J. Hydrol. Hydromech., *60*, 2012, 1, 45–56
DOI: 10.2478/v10098-012-0004-5

PREDICTIVE CAPABILITY OF BEDLOAD EQUATIONS USING FLUME DATA

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The study on bedload transport behaviour is widely explored from the last few decades and many semi-empirical or empirical equilibrium transport equations are developed. The phenomenon is a very complex due to its varied physical properties like velocity, depth, slope, particle size in the alluvial system. In practical applications, these formulae have appreciable deviation from each other in derivation and also their ranges of applications are different. Here, bedload transports have been categorized into moderate bedload transport and intense bedload transport depending upon the Einstein bedload transport parameter. Based on large database of different bedload measurements, a comparative analysis has been performed to ascertain prediction ability of different bedload equations based on various statistical criteria such as the coefficient of determination, Nash–Sutcliffe coefficient and index of agreement. It has been found that equations based on shear stress have worked better than other approaches (discharge, probabilistic and regression) for flume observations.

KEY WORDS: Alluvial Channel, Bedload Transport, Flume Data, Predictive Capability, Shear Stress.

S. Talukdar, Bimlesh Kumar*), S. Dutta: PRESNOSŤ ROVNÍC TRANSPORTU SPLAVENÍN S VYUŽITÍM ÚDAJOV Z HYDRAULICKÉHO ŽĽABU. J. Hydrol. Hydromech., *60*, 2012, 1; 69 lit., 2 obr., 3 tab.

Výskum transportu splavenín počas posledného obdobia bol relatívne intenzívny; jeho výsledkom bolo množstvo empirických a poloempirických rovníc kvantifikujúcich rovnovážny transport splavenín. Je to zložitá problematika; je to tak v dôsledku meniacich sa fyzikálnych vlastností ako je rýchlosť, hĺbka, sklon, zrnitostné zloženie splavenín v aluviálnom systéme. Výsledky výpočtu z týchto rovníc sa významne líšia a líši sa tiež oblasť ich možnej aplikácie. V tejto štúdii je transport splavenín rozdelený na priemerný a intenzívny, podľa Einsteinovho parametra transportu splavenín. S využitím štatistických metód sme uskutočnili komparatívnu analýzu presnosti rozdielnych rovníc transportu splavenín. Pri analýze bola použitá rozsiahla databáza výsledkov meraní. Výsledkom je, že rovnice založené na informácii o tangenciálnom napätí dávajú lepšie výsledky ako tie, ktoré využívajú pre výpočet transportu splavenín prietoky, pravdepodobnostný prístup a regresie.

KĽÚČOVÉ SLOVÁ: aluviálny kanál, transport splavenín, hydraulický žľab, presnosť výpočtu, tangenciálne napätie.

1. Introduction

Bedload transport in alluvial rivers is the principle link between river hydraulics and river form and is responsible for building and maintaining the channel geometry (*Parker*, 1979; *Leopold*, 1994; *Goodwin*, 2004). Bedload prediction is of primary importance for river engineering, fluvial geomorphology, eco-hydrology, environmental surveys and management, and hazard prediction (*Recking*, 2009). Bedload transport provides the major process linkage between the hydraulic and material conditions that govern river-channel morphology and knowledge of bedload movement is required not only to elucidate the causes and consequences of changes in fluvial form but also to make informed management decisions that affect a river's function. Bedload transport can be described as a random phenomenon that is generated by the interaction of turbulent flow structure with the materials of the bed surface (*Einstein*, 1950). This interaction is very complex and as a result, attempts to model this process have largely resulted in limited or qualified success. There have been two general approaches towards the concept of bedload transport. The first and most popular approach is through the use of a critical variable such as shear stress, stream power, discharge, or velocity. This approach assumes that there is no bedload transport until the critical variable has been exceeded by the flow conditions and that the bedload transport rate increases in proportion to the increase in the flow condition beyond the critical value. There have been numerous studies, both in natural channels and in flumes, based on this concept and there are a large number of these transport rate equations in the literature. Use of these equations depends on the conditions under which the equations were developed. The second approach is based on a probabilistic approach to bedload movement (*Einstein*, 1950). This approach offered new insight into bedload transport processes. However, the level of complexity made application of this method to natural channels very difficult (*Yang*, 1996).

