previous baskets. Based on Equation (8), it can be written as a product

of direct Lowe indexes as follows:

$$\begin{aligned}
 P_{\text{ChLo}_2}(2010, 1) &= P_{\text{Lo}}(p^{2002,1}, p^{2004,1}, q^{2000}) P_{\text{Lo}}(p^{2004,1}, p^{2006,1}, q^{2002}) \\
 &P_{\text{Lo}}(p^{2006,01}, p^{2008,1}, q^{2004}) P_{\text{Lo}}(p^{2008,1}, p^{2010,1}, q^{2006})
 \end{aligned}
 \tag{16}$$

The direct Lowe index on the right-hand side of (15) can be compiled based on Equation (3) as follows:

$$P_{\text{Lo}}(p^{2010,1}, p^{2011,8}, q^{2008}) = \frac{\sum_i p_i^{2011,8} q_i^{2008}}{\sum_i p_i^{2010,1} q_i^{2008}}
 \tag{17}$$

Next, the chain-linked Lowe index for the same month, August 2011, but with different weight-updating frequency,  $x = 3$ , denoted by  $P_{\text{ChLo}_3}(2011, 8)$ , is considered. It can be compiled based on the process described in the case of a weight update every two years (refer to Equation (9) to (14)), as follows:

$$P_{\text{ChLo}_3}(2011, 8) = P_{\text{ChLo}_3}(2011, 1) P_{\text{Lo}}(p^{2011,1}, p^{2011,8}, q^{2009})
 \tag{18}$$

With the two CPI index values associated with different frequencies of weight updates, the commodity-substitution bias can be then estimated by comparing the chain-linked Lowe index with the same target index. For example, let  $Bias_{\text{ChLo}_2}(2011, 8)$  and  $Bias_{\text{ChLo}_3}(2011, 8)$  denote the commodity-substitution bias, measured in terms of index level, of the chain-linked Lowe index for August 2011, with weight-updating frequencies equal to every two and every three years, respectively. They can be defined as follows

$$Bias_{\text{ChLo}_2}(2011, 8) = P_{\text{ChLo}_2}(2011, 8) - P_{\text{Target}}(2011, 8)
 \tag{19}$$

$$Bias_{\text{ChLo}_3}(2011, 8) = P_{\text{ChLo}_3}(2011, 8) - P_{\text{Target}}(2011, 8)
 \tag{20}$$

To compare the magnitude of the bias generated by different weight-updating frequencies, the following procedure is employed:

$$\begin{aligned}
 &Bias_{\text{ChLo}_2}(2011, 8) - Bias_{\text{ChLo}_3}(2011, 8) \\
 &= [P_{\text{ChLo}_2}(2011, 8) - P_{\text{Target}}(2011, 8)] - [P_{\text{ChLo}_3}(2011, 8) - P_{\text{Target}}(2011, 8)] \\
 &= P_{\text{ChLo}_2}(2011, 8) - P_{\text{ChLo}_3}(2011, 8) \\
 &= [P_{\text{ChLo}_2}(2010, 1) P_{\text{Lo}}(p^{2010,1}, p^{2011,8}, q^{2008})] - [P_{\text{ChLo}_3}(2011, 1) P_{\text{Lo}}(p^{2011,1}, p^{2011,8}, q^{2009})] \\
 &= P_{\text{Lo}}(p^{2002,1}, p^{2004,1}, q^{2000}) P_{\text{Lo}}(p^{2008,1}, p^{2010,1}, q^{2006}) \\
 &\quad \left\{ \begin{aligned} &\left[ \begin{aligned} &P_{\text{Lo}}(p^{2004,1}, p^{2005,1}, q^{2002}) P_{\text{Lo}}(p^{2005,1}, p^{2006,1}, q^{2002}) P_{\text{Lo}}(p^{2006,1}, p^{2008,1}, q^{2004}) \\ &P_{\text{Lo}}(p^{2010,1}, p^{2011,1}, q^{2008}) P_{\text{Lo}}(p^{2011,1}, p^{2011,8}, q^{2008}) \end{aligned} \right] - \\ &\left[ \begin{aligned} &P_{\text{Lo}}(p^{2004,1}, p^{2005,1}, q^{2000}) P_{\text{Lo}}(p^{2005,1}, p^{2006,1}, q^{2003}) P_{\text{Lo}}(p^{2006,1}, p^{2008,1}, q^{2003}) \\ &P_{\text{Lo}}(p^{2010,1}, p^{2011,1}, q^{2006}) P_{\text{Lo}}(p^{2011,1}, p^{2011,8}, q^{2009}) \end{aligned} \right] \end{aligned} \right\}
 \end{aligned}
 \tag{21}$$

To facilitate the comparison, all the direct Lowe indexes in Equation (21) are written in terms of the indexes with the same price comparison periods. From the right-hand side of

Equation (21), it can be seen that the two pairs of Lowe indexes,  $P_{Lo}(p^{2002,1}, p^{2004,1}, q^{2000})$  and  $P_{Lo}(p^{2008,1}, p^{2010,1}, q^{2006})$ , are identical; whereas, the other five pairs of Lowe indexes measure the price movement over the same periods but use different quantity weight vectors:

- In three pairs of Lowe indexes representing four years of price change – from January 2004 to January 2005  $P_{Lo}(p^{2004,1}, p^{2005,1}, q^{2002})$  and  $P_{Lo}(p^{2004,1}, p^{2005,1}, q^{2000})$ , from January 2006 to January 2008  $P_{Lo}(p^{2006,1}, p^{2008,1}, q^{2004})$  and  $P_{Lo}(p^{2006,1}, p^{2008,1}, q^{2003})$ , and from January 2010 to January 2011  $P_{Lo}(p^{2010,1}, p^{2011,1}, q^{2008})$  and  $P_{Lo}(p^{2010,1}, p^{2011,1}, q^{2006})$  – those with a more frequent weight-updating schedule ( $x = 2$ ) use relatively more up-to-date quantity weight vectors.
- Whereas, of the other two pairs of indexes corresponding to less than two years’ price movement – one from January 2005 to January 2006,  $P_{Lo}(p^{2005,1}, p^{2006,1}, q^{2002})$  and  $P_{Lo}(p^{2005,1}, p^{2006,1}, q^{2003})$ , and the other from January 2011 to August 2011,  $P_{Lo}(p^{2011,1}, p^{2011,8}, q^{2008})$  and  $P_{Lo}(p^{2011,1}, p^{2011,8}, q^{2009})$  – those with a less frequent weight-updating process ( $x = 3$ ) use more up-to-date quantity weight vectors.

