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BASSET FORCE IN NUMERICAL MODELS OF SALTATION

NIKOLAY LUKERCHENKO†, JINDRICH DOLANSKY, PAVEL VLASAK

Institute of Hydrodynamics of Academy of Sciences of the Czech Republic, v. v. i., Pod Patankou 30/5, 166 12 Prague 6, Czech Republic; Mailto: vlasak@ih.cas.cz

In numerical models of fluid flow with particles moving close to solid boundaries, the Basset force is usually calculated for the particle motion between particle-boundary collisions. The present study shows that the history force must also be taken into account regarding particle collisions with boundaries or with other particles. For saltation – the main mode of bed load transport – it is shown using calculations that two parts of the history force due to both particle motion in the fluid and to particle-bed collisions are comparable and substantially compensate one another. The calculations and comparison of the Basset force with other forces acting on a sand particle saltating in water flow are carried out for the different values of the transport stage. The conditions under which the Basset force can be neglected in numerical models of saltation are studied.

KEY WORDS: Basset Force, Bed Load Transport, Numerical Model, Particle-Bed Collision.

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V numerických modelech proudění tekutin s pevnými částicemi v blízkosti pevné stěny je Bassetova historická síla obvykle počítána pro pohyb částice mezi jejími jednotlivými kolisemi se dnem. Předložená studie ukazuje, že při výpočtu Bassetovy historické síly je nutné brát v úvahu kolisi částice s pevným dnem nebo s jinými částicemi. Pro saltaci, hlavní typ pohybu splavenin u dna koryta, je na základě použitých výpočtů ukázáno, že dvě části Bassetovy historické síly, tj. síly způsobené pohybem částice v tekutině a kolisí částice se dnem, jsou srovnatelné a mohou se vzájemně významně kompensovat. Výpočet Bassetovy historické síly a její srovnání s ostatními silami působícími na písčitou částici při jejím saltačním pohybu ve vodě je uskutečněn pro různé hodnoty tzv. transport stage (poměr aktuálního a kritického smykového napětí na dně). Zároveň byly studovány podmínky, za nichž může být Bassetova historická síla v numerických modelech zanedbána.

KLÍČOVÁ SLOVA: Bassetova historická síla, pohyb splavenin, numerický model, kolise částice se dnem.

1. Introduction

For the transport of sediment particles by a water flow three modes of particle motion are usually distinguished, depending on the flow conditions, size and density of the sediment particles, and size of particles forming the bed, (e.g., *van Rijn*, 1984): (1) rolling and (or) sliding motion; (2) saltation motion; and (3) suspended particle motion. When the value of the shear velocity u_* just exceeds the critical value u_{*c}, the particles initiate a motion in the form of rolling and (or) sliding. The dimensionless parameter τ_*, named the Shields parameter or the dimensionless bed shear stress, corresponds to the shear velocity u_*: $\tau_* = u_*^2/(gRd_p)$, where d_p is the particle diameter, $R = \rho_p/\rho_f - 1$, ρ_p is the particle density, ρ_f – the fluid density, and g is the acceleration due to gravity. The critical Shields parameter τ_{*c} corresponds to the critical shear velocity u_{*c}. The value of τ_{*c} can be obtained from the Shields diagram (e.g., *van Rijn*, 1984). The dimensionless parameter $T_* = \tau_*/\tau_{*c} = (u_*/u_{*c})^2$ is named the transport stage, $T_* = 1$ corresponds to the particle's initiation of motion. For increasing values of the transport stage, the particles will move along the bed by jumps; this particle motion is referred to as saltation.

When the value of the transport stage exceeds the critical value T_{*c} that corresponds to the condition under which the value of the shear velocity exceeds the fall velocity w_s of the particle, i.e., $u_* \geq w_s$, the upward turbulent forces are comparable with or of higher order than the submerged weight of the particles and as result the sediment particles may go into suspension (*van Rijn*, 1984; *Vlasak* et al., 2012). The values of the transport stage considered in the present study are considerably less than the critical values or values for suspended load (*Galia* and *Hradecky*, 2011). In this case the particle motion occurs in the saltation mode.

The history force evaluation requires significant computer resources. The relative accelerations of the particle have to be stored for integration over the entire lifetime of the particle what can make the calculation very time and memory consuming. In the case of large numbers of particles simulations become even more demanding. Search for the solution to this problem results in different approaches. First approach employs the fact that the history force can be entirely neglected under the condition that it is small in comparison with the other forces acting on the particle (e.g., *Schmeeckle* and *Nelson*, 2003; *Kholpanov* and *Ibyatov*, 2005; *Lee* et al., 2000, 2006). The other development is based on a resource-saving method of history force calculation (e.g., *Michaelides*, 1992; *Bombardelli* et al., 2008), which was successfully used e. g. by *Bialik* (2011a). Finally, there have been proposed new approaches for the evaluation of the history force which allow reducing of the computational costs (e.g., *Hinsberg* et al., 2011).

Bombardelli et al. (2008) using calculations and the findings of *Mordand* and *Pinton* (2000) and *Niño* and *Garcia* (1994, 1998) arrived at the following conclusion: the history force must be included in Lagrangian models of bed-load transport for particles whose fall velocity w_s is such that the particle Reynolds number Re_p (defined by the fall velocity w_s) is smaller than about 4000; $\mathrm{Re}_p = w_s d_p/\nu$, where ν is the kinematic viscosity of the fluid. This condition is examined in the present study for saltation of a sand particle in channel with a rough fixed bed. The forces acting on the particle are calculated for different particle sizes and flow conditions. The history force is compared with other forces and the conditions under which the history force is negligible are discussed.