A large number of sediment transport models designed to describe bedload have been formulated e.g. *Meyer-Peter* and *Müller* (1948), *Bagnold* (1956), *Bagnold* (1966), *Einstein* (1950), *Yalin* (1963), *Chang* et al., (1967). Some of the many other transport models which can be mentioned in this context are: *Engelund* and *Hansen* (1967), *Ackers-White* (1973), *Engelund* and *Fredsøe* (1976) and *Van Rijn* (1984). Several researchers have reviewed the bedload formula based on laboratory or field data. *Mahmood* (1980) has compared of twelve bedload functions to study their predictive accuracy. The comparison was based on 97 field measurements of sediment and hydraulic variables in Missouri River. In comparing the results, *Mahmood* (1980) found that Toffaleti's method yields best results. *Bathurst* et al. (1987) assessed the applicability of six bedload equations for steep mountain streams affected by limited sediment availability and bed armoring. They concluded that bedload discharge is best predicted by the Schoklitsch equation. *Gomez* and *Church* (1989) used a set of 410 bedload events from field and flume measurements to test 12 bedload equations for gravel-bed channels. They concluded that none of the tested equations performed consistently because of limitations of the data used and the complexity of the transport phenomena. They noted that the prediction of bedload transport under limited hydraulic information is best accomplished by using equations based on the stream-power concept, whereas the *Einstein* (1950) and *Parker* et al. (1982) equations should be used when local hydraulic information is available. *Reid* et al. (1996) assessed the performance of several popular bed load formulae in the Negev Desert, Israel, and found that the *Meyer-Peter* and *Müller* (1948) and *Parker* (1990a) equations performed best, but their analysis considered only one gravel bed river. *Almedeij* and *Diplas* (2003) considered the performance of the *Meyer-Peter* and *Müller* (1948), *Einstein* (1950), *Parker* (1979) and *Parker* et al. (1982) bed load transport equations on three natural gravel bed streams, using a total of 174 transport observations. They found that formula performance varied between sites, in some cases over predicting observed bed load transport rates by one to three orders of magnitude, while at others under predicting by up to two orders of magnitude. *Bravo-Espinosa* et al. (2003) ascertained that the equations by *Parker* et al. (1982) and *Meyer-Peter* and *Müller* (1948) adequately predicted bedload transport in flumes.

Though a number of field observations and flume experimental results are available, it is difficult to find good data sets to calibrate bedload transport models. For mechanism analyses, flume experimental data are usually preferred because of more control over flow properties and bed materials (*Chen* and *Stone*, 2008). Field observations typically include many complicating factors such as measurement of bed material grain size distributions, variable channel geometry and variable flow conditions that affect the quality of the data. A carefully designed flume experiment can avoid most of the aforementioned difficulties and provide a more comprehensive data set. *Fang* (1998) remarked on the need for a critical evaluation and comparison of the plethora of sediment transport formulae currently available.Thus, present work does an attempt to analyze some of the most used equations for their prediction capability based on different statistical criteria by using a comprehensive database of flume experiments.

2. Basic data

All data sources are not cited in the reference of this paper as they could be found in *Recking* (2006). Experimental procedure and ranges of all observations are detailed in *Recking* (2009, 2010). Observations conform to uniform flow with slope ≥ 0.1%, minimum bed particle diameter is 1mm and particles suspension and bed formation are negligible. The data are distributed into two distinct groups according to whether Φ (Einstein transport parameter) is lower or higher than approximately 0.4 (*Graf*, 1998) and has been termed as *moderate* bedload transport and *intense* bedload transport respectively.

Φ (Einstein transport parameter) is defined as: $\Phi = \frac{q_b}{\gamma_s}\sqrt{\frac{\rho}{\rho_s-\rho}}\left(\frac{1}{gd^3}\right)^{1/2}$. q_b represents the volume of the transported bed material per unit width per unit time, g_s and r_s – the specific weight and specific density of bed particles of diameter (d) respectively and ρ is specific density of water. Total 1282 values have been considered in the present analysis out of which 940 values are identified as *moderate* condition and 342 values as *intense* condition. The range of particle diameter varies between 0.13 mm to 44.3 mm. The flow condition comprises of both sub critical and super critical (Froude number = 0.41 to 5.19). Observed variation of sediment concentration is from 0.1 – 1356000 g m^{-3}. Tab. 1 and 2 shows the ranges of the observations used in the present work.

T a b l e 1a) Summary of hydraulic properties (moderate bedload transport).

Variable	Average	Minimum	Maximum	Std. Dev.
W [m]	0.69	0.15	2	0.37
d [m]	0.0074	0.000506	0.0443	0.009
σ	1.52	1	5.71	0.81
ρ_s	2661.56	1250	2810	104.52
S_o	0.008	0.00099	0.09	0.008
u [m s^{-1}]	0.81	0.221	2.88	0.40
H [m]	0.12	0.0091	1.0921	0.13
C [g m^{-3}]	750.53	0.1	32222.22	2853.26
Φ	0.0071	0.00001	0.04	0.009

T a b l e 1b) Summary of hydraulic properties (intense bedload transport).

Variable	Average	Minimum	Maximum	Std. Dev.
W [m]	0.55	0.10	2.00	0.38
d [m]	0.01	0.00	0.04	0.01
σ	1.58	1.00	8.46	1.25
ρ_s	2650.37	1250.00	2810.00	127.35
S_o	0.02	0.00	0.20	0.04
u [m s^{-1}]	0.94	0.22	2.88	0.47
H [m]	0.07	0.01	0.86	0.08
C [g m^{-3}]	82826.03	314.20	1356000.00	183492.42
Φ	4.58	0.04	264.05	17.96

3. Bedload transport equations

Einstein (1942) defined a bedload formula as "an equation linking the rate of bedload transportation with the properties of the grain and of the flow causing the movement". It is very difficult to accurately measure the bedload transport rate. At present, no universal bedload transport equation exists (*Reid* et al., 1996). *Einstein* (1950) sums up one main reason: "sediment movement and river behavior are inherently complex natural phenomena involving a great many variables". Despite the plethora of available bedload transport relations, there is still considerable controversy over their performance. Using data compiled from literature, it is planned to determine which model best predicts bedload transport. Tab. 3 details about the various equations analyzed in the present work out of which some of the representative formulations have been discussed as follows.

