

A numerical investigation of a buoyancy driven flow in a semi-porous cavity: comparative effects of ramped and isothermal wall conditions

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Abstract: Steady two-dimensional natural convection taking place in a rectangular cavity, partially filled with an isotropic porous material, has been investigated numerically using an ADI method. It is assumed that one of the vertical walls of the cavity has a ramped temperature distribution. The vorticity-stream function formulation has been used to solve the set of nonlinear partial differential equations governing the flows in the clear region and the adjoining porous region. The effects of Darcy number and Rayleigh number have been discussed in detail.

Keywords: Rectangular cavity; Free convection; Brinkman model; Coupled flow; Ramped boundary condition; Numerical solution.

INTRODUCTION

Theoretical investigations of transport mechanisms at the interface region of a fluid-saturated porous medium and an adjoining fluid layer have gathered great pace during the last four decades or so. The coupling of fluid motions in the clear region and the porous medium typically takes place as a result of fluid transport to either region through the interface. These coupled flows have been reported in the literature to have numerous engineering and industrial applications in fields such as transpiration cooling, geothermal energy systems, crude oil extraction, thermal insulation, cooling and fumigation of stored food grains, drying processes, to name a few. The pioneer contribution of Beavers and Joseph (1967) and, soon after, by a number of other researchers led to a solid foundation being made in the theoretical development of such flows, particularly with regard to the all-important condition at the clear region – porous medium interface. Subsequent studies, both analytical and numerical, in this ever growing field (Alazmi and Vafai, 2001; Bhatt and Sacheti, 1994; Nield, 1977; Sacheti and Bhatt, 1988; Singh et al., 2006), have covered wide-ranging aspects of viscous flows, including natural convection.

The study of laminar natural convection and heat transfer from an infinite plane vertical wall in the presence or absence of a porous medium has been a subject of intense and comprehensive investigations in the literature due to real life applications in a number of areas. Typical examples of such free convective steady or unsteady flows include the physical configurations comprising bounding vertical flat surface(s) subjected to isothermal or adiabatic temperature conditions or constant heat flux (Callahan and Marner, 1976; Chandran et al., 2001; Magyari and Keller, 2003; Paul et al., 1996; Singh and Singh, 1983). Moreover, there have also been a number of works involving buoyancy-driven flows where researchers have discussed natural convection from vertical walls subjected to non-uniform wall temperature conditions, e.g., step discontinuities or ramped profiles (temporal or spatial) for the surface temperature (Kao, 1975; Lee and Yovanovich, 1991; Hayday et al., 1967; Chandran et al., 2005; Singh et al., 2008). Such unbounded flow problems have also been extended to confined

regions (Nishimura et al., 1986; Singh et al., 1993; Valencia-López and Ochoa-Tapia, 2001).

In the present work, we have considered a steady two-dimensional free convective coupled flow in a rectangular cavity comprising a fluid-saturated porous medium of high permeability underlying a viscous fluid layer in the clear region. It is assumed that the porosity of the porous medium is close to unity. The walls of the cavity are taken to be rigid. Furthermore, one of the vertical walls of the cavity is assumed to be subjected to a spatially ramped temperature profile.

It is well known that the classical Darcy law neglects the effects of a solid boundary and the inertial forces on fluid flow and heat transfer through porous media (Collins, 1961; Scheidegger, 1974; Vafai and Tien, 1981). As reported by several researchers (Chandrasekhara and Vortmeyer, 1979; Palm et al., 1972; Vafai and Tien, 1981), these effects become significant near the boundary and in high porosity media, rendering the Darcy law invalid. However, modern applications involving high porosity materials necessitate a comprehensive understanding of the boundary and inertia effects. In order to account for them in our present analysis, we have employed the widely used Brinkman model (often referred to in the literature as Darcy-Brinkman model) to describe the flow in the porous medium of high permeability. In fact, a number of researchers have strongly advocated the appropriateness of this model for high porosity bounded media (Beckermann et al., 1988; Hill and Straughan, 2009a,b; Neale and Nader, 1974; Nield and Bejan, 2006; Nishimura et al., 1986; Singh et al., 1993; Singh et al., 2009; Straughan, 2002; Zhao et al., 2004). Porous materials with porosity close to unity are encountered in several industrial and geophysical applications. For example, storage systems and metallic foams (foamets) belong to this category. The latter (foamets) are used extensively in numerous industrial applications such as lightweight structures, biomedical implants, fluid filters, electrodes, heat exchangers and chemical reactors (Lefebvre et al., 2008; Straughan, 2002). As mentioned in the literature (see, e.g., Zhao et al., 2004), high porosity materials requiring the use of Brinkman model are also of much current interest in industry in the design of heat transfer devices.

The governing sets of partial differential equations for each region – clear region and porous medium – have been presented in the next section, along with the corresponding boundary and interface conditions. The resulting boundary value problem has been solved numerically using a suitable ADI method (Mallinson and de Vahl Davis, 1973; Singh et al., 2000). Illustrative sketches for streamlines and isotherms have been shown for both ramped and constant temperature profiles at a side wall. They have been analyzed in relation to the Darcy and Rayleigh numbers with a view to assess the extent of the natural convection process in both clear and permeable regions of the cavity.