This simple comparison indicates that the chain-linked series with more frequent weight updates applies up-to-date quantity weights more often than those series with less frequent basket updates. Generally speaking, the price index compiled using a more outdated basket tends to exceed that which uses more up-to-date baskets due to price-induced commodity substitution. Thus, through this rough comparison, it is intuitively believed that more frequent weight updates would generate lower commodity-substitution bias in general.

To identify conditions under which more frequent weight updates would generate lower commodity-substitution bias, we compare one of the pairs of the Lowe indexes in Equation (21):

$$\begin{aligned}
 & P_{Lo}(p^{2004,1}, p^{2005,1}, q^{2002}) - P_{Lo}(p^{2004,1}, p^{2005,1}, q^{2000}) \\
 &= \frac{\sum_i p_i^{2005,1} q_i^{2002}}{\sum_i p_i^{2004,1} q_i^{2002}} - \frac{\sum_i p_i^{2005,1} q_i^{2000}}{\sum_i p_i^{2004,1} q_i^{2000}} \\
 &= \frac{\sum_i \left( \frac{p_i^{2005,1}}{p_i^{2004,1}} - P_{Lo}(p^{2004,1}, p^{2005,1}, q^{2002}) \right) \left( \frac{q_i^{2002}}{q_i^{2000}} - Q_{Lo}(p^{2004,1}, q^{2000}, q^{2002}) \right)}{Q_{Lo}(p^{2004,1}, q^{2000}, q^{2002})} s_i^{2004,1:2000}
 \end{aligned} \tag{22}$$

where the Lowe quantity index,  $Q_{Lo}(p^{2004,1}, q^{2000}, q^{2002})$ , is defined as:

$$Q_{Lo}(p^{2004,1}, q^{2000}, q^{2002}) = \frac{\sum_i p_i^{2004,1} q_i^{2002}}{\sum_i p_i^{2004,1} q_i^{2000}} \tag{23}$$

and the hybrid expenditure shares  $s_i^{2004,1:2000}$  are defined in terms of the year 2000 quantity vector evaluated at January 2004 prices:

$$s_i^{2004,1:2000} = \frac{P_i^{2004,1} q_i^{2000}}{\sum_i P_i^{2004,1} q_i^{2000}} \quad (24)$$

The last line of Equation (22) indicates that the price deviations and quantity deviations are for two *different* periods; the former is pertaining to the period from January 2004 to January 2005, while the latter is for the period from year 2000 to 2002. Provided that the price and quantity changes were for the same period (e.g., from 2000 to 2002), the right-hand side of Equation (22) would be regarded as the covariance between the price deviations of price relatives from their mean,  $\frac{P_i^{2002}}{P_i^{2000}} - P_{Lo}(p^{2000}, p^{2002}, q^{2002})$ , and the corresponding quantity deviations of quantity relatives from their mean,  $\frac{q_i^{2002}}{q_i^{2000}} - Q_{Lo}(p^{2004,1}, q^{2000}, q^{2002})$ . If this covariance is negative (which is the usual case in the consumer context) and the price trend from 2000 to 2002 on average is in the same direction as those going from January 2004 to January 2005, the difference between the two Lowe indexes, shown in Equation (22), would be negative, which implies that the Lowe index using the up-to-date basket,  $P_{Lo}(p^{2004,1}, p^{2005,1}, q^{2002})$ , will be lower than that using the out-dated basket,  $P_{Lo}(p^{2004,1}, p^{2005,1}, q^{2000})$ .

In short, the relationship between  $P_{Lo}(p^{2004,1}, p^{2005,1}, q^{2002})$  and  $P_{Lo}(p^{2004,1}, p^{2005,1}, q^{2000})$  depends upon the persistent tendency of price change and the associated change in consumers' expenditure patterns. This conclusion will also be true for the comparison of the other pairs of the Lowe indexes in Equation (21). However, the determination of the sign of Equation (21), which represents the relationship between the commodity-substitution biases in the Lowe indexes calculated with different frequencies of weight-updates, is far more complicated than what we have discussed here as it is affected by the interaction of the different time periods involved in the calculation. Despite this, from this simple example, we can still find that the impact of the frequency of weight updates on the upper-level commodity-substitution bias depends on the relationship between the price trend and the expenditure pattern of different time periods.

Intuitively, the more frequent the weights are updated, the more up-to-date weights would be employed in the index calculation. This is true for the comparison among other weight-updating frequencies. In addition, if persistent long-term price trends and consumers' price-induced commodity-substitution behaviour are present, then increasing the frequency of weight updates would lower the commodity-substitution bias.

#### 4.1.2. Empirical Results: Impact of the Basket-Update Frequency on the Canadian CPI

Using the constructed data set, we compiled different CPI series by assuming different frequencies of updating the CPI basket while fixing the implementation lag equal to 13 months. Figure 2 shows the CPI series constructed with different frequencies of basket updates – from every year to every five years, and also with no basket updates at all, for the period from January 2002 to December 2013.