3.1 Shear stress approach

The basic assumption is that, when the flow conditions exceed the criteria for incipient motion, sediment particles on the streambed start to move. The transport of bed particles in a stream is a function of the fluid forces per unit area (the tractive force or shear stress) acting on the streambed. The initiation of motion of bed particles will occur when the Shield's parameter θ is greater than critical value, θ_c, which can be expressed as:

$$\left.\begin{aligned}\theta &= \frac{\tau}{(\gamma_s-\gamma)d}\\ \theta_c &= \frac{\tau_c}{(\gamma_s-\gamma)d}\end{aligned}\right\} \tag{1}$$

where τ is shear stress and τ_c – the critical shear stress. Here, equation given by *Paphitis* (2001) has been used to calculate the value of Shields' critical shear stress θ_c:

$$\theta_c = \frac{0.273}{1+1.2d_*} + 0.046\left(1-0.576e^{-0.05d_*}\right), \tag{2}$$

where d_* is calculated as $d\left[\frac{g(s-1)}{\upsilon^2}\right]^{1/3}$ (*Yalin*, 1977).

Shields (1936) has been the pioneer to describe the threshold shear stress at which the individual particles on a sedimentary bed, comprising nearly spherical shaped and uniform sediments, are on the verge of motion by a unidirectional stream flow and formulated as:

$$\frac{q_b\gamma_s}{q\gamma S} = 10\frac{\tau-\tau_c}{(\gamma_s-\gamma)d}, \tag{3}$$

where q is the discharge per unit width of flow, q_b – the bedload discharge per unit width and S is slope.

T a b l e 2. Selected bedload equations and their range of applicability proposed by different authors.

Sl. No.	Name of author	Formula
		SHEAR STRESS APPROACH
1.	*Shields* (1936)	$\frac{q_b \gamma_s}{q \gamma S} = 10 \frac{\tau_0 - \tau_c}{(\gamma_s - \gamma) d}$
2.	*Wilson* (1966)	$\Phi = 12(\theta - \theta_c)^{3/2}$
3.	*Fernandez-Luque* and *van Beck* (1976)	$\Phi = 5.7(\theta - \theta_c)^{3/2}$
4.	*Graf* and *Suszka* (1987)	$\Phi = \begin{cases} 10.4\theta^{1.5}(1 - \frac{0.045}{\theta})^{2.5} & \theta < 0.068 \\ 10.4\theta^{2.5} & \theta \geq 0.068 \end{cases}$
5.	*Ashmore* (1988)	$\Phi = 3.11(\theta - 0.045)^{1.37}$
6.	*Low* (1989)	$q_b = \frac{6.42}{(s-1)^{0.5}} (\theta - \theta_c)\ d\ u\ S^{0.5}$
7.	*Wiberg* and *Smith* (1989)	$\Phi = \alpha_s\ (\theta - \theta_c)^{3/2}$ $\alpha_s = 9.64(\theta^{0.166})$
8.	*Soulsby* (1997)	$\Phi = 5.1(\theta - \theta_c)^{3/2}$
9.	*Graf* (1998)	$\Phi = 8(\theta - \theta_c)^{3/2}$
10.	*Ribberink* (1998)	$\Phi = 11(\theta - \theta_c)^{1.65}$
11.	*Wong* and *Parker* (2006a)	$\Phi = 4.93(\theta - 0.047)^{1.6}$
12.	*Wong* and *Parker* (2006b)	$\Phi = 3.97(\theta - 0.0495)^{3/2}$
		ENERGY SLOPE APPROACH
13.	*Meyer-Peter* et al. (1934)	$\frac{0.04 q_b^{2/3}}{d} = \frac{q^{2/3} S}{d} - 17$
14.	*Meyer-Peter* and *Müller* (1948)	$\Phi = \begin{cases} 8(\theta - \theta_c)^{3/2} & \theta \geq \theta_c \\ 0 & \theta < \theta_c \end{cases}$
		DISCHARGE APPROACH
15.	*Schoklitsch* (1934)	$q_b = \frac{2.5}{s} S^{3/2} (q - q_c)$ $q_c = 0.26(s-1)^{5/3} \left(\frac{d^{3/2}}{S^{7/6}} \right)$
		PROBABILISTIC APPROACH
16.	*Einstein* (1950)	$\Phi = \begin{cases} \frac{K \exp(-0.391/\theta)}{0.465} & \theta < 0.182 \\ 40 K \theta^3 & \theta \geq 0.182 \end{cases}$ where $K = \sqrt{\frac{2}{3} + \frac{36}{d_*^3}} - \sqrt{\frac{36}{d_*^3}}$
17.	*Parker* (1979)	$\Phi = 11.20 \frac{(\theta - 0.03)^{4.5}}{\theta^3}$
		REGRESSION APPROACH