FLOW CONFIGURATION AND GOVERNING EQUATIONS

We consider a steady two-dimensional free convective flow of a viscous incompressible fluid in a rectangular cavity comprising a clear region overlying a porous medium of large permeability and bounded by impermeable walls. The physical situation is shown in Fig 1. The x' -axis is taken along the vertical direction while the y' -axis is taken along the horizontal direction, the positive direction of y' -axis being directed into the fluid. The clear and porous regions are separated by a horizontal interface at $x'=d'$, allowing the fluid in either region to migrate to the other. As regards the temperature T' on the walls of the cavity, the left vertical wall of the enclosure is assumed to have a spatially ramped temperature distribution while the right vertical wall is isothermal with constant temperature T_c . The ramped temperature on the left vertical wall is assumed in the forms as given in Eqs (10) and (11), below. It is known from physical considerations that the consequence of keeping the walls $y'=0$ and $y'=L$ at different temperatures is to have density variation in the fluid, leading to buoyancy. Such buoyancy effects are known to cause free convection currents. Furthermore, the horizontal upper and lower walls of the enclosure are assumed to be perfectly thermally insulated. We assume that all physical properties of the fluid are constant except the density in the buoyancy term, where the Boussinesq approximation is assumed to hold.

We shall next introduce the governing equations of the two-dimensional flow for each region. We denote the equations in the clear region by the subscript f and those in the porous region by the subscript p . The equations are

Clear region ($0 \leq x' \leq d', 0 \leq y' \leq L$):

$$\frac{\partial u'_f}{\partial x'} + \frac{\partial v'_f}{\partial y'} = 0, \quad (1)$$

$$\begin{aligned} \rho'_f \left(u'_f \frac{\partial u'_f}{\partial x'} + v'_f \frac{\partial u'_f}{\partial y'} \right) = -\frac{\partial P'_f}{\partial x'} + \\ + \mu_f \left(\frac{\partial^2 u'_f}{\partial x'^2} + \frac{\partial^2 u'_f}{\partial y'^2} \right) + g\rho'_f, \end{aligned} \quad (2)$$

$$\begin{aligned} \rho'_f \left(u'_f \frac{\partial v'_f}{\partial x'} + v'_f \frac{\partial v'_f}{\partial y'} \right) = -\frac{\partial P'_f}{\partial y'} + \\ + \mu_f \left(\frac{\partial^2 v'_f}{\partial x'^2} + \frac{\partial^2 v'_f}{\partial y'^2} \right), \end{aligned} \quad (3)$$

$$\begin{aligned} c_f \rho'_f \left(u'_f \frac{\partial T'_f}{\partial x'} + v'_f \frac{\partial T'_f}{\partial y'} \right) = \\ = \kappa_f \left(\frac{\partial^2 T'_f}{\partial x'^2} + \frac{\partial^2 T'_f}{\partial y'^2} \right). \end{aligned} \quad (4)$$

Porous region ($d' \leq x' \leq H, 0 \leq y' \leq L$):

$$\frac{\partial u'_p}{\partial x'} + \frac{\partial v'_p}{\partial y'} = 0, \quad (5)$$

$$\begin{aligned} \rho'_p \left(u'_p \frac{\partial u'_p}{\partial x'} + v'_p \frac{\partial u'_p}{\partial y'} \right) = -\frac{\partial P'_p}{\partial x'} + \\ + \mu_p \left(\frac{\partial^2 u'_p}{\partial x'^2} + \frac{\partial^2 u'_p}{\partial y'^2} \right) + g\rho'_p - \frac{\mu_p}{K} u'_p, \end{aligned} \quad (6)$$

$$\begin{aligned} \rho'_p \left(u'_p \frac{\partial v'_p}{\partial x'} + v'_p \frac{\partial v'_p}{\partial y'} \right) = -\frac{\partial P'_p}{\partial y'} + \\ + \mu_p \left(\frac{\partial^2 v'_p}{\partial x'^2} + \frac{\partial^2 v'_p}{\partial y'^2} \right) - \frac{\mu_p}{K} v'_p, \end{aligned} \quad (7)$$

$$\begin{aligned} c_p \rho'_p \left(u'_p \frac{\partial T'_p}{\partial x'} + v'_p \frac{\partial T'_p}{\partial y'} \right) = \\ = \kappa_p \left(\frac{\partial^2 T'_p}{\partial x'^2} + \frac{\partial^2 T'_p}{\partial y'^2} \right), \end{aligned} \quad (8)$$

where u' and v' are, respectively, the components of fluid velocity in the x' and y' directions, P' is the pressure, T' is the fluid temperature, ρ' is the density, μ is the viscosity, g is the acceleration due to gravity, K is the permeability of the porous medium, C is the specific heat at constant pressure and κ is the thermal conductivity of the fluid.