The main trait of bed load transport that distinguishes it from the other particle-laden flows and must be taken into account in numerical models is that the particle motion occurs near a solid rough boundary – the channel bed with irregular and random configuration. It results in random process of particle-bed collisions.

In its classical definition, the integral associated with the history force (hereafter called the history integral) must be calculated between the initial and current time. In the case of particle saltation in fluid flow over a rough bed, a few particle collisions with the bed are possible during this period. From a mathematical point of view, the collision model usually supposes that during the infinitesimal time of the collision, an infinite force acts on the particle so that the impulse of the force remains finite. The infinitely large force corresponds to an infinitely large acceleration. Since the relative particle acceleration is contained in the integrand of the history integral, the particle acceleration due to the collision must be taken into account – although theoretically its period of activity is zero. The present study shows that for a particle saltating in fluid flow over a rough bed, the part of the history force related to particle-bed collisions and that related to particle motion in fluid are comparable and substantially compensate one another.

This paper is organized as follows. The used numerical model of particle saltation is briefly described in Section 2. In Section 3 the history force related to the particle-bed collision is introduced. In Section 4 it is shown that two parts of the history force – due to both particle motions in fluid and particle-bed collisions – can be under some conditions comparable and can substantially compensate one another. In Section 5 the comparison of the forces acting on the saltating particle are realized. In Section 6 are obtained the conditions under which the Basset force can be neglected. The final section concludes with a summary of the work. In Appendix A the formula for the history force due to the particle collision is derived for arbitrary continuous kernels and in Appendix B the correlation between the particle Reynolds number and critical transport stage for sand particles in water is considered.

2. Numerical model of the particle saltation

For the calculations, the 3D numerical model of *Lukerchenko* et al. (2009a) is used. It can be briefly described as follows.

Using the *Mei* et al. (1991) form of the governing equation (introduced by *Maxey* and *Riley*, 1983) for the motion of a small spherical particle in an unbounded fluid, *Niño* and *Garcia* (1994) proposed equations for the 2D mean trajectory of the saltating particle in a turbulent boundary layer. Extending these equations to the 3D model and taking into account also particle rotation, the system of the governing equations for the particle saltation motion in the channel with a rough bed can be written as

$$\rho_p \Omega \frac{d\boldsymbol{v}}{dt} = \boldsymbol{F_d} + \boldsymbol{F_m} + \boldsymbol{F_g} + \boldsymbol{F_h} + \boldsymbol{F_M}, \tag{1}$$

$$J \frac{d\omega}{dt} = \boldsymbol{T}, \tag{2}$$

where t is time, $\boldsymbol{v}$ – the vector of the velocity of the particle's centre of mass, $\boldsymbol{\omega}$ – the vector of angular velocity of the particle's rotation around its diameter, J – the particle's moment of inertia, $\Omega = 4\pi r^3/3$ is the particle's volume, $r = 0.5\ d_p$ is the particle's radius; $\boldsymbol{F_d}$, $\boldsymbol{F_m}$, $\boldsymbol{F_g}$, $\boldsymbol{F_h}$, $\boldsymbol{F_M}$ are the following forces: drag force, force due to added mass, submerged gravitational force, history force, and Magnus force per unit volume, respectively, and $\boldsymbol{T}$ is the drag torque of viscous forces acting on the particle.

Let $\boldsymbol{v_r} = \boldsymbol{v} - \boldsymbol{v_f}$ be the vector of the particle's relative velocity, $\boldsymbol{v_f}$ be the vector of fluid velocity; $\boldsymbol{\omega_r} = \boldsymbol{\omega} - 0.5 \text{ rot } \boldsymbol{v_f}$ be the vector of the particle's relative angular velocity, and $\mathbf{g}$ be the vector of the acceleration due to gravity.

The expression for the drag force is

$$\boldsymbol{F_d} = -C_d \rho_f |\boldsymbol{v_r}| \boldsymbol{v_r} \pi d_p^2/8. \tag{3}$$

The force due to added mass can be written as

$$\boldsymbol{F_m} = \rho_f \Omega C_m \left[(\boldsymbol{v}\nabla)\boldsymbol{v_f} - \frac{d\boldsymbol{v}}{dt} \right], \tag{4}$$

where $C_m = 0.5$ is the added mass coefficient.

The submerged gravitational force is

$$\boldsymbol{F_g} = \Omega (\rho_p - \rho_f)\, \mathbf{g}. \tag{5}$$

The expression for the history force is discussed in the next section.

The Magnus force is

$$\boldsymbol{F_M} = C_M \Omega \rho_f [\boldsymbol{\omega_r}, \boldsymbol{v_r}]. \tag{6}$$

The drag torque is

$$\boldsymbol{T} = -C_\omega \frac{\rho}{2} \omega_{\boldsymbol{r}} |\omega_{\boldsymbol{r}}| r^5. \tag{7}$$

In (3), (6), and (7), C_d, C_M, and C_ω are the dimensionless drag force, Magnus force and drag torque coefficients, respectively. These coefficients are the functions of two dimensionless parameters: the Reynolds number $\text{Re} = |\boldsymbol{v_r}|\ d_p/\nu$ (based on the relative particle-fluid velocity $|v_r|$, similarly as *Niño* and *Garcia*, 1994) and the rotational Reynolds number $\text{Re}_\omega = |\boldsymbol{\omega_r}|\ r^2/\nu$ (based on the relative angular velocity $|\omega_r|$, e.g., *Oesterle* and *Dinh*, 1998; *Michaelides*, 2003; *Lukerchenko* et al., 2008).