18.	*Yalin* (1963)	$\Phi = 0.635 r\sqrt{\theta}\left[1-\frac{1}{\sigma r}\ln(1+\sigma r)\right]$ $r = \frac{\theta}{\theta_c}-1,\quad \sigma = 2.45\frac{\sqrt{\theta_c}}{s^{0.4}}$
19.	*Engelund* and *Hansen* (1967)	$\Phi = 0.05(\frac{u}{u_*})^2\theta^{5/2}$
20.	*Ashida* and *Michiue* (1972)	$\Phi = 17(\theta-\theta_c)\left[(\theta)^{1/2}-(\theta_c)^{1/2}\right]$
21.	*Engelund-Fredsoe* (1976)	$\Phi = 18.74(\theta-\theta_c)\left[(\theta)^{1/2}-0.7(\theta_c)^{1/2}\right]$
22.	*Nelson* (1988)	$\Phi = 12\theta^{1/2}(\theta-\theta_c)$
23.	*Madsen* (1991)	$\Phi = (\theta^{1/2}-0.7\theta_c^{1/2})(\theta-\theta_c)$
24.	*Fredsoe* and *Deigaard* (1992)	$q_b = \frac{30}{\pi}(\theta-\theta_c)\left[\theta^{0.5}-0.7\theta_c^{0.5}\right]$
25.	*Van Rijn* (1993)	$\Phi = \frac{0.053}{d_*^{0.3}}\left(\frac{\theta}{\theta_c}-1\right)^{2.1}$
26.	*Nino* and *Garcia* (1994)	$\Phi = \frac{12}{\mu_d}(\theta^{1/2}-0.7\theta_c^{1/2})(\theta-\theta_c)\ ,\qquad \mu_d = 0.23$
27.	*Julien* (1995)	$\Phi = \begin{cases} 2.15e^{-0.391/\theta} & \theta < 0.18 \\ 40\theta^3 & 0.18 \le \theta \le 0.52 \\ 15\theta^{1.5} & \theta > 0.52 \end{cases}$
28.	*Julien* (2002)	$\Phi = \frac{18\sqrt{gd}^{3/2}\theta^2}{\sqrt{g(s-1)d^3}}$
29.	*Abrahams* and *Gao* (2006)	$\Phi = \theta^{1.5}(1-\frac{\theta_c}{\theta})^{3.4}\frac{u}{u_*}$
30.	*Camenen* et al. (2006)	$\Phi_b = 12\ \theta^{0.5}\exp(-\ 4.5\frac{\theta_c}{\theta})$
		EQUAL MOBILITY APPROACH
31.	*Wilcock* (2001)	$q_b = \frac{W_s^*\ u_*^3\ \rho_s}{(s-1)g}\qquad W_s^* = 11.2(1-.846\theta_{50}^{-1/2})^{4.5}$ $\theta = \frac{u_*^2}{(s-1)gd},\quad \theta_{50} = \frac{\theta}{\theta_c}$
32.	*Wilcock-Crowe* (2003)	$q_b = \frac{W^*\ u_*^3\ \rho_s}{(s-1)g}\qquad \theta_{50} = \frac{\theta}{\theta_c},\quad \theta = \frac{u_*^2}{(s-1)gd}$ $W^* = \begin{cases} 14(1-\frac{0.846}{\theta_{50}^{0.5}})^{4.5} & \theta_{50} \ge 1.35 \\ 0.002(\theta_{50})^{7.5} & \theta_{50} < 1.35 \end{cases}$

3.2 *Energy slope approach*

Sediment transport phenomenon is the result of many interacting parameters and one or the other parameter can be replaced by a function of some other ones. One example is the wall shear stress, which depends directly on the slope and the bed roughness. One of the formulae most widely used in laboratory and field investigations as well as in numerical simulations of bedload transport is the empirical relation proposed by *Meyer-Peter* and *Müller* (1948). It allows estimation of the bedload transport rate in an open-channel, as a function of the excess bed shear stress applied by the flowing water. *Meyer-Peter* and *Müller* (1948) formula is expressed as:

T a b l e 3a) Comparison of different efficiency criteria for predicting *moderate* bedload rate.