Eqs (1) – (8) are to be solved subject to a set of boundary conditions at $x'=0, H$; $y'=0, L$ and a physically plausible set of matching conditions at the interface $x'=d'$. The boundary conditions are given as

$$\frac{\partial T'}{\partial x'} = 0 \quad \text{on } x' = 0, H, \quad (9)$$

$$T' = T_H \quad \text{for } 0 \leq x' \leq d', y' = 0, \quad (10)$$

$$\begin{aligned} T' = T_c - \frac{T_H - T_c}{H - d'} (x' - H) \\ \text{for } d' \leq x' \leq H, y' = 0, \end{aligned} \quad (11)$$

$$u' = v' = 0 \quad \text{on } x' = 0, H \quad \text{and} \quad y' = 0, L. \quad (12)$$

Next, we shall present the matching conditions at the clear fluid-porous medium interface. The assumptions of the continuity of temperature and heat flux at the interface $x' = d'$ lead to

$$T'_f = T'_p, \quad \kappa_f \frac{\partial T'_f}{\partial x'} = \kappa_p \frac{\partial T'_p}{\partial x'} \quad (13)$$

while the continuity of velocity and shear stress at $x' = d'$ yield

$$u'_f = u'_p, \quad v'_f = v'_p, \quad (14)$$

$$\mu_f \frac{\partial v'_f}{\partial x'} = \mu_p \frac{\partial v'_p}{\partial x'}. \quad (15)$$

In the momentum Eqs (2), (3), (6), (7) and the matching condition (15), μ_f and μ_p are, in general, not equal. However, as reported by Neale and Nader (1974), and used by a number of researchers in later works, (e.g., Kuznetsov, 1996; Bhatt and Sacheti, 1994; Singh and Thorpe, 1995), in certain situations, the assumption $\mu_f = \mu_p$ yields reasonably accurate results. Accordingly, we shall use this assumption in our analysis. We shall also assume that $\rho_f = \rho_p$ and $c_f = c_p$. Furthermore, the commonly used Boussinesq approximation is

$$\rho' = \rho'_0 [1 - \beta(T' - T'_0)], \quad (16)$$

where β is the coefficient of thermal expansion, and ρ'_0 and T'_0 are the reference values of density and temperature, respectively. The above approximation presumes that the fluid properties are constant except that the influence of the density variation arising from the temperature differential can be incorporated in a body force term.

In order to solve the set of governing Eqs (1) – (8), subject to the boundary and matching conditions (9) – (15), we, as a first step, re-formulate our problem in terms of the new dependent variables Ψ' , the stream function and ξ , the vorticity. The continuity equation for the incompressible flow – Eq. (1) or Eq. (5) – is now used to introduce Ψ' as

$$u' = \frac{\partial \Psi'}{\partial y'}, \quad v' = -\frac{\partial \Psi'}{\partial x'}. \quad (17)$$

On the other hand, the vorticity function ξ is defined as

$$\xi = \frac{\partial v'}{\partial x'} - \frac{\partial u'}{\partial y'}. \quad (18)$$

Having introduced Ψ' and ξ , one can easily eliminate the pressure gradient terms in either set of momentum equations – Eqs (2), (3) or Eqs (6), (7) – using the Eqs (16), (17) and (18). This finally leads to our formulation of the problem in terms of Ψ' and ξ , as

$$\frac{\partial^2 \Psi'_f}{\partial x'^2} + \frac{\partial^2 \Psi'_f}{\partial y'^2} = -\zeta'_f, \quad (19)$$

$$u'_f \frac{\partial \zeta'_f}{\partial x'} + v'_f \frac{\partial \zeta'_f}{\partial y'} = v_f \left(\frac{\partial^2 \zeta'_f}{\partial x'^2} + \frac{\partial^2 \zeta'_f}{\partial y'^2} \right) + g\beta \frac{\partial T'_f}{\partial y'}, \quad (20)$$

$$\frac{\partial^2 \Psi'_p}{\partial x'^2} + \frac{\partial^2 \Psi'_p}{\partial y'^2} = -\zeta'_p, \quad (21)$$

$$u'_p \frac{\partial \zeta'_p}{\partial x'} + v'_p \frac{\partial \zeta'_p}{\partial y'} = v_p \left(\frac{\partial^2 \zeta'_p}{\partial x'^2} + \frac{\partial^2 \zeta'_p}{\partial y'^2} \right) + g\beta \frac{\partial T'_p}{\partial y'} - \frac{v_p}{K} \zeta'_p. \quad (22)$$

In the above, $\nu = (\mu / \rho)$ is the kinematic viscosity of the fluid.

The boundary and matching conditions corresponding to the Eqs (12), (14) and (15) now become

$$\Psi' = 0 \quad \text{on } x' = 0, H \quad \text{and} \quad y' = 0, L, \quad (23)$$

$$\zeta' = -\frac{\partial^2 \Psi'}{\partial x'^2} \quad \text{on } x' = 0, H, \quad (24)$$

$$\zeta' = -\frac{\partial^2 \Psi'}{\partial y'^2} \quad \text{on } y' = 0, L, \quad (25)$$

$$\zeta'_f = \zeta'_p, \quad \frac{\partial \zeta'_f}{\partial x'} = \frac{\partial \zeta'_p}{\partial x'}, \quad (26)$$

$$\Psi'_f = \Psi'_p, \quad \frac{\partial \Psi'_f}{\partial x'} = \frac{\partial \Psi'_p}{\partial x'}. \quad (27)$$

It may be noted that the condition (24) follows from the fact that the streamlines near the boundaries $x' = 0$ and $x' = H$ are parallel to the y' -axis, hence showing negligible variation with respect to the y' -coordinate. A similar argument applies to Eq. (25) also.