Series “Freq\_ $x$ ” ( $x = 1, 2, \dots, 5$ ) in Figure 2 denotes CPI series compiled with the basket updated every  $x$  years. It illustrates that the index level for a given time period gradually decreases as the frequency of basket updates is accelerated. The index levels of

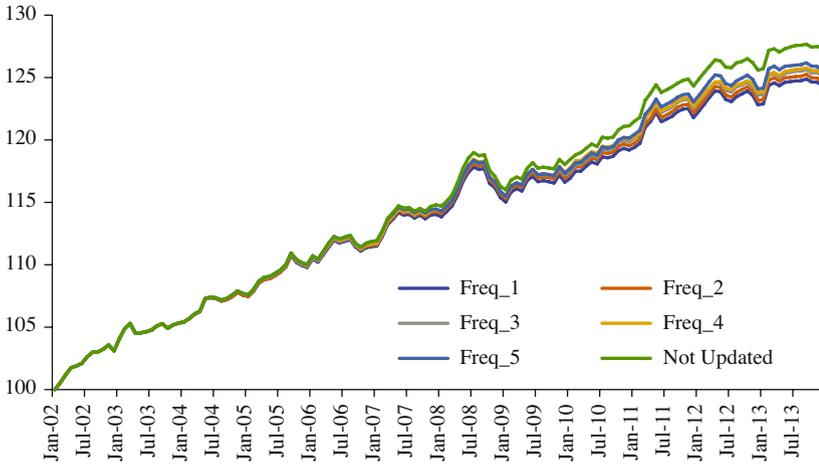


Fig. 2. Comparisons among the CPI series compiled with different frequencies of updating the CPI basket (January 2002 = 100).

the CPI series with no basket updates are considerably higher than levels of the other series. It is also noted that the differences in the index values are not obvious within the first five or six years. The impact of weight-updating frequency can be shown more explicitly in Table 2 in which the corresponding annual index levels were compared with the chain-linked Fisher index. The official CPI, compiled using a different data set, is not comparable to the other series reported in the table and is cited purely for reference.

Examining these results, we find that the commodity-substitution bias could be reduced by increasing the frequency of updating the CPI basket in the examined period; however, the magnitude of the marginal reduction in commodity-substitution bias for each additional increase in the frequency of basket updates varied. If we increased the frequency of updating the CPI basket from every two years to every year, we could reduce the commodity-substitution bias, measured by the difference of index growth rate, from

Table 2. Comparisons of different CPIs, compiled with various frequencies of basket updates and the Fisher index (2003–2011).

	Indexes (2003 = 100)	Difference in the indexes	Annual growth rate	Difference in the growth rate
	2003 to 2011		(%)	(%)
<b>Fisher – Target index</b>	<b>114.389</b>	<b>0.000</b>	<b>1.695</b>	<b>0.000</b>
Low index-every 1 year	115.857	1.468	1.857	0.162
Low index-every 2 years	116.153	1.764	1.889	0.195
Low index-every 3 years	116.547	2.157	1.932	0.238
Low index-every 4 years	116.645	2.256	1.943	0.249
Low index-every 5 years	116.918	2.528	1.973	0.278
Low index-no updates	118.009	3.620	2.091	0.397
Official CPI	116.70	2.3	1.944	0.249

0.195 percentage points to 0.162 percentage points on average. The impact was more significant when we changed the frequency from every four years to every two years, in which case the commodity-substitution bias was reduced from 0.249 percentage points to 0.195 percentage points on average for the sample period.

A similar impact on the CPI of increasing the frequency of weight updates was also shown in other studies, such as in [Greenlees and Williams \(2009\)](#) and in [Ho et al. \(2011\)](#). More recent research conducted by Australia, ([Australian Bureau of Statistics 2016](#)), found that the bias declined from 0.24% per year with six-year weight updates to 0.09% per year with one- year updates for the period between September 2005 and September 2011. Despite the magnitude of change being different between the two countries, we observe a similar impact of a reduced bias on the CPI through increasing the frequency of basket updates.

#### 4.2. Commodity-Substitution Bias and the Implementation Lag of a New Basket

It is impossible to implement a new CPI basket in the weight reference period it refers to because of the time needed to conduct and process the Survey of Household Spending (SHS). This fact results in a certain time lag between the weight reference period and the implementation time of the basket. In this article, this time lag is referred to as the implementation lag. It is widely recognized that shortening the implementation lag of a new CPI basket can lower the upward bias in a Lowe price index. In this section, we will revisit this common belief and verify how this lag influences the CPI.

##### 4.2.1. Conceptual Impact of the Implementation Lag on the CPI

If, for example, two baskets – the 2005 and 2009 baskets – are available for the period from January 2009 to December 2012, to implement the latter, we need a link month that chains indexes across the two baskets. To identify the impact of the implementation lag on the CPI, we assume that there are two possible link months, say December 2010 and April 2011, for introducing the 2009 basket. One has a shorter implementation lag (twelve months) while the other has a longer one (16 months). To assess the common belief in this simple setting, where a chain-linked Lowe index, defined in Equation (6), will be calculated, we compare the difference in the CPI series calculated using the two possible link months. Because of the inherent limitations of the Lowe formula, we believe that it will generate upward bias in most cases. Therefore, only upward bias will be taken into consideration.