Sl. No.	Formula	R^2	σ	Distribution Slope	E	I_d
1.	*Meyer-Peter* et al.(1934)	0.942	0.002	0.100	0.028	0.239
2.	*Schoklitsch* (1934)	0.927	0.067	2.450	–0.242	0.326
3.	*Shields* (1936)	0.873	0.003	0.001	–0.242	0.326
4.	*Meyer-Peter* and *Müller* (1948)	0.919	0.054	2.041	–1.030	0.793
5.	*Einstein* (1950)	0.797	0.047	1.064	0.328	0.865
6.	*Yalin* (1963)	0.900	0.040	1.394	0.322	0.894
7.	*Wilson* (1966)	0.920	0.080	3.062	–5.819	0.635
8.	*Engelund* and *Hansen* (1967)	0.762	0.010	0.228	0.227	0.532
9.	*Ashida* and *Michue* (1972)	0.901	0.062	2.239	–2.082	0.746
10.	*Engelund-Fredsoe* (1976)	0.923	0.085	3.610	–9.674	0.573
11.	*Fernandez-Luque* and *van Beck* (1976)	0.919	0.038	1.454	0.396	0.901
12.	*Parker* (1979)	0.906	0.052	1.904	–0.805	0.809
13.	*Graf* and *Suszka* (1987)	0.869	0.025	0.770	0.751	0.923
14.	*Ashmore* (1988)	0.912	0.029	1.017	0.815	0.952
15.	*Nelson* (1988)	0.919	0.086	3.562	–9.429	0.571
16.	*Low* (1989)	0.849	0.00283	0.0007	–0.242	0.326
17.	*Wiberg* and *Smith* (1989)	0.914	0.051	1.858	–0.530	0.824
18.	*Madsen* (1991)	0.923	0.005	0.193	0.184	0.507
19.	*Fredsoe-Deigaard* (1992)	0.712	0.004	0.080	–0.057	0.380
20.	*Van Rijn* (1993)	0.907	0.023	0.763	0.799	0.936
21.	*Julien* (1995)	0.894	0.198	6.152	–43.974	0.338
22.	*Soulsby* (1997)	0.928	0.031	1.240	0.709	0.941
23.	*Nino* and *Garcia* (1994)	0.923	0.237	10.052	–117.902	0.253
24.	*Graf* (1998)	0.910	0.057	1.965	–3.474	0.802
25.	*Ribberink* (1998)	0.835	0.057	2.085	–1.241	0.783
26.	*Wilcock* (2001)	0.901	0.005	0.160	0.108	0.434
27.	*Julien* (2002)	0.840	0.093	2.604	–4.811	0.628
28.	*Wilcock* and *Crowe* (2003)	0.856	0.002	0.050	–0.121	0.336
29.	*Abrahams* and *Gao* (2006)	0.882	0.037	1.084	0.668	0.927
30.	*Camenen* et al. (2006)	0.932	0.191	8.500	–80.060	0.289
31.	*Wong* and *Parker* (2006a)	0.820	0.030	0.991	0.812	0.951
32.	*Wong* and *Parker* (2006b)	0.907	0.028	0.945	0.835	0.954

$$\left[\frac{q_s\left(\gamma_s-\gamma\right)}{\gamma_s}\right]^{2/3}\left(\frac{\gamma}{g}\right)^{1/3}\frac{0.25}{\left(\gamma_s-\gamma\right)d}= \\ =\frac{\left(K_s/K_r\right)^{3/2}\gamma RS}{\left(\gamma_s-\gamma\right)d}-0.047. \tag{4}$$

The slope S and energy loss due to grain resistance S_r can be obtained from Strickler's formula:

$$S=\frac{u^2}{K_s^2R^{4/3}}\ ; \qquad S_r=\frac{u^2}{K_r^2R^{4/3}}\ ; \\ K_s=\frac{21.1}{d^{1/6}} \qquad K_r=\frac{26}{d_{90}^{1/6}}\ , \tag{5}$$

where d_{90} is the size of the sediment for which 90% of the material is finer, K_s and K_r – the Strickler and Müller coefficient respectively.

3.3 *Discharge approach*

Schoklitsch (1934) stated that the use of water depth (shear stress) is not appropriate for the determination of the initiation of motion in natural rivers with steep slopes. *Schoklitsch* (1934) suggested that better results are attained by the use of specific discharge, which is formulated as follows:

$$g_b=2500S^{3/2}\left(q-q_c\right)\ ; \\ q_c=0.26\left(\frac{\gamma_s}{\gamma}-1\right)^{5/3}\frac{d^{3/2}}{S^{7/6}} \tag{6}$$

where g_b and q_c are the bedload rate in weight per unit width per unit time and critical discharge at which sediments began to move.

T a b l e 3b) Comparison of different efficiency criteria for predicting *intense* bedload rate.

Sl. No.	Formula	R^2	σ	Distribution Slope	E	I_d
1.	*Meyer-Peter* et al.(1934)	0.915	0.267	0.027	–0.042	0.175
2.	*Schoklitsch* (1934)	0.916	0.064	0.0014	–0.100	0.157
3.	*Shields* (1936)	0.936	0.005	0.005	–0.099	0.157
4.	*Meyer-Peter* and *Müller* (1948)	0.940	2.606	0.275	0.407	0.630
5.	*Einstein* (1950)	0.965	24.448	3.775	–8.735	0.615
6.	*Yalin* (1963)	0.924	3.426	0.343	0.501	0.718
7.	*Wilson* (1966)	0.940	3.910	0.412	0.588	0.785
8.	*Engelund* and *Hansen* (1967)	0.930	12.512	1.358	0.527	0.922
9.	*Ashida* and *Michue* (1972)	0.944	4.275	0.507	0.694	0.862
10.	*Engelund-Fredsoe* (1976)	0.941	5.257	0.585	0.752	0.897
11.	*Fernandez-Luque* and *van Beck* (1976)	0.940	1.857	0.196	0.281	0.502
12.	*Parker* (1979)	0.941	3.338	0.375	0.547	0.754
13.	*Graf* and *Suszka* (1987)	0.960	7.202	1.084	0.882	0.974
14.	*Ashmore* (1988)	0.930	1.060	0.095	0.096	0.285
15.	*Nelson* (1988)	0.939	4.032	0.417	0.592	0.788
16.	*Low* (1989)	0.890	0.0018	0.0016	-0.100	0.157
17.	*Wiberg* and *Smith* (1989)	0.940	1.857	0.196	0.573	0.776
18.	*Madsen* (1991)	0.941	0.281	0.031	-0.032	0.162
19.	*Fredsoe-Deigaard* (1992)	0.902	5.628	0.528	0.687	0.875
20.	*Van Rijn* (1993)	0.954	5.660	0.832	0.901	0.971
21.	*Nino* and *Garcia* (1994)	0.941	14.627	1.629	0.111	0.873
22.	*Julien* (1995)	0.932	5.123	0.531	0.702	0.868
23.	*Soulsby* (1997)	0.940	1.681	0.176	0.248	0.465
24.	*Graf* (1998)	0.940	2.590	0.275	0.408	0.631
25.	*Ribberink* (1998)	0.948	3.510	0.438	0.627	0.816
26.	*Wilcock* (2001)	0.941	0.326	0.037	-0.020	0.166
27.	*Julien* (2002)	0.957	5.517	0.837	0.906	0.972
28.	*Wilcock* and *Crowe* (2003)	0.950	0.206	0.028	-0.038	0.163
29.	*Abrahams* and *Gao* (2006)	0.928	4.582	0.512	0.694	0.865
30.	*Camenen* et al. (2006)	0.768	5.297	0.165	0.173	0.316
31.	*Wong* and *Parker* (2006a)	0.945	1.568	0.187	0.268	0.494
32.	*Wong* and *Parker* (2006b)	0.940	1.278	0.136	0.176	0.387