NON-DIMENSIONAL EQUATIONS

We introduce the non-dimensional quantities

$$(x, y) = (x', y') / L, \quad u = Lu' / \alpha_f, \quad v = Lv' / \alpha_f, \\ \zeta = L^2 \zeta' / \alpha_f, \quad \Psi = \Psi' / \alpha_f, \\ \theta = (T' - T_c) / (T_H - T_c), \quad (28)$$

where $\alpha_f = \kappa_f / (\rho_f c_f)$ and the non-subscripted quantities refer to either the clear fluid or the porous medium. Using Eq. (28) together with the non-dimensional clear-region depth parameter $d = d' / L$ and the aspect ratio $Ar = H / L$ in Eqs

(17) – (22), (4), (8), (9) – (11), (13) and (23) – (27), the non-dimensional equations are obtained as

$$u = \frac{\partial \Psi}{\partial y}, \quad v = -\frac{\partial \Psi}{\partial x}, \quad (29)$$

$$\xi = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}. \quad (30)$$

Clear region ($0 \leq x \leq d, 0 \leq y \leq 1$):

$$\frac{\partial^2 \Psi_f}{\partial x^2} + \frac{\partial^2 \Psi_f}{\partial y^2} = -\zeta_f, \quad (31)$$

$$u_f \frac{\partial \zeta_f}{\partial x} + v_f \frac{\partial \zeta_f}{\partial y} = \text{Pr} \left(\frac{\partial^2 \zeta_f}{\partial x^2} + \frac{\partial^2 \zeta_f}{\partial y^2} \right) + \text{Ra} \text{Pr} \frac{\partial \theta_f}{\partial y}, \quad (32)$$

$$u_f \frac{\partial \theta_f}{\partial x} + v_f \frac{\partial \theta_f}{\partial y} = \frac{\partial^2 \theta_f}{\partial x^2} + \frac{\partial^2 \theta_f}{\partial y^2}. \quad (33)$$

Porous region ($d \leq x \leq \text{Ar}, 0 \leq y \leq 1$):

$$\frac{\partial^2 \Psi_p}{\partial x^2} + \frac{\partial^2 \Psi_p}{\partial y^2} = -\zeta_p, \quad (34)$$

$$u_p \frac{\partial \zeta_p}{\partial x} + v_p \frac{\partial \zeta_p}{\partial y} = \text{Pr} \left(\frac{\partial^2 \zeta_p}{\partial x^2} + \frac{\partial^2 \zeta_p}{\partial y^2} \right) + \text{Ra} \text{Pr} \frac{\partial \theta_p}{\partial y} - \frac{\text{Pr}}{\text{Da}} \zeta_p, \quad (35)$$

$$u_p \frac{\partial \theta_p}{\partial x} + v_p \frac{\partial \theta_p}{\partial y} = \lambda \left(\frac{\partial^2 \theta_p}{\partial x^2} + \frac{\partial^2 \theta_p}{\partial y^2} \right). \quad (36)$$

Boundary conditions and matching conditions:

$$\frac{\partial \theta}{\partial x} = 0 \quad \text{on} \quad x = 0, \text{Ar}, \quad (37)$$

$$\theta = 1 \quad \text{for} \quad 0 \leq x \leq d, \quad y = 0, \quad (38)$$

$$\theta = \frac{x - \text{Ar}}{d - \text{Ar}} \quad \text{for} \quad d \leq x \leq \text{Ar}, \quad y = 0, \quad (39)$$

$$\Psi = 0 \quad \text{on} \quad x = 0, \text{Ar} \quad \text{and} \quad y = 0, 1, \quad (40)$$

$$\zeta = -\frac{\partial^2 \Psi}{\partial x^2} \quad \text{on} \quad x = 0 \quad \text{and} \quad x = \text{Ar}, \quad (41)$$

$$\zeta = -\frac{\partial^2 \Psi}{\partial y^2} \quad \text{on} \quad y = 0 \quad \text{and} \quad y = 1, \quad (42)$$

$$\theta_f = \theta_p, \quad \frac{\partial \theta_f}{\partial x} = \lambda \frac{\partial \theta_p}{\partial x} \quad \text{on} \quad x = d, \quad (43)$$

$$\zeta_f = \zeta_p, \quad \frac{\partial \zeta_f}{\partial x} = \frac{\partial \zeta_p}{\partial x} \quad \text{on} \quad x = d, \quad (44)$$

$$\Psi_f = \Psi_p, \quad \frac{\partial \Psi_f}{\partial x} = \frac{\partial \Psi_p}{\partial x} \quad \text{on} \quad x = d. \quad (45)$$

The non-dimensional physical parameters appearing in Eqs (32), (35) and (36) are defined as

$$\text{Pr} = \frac{\nu_f}{\alpha_f}, \quad \text{Ra} = \frac{g\beta L^3 (T_H - T_c)}{\nu_f \alpha_f}, \quad \text{Da} = \frac{K}{L^2}, \quad \lambda = \frac{\kappa_p}{\kappa_f}, \quad (46)$$

and these are the well-known parameters: Prandtl number Pr, Rayleigh number Ra, Darcy number Da, and the ratio λ of the thermal conductivities of porous and fluid layers.

RESULTS

To solve the system of partial differential equations in the above section, we have employed a false transient method (Mallinson and de Vahl Davis, 1973) by which the nonlinear equations are transformed to parabolic form, and the transformed equations are then discretized on a non-uniform grid. The resulting finite difference equations have been solved by a well-known alternating direction implicit (ADI) method, following Singh et al. (2000).