For instance, the CPI from January 2009 to December 2012 using a shorter implementation lag, with December 2010 as the link month, denoted by  $P_{ChLo}^{2010,12}(2012, 12)$ , can be compiled as follows:

$$\begin{aligned}
 P_{ChLo}^{2010,12}(2012, 12) &= P_{Lo}(p^{2009,01}, p^{2010,12}, q^{2005}) P_{Lo}(p^{2010,12}, p^{2012,12}, q^{2009}) \\
 &= \frac{\sum_n p_n^{2010,12} q_n^{2005}}{\sum_n p_n^{2009,01} q_n^{2005}} \frac{\sum_i p_i^{2012,12} q_i^{2009}}{\sum_i p_i^{2010,12} q_i^{2009}} \tag{25}
 \end{aligned}$$

The CPI for the same comparison periods using a longer lag, with April 2011 as the link month, denoted by  $P_{ChLo}^{2011,04}(2012, 12)$ , can be compiled as follows:

$$\begin{aligned}
 P_{ChLo}^{2011,04}(2012, 12) &= P_{Lo}(p^{2009,01}, p^{2011,04}, q^{2005}) P_{Lo}(p^{2011,04}, p^{2012,12}, q^{2009}) \\
 &= \frac{\sum_n p_n^{2011,04} q_n^{2005} \sum_i p_i^{2012,12} q_i^{2009}}{\sum_n p_n^{2009,01} q_n^{2005} \sum_i p_i^{2011,04} q_i^{2009}} \tag{26}
 \end{aligned}$$

The difference in the magnitude of the commodity-substitution bias in the two CPIs can be derived from the following expression:

$$\begin{aligned}
 & \left[ P_{ChLo}^{2010,12}(2012, 12) - P_{target}(2012, 12) \right] - \left[ P_{Ch-Lo}^{2011,04}(2012, 12) - P_{target}(2012, 12) \right] \\
 &= P_{ChLo}^{2010,12}(2012, 12) - P_{ChLo}^{2011,04}(2012, 12) \\
 &= \left[ \frac{\sum_i p_i^{2012,12} q_i^{2009} \sum_n p_n^{2010,12} q_n^{2005}}{\sum_i p_i^{2010,12} q_i^{2009} \sum_n p_n^{2009,01} q_n^{2005}} \right] - \left[ \frac{\sum_i p_i^{2012,12} q_i^{2009} \sum_n p_n^{2011,04} q_n^{2005}}{\sum_i p_i^{2011,04} q_i^{2009} \sum_n p_n^{2009,01} q_n^{2005}} \right] \tag{27} \\
 &= \frac{\sum_i p_i^{2012,12} q_i^{2009} \sum_n p_n^{2010,12} q_n^{2005}}{\sum_n p_n^{2009,01} q_n^{2005} \sum_i p_i^{2011,04} q_i^{2009}} \left( \frac{\sum_i p_i^{2011,04} q_i^{2009} \sum_n p_n^{2011,04} q_n^{2005}}{\sum_i p_i^{2010,12} q_i^{2009} \sum_n p_n^{2010,12} q_n^{2005}} \right)
 \end{aligned}$$

A negative sign resulting from Equation (27) would imply that a shorter implementation lag leads to a lower commodity-substitution bias. Furthermore, the last line of Equation (27) indicates that the sign is determined by the difference between  $\left( \frac{\sum_i p_i^{2011,04} q_i^{2009}}{\sum_i p_i^{2010,12} q_i^{2009}} \right)$  and  $\left( \frac{\sum_n p_n^{2011,04} q_n^{2005}}{\sum_n p_n^{2010,12} q_n^{2005}} \right)$ , the two price indexes that measure price changes between the two link months (December 2010 and April 2011) with different baskets (the 2005 basket and 2009 basket). As mentioned before, generally speaking, price indexes using a more obsolete basket tend to exceed those using a more up-to-date basket due to consumers' substitution behaviour. If this is the case, the above difference would be negative, which leads to the conclusion that a shorter time lag would generate a lower bias as is commonly believed. However, is this intuition always true? To verify this, the difference between these two indexes is further examined.

To simplify the problem, we fix the products and services belonging to the two baskets. Decomposing the index difference yields the following expression:

$$\begin{aligned}
 & \frac{\sum_i p_i^{2011,04} q_i^{2009}}{\sum_i p_i^{2010,12} q_i^{2009}} - \frac{\sum_i p_i^{2011,04} q_i^{2005}}{\sum_i p_i^{2010,12} q_i^{2005}} \\
 &= \sum_i \frac{\overbrace{\left( \frac{p_i^{2011,04}}{p_i^{2010,12}} - P_{Lo}(p^{2010,12}, p^{2011,04}, q^{2009}) \right)}^{\text{price deviation}} \overbrace{\left( \frac{q_i^{2009}}{q_i^{2005}} - Q_{Lo}(p^{2010,12}, q^{2005}, q^{2009}) \right)}^{\text{quantity deviation}}}{Q_{Lo}(p^{2010,12}, q^{2005}, q^{2009})} s_i^{2010,12:2005}
 \end{aligned}
 \tag{28}$$

where the Lowe quantity index is defined as:

$$Q_{Lo}(p^{2010,12}, q^{2005}, q^{2009}) = \frac{\sum_i p_i^{2010,12} q_i^{2009}}{\sum_i p_i^{2010,12} q_i^{2005}}
 \tag{29}$$

and the hybrid expenditure shares are defined as:

$$s_i^{2010,12:2005} = \frac{p_i^{2010,12} q_i^{2005}}{\sum_i p_i^{2010,12} q_i^{2005}}
 \tag{30}$$

Thus, Equation (28) demonstrates that which link month yields lower commodity-substitution bias is determined by both price and quantity variations. It is, however, not easy to determine its sign, because the price and quantity deviations are for two different periods. If the deviations in both prices and quantities are for the same period, it could be regarded as the covariance between price relatives and the corresponding quantity relatives. In typical consumer theory, this covariance is negative – the price deviation  $\left( \frac{p_i^{2009}}{p_i^{2005}} - P_{Lo}(p^{2005}, p^{2009}, q^{2009}) \right)$  and the quantity deviation  $\left( \frac{q_i^{2009}}{q_i^{2005}} - Q_{Lo}(p_i^{2010,12}, q^{2005}, q^{2009}) \right)$  are negatively correlated. If the price trend between the two possible link months (December 2010 and April 2011), represented by  $\left( \frac{p_i^{2011,04}}{p_i^{2010,12}} - P_{Lo}(p^{2010,12}, p^{2011,04}, q^{2009}) \right)$  is, on average, in the same direction as those between the two weight reference years (2005 and 2009), then we would expect that  $\frac{\sum_i p_i^{2011,04} q_i^{2005}}{\sum_i p_i^{2010,12} q_i^{2005}}$  exceeds  $\frac{\sum_i p_i^{2011,04} q_i^{2009}}{\sum_i p_i^{2010,12} q_i^{2009}}$ . As a result, shortening the implementation lag could reduce the commodity-substitution bias.