3.4 *Probabilistic approach*

A major change in the approach to predicting sediment transport was proposed by *Einstein* (1942, 1950) when he presented a bed-load formula based on probability concepts. *Einstein* (1942) assumed that a sediment particles moves if the instantaneous hydrodynamic lift on the particle exceeds the submerged weight of the particle (*Garde*, 2005). Once the particle is in motion, the probability of the particle being re-deposited us assumed to be equal at all points on the bed. Also, the average distance traveled by any particles moving as bedload is assumed to be constant. Thus, *Einstein* (1942) has obtained a relationship between bedload parameter (also called as Einstein transport parameter) Φ with flow parameter ψ. Flow parameter (ψ) is defined as $\Psi = \frac{(\gamma_s - \gamma) d}{\tau} = \frac{1}{\theta}$. Later *Brown* (1950) has modified the *Einstein* (1942) formulation for bedload (known as Einstein – Brown formula) as follows:

$$\Phi = f(\psi) \tag{7}$$

where,

$$\Phi = \frac{q_b}{K\left[g(\gamma_s - \gamma) d^3\right]^{1/2}} \quad \text{and}$$

$$K = \left[\frac{2}{3} + \frac{36\upsilon^2}{gd^3\left(\frac{\gamma_s}{\gamma} - 1\right)}\right]^{1/2} - \left[\frac{36\upsilon^2}{gd^3\left(\frac{\gamma_s}{\gamma} - 1\right)}\right]^{1/2}.$$

The parameter, *K* accounts the fall velocity of sediment particles. When ψ ≤ 5.5, $\Phi = \frac{40}{\psi^3}$ and $\Phi = 2.15 e^{-0.39\psi}$ *for* $\psi >$ 5.5.

3.5 *Regression approach*

Limitation of defining complex process into precise mathematical expression, which governs the bedload transport, has compelled researchers to use the data driven model (regression, neural network) for explaining bedload transport process. Using regression approach, *Rottner* (1959) has devised the following expression:

$$g_b = \gamma_s \left(\frac{\gamma_s - \gamma}{\gamma} g h^3 \right)^{1/2} \left[\frac{u}{\left(\frac{\gamma_s - \gamma}{\gamma} g h \right)^{1/2}} \left\{ 0.667 \left(\frac{d}{h} \right)^{2/3} + 0.14 \right\} - 0.778 \left(\frac{d}{h} \right)^{2/3} \right]^3, \tag{8}$$

where h and u are the depth and velocity of flow respectively.

3.6 Equal-mobility approach

Based on field data from Oak Creek (*Milhous*, 1973), *Parker* et al. (1982) introduced the equal-mobility theory, which states that the threshold conditions for each size fraction is independent of grain size. As a consequence of equal mobility, the bedload transport rate of a certain flow can be calculated by a single representative grain diameter (*Parker* et al., 1982). Representative one is as follows:

$$q_b = \frac{W^* u_*^3 \rho_s}{\left(\frac{\rho_s}{\rho} - 1 \right) g} \tag{9}$$

$$W^* = \begin{cases} 11.2 \left(1 - \frac{0.822}{\theta_{50}} \right)^{4.5} & \theta_{50} > 1.65 \\ 0.0025 \theta_{50}^{14.2} & \theta_{50} < 0.95 \\ 0.0025 e^{\left(14.2(\theta_{50} - 1) - 9.28(\theta_{50} - 1)^2 \right)} & 0.95 \le \theta_{50} \le 1.65 \end{cases} \qquad \theta_{50} = \frac{\theta}{\theta_c} \tag{10}$$

where W^* is the sediment transport rate and u_* is the shear velocity.