For the calculation of particle-bed collisions the contact zone stochastic method is used. Immediately before the collision the contact zone is calculated as the set of points on the particle surface in which contact with the bed is possible. The contact zone depends on the direction of the particle velocity immediately before the collision. The contact point is chosen from the points of the contact zone in random manner using a random-number generator. Then the coordinate system is transformed to the collision coordinate system, in which the impulse equations can be written in the simplest form. The system of impulse equations is used to calculate the translational and angular velocities immediately after the collision. The values of these velocities are the initial conditions for the calculation of the next particle trajectory.

3. History force related to the collision

If a small spherical particle moves in a viscous fluid flow, the history force $\boldsymbol{F_h}$ acting on the particle can be written in the following form (*Mei* and *Adrian*, 1992):

$$\boldsymbol{F_h} = -6\pi \mu_f r \int_{-\infty}^{t} \frac{d\boldsymbol{v_r}}{d\tau} K(t-\tau, \tau)\, d\tau, \tag{8}$$

where t is the current moment and μ_f – the fluid dynamic viscosity.

At Reynolds number Re << 1 the history force is known as the Basset force $\boldsymbol{F_B}$ with the kernel (*Basset*, 1888)

$$K_B(t-\tau, \tau) = [\rho_f r^2/\pi\mu_f(t-\tau)]^{1/2} \tag{9}$$

and the collision history force is then (see Appendix A)

$$\boldsymbol{F_{Bc}} = -6r^2 \sqrt{\pi \mu_f \rho_f} \frac{\boldsymbol{v_p^+} - \boldsymbol{v_p^-}}{\sqrt{t - t_c}}. \tag{10}$$

For saltation of the sand particle in water flow over a rough bed the values of the Reynolds number can reach a few hundreds or even thousands. Moreover, the particle rotation also can influence on the forces, including the history force, acting on the particle. Formulas for the history force, which are valid for these conditions, have up to now not been obtained. Seemingly, the expression of the history force kernel supposed by *Mei* and *Adrian* (1992).

$$K_{MA}\left(t-\tau,\tau\right)=\left\{\left[\frac{\pi\nu}{r^2}\left(t-\tau\right)\right]^{\frac{1}{4}}+\left[\frac{\pi}{2r\nu}\left(\frac{\left|\boldsymbol{v_r}\left(\tau\right)\right|}{0.75+0.105\mathrm{Re}\left(\tau\right)}\right)^3\left(t-\tau\right)^2\right]^{\frac{1}{2}}\right\}^{-2}, \tag{11}$$

which is valid also for finite Reynolds number, reflects the behaviour of the history force more adequately than kernel (9). The expression (11) shows that the history kernel decays for small times as $t^{-1/2}$ but for large times as t^{-2}.

Lovalenti and *Brady* (1993a,b) derived an expression for the hydrodynamic force acting on a rigid spherical particle translating with arbitrary time-dependent motion in an unsteady flow also for small Reynolds numbers Re < 1.

Lawrence and *Mei* (1995) and *Lovalenti* and *Brady* (1995) shown that the asymptotic behavior of the kernel at large times may be as t^{-2} or t^{-1} or even exponential, depending on the type of motion (sudden stop, sudden increase, reverse motion, etc.)

Kim et al. (1998) using a three-dimensional numerical solution of the Navier–Stokes equations proposed a modified history kernel that is valid for Re up to 150 and particle to fluid density ratios from 5 to 200.

In numerical models of saltation the history force is usually used in the Basset form (e.g., *Niño* and *Garcia*, 1994; *Lukerchenko* et al., 2006, 2009a, *Bialik*, 2011a), i.e., with the history kernel (9). For any kernel that decays faster than the kernel (9), the magnitude of the history force is less than the magnitude of the Basset force. Therefore the estimations obtained below of the particle Reynolds numbers Re_p, above which the Basset force can be neglected, can be considered as upper estimations. In other words, for these and larger values of Re_p, the history force can be also neglected for any kernel that decays faster than (9), including the kernel (11).

4. Basset force calculation

Let us consider a spherical particle saltating in fluid flow over a rough bed. The *x*-axis coincides with the downstream direction, the *y*-axis is normal to the bed and upwards, and the *z*-axis is in the lateral direction. For example, let us consider the *y*-component of the Basset history force $F_{By} = F_{Bmy}+F_{Bcy}$, where F_{Bmy} is the part of this force's component related to the particle motion in the fluid and F_{Bcy} is that related to particle-bed collisions. This component of the Basset force has an influence on the height of the particle's trajectory. Let us show that $F_{Bmy} > 0$ and $F_{Bcy} < 0$. It follows from physical considerations that the particle has a *y*-component of instantaneous acceleration $a_y \leq 0$ during the entire period of its motion in the fluid. The integrand of the Basset integral is negative and $F_{Bmy} > 0$.

The *y*-component of the particle velocity immediately before a collision with the bed is $u^-_{py} < 0$ and immediately after the collision, $u^+_{py} > 0$. According to (10), $F_{Bcy} < 0$ for each particle-bed collision.

The calculations were carried out for sand particles (density ρ_p = 2650 kg m^{-3}) saltating in water (density ρ_f = 1000 kg m^{-3}; ν = 10^{-6} m^2 s^{-1}) in a channel with a rough bed. The bed roughness k_s was taken to be equal to the saltating particle diameter, i.e., $k_s = d_p$.

Fig. 1 shows the calculated changes of the Basset force components and its modulus during five successive jumps of the saltating particle. The modulus of the submerged gravitational force $|\boldsymbol{F}_\mathbf{g}|$ = const is chosen as a force unit, with the second the unit of time. The value of the particle Reynolds number Re_p = 3970 corresponds to a particle diameter of d_p = 7 mm. (the correlation between the parameters Re_p and d_p is given in the Appendix B. The value of the Shields parameter τ_* = 0.2 corresponds to the shear velocity u_* = 0.15 m s^{-1}.