The free convective flow considered here is governed by a set of non-dimensional parameters Ar, Pr, Da, Ra, λ and d . In this work, we have examined the influence of mainly two key parameters – the Darcy number Da and the Rayleigh number Ra – on the streamlines and temperature, keeping the remaining parameters fixed. Furthermore, in studies involving ramped temperature at a boundary, it is desirable to compare the effects of ramped temperature profile vis-à-vis constant temperature profile (Chandran et al., 2005; Singh et al., 2008). We have thus computed the streamlines as well as isotherms for a range of values of Da and Ra. These have been illustrated in Figs 2 – 5. Furthermore, the variation of temperature in the cavity has also been shown in Figs 6 and 7. In each figure, the effects of ramped wall temperature in comparison to isothermal surface have been illustrated. In Figs 2 – 7, we have assumed the values Ar = 1, Pr = 0.71 and $\lambda = 1$, while the thickness parameter d has been set at 0.5, indicating that the clear region and the porous region occupy equal space of the cavity.

In Figs 2 and 3, we have shown the effects of permeability on streamlines, for Ra = 10^5 and 10^6 , respectively. It can be seen that the streamlines for Da = 10^{-5} and Da = 10^{-6} are very similar. However, as Da increases, the innermost streamlines undergo changes. This is particularly manifested for high-

er values of Ra , as can be seen from Fig. 3 for $Da = 10^{-4}$ ramped temperature case; here the innermost streamline breaks into two loops. One can also notice the formation of boundary layers in the clear region of the cavity for higher values of the buoyancy parameter Ra ($= 10^6$). As regards the comparison of streamlines for the ramped wall temperature case with the constant profile case, we notice that for $Ra = 10^5$, the pattern of innermost streamlines is more sensitive to changes in Da . In this case, the effect of Da on the innermost streamlines is seen to be opposite in nature.

The variations of isotherms with the parameters Da and Ra have been shown in Figs 4 and 5. It can be noted that the isotherms, in general, are not too sensitive to variations in these parameters. However, they are greatly influenced by the temperature profile at the wall $y = 0$. The effect of ramped wall temperature on the isotherms is conspicuous if one compares any of the profiles in these figures with their constant temperature profile counterparts. The comparative effects of ramped and isothermal wall temperatures have been further illustrated in Figs 6 and 7, and are self-explanatory.

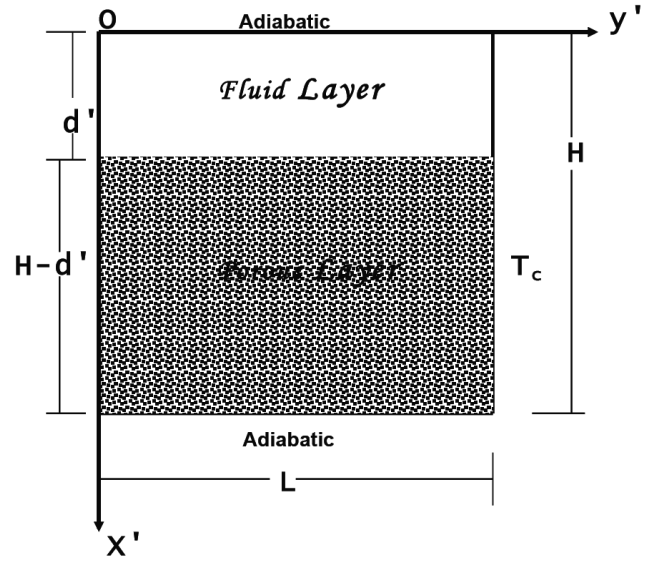


Fig 1. Physical description.