In summary, this simplified case shows that a shorter implementation lag is associated with lower commodity-substitution bias as long as (i) the price trend between the two weight reference years is in the same direction as those between the two possible link months, and (ii) price-induced consumers' commodity-substitution behavior exists.

Price trends between the two weight reference years, in general, represent long-term price movements, whereas the price trends between two possible link months, if not too far from each other, normally reflect unpredictable price changes that are not necessarily in line with the long-term price movements, especially considering seasonal items. This implies that the impact on the CPI of shortening the implementation lag is not predictable.

It depends on the consistency between the long-term price trends and short-term price fluctuations, and on the presence of consumer’s commodity-substitution behaviour. If prices of the majority of goods and services move persistently in the same direction for a long period, such as in an inflation context, this condition is more likely to be satisfied.

4.2.2. Empirical Results: Impact of the Implementation Lag on the Canadian CPI

In the first part of this section, we use the CPI series and apply the official CPI baskets without any adjustments, to examine whether shortening the implementation lag for introducing the 2005 basket, the 2009 basket, and the 2011 basket could reduce the commodity-substitution bias in the Canadian CPI.

The 2005 CPI basket was officially implemented in May 2007. Here we assume that it could have been implemented in any month from January 2007 to April 2007. Under operational constraints, we assume it is infeasible to implement the 2005 baskets earlier than January 2007. A negative difference would be shown in the fifth column of Table 3 if introducing the 2005 basket earlier than May 2007 could reduce the commodity-substitution bias. However, the numerical results reported in Table 3 imply that implementing the 2005 basket earlier than May 2007 would not yield a lower CPI bias.

Similarly the sign of the difference between  $\frac{\sum_n p_n^{2011,4} q_n^{2009}}{\sum_n p_n^{link} q_n^{2009}}$  and  $\frac{\sum_n p_n^{2011,4} q_n^{2005}}{\sum_n p_n^{link} q_n^{2005}}$  listed in the fifth column of Table 4 would determine whether the commodity-substitution bias would have decreased or increased by introducing the 2009 basket earlier than May 2011. The sign of the difference between  $\frac{\sum_n p_n^{2013,1} q_n^{2011}}{\sum_n p_n^{link} q_n^{2011}}$  and  $\frac{\sum_n p_n^{2013,1} q_n^{2009}}{\sum_n p_n^{link} q_n^{2009}}$  in the fifth column of Table 5 determines whether the commodity-substitution bias in the Canadian CPI could have decreased or increased by shortening the implementation lag of the 2011 basket. The results in both Table 4 and Table 5 show that reducing the implementation lag of the 2009 and 2011 baskets would not yield a lower CPI bias under the time constraints of the availability of the SHS.

In the following part of this section, the different CPI series calculated with different implementation lags using the constructed data set are reported. To isolate the impact of this phenomenon as opposed to the impact of weight-updating frequency, we fix the frequency of updating weights at every two years, and vary only the implementation lag to somewhere between twelve and 24 months. We also show the results of using one month as the implementation lag; although this is currently operationally impossible as the

Table 3. Different link months for introducing the 2005 CPI basket.

Possible implementing month	Possible link month	$\frac{\sum_n p_n^{2007,4} q_n^{2005}}{\sum_n p_n^{link} q_n^{2005}}$ (A)	$\frac{\sum_i p_i^{2007,4} q_i^{2001}}{\sum_i p_i^{link} q_i^{2001}}$ (B)	Difference (A)–(B)
Jan. 2007	Dec. 2006	102.0116	101.9890	0.0226
Feb. 2007	Jan. 2007	101.9928	101.9447	0.0481
Mar. 2007	Feb. 2007	101.2472	101.2451	0.0021
Apr. 2007	Mar. 2007	100.3856	100.3813	0.0043

Table 4. Different link months for introducing the 2009 CPI basket.

Possible implementing month	Possible link month	$\frac{\sum_n p_n^{2011,4} q_n^{2009}}{\sum_n p_n^{link} q_n^{2009}}$ (A)	$\frac{\sum_i p_i^{2011,4} q_i^{2005}}{\sum_i p_i^{link} q_i^{2005}}$ (B)	Difference (A)-(B)
Jan. 2011	Dec. 2010	102.0339	102.0011	0.0329
Feb. 2011	Jan. 2011	101.7826	101.7540	0.0287
Mar. 2011	Feb. 2011	101.4966	101.4701	0.0266
Apr. 2011	Mar. 2011	100.4137	100.4076	0.0062

finalized expenditure data, taken mainly from the SHS, can be obtained only as early as eleven months after the weight reference year.

Figure 3 shows the cumulative impact of the implementation lag, which are kept unchanged for each CPI series, on the index values. In general, there are minor differences in index values when the implementation lags are not significantly different from each other; this explains why the ten CPI series cannot be distinguished separately in Figure 3. However, over time, the series with longer implementation lags clearly begin to diverge from the series with shorter lags (for example, 24 months compared to twelve months). It can also be demonstrated by the fact that the CPI series with a one-month implementation lag is significantly lower than the other CPI series.