Fig. 1b) depicts the typical dependence of the *y*-component of the Basset force F_{By} and its parts F_{Bmy} and F_{Bcy} on *t*. The particle motion was calculated during 100 particle jumps. The Basset force related to particle-bed collisions was calculated as the sum of all collisions that occurred from the initial to the current moment using (10).

In Fig. 1b) F_{By} is completely different from F_{Bmy}. The parts F_{Bmy} and F_{Bcy} substantially compensate one another, while the total value of the Basset force *y*-component, F_{By} is significantly closer to zero than the value of F_{Bmy}. The same is true for the *x*- and *z*-components of the Basset force.

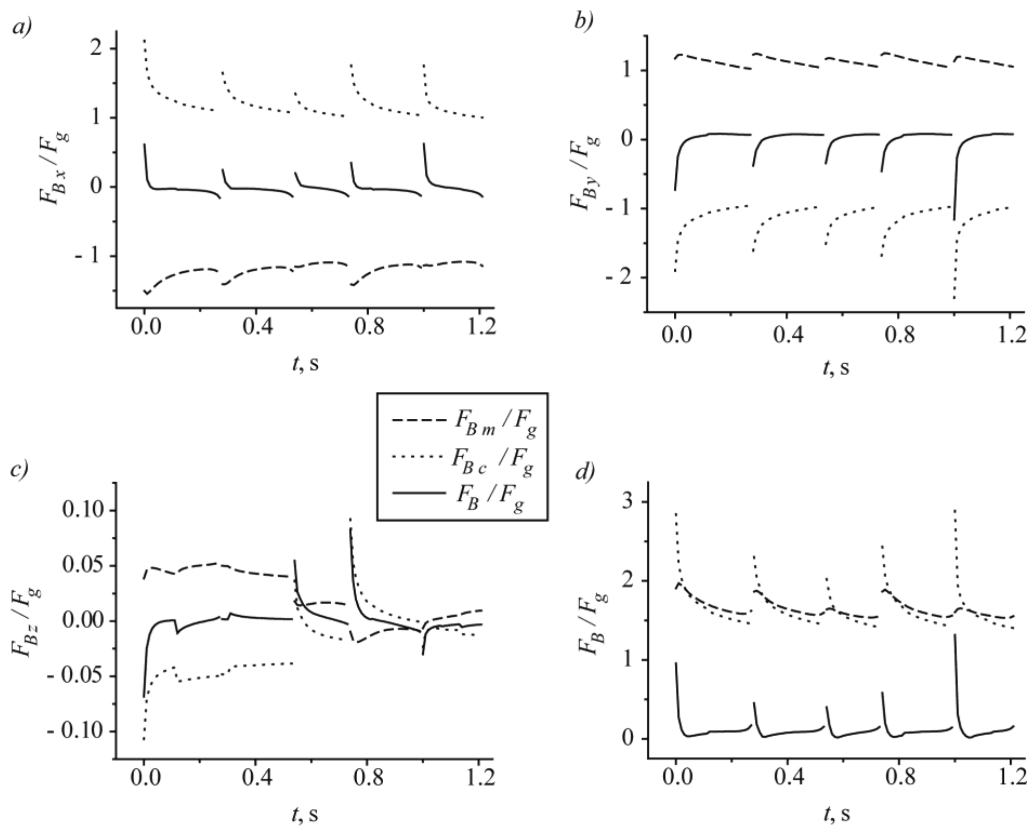

Fig. 1. Typical dependences of the components and modulus of the Basset force F_B and its parts F_{Bm} and F_{Bc} on time, during five successive jumps of the saltating particle ($Re_p = 3970$; $\tau_* = 0.2$).

5. Comparison of the forces acting on the particle

According to *Bombardelli* et al. (2008), the Basset force must be included in Lagrangian models of bed-load transport for particle Reynolds numbers Re_p smaller than about 4000. Let us examine this statement, using the calculation and comparison of the Basset force with other forces acting on a particle saltating in water flow. A sand particle of diameter $d_p = 7$ mm in water corresponds to the value of the Reynolds number $Re_p = 3970 \approx 4000$ (Appendix B).

The calculation is carried out under the following conditions: the flow shear velocity is $u_* = 0.15$ m s^{-1}, corresponding to the transport stage $T_* = 3.6$.

In Fig. 2 the components and moduli of the drag, Magnus, Basset and resultant forces versus time t are shown during the five successive jumps of the saltating particle. The resultant force is the vector sum of all forces acting on the particle: the drag force, Magnus force, submerged gravitational force, force due to added mass and Basset force. The absolute values of the Basset force components are significantly less than the absolute values of the components of the drag and Magnus forces. However, a mathematically-exact evaluation of the force's contribution to the particle motion is necessary to answer the question: "How large can the error due to neglect of the Basset force be?"

The contribution of a force to particle motion during time t can be characterized by the average absolute values of the force components and average value of the force modulus:

$$\hat{F}_i = t^{-1}\int_0^t \left|F_i(t)\right| dt \quad \text{and} \quad \hat{F} = t^{-1}\int_0^t \left|\boldsymbol{F}(t)\right| dt . \qquad (12)$$

These quantities make possible to compare relative importance of particular forces acting on the particle. It is documented in Fig. 3.