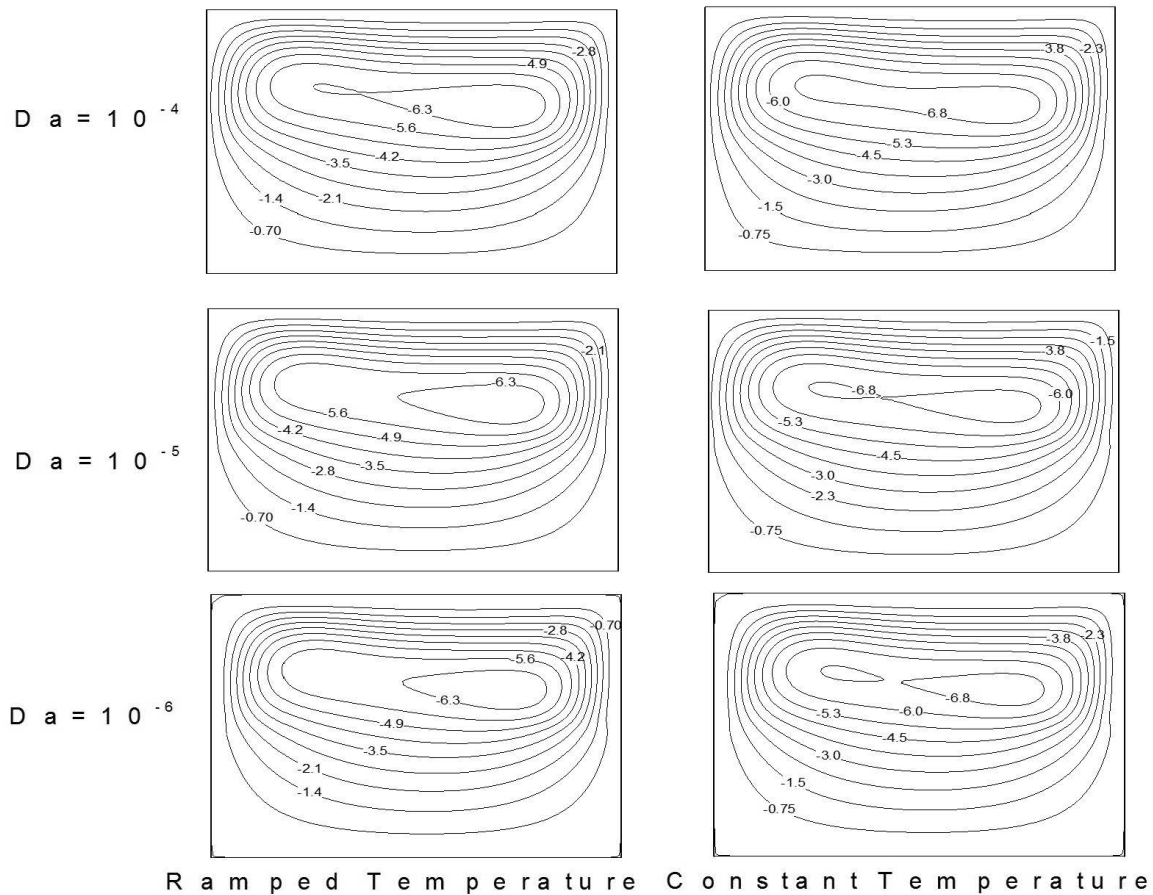


Fig 2. Streamlines. ($Ra = 10^5$).

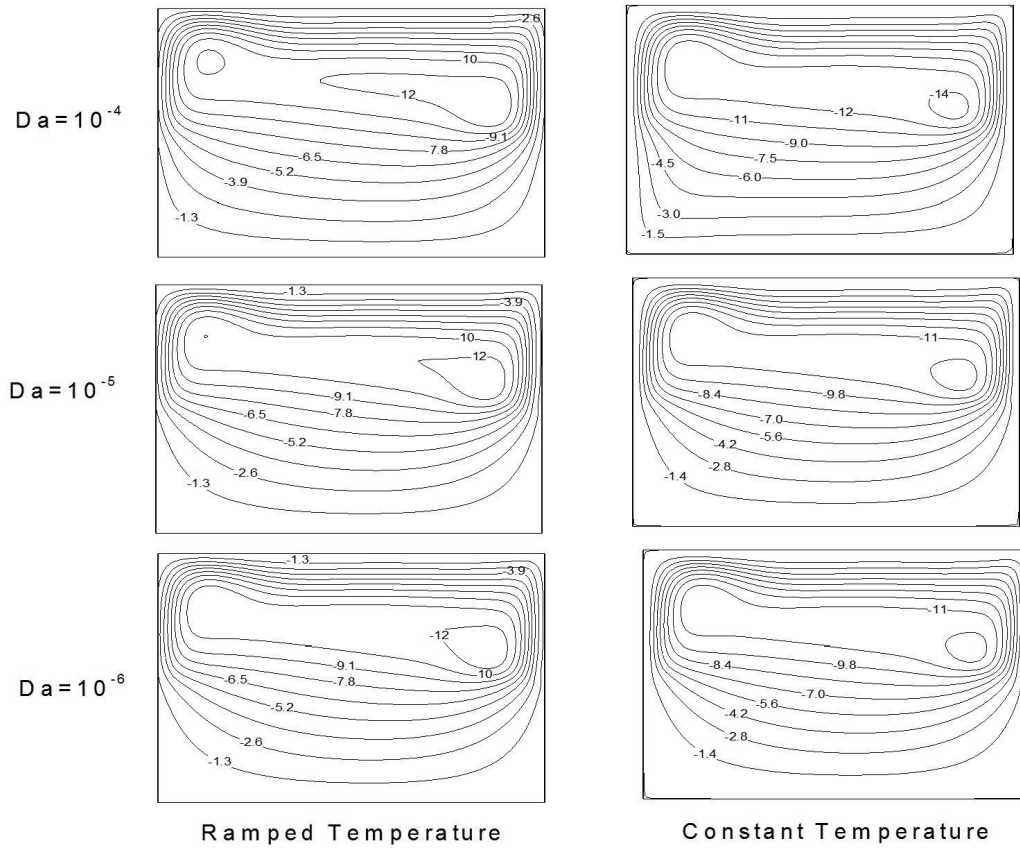


Fig 3. Streamlines. ($Ra = 10^6$).