Table 6 shows the comparison between the annual chained Fisher index and the annual chained Lowe price indexes compiled using different implementation lags for the period from 2003 to 2011, as well as the corresponding geometric average growth rates. The annual chained Lowe indexes are derived by taking a simple arithmetic average of monthly chained Lowe indexes of a calendar year. Even though the ways of calculating an annual chain Fisher index and annual chain Lowe index are different, the comparison still provides insights when comparing with the same target index. It clearly indicates how the index value and the average inflation rate change with the implementation lags. Among the chained-CPI series that can be compiled in a timely manner, using twelve months as the implementation lag yielded the lowest inflation rate; however, the difference in the average inflation rate between using twelve months and 14 months as the implementation lag was only 0.01 percentage points for the sample period. As expected from the conceptual framework, the impact of the implementation lag on the CPI is not predictable, especially when we shorten or increase the lags by increments of one or two months.

Table 5. Different link months for introducing the 2011 CPI basket.

Possible implementing month	Possible link month	$\frac{\sum_n p_n^{2013,1} q_n^{2011}}{\sum_n p_n^{link} q_n^{2011}}$ (A)	$\frac{\sum_i p_i^{2013,1} q_i^{2009}}{\sum_i p_i^{link} q_i^{2009}}$ (B)	Difference (A)-(B)
Jan. 2013	Dec. 2012	100.0678	100.0567	0.0111
Feb. 2013	Jan. 2013	100.0000	100.0000	0.0000
Mar. 2013	Feb. 2013	98.8240	98.8067	0.0173

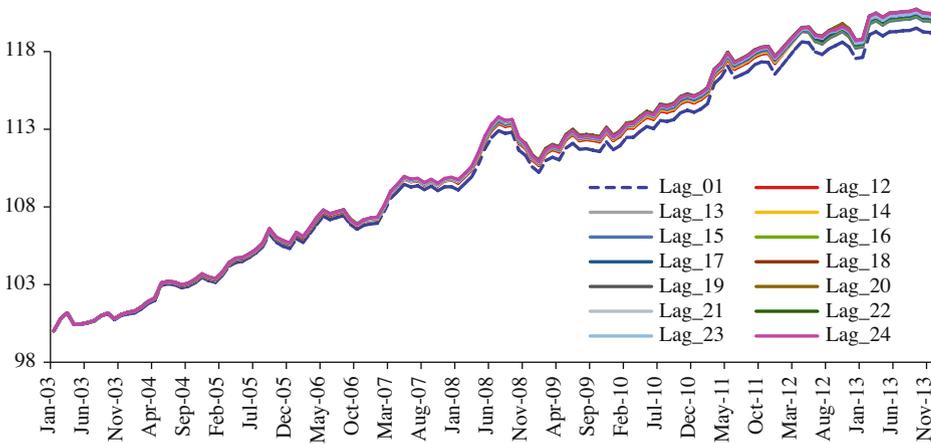


Fig. 3. Different CPI series corresponding to various implementations lags.

However, the commodity-substitution bias could generally be reduced if the implementation lag were substantially shortened. This can be shown from the difference in the growth rates between implementation lags of one month and twelve months, as well as twelve months and 24 months. Table 6 indicates that the substitution bias can be reduced from 0.221 percentage points to 0.176 percentage points if the implementation lag is shortened from 24 months to twelve months. ABS (2016) found a similar impact of the weight implementation lag on the CPI. The bias declined from 0.15% per year with a two-year implementation lag to 0.09% per year with a one-year implementation lag for the period from September 2005 to September 2011.

Table 6. Comparison of the geometric average growth rates of the different CPI series using various implementation lags and the Fisher index.

	Indexes (2003 = 100) 2003–2011	Differences in indexes	Annual growth rate (%)	Difference in growth rate (%)
Fisher	114.389	0.000	1.695	0.000
Low index, 1 month lag	115.484	1.095	1.816	0.121
Low index, 12 month lag	115.980	1.591	1.870	0.176
Low index, 13 month lag	116.153	1.764	1.889	0.195
Low index, 14 month lag	116.075	1.686	1.881	0.186
Low index, 15 month lag	116.164	1.775	1.891	0.196
Low index, 16 month lag	116.300	1.911	1.905	0.211
Low index, 17 month lag	116.282	1.893	1.903	0.209
Low index, 18 month lag	116.340	1.951	1.910	0.215
Low index, 19 month lag	116.432	2.043	1.920	0.225
Low index, 20 month lag	116.413	2.023	1.918	0.223
Low index, 21 month lag	116.348	1.959	1.911	0.216
Low index, 22 month lag	116.405	2.016	1.917	0.222
Low index, 23 month lag	116.316	1.926	1.907	0.213
Low index, 24 month lag	116.393	2.004	1.916	0.221

Table 7. Different link months for introducing the 2010 CPI basket.

Possible implementing month	Possible link month	$\frac{\sum_n p_i^{2012,4} q_i^{2010}}{\sum_n p_i^{link} q_i^{2010}}$	$\frac{\sum_i p_i^{2012,4} q_i^{2008}}{\sum_i p_i^{link} q_i^{2008}}$	Difference (A)–(B)
		(A) 2010 basket	(B) 2008 basket	
Jan. 2012	Dec. 2011	101.707	101.589	0.118
Feb. 2012	Jan. 2012	101.263	101.192	0.071
Mar. 2012	Feb. 2012	100.821	100.773	0.048
Apr. 2012	Mar. 2012	100.382	100.370	0.011
May 2012	Apr. 2012	100.000	100.000	0.000
June 2012	May 2012	100.027	99.974	0.053
July 2012	June 2012	100.500	100.399	0.102
Aug. 2012	July 2012	100.629	100.470	0.159
Sep. 2012	Aug. 2012	100.338	100.167	0.171
Oct. 2012	Sep. 2012	100.170	100.083	0.087

From these empirical results, we cannot infer the impact on the CPI of a given link month of a particular CPI basket. To identify and illustrate this impact, we examine the introduction of a specific CPI basket. If, for example, the 2010 basket could be possibly implemented between January 2012 and October 2012, any month from December 2011 to September 2012 could, therefore, be chosen as the link month. Using Equation (28), we can determine retrospectively which month is the optimal link month for introducing the 2010 CPI basket. Table 7 shows the comparison between April 2012 and all the other possible link months, which are within the timeline of the SHS.