The plots represent the contributions of the particular forces to the resultant force for the given particle Reynolds number $Re_p = 3970$ ($d_p = 7$ mm). More accurately, the ratios of the average values of the components and moduli of the drag, Magnus and Basset forces to the average values of the components and modulus of the resultant force are depicted as functions of the transport stage T_*. It was

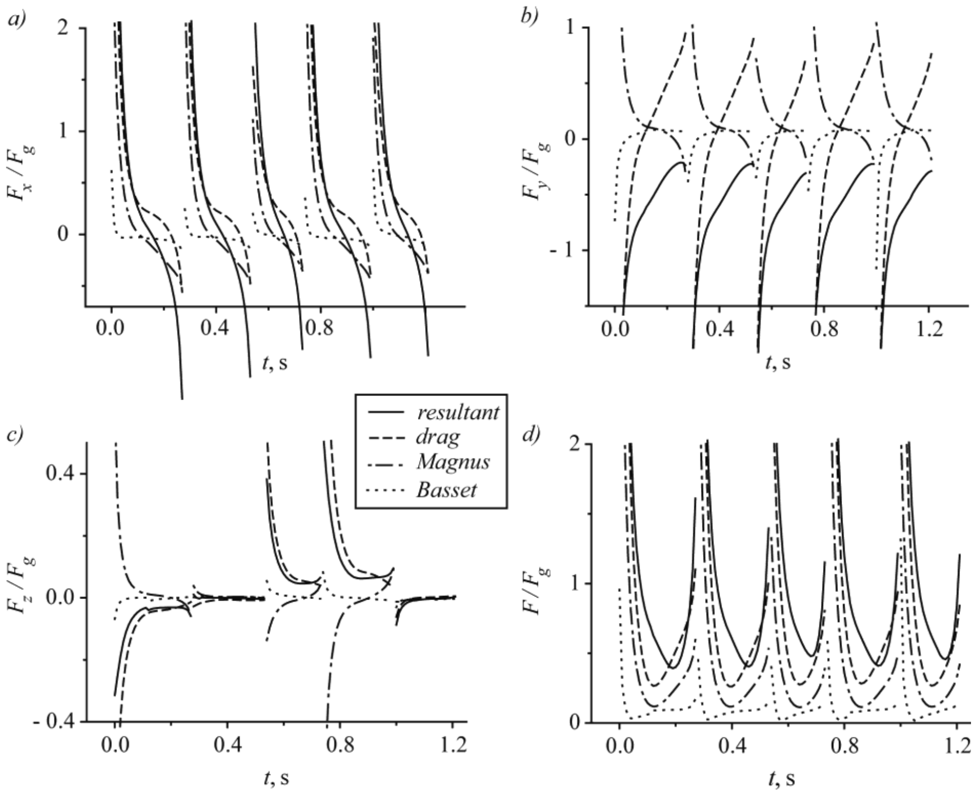

Fig. 2. The components and moduli of the drag, Magnus, Basset and resultant forces during five successive jumps of the particle ($\mathrm{Re}_p = 3970$; $\tau_* = 0.2$).

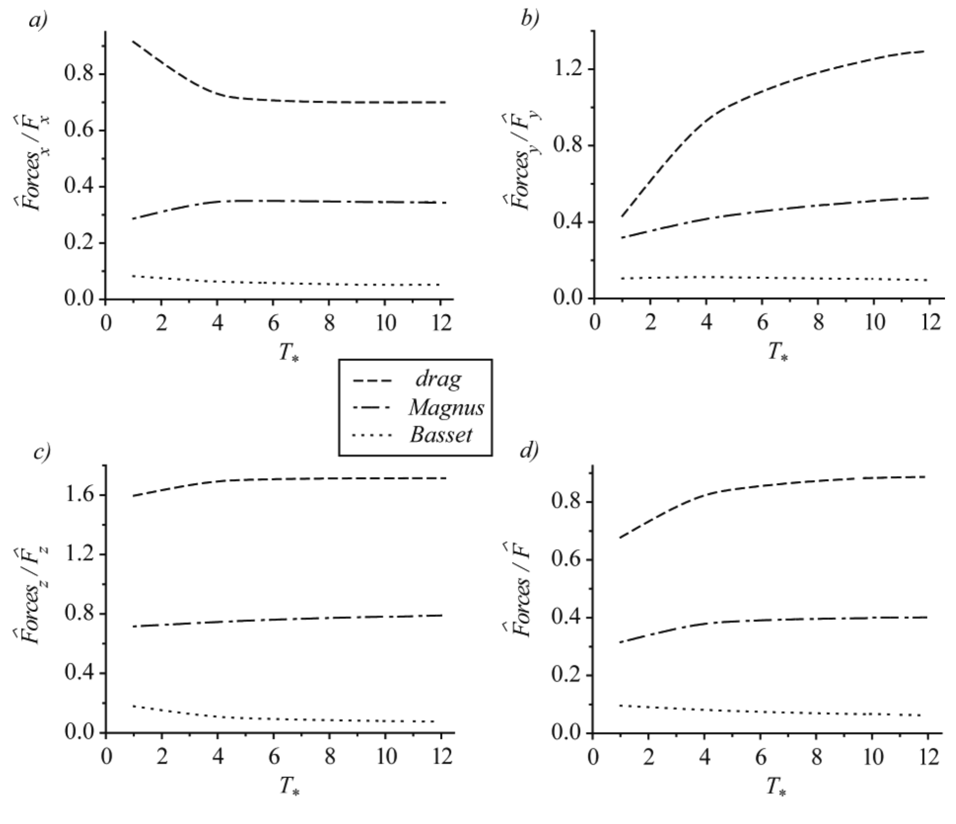

Fig. 3. The ratios of the average values of the components and moduli of the drag, Magnus and Basset forces, to the average values of the components and modulus of the resultant force, versus transport stage ($\mathrm{Re}_p = 3970$).

conducted that the Basset force modulus remains small in comparison with other forces. However, for $T_* = 1$ the Basset force modulus is about 17% of the drag force modulus, and thus we can infer that especially in initiation of motion the Basset force contributes to motion of the particle by a certain amount.