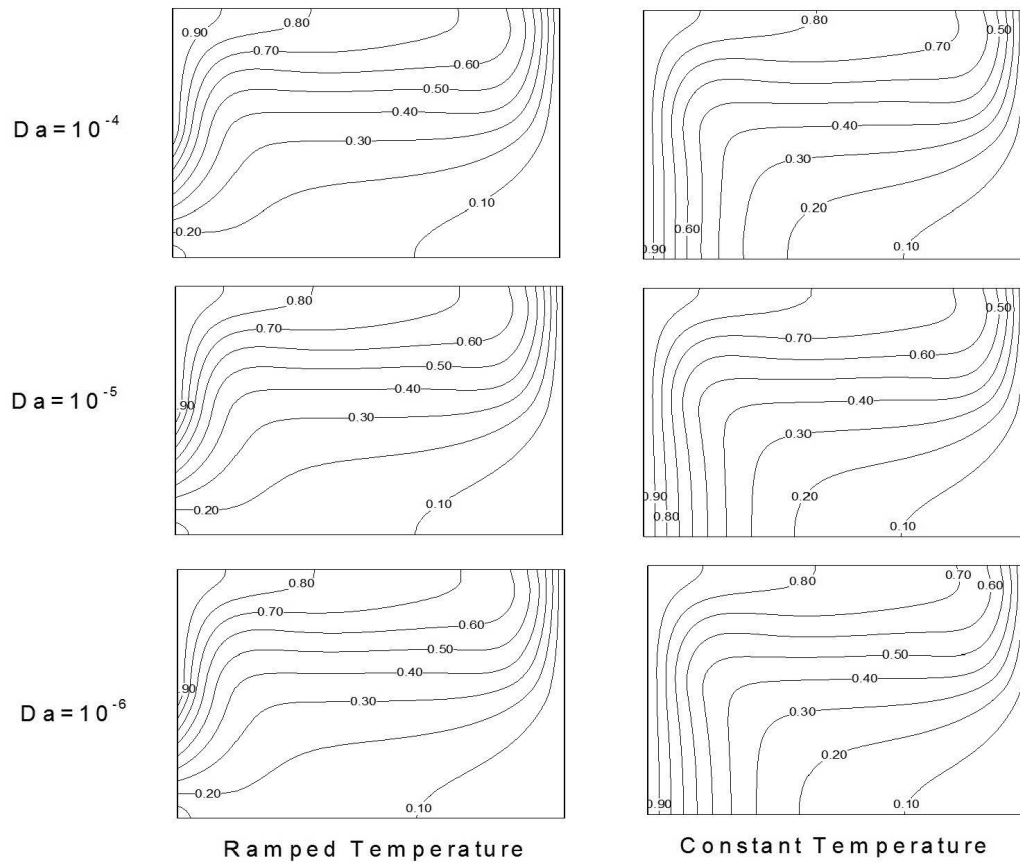


Fig 4. Isotherms. ($Ra = 10^5$).

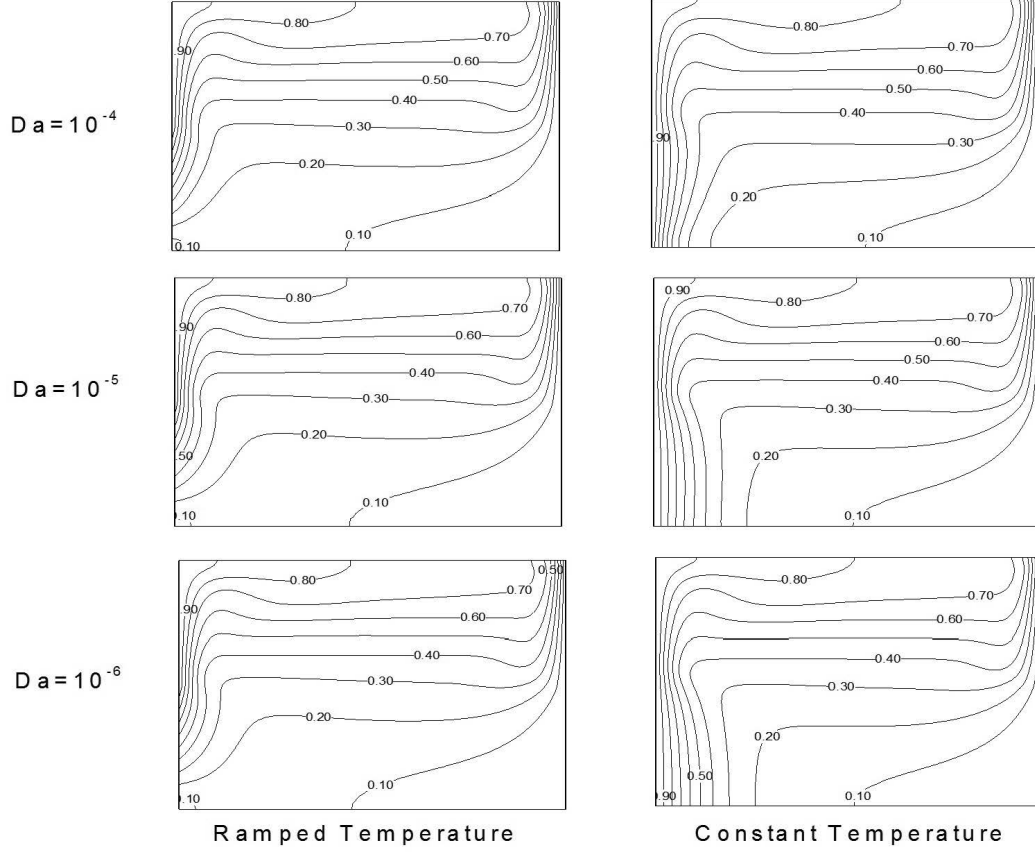


Fig 5. Isotherms. ($Ra = 10^6$).

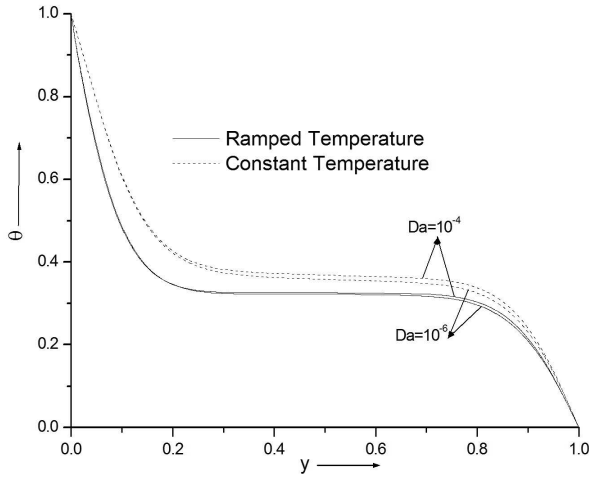


Fig 6. Temperature θ . ($Ra = 10^5$).