We obtained positive differences in the fifth column of Table 7, implying that using months either earlier or later than April 2012 as the link month cannot reduce commodity-substitution bias in the CPI based on Equation (28). Although using April 2012 as the link month to introduce the 2010 basket generates the lowest index level, it might not necessarily be true for introducing other new baskets. We therefore perform the same exercise (results are available on request) for the introduction of other baskets, and find that the optimal month for different baskets varies with the price fluctuation.

The empirical results illustrate that the impact of shortening the implementation lag on the commodity-substitution bias is not predictable, especially when the price trends are not persistent over time. However, in the case that a country's economy exhibited persistent and predictable inflation, the conditions implied by Equation (28) might very likely be satisfied. This could result in the observance of a relatively significant impact of a shortened implementation lag on the substitution bias.

Recently, as a result of operational constraints, Statistics Canada used 13 months as the implementation lag to introduce the 2011 basket. The empirical results from this study suggest that shortening the implementation lag to twelve months may not have a significant impact on further reducing the commodity-substitution bias. Moreover, the link month that yields the lowest commodity-substitution bias may not always be the same because of different monthly price fluctuations over time. As a result, it is not meaningful

to fix the link month of implementing a new basket for the purpose of reducing the commodity-substitution bias; in addition, the optimal link month of introducing a new CPI basket cannot be determined in advance. However, since Statistics Canada also compiles the CPI annual table based on the calendar year, we recommend that the new baskets be introduced in January to have a consistent annual index.

#### 4.3. Alternative Data Sources and Substitution Bias

Many retailers, including nearly all major retailers, collect data through automated point-of-sale scanners. Scanner data is becoming an increasingly important source of information for statistical agencies, providing them with the prices and quantities of a large number of actual transactions in a timely manner. Several national statistical agencies currently make use of this data, including the Netherlands, Norway, Sweden, Switzerland, and New Zealand. Meanwhile, with the development of electronic commerce, online shopping has become more popular. Accompanying this growth, public information on product prices and characteristics is also available online. Automated data collection (“web-scraping”) can replace traditional price collection for some product categories. With these “big data” sources, statistical agencies and academic researchers have an opportunity to study many research issues that used to be operationally infeasible and purely theoretical, and explore new methods to solve these issues.

With the availability of scanner data, it seems that the commodity-substitution bias issue raised in this article can more easily be addressed by using the prices and quantities to construct weighted (preferably superlative) price indexes. Research using scanner data to either estimate the substitution bias or to produce a superior estimate of the CPI has been ongoing for more than thirty years. New challenges and problems also arise with the use of scanner data, such as more volatile estimates of the CPI and chain-drift caused by the use of high-frequency scanner data. To overcome these new problems with the use of scanner data, [Ivancic et al. \(2011\)](#) proposed an innovative method, described as a rolling year GEKS method (RYGEKS), which adapts multilateral index number theory to making comparisons between multiple time periods. The GEKS method, described by [Gini \(1931\)](#), [Eltető and Köves \(1964\)](#), and [Szulc \(1964\)](#), was originally used to conduct multilateral comparison, involving two stages of aggregation. The RYGEKS method makes maximum use of all matches in the scanner data to compile non-revisable CPIs that are approximately free from chain drift. Since then, this novel approach has been tested by many countries’ numerical experiments, such as [de Haan and van der Grient \(2011\)](#) using Dutch data, [Johansen and Nygaard \(2011\)](#) using Norwegian data, and [Krsinich \(2011\)](#) using scanner data from New Zealand. Extensions to the RYGEKS have also been made; for instance, [de Haan and Krisinich \(2014\)](#) used an imputation Törnqvist rolling year GEKS procedure (ITRYGEKS) to derive quality-adjusted and chain-drift free price indexes. This method was applied by [Statistics New Zealand \(2014\)](#) to produce a CPI for its electronics products beginning in the September 2014 quarter.

Following the advancement of these new data processing techniques, the availability of large-volume data sources could provide statistical agencies with new practical solutions to primary questions covered in this paper. It might be feasible to obtain timely information on quantities purchased by households and to dramatically shorten the basket

implementation lag with the arrival of new data processing systems that could eliminate or suppress some of the current operational constraints. However, more research is needed before incorporating these data sources into the CPI.

## **5. Conclusion**

The Lowe index number formula, one of the fixed-basket concept indexes, is widely used by statistical agencies to compile their Consumer Price Index (CPI). However, because of its limitations associated with the fixed-basket concept, some concern arises from the use of this formula, in particular the issue of commodity-substitution bias. Because of the importance of the CPI to its different users (such as central banks, policy makers, and the general population as a whole), researchers have devoted, and continue to devote, much work into investigating the issue of commodity-substitution bias in the CPI.

In this article, we constructed a comprehensive data set by using information taken from Statistics Canada's Survey of Household Spending (SHS) for the years from 2000 to 2011, and the monthly CPI data for Canada at the basic class level for the period from January 2000 to December 2013.

This study focused on the investigation of approaches to reducing the commodity-substitution bias in the Canadian CPI based on two key aspects associated with the introduction of new CPI baskets. Namely, updating the CPI basket more frequently, and introducing a new CPI basket in a more timely manner. The empirical results found in this paper for the examination period indicate that increasing the frequency of updating the CPI basket could reduce the commodity-substitution bias. This finding is consistent with what has been shown in [Greenlees and Williams \(2009\)](#) and in other studies. In addition this paper's results reveal that the marginal gains from moving from basket-updates every four years to every two years are more significant than those from moving from basket-updates every two years to every year.