6. Conditions under which the Basset force can be neglected

The ratios of the average Basset force components and modulus to the average resultant force components and modulus versus the transport stage is shown in Fig. 4. The values of the particle Reynolds numbers 530, 1650, 3970 and 8000 in the plots correspond to particle diameters of 2, 4, 7 and 11 mm respectively, for sand particle saltation in water flow. These plots represent the contribution of the Basset force to the resultant force.

With a decrease in the particle Reynolds number, the contribution of the Basset force increases. For $\mathrm{Re}_p = 3970$, the contribution of the x-component is less than about 8%, the y-component less than 11%, the z-component less than 18% and modulus less than 10%. Let us suppose that the Basset force can be neglected in the numerical model if its contribution to the resultant force does not exceed about 10%. Then for 2D numerical models, in which the z-component is not considered, the previously-mentioned conclusion of *Bombardelli* et al. (2008) can be reformulated a little: The Basset force can be neglected in Lagrangian models of bed-load transport if the particle Reynolds number Re_p is larger than about 4000. For the saltation of sand particles in water this corresponds to the condition $d_p \geq 7$ mm.

However for 3D numerical models, in which the z-component is important – for example, in the case of the calculation of the lateral dispersion of particles (e.g., *Lukerchenko*, 2009b; *Bialik*, 2011b; *Bradley* et al., 2010; *Nikora* et al., 2001) – the Basset force can be neglected only if the particle Reynolds number Re_p is larger than about 8000.

As was noted above, these evaluations of the Re_p are upper bounds. The kernel of the Basset force was derived for vanishing values of the Reynolds number Re. The kernels of the history force mentioned in Section 3 that are valid for finite values of the Re, decay faster than the kernel of the Basset force. Therefore, under the same conditions the magnitude of the history force is less than the magnitude of the Basset force and the history force can be neglected also for less values of the Re_p.

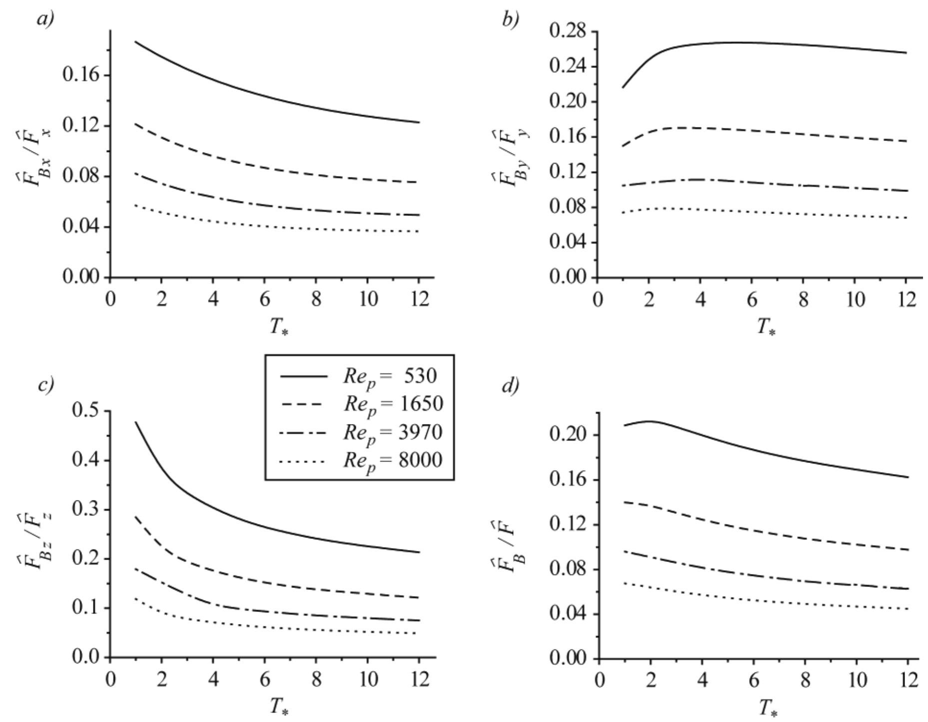

Fig. 4. The ratio of the average Basset force components and modulus to the average resultant force components and modulus, versus the transport stage.

7. Conclusions

The contribution of particle collisions to the Basset history force has been taken into account in the numerical modeling of spherical particle saltation. A particle collision brings to a peak the increase in the Basset force during a short time period so that its impulse remains finite.

It was shown that for a particle saltating in fluid flow over a rough bed the two parts of the Basset force (the first related to the particle motion in fluid and the second to the particle-bed collisions) are comparable and substantially compensate one another.

Calculations for the example Re_p = 3970 show that the Basset force components and its modulus are significantly less than the components and modulus of the drag and Magnus forces.

The ratios of the average Basset force components and modulus to the average resultant force components and modulus were calculated as functions of the transport stage for the values 530, 1650, 3970 and 8000 of the particle Reynolds numbers.

For 2D numerical models of saltation, the Basset force can be neglected if the particle Reynolds number Re_p is larger than about 4000; for 3D numerical models of saltation in the cases where the lateral component is important, the Basset force can be neglected if $\mathrm{Re}_p \geq 8000$. In these cases the inaccuracy of the resultant force calculations, connected to the neglect of the Basset force, is less than about 10%.

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Appendix A.

Let us suppose that the kernel $K(t-\tau, \tau)$ in (8) is a continuous function. The integral must be performed, by definition, between the beginning of time and the current time t. The history of the particle motion during this period must therefore be taken completely into account, including particle collisions with boundaries and with other particles.