Finally, in Table 1, we have compared the maximum absolute value of the stream function, $|\Psi|_{\max}$, and the average value of Nusselt number, Nu_{av} . The importance of $|\Psi|_{\max}$ arises from the fact that it is directly related to the intensity of the natural convection inside the cavity while the average Nusselt number represents the overall rate of heat transfer. The average Nusselt number is defined as

$$Nu_{av} = \frac{1}{Ar} \int_0^{Ar} Nu(x) dx, \quad (47)$$

where $Nu(x)$ is the local Nusselt number on the hot wall, defined by

$$Nu(x) = \begin{cases} \frac{\partial \theta_f}{\partial y}, & 0 \leq x \leq d \\ \frac{1}{2} \left(\frac{\partial \theta_f}{\partial y} + \lambda \frac{\partial \theta_p}{\partial y} \right), & x = d \\ \lambda \frac{\partial \theta_p}{\partial y}, & d \leq x \leq Ar \end{cases} \quad (48)$$

From Table 1, we observe that the effect of increase of Da is to enhance $|\Psi|_{\max}$ and Nu_{av} for both ramped and constant temperature wall conditions. Qualitatively, similar behaviour is also observed for Ra . However, the effect of Ra is more pronounced than that of Da . We further observe that $|\Psi|_{\max}$ increases with the aspect ratio Ar for both ramped as well as constant temperature wall conditions. In contrast, the behaviour of Nu_{av} with regard to increase in Ar is noteworthy in that this parameter attains its maximum for only a square cavity. Finally, with regard to the relative influence of ramped temperature vis-à-vis constant temperature, we notice that the former causes moderate reduction in both convection currents and the rate of heat transfer.

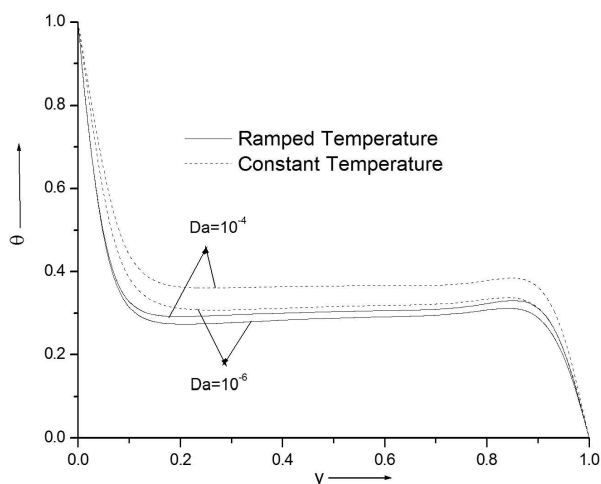


Fig 7. Temperature θ . ($Ra = 10^6$).

Table 1. Values of $|\Psi|_{\max}$ and Nu_{av} . ($Pr = 0.71$, $\lambda = 1.0$, $d = 0.5$).

Ar	Ra	Da	$ \Psi _{\max}$		Nu_{av}	
			T Constant	T Ramped	T Constant	T Ramped
0.5	10^5	10^{-4}	2.7704	2.5812	2.1609	1.8541
0.5	10^5	10^{-6}	2.6217	2.4484	2.0388	1.7627
0.5	10^6	10^{-4}	8.2740	7.9133	6.5143	5.8016
0.5	10^6	10^{-6}	7.9818	7.7480	6.0182	5.5546
1.0	10^5	10^{-4}	7.2246	6.8621	3.3184	3.0473
1.0	10^5	10^{-6}	7.1065	6.7963	3.2458	3.0073
1.0	10^6	10^{-4}	14.001	12.866	6.6394	6.0775
1.0	10^6	10^{-6}	13.145	12.585	6.1202	5.8516
2.0	10^4	10^{-4}	6.6426	6.2925	1.7641	1.6000
2.0	10^4	10^{-6}	6.6043	6.2730	1.7597	1.5942
2.0	10^5	10^{-4}	11.547	10.887	3.0615	2.9035
2.0	10^5	10^{-6}	11.306	10.796	3.0145	2.8765

CONCLUSIONS

This paper deals with a steady two-dimensional free convective coupled flow taking place in the confines of a rectangular vertical cavity with impermeable bounding walls. The cavity space is assumed to consist of a clear fluid region overlying a fluid-saturated porous medium, with Brinkman model governing the flow in the permeable region. It is further assumed that the left vertical wall of the cavity is subjected to ramped temperature distribution. The governing flow equations for each region, allowing for Boussinesq approximation and formulated in terms of vorticity and stream function, have been solved numerically subject to a host of boundary and matching conditions using an ADI method. The influence of two key parameters – Darcy number and the Rayleigh number – on the streamlines, isotherms and fluid temperature, has been discussed in detail. The results for ramped temperature case have been compared with the corresponding results for isothermal temperature profile. Some quantities of interest, arising in a number of applications, have also been computed.

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