4. Results and discussions

Although based on different approaches, these equations rely primarily on the same general assumptions: (i) the fluid and sediment properties are steady and uniform; (ii) there is an infinite and continuous supply of sediment particle sizes represented for some component of the bed material; (iii) there is a specific relation between hydraulic and sedimentological parameters and the rate at which the bedload is transported; and (iv) the sediment stored in a reach can be neglected (*Gomez* and *Church*, 1989; *Reid* and *Dunne*, 1996; *Graf*, 1998). Results of comparisons of bedload-transport predictions in moderate and intense bedload transport cases are based on different efficiency criteria:

The efficiency criteria are defined as mathematical measures of how a model simulation fits the available observations (*Beven*, 2001). In general, many efficiency criteria contain a summation of the error term (difference between the simulated and the observed variable at each time step) normalized by a measure of the variability in the observations. To avoid the canceling of errors of opposite sign, the summation of the absolute or squared errors is often used for many efficiency criteria. Further, different efficiency criterion like Nash-Sutcliffe efficiency (E) and Index of Agreement (I_d) may be used to provide more information on the systematic and dynamic errors present in the model simulation.

4.1 Coefficient of determination R^2

The coefficient of determination R^2 can be expressed as the squared ration between the covariance and the multiplied standard deviations of the observed and predicted values. Therefore it estimates the combined dispersion against the single dispersion of the observed and predicted series. The range of R^2 lies between 0 and 1.0 which describes how much of the observed dispersion is explained by the prediction. A value of zero means no correlation at all whereas a value of 1 means that the dispersion of the prediction is equal to that of the observation. The fact that only the dispersion is quantified is one of the major drawbacks of R^2 if it is considered alone. It is advisable that gradient of fitted regression and root mean squared errors (σ) should be given along with R^2.

4.2 Nash-Sutcliffe efficiency E

The efficiency E proposed by *Nash* and *Sutcliffe* (1970) is defined as:

$$E = 1 - \frac{\sum_{i=1}^{n}\left(O_i - P_i\right)^2}{\sum_{i=1}^{n}\left(O_i - \bar{O}\right)^2}, \qquad (11)$$

where *O* represents the calculated and *P* represents the predicted values. The range of E lies between 1.0 (perfect fit) and $-\infty$. An efficiency of lower than zero indicates that the mean value of the observed time series would have been a better predictor than the model.

4.3 Index of agreement d

The index of agreement d was proposed by *Willmot* (1981) to overcome the insensitivity of E and R^2 to differences in the observed and predicted means and variances (*Legates* and *McCabe*, 1999). The index of agreement represents the ratio of the mean square error and the potential error (*Willmot*, 1981) and is defined as:

$$I_d = 1 - \frac{\sum_{i=1}^{n}\left(O_i - P_i\right)^2}{\sum_{i=1}^{n}\left(\left|P_i - \bar{O}\right| + \left|O_i - \bar{O}\right|\right)^2}. \qquad (12)$$

The potential error in the denominator represents the largest value that the squared difference of each pair can attain. The range of I_d is similar to that of R^2 and lies between 0 (no correlation) and 1.0 (perfect fit).

The ability of each approaches (Tab. 2) in predicting bedload rate are calculated against experimental data and the correlation coefficients (R^2), root mean squared errors (*RMSE*, σ), slope of lines of best fit (*slope*), Nash-Sutcliffe efficiency (E) and Index of Agreement (I_d) are shown in Tab. 3a) and 3b) for moderate and intense respectively.

4.4 Moderate load

Fig. 1 shows the best comparison curves (Φ_{obs} versus Φ_{cal}) proposed by respective investigators with data reported in the various studies for sediment bedload transport at moderate condition. The figure shows considerable compatibility with the observed Φ. The experimental data shows a marginally scatter and could be explained as a set rather than well defined curves. The discrepancy is primarily due to particle shape, random nature of the entrainment process, and the difficulty with defining criteria that adequately capture this feature, but other factors may also play a role. These comparisons show that the estimation for bedload transport may be a standardize phenomenon for studying the transport of sediments over the surface. The prediction made by *Ashmore* (1988) and *Wong* and *Parker* (2006b) are found to be optimal on above criterion. The higher values of R^2, *σ, slope,* E and I_d of *Ashmore* (1988) method shows the best predicting approach among all, having Φ values consistently close to the experimental values.

4.5 Intense load

Similar performance analysis of each approach (Tab. 2) has been worked out at intense bedload condition. Two best qualitative performances are *Graf* and *Suszka* (1987) and *Julien* (2002) in the shear stress and regression approaches respectively. *Graf* and *Suszka* (1987) method has the optimum values of R^2, σ, *slope*, E and I_d which predicts best Φ values than other formulae.

Literature review suggests that equation conforming to discharge type and probabilistic approach work better than shear stress approach. However, present work establishes that based on several statistical criteria, formula of shear stress approach works better than others for moderate load condition. Same trend has been also found in case of intense bedload. Here regression equations developed by *Julien* (2002) and *Van Rijn* (1993) have worked better than others. Equations of *Julien* (2002) and *Van Rijn* (1993) have been developed through shear stress approach. Thus, in general it can be said that shear stress approach has worked better. Furthermore, *Du Boys* (1879), *Einstein* (1942), *Rottner* (1959), *Paintal* (1971), *Gill* (1972), *Pica* (1972), *Parker* et al. (1982), *Parker* (1990a,b) and *Bhattacharya* et al. (2007) are also compared and found that their prediction with existing database contain disorderliness of several degree.