Let us consider a single particle collision, which occurs at the moment $t_c < t$. Let us denote by δt_c the small period of time during which the collision continues, and then the contribution of the collision to the integral in (8) is:

$$I_{hc} = \int_{t_c}^{t_c+\delta t_c} \frac{d\boldsymbol{v_r}}{d\tau} K(t-\tau,\tau)\,d\tau = \int_{t_c}^{t_c+\delta t_c} \frac{d\boldsymbol{v_p}}{d\tau} K(t-\tau,\tau)\,d\tau - \int_{t_c}^{t_c+\delta t_c} \frac{d\boldsymbol{v_f}}{d\tau} K(t-\tau,\tau)\,d\tau. \tag{A.1}$$

The second integral in (A.1) tends to zero if $\delta t_c \to 0$ from physical considerations because the fluid acceleration is finite – the velocity of a viscous fluid is a continuous function of time and coordinates. Let us suppose that the first integral can be written using the theorem of the mean as

$$\int_{t_c}^{t_c+\delta t_c} \frac{d\boldsymbol{v_p}}{d\tau} K(t-\tau,\tau)\,d\tau \approx \frac{\delta \boldsymbol{v_p}}{\delta t_c} K(t-\tau_0,\tau_0) \int_{t_c}^{t_c+\delta t_c} d\tau, \tag{A.2}$$

where $t_c \leq \tau_0 \leq t_c + \delta t_c$, $\delta \boldsymbol{v}_p = \boldsymbol{v}_p^+ - \boldsymbol{v}_p^-$ is the change in the particle velocity vector owing to the collision and $\boldsymbol{v}_p^-$ and $\boldsymbol{v}_p^+$ are the particle velocity vectors immediately before and after the collision, respectively. For $\delta t_c \to 0$ we obtain finally

$$I_{hc} = \left(\boldsymbol{v_p^+} - \boldsymbol{v_p^-}\right) K(t-t_c, t_c), \tag{A.3}$$

and the contribution of the particle collision to the history force (hereafter called the "collision history force") can be written as

$$\boldsymbol{F_{hc}} = -6\pi\mu_f r \left(\boldsymbol{v_p^+} - \boldsymbol{v_p^-}\right) K(t-t_c, t_c). \tag{A.4}$$

The similar formulas were obtained by *Lawrence* and *Mei* (1995) for a step change of the fluid velocity in the case of the Basset force (Re << 1) and by *Kim* et al. (1998) for the case where there is a steady flow prior to moment $t = 0$ with an impulse at $t = 0$. It seems that the present derivation is closer to the classical model of the particle collision.

The formula for the calculation of the full history force is

$$\boldsymbol{F_h} = \boldsymbol{F_{hm}} + \boldsymbol{F_{hc}} = -6\pi\mu_f r \left\{ \begin{array}{l} \int_{-\infty}^{t} \frac{d\boldsymbol{v_r}}{d\tau} K(t-\tau,\tau)\,d\tau + \\ \sum_i \left(\boldsymbol{v_{pi}^+} - \boldsymbol{v_{pi}^-}\right) K(t-t_{ci}, t_{ci}) \end{array} \right\}, \tag{A.5}$$

where t_{ci} is the time of the i-th particle collision, and $\boldsymbol{v}_{pi}^{-}$ and $\boldsymbol{v}_{pi}^{+}$ – the particle velocity vectors immediately before and after the i-th collision, respectively.

At Reynolds number Re << 1 the history force is known as the Basset force $\boldsymbol{F_B}$ with the kernel (*Basset*, 1888)

$$K_B(t-\tau,\tau) = [\rho_f r^2/\pi\mu_f(t-\tau)]^{1/2} \tag{A.6}$$

and the collision history force is then (see also *Lukerchenko*, 2010)

$$\boldsymbol{F_{Bc}} = -6r^2\sqrt{\pi\mu_f\rho_f}\,\frac{\boldsymbol{v}_p^{+}-\boldsymbol{v}_p^{-}}{\sqrt{t-t_c}}. \tag{A.7}$$

It follows from (A.6) that at the moment of the collision, the value of the Basset force becomes infinitely large. In moments just after the collision, $t = t_c + \Delta t$ ($\Delta t << t_c$) , the value of $\boldsymbol{F_{hc}}$ is great but the impulse of the Basset force has the order of $\sqrt{\Delta t}$, i.e., is small. Thus a particle collision brings to a peak the increase of the Basset force in a short period of time, so that its impulse remains finite.

Appendix B. Particle Reynolds number and critical transport stage for sand particles in water

In the opinion of the *Bombardelli* et al. (2008), the Basset force must be included in Lagrangian models of bed-load transport for particle Reynolds numbers $\mathrm{Re}_p = w_s d_p/\nu$ smaller than about 4000. Let us find the values of Re_p as a function of particle size for a sand particle (ρ_p = 2650 kg m^{-3}) moving in water (ρ_f =1000 kg m^{-3}; $\nu = 10^{-6}$ m^2 s^{-1}). The value of the particle fall velocity can be calculated from the balance of the forces acting on the particle when it falls with the constant velocity w_s in water – the submerged gravitational force and drag force:

$$\Omega g(\rho_p - \rho_f) = C_d\,(\rho_f w_s^2/2)(\pi d_p^2/4). \tag{B.1}$$

The drag force coefficient C_d can be calculated from the expression (*Niño* and *Garcia*, 1994):

$$C_d = \frac{24}{\mathrm{Re}_p}(1+0.15(\mathrm{Re}_p)^{0.5}+0.017\mathrm{Re}_p) - \frac{0.208}{1+10^4\mathrm{Re}_p^{-0.5}}. \tag{B.2}$$

The system (B.1) and (B.2) was solved using an iteration method. Fig. B1 shows the dependence of the particle Reynolds number on particle size in the case of particle motion in water.