The impact of shortening the implementation lag for a new CPI basket on the commodity-substitution bias is unpredictable because it depends on the consistency between the long-term price trends (between the two basket reference periods) and the short-term price movements (between the possible link months), as well as the existence of consumers' price-induced commodity-substitution behaviour. Clear differences can be perceived in the price indexes compiled by using a twelve-month implementation lag versus an 18-month or longer implementation lag, while the differences in the indexes are largely reduced when they are constructed using twelve months compared to 14 months as implementation lags. Therefore, based on both the decomposition of index differences and the empirical results in this article, it is believed that the conclusion in [Généreux \(1983\)](#) would hold only when the conditions illustrated above were satisfied. Consequently, it is worthwhile for a statistical agency to pursue ways to dramatically shorten the implementation lag; however, taking great effort to slightly improve the timeliness of implementing a new basket may not provide meaningful returns.

In this article, we presented the empirical results using Canadian data for the period between 2003 and 2011. These results do not provide direct answers for choosing the most effective approach to reducing the commodity-substitution bias in a CPI. Statistical agencies in other countries can draw inferences from this empirical work but should be

cautious in generalizing these results to other CPIs because of the time dependence of the empirical results. Finally, new practical solutions associated with the incorporation of large-volume data sources in the CPI is worth further investigation from statistical agencies.

## 6. References

- Australian Bureau of Statistics. 2016. "Increasing the Frequency of Consumer Price Index Expenditures Class Weight Updates." *Australian Bureau of Statistics Information Paper* 6401.0.60.002. July.
- Bérubé, C. 1996. "Selecting a Formula for the Canadian CPI, 1962–1994." Price Division Analytical Series, Statistics Canada. No. 7, Catalogue No. 62F0014MPB, p 7.
- Boskin, Michael J., Ellen R. Dullberger, Robert J. Gordon, Zvi Griliches, and Dale W. Jorgenson. 1996. "Toward a More Accurate Measure of the Cost of Living: Final Report to the Senate Finance Committee from the Advisory Commission to Study the Consumer Price Index." Washington, D.C.: US Government Printing Office.
- Crawford, A. 1998. "Measurement Biases in the Canadian CPI: An Update." *Bank of Canada Review*. Spring. Page 39–56.
- De Haan, Jan and Heymerik A. van der Grient. 2011. "Eliminating Chain Drift in Price Indexes Based on Scanner Data." *Journal of Econometrics* 161(1): 36–46.
- De Haan, Jan and Frances Krisnich. 2014. "Scanner Data and the Treatment of Quality Change in Non-Revisable Price Indexes." *Journal of Business & Economic Statistics* 32(3): 341–358.
- Diewert, W. Erwin. 1976. "Exact and Superlative Index Numbers." *Journal of Econometrics*. May. Page 115–145.
- Eltető, Ö. and P. Köves. 1964. "On a Problem of Index Number Computation Relating to International Comparisons." *Statisztikai Szemle* 42: 507–518. (in Hungarian).
- Généreux, Pierre A. 1983. "Impact of the Choice of Formulae on the Canadian Consumer Price Index." *Price Level Measurement: Proceedings from a Conference* Sponsored by Statistics Canada. Erwin Diewert and Claude Montmarquette (eds.).
- Gini, C. 1931. "On the Circular Test of Index Numbers." *Metron* 9(9): 3–24.
- Greenlees, John S. and Elliot Williams. 2009. *Reconsideration of Weighting and Updating Procedure in the US CPI*. Paper presented at the 11th Meeting of the International Working Group on Price Indexes (Ottawa group) in Neuchâtel, Switzerland.
- Ho, Ricky, Peter Champion, and Chris Pike. 2011. *New Zealand Consumer Price Index—an empirical analysis of the frequency and level of weight updates*. Room document at the 12th Meeting of the International Working Group on Price Indexes (Ottawa group) in Wellington, New Zealand.
- ILO / IMF / OECD / IMECE / Eurostat / World Bank. 2004. *Consumer Price Index Manual: Theory and Practice*. Geneva: ILO.
- Ivancic, Lorraine, W. Erwin Diewert, and Kevin J. Fox. 2011. "Scanner Data, Time Aggregation and Construction of Price Indexes." *Journal of Econometrics* 161(1): 24–35.
- Johansen, Ingvild, and Ragnhild Nygaard. 2011. *Dealing with bias in the Norwegian superlative price index of food and non-alcoholic beverages*. Paper presented at the 12th

- Meeting of the International Working Group on Price Indexes (Ottawa group) in Wellington, New Zealand.
- Krsinich, Frances. 2011. *Price indexes from scanner data*. Paper presented at the 12th Meeting of the International Working Group on Price Indexes (Ottawa group) in Wellington, New Zealand. May 2011.
- Rossiter, James. 2005. *Measurement bias in the Canadian Consumer Price Index*. Bank of Canada Working Paper 2005-39. December.
- Sabourin, Patrick. 2012. "Measurement bias in the Canadian Consumer Price Index: An update." *Bank of Canada Review*. Summer.
- Shiratsuka, Shigenori. 2006. "Measurement Errors in Japanese Consumer Price Index." *Monetary and Economic Studies* 17(3). Institute for Monetary and Economic Studies, Bank of Japan, 69–102.
- Statistics Canada. 2014. "The Canadian Consumer Price Index Reference Paper." Catalogue No. 62-553-X.
- Statistics New Zealand. 2014. *Measuring price change for consumer electronics using scanner data*. Available from [www.stats.govt.nz](http://www.stats.govt.nz).
- Szulc, B. 1964. "Indexes for Multiregional Comparisons." *Przegląd Statystyczny* 3: 239–254. (in Polish).
- Wynne, Mark A. and Diego Rodriguez-Palenzuela. 2002. *Measurement Bias in the HICP: What do we know, and what do we need to know?* European Central Bank Working Paper Series, No. 131. Frankfurt: European Central Bank.

Received January 2016

Revised October 2016

Accepted January 2017