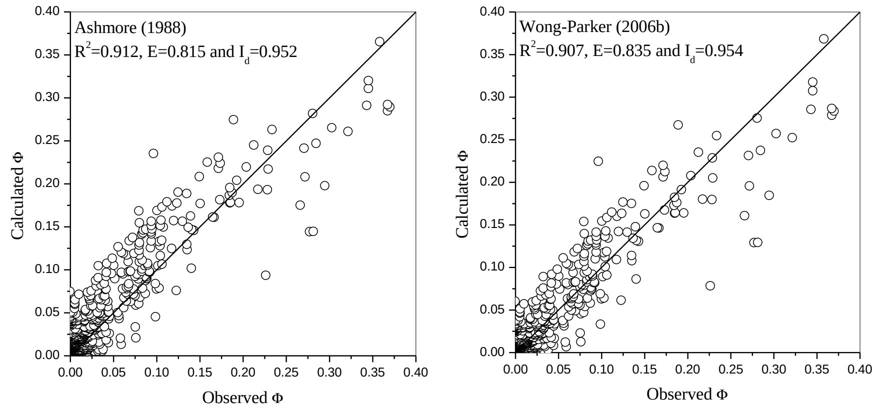

Fig. 1. Best predictors for bedload transport rate at moderate condition.

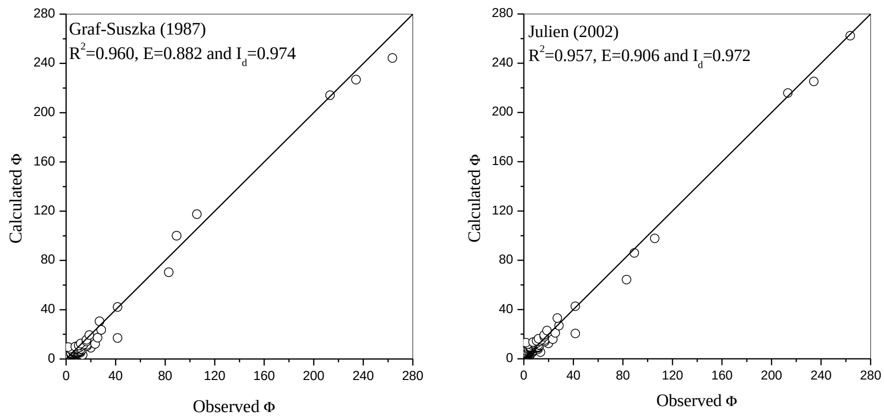

Fig. 2. Best predictors for bedload transport rate at intense condition.

6. Conclusions

Bedload transport formulae are based on the idea that a relationship exists between hydraulic conditions, the sediments present and the sediment transport rate (*Gomez* and *Church*, 1989). However, there are many factors, primarily related to the temporal and spatial resolution and accuracy of observations in real rivers that confound this relationship for example local shear stress and description of sediment and surface structure. The primary aim of the present investigation is to study the degree of deviancy of calculated bedload rate with the measured value. Towards this objective, an extensive set of existing experimental data set on bedload sediment are used to find the most successful prediction for measured bedload transport rate both at *moderate* and *intense* conditions. Analyses have shown that shear stress based approach works better for flume observations. Results show that *Ashmore* (1988) formula is useful for prediction of Φ at *moderate* condition (R^2 = 0.912, σ = 0.029, slope = 1.017, E = = 0.815 and I_d = 0.952) compared to other empirical curves. Similarly *Graf* and *Suszka* (1987) formula works better for *intense* condition having R^2 = = 0.960, σ = 7.2016, slope = 1.084, E = 0.882 and I_d = 0.974. It may be concluded that the proposed methods are statistically precise with the observed (measured) bedload sediment transport.

Acknowledgements. The authors gratefully acknowledge the financial support that was received from the Department of Science and Technology, Govt. of India (SERC-DST: SR/S3/MERC/ 005/2010) to carry out the research work presented in this paper.

List of symbols

C	– sediment concentration in ppm [M L^{-3}],
d	– sediment particle size [L],
d_*	– dimensionless particle parameter [–],
E	– Nash–Sutcliffe coefficient [–],
g_b	– bedload rate in weight per unit width per unit time [ML^{-1} T^{-1}],
h	– depth of flow [L],
I_d	– index of agreement [–],
q	– discharge per unit width of flow [L^2 T^{-1}],
q_b	– bedload transport rate per unit width [L^2 T^{-1}],
q_c	– critical discharge at which sediments began to move [L^2 T^{-1}],
R^2	– coefficient of determination [–],
S	– slope of the channel [L L^{-1}],
u	– mean flow velocity [L T^{-1}],
u_*	– shear velocity [L T^{-1}],
W	– width of channel [L],
γ_s	– specific weight of sediment [ML T^{-2}],
υ	– kinematic viscosity [L^2 T^{-1}],
Φ	– dimensionless intensity of the bedload rate [–],
ρ_s	– density of sediment [M L^{-3}],
ρ	– density of water [M L^{-3}],
$S = \frac{\rho_s}{\rho}$	– specific gravity of bed material [–],
σ	– standard deviation [–],
τ	– shear stress at the bed [ML^{-1} T^{-2}],
τ_c	– critical shear stress at incipient motion [ML^{-1} T^{-2}],
θ	– Shield's parameter for initiation of motion [–],
θ_c	– critical Shield's parameter for initiation of motion [–].

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Received 18 April 2011
Accepted 19 October 2011