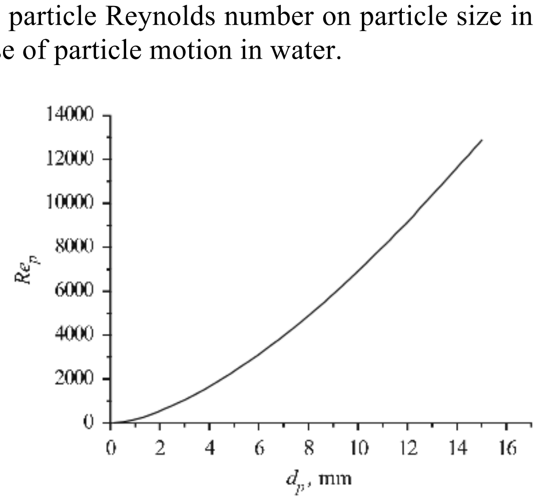

Fig. B1. The particle Reynolds number versus the particle diameter for a spherical sand particle moving in water.

According to Fig. B1, the value of the Reynolds number 4000 corresponds to the particle diameter d_p= 7 mm.

Depending on the size of the bed material and on the flow conditions, the transport of sediment particles by a flow of water can take place as either bed load or suspended load. When the value of the shear velocity exceeds the fall velocity of the particles, i.e., $u_* > w_s$, the particle may go into suspension (*van Rijn*, 1984; *Ramsankaran* et al., 2010). Using this condition, the value of the critical transport stage T_{*c} under which the bed load is transformed into suspension can be found.

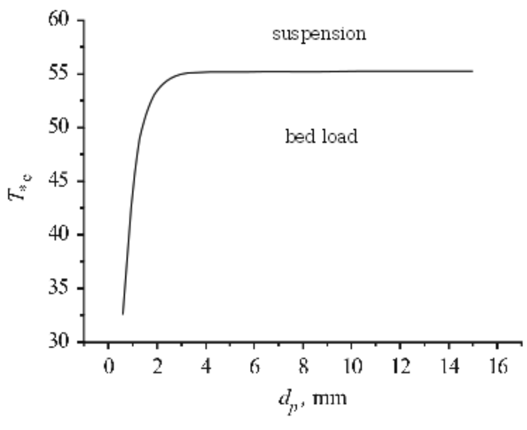

Fig. B2. The critical transport stage versus the particle diameter for a spherical sand particle moving in water.

Fig. B2 shows the approximate plot of the critical transport stage T_{*c} versus the particle diameter d_p in the case of a spherical particle with the density of sand in water flow. For the values of the particle diameter d_p > 2 mm (Re_p >530) used in the present work, the critical transport stage was $T_{*c} > 50$. In these calculations, the values of the transport stage were in the range of 1 to 12, i.e., far from the suspension mode.

List of symbols

C_d – drag force coefficient [–],
C_m – force due to added mass coefficient [–],
C_M – Magnus force coefficient [–],
C_ω – drag rotation coefficient [–],
d_p – diameter of the moving particle [m],
$\boldsymbol{F_B}$ – Basset force [N],
$\boldsymbol{F_h}$ – history force [N],
$\boldsymbol{F_d}$ – drag force [N],
$\boldsymbol{F_g}$ – submerged gravitational force [N],
$\boldsymbol{F_m}$ – force due to added mass [N],
$\boldsymbol{F_M}$ – Magnus force [N],
g – gravitational acceleration [m s^{-2}],
J – particle moment of inertia [kg m^{-2}],
k_s – bed roughness [m],
K – kernel of the history force integral [–],
$\boldsymbol{T}$ – drag torque of viscous forces acting on a rotating particle in fluid [N m],
T_* – transport stage [–],
T_{*c} – critical transport stage [–],
R – dimensionless parameter, which characterizes the relative particle density [–],
r – radius of the moving particle [m],
Re – translational Reynolds number [–],
Re_ω – rotational Reynolds number [–],
Re_p – particle Reynolds number defined by the fall velocity [–],
t – time [s],
t_c – moment of a collision [s],
u_* – fluid shear velocity [m s^{-1}],
u_{*c} – critical fluid shear velocity [m s^{-1}],
$\boldsymbol{v_f}$ – vector of the fluid velocity [m s^{-1}],
$\boldsymbol{v}$ – vector of velocity of the particle centre of mass [m s^{-1}],
$\boldsymbol{v_r}$ – vector of the particle relative velocity [m s^{-1}],
$\boldsymbol{v}^-_p$ – vector of the particle translational velocity immediately before a collision [m s^{-1}],
$\boldsymbol{v}^+_p$ – vector of the particle translational velocity immediately after a collision [m s^{-1}],
w_s – particle fall velocity [m s^{-1}],
δt_c – period of time during which the collision continues [s],
$\delta \boldsymbol{v}_p$ – change in the particle velocity vector owing to the collision [m s^{-1}],
μ_f – dynamic viscosity [Pa s],
ν – kinematic viscosity [m^2 s^{-1}],
ρ_f – fluid density [kg m^{-3}],
ρ_p – density of the moving particle [kg m^{-3}],
τ_* – dimensionless bed shear stress or Shields parameter [–],
τ_{*c} – dimensionless critical shear stress for sediment motion [–],
Ω – particle volume [m^3],
$\boldsymbol{\omega}$ – vector of angular velocity of the particle rotation around its diameter [s^{-1}],
$\boldsymbol{\omega_r}$ – vector of the particle relative angular velocity [s^{-1}].

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