ON EQUIVALENT ROUGHNESS OF MOBILE BED AT HIGH SHEAR STRESS

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The friction conditions at the top of a mobile bed are discussed for flows in the upper-plane-bed regime, i.e. for the flows with values of the bed Shields parameter larger than approximately 0.6. A special attention is devoted to flows of the bed Shields parameter bigger than 2. Experimental data for flows at high bed shear are collected from literature and new data are added from own measurements of flows of a 1.36-mm sand slurry in the 100-mm pipe loop of the Institute of Hydrodynamics. The database represents flows of very different solids and covers friction conditions within a broad range of Shields parameters up to the maximum value of about 23. The paper analyses the data in order to evaluate a relationship among the equivalent roughness of the top of the bed and other relevant parameters. A semi-empirical formula is proposed that relates the equivalent roughness to the bed Shields parameter, the ratio of flow velocity and particle settling velocity, and the ratio of flow hydraulic radius and particle diameter. The formula is applicable primarily to flows of combined load (contact- and suspended loads together).

KEY WORDS: Bed Shear, Experiment, Hydraulic Transport, Sediment Transport.

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Příspěvek se zabývá mechanismem tření na povrchu erodovatelného dna za podmínek horního režimu pohybu splavenin nad plochým dnem, to jest při hodnotách Shieldsova parametru větších než přibližně 0,6. Zvláštní pozornost je věnována proudění při Shieldsově parametru větším než 2. Z literatury byla sebrána experimentální data pro proudění za vysokých smykových napětí, ke kterým byla přidána nová data z vlastních měření s pískem velikosti středního zrna 1,36 mm v potrubí průměru 100 mm trubního okruhu Ústavu pro hydrodynamiku AV ČR. Databáze obsahuje výsledky měření pro částice různých vlastností v širokém rozsahu hodnot Shieldsova parametru až do hodnoty okolo 23. Příspěvek analyzuje data za účelem stanovení závislosti hydraulické drsnosti povrchu sedliny na ostatních parametrech proudění. Je navržen semiempirický vztah, který předpokládá závislost hydraulické drsnosti na Shieldsově parametru, poměru rychlosti řástic. Tento vztah se doporučuje především pro proudění nesoucí jak dnové splaveniny, tak suspendované částice.

KLÍČOVÁ SLOVA: tření o dno, experiment, hydraulická doprava, pohyb splavenin.

1. Introduction

A lot of experimental work has been devoted to the investigation of mobile-bed roughness at values of the bed shear stress that are typical for usual flow conditions in open channels (for recent literature survey see e.g. *van Rijn*, 2007). In such conditions, the bed shear stress produces values of the bed Shields parameter,

$$\theta_b = \frac{\tau_b}{\left(\rho_s - \rho_f\right) \cdot \mathbf{g} \cdot d_{50}},\tag{1}$$

where $\tau_{\rm b}$ – bed shear stress, $\rho_{\rm s}$ – density of solids, $\rho_{\rm f}$ – density of liquid, g – gravitational acceleration, d_{50} – mass-median diameter of solid particle, that are usually smaller than 0.6. Weak sediment transport and bed undulation are associated with the flow conditions at low shear stress. However, if $\theta_{\rm b}$ exceeds 0.6 the upper-plane-bed regime develops that is typical for flows at high bed shear. At high shear stress, a flow erodes the top of the mobile bed and prevents a development of bed forms. As a result the bed is flat and intense transport of solid particles takes place. The transported particles in-fluence friction conditions at the top of the bed.

Fairly less information is available on bed roughness for flows under these conditions (*Wilson*, 1989, 2005; *Nnadi* and *Wilson*, 1992, 1995; *Sumer* et al., 1996; *Pugh* and *Wilson*, 1999; *Matoušek*, 2005, 2007a); *Camenen* et al., 2006) than for flows at low shear stress.

The flow in the upper-plane-bed regime is typical for e.g. open-channel flows of steep slopes and/or high discharges (flood conditions) or enclosed-pipe flows above stationary deposits in slurry pipelines. Laboratory pipe loops manufactured to convey slurries are appropriate for testing phenomena related to friction at the top of a granular bed. This is because a broad range of flow conditions can be installed in the loop. Currently, only a limited number of experimental data suitable to investigation of resistance of a mobile bed is available for high-shear stress flows, particularly for flows of Shields-parameter values larger than 2 (*Nnadi*, 1992; *Nnadi* and *Wilson*, 1992; *Pugh*, 1995; *Sumer* et al., 1996; *Matoušek*, 2007a)).

2. Experimental database

2.1 Literature data

2.1.1 Queen's University

The circuit of the Oueen's University in Kingston, Canada has been used for experiments with slurry flows above a stationary bed (*Nnadi*, 1992; Nnadi and Wilson, 1992; Wilson, 2005). The circuit contains a rectangular enclosed conduit with a square cross section of the internal size 98 millimeter. The circuit is described in details in Nnadi's PhD thesis (Nnadi, 1992). Nnadi tested three fractions of sand ($d_{50} = 0.35$, 0.55, and 1.10 mm respectively; $\rho_{\rm s} = 2670$, 2660, and 2660 kg m⁻³ respectively), two fractions of bakelite ($d_{50} = 0.67$, and 1.05 mm; $\rho_s = 1570$, and 1540 kg m⁻³), and one fraction of nylon particles ($d_{50} = 3.94$ mm; $\rho_s = 1140$ kg m⁻³). Information on the particle size distribution of the tested solids are collected in Fig. 1 and Tab. 1. Wilson (2005) presented the bed roughness results from his 0.11-mm sand slurry tests (Fig. 5 in Wilson, 2005), but no source data.

2.1.2 Saskatchewan Research Council (SRC)

Pugh (1995) used one of laboratory slurry-pipe circuits at the Saskatchewan Research Council in Saskatoon, Canada (the one with the inner diameter of 105 mm) to carry out tests for flows of different

slurries above a stationary bed. The circuit is described in details in Pugh's PhD thesis (*Pugh*, 1995). Three sediments were tested: two fractions of sand ($d_{50} = 0.30$, and 0.56 mm; $\rho_s = 2650$ kg m⁻³) and one fraction of bakelite ($d_{50} = 1.05$ mm; $\rho_s = 1530$ kg m⁻³), see Fig. 1 and Tab. 1.



Fig. 1 Particle size distribution of different tested solids. Legend: • – 1.36-mm sand, IAHSCR (new data), ∇ – 0.37-mm sand, DUT (*Matoušek*, 2007a), Δ – 0.35-mm sand, Queen's University (*Nnadi*, 1992), \Box – 0.56-mm sand, Queen's University (*Nnadi*, 1992), \circ – 1.1- mm sand, Queen's University (*Nnadi*, 1992), \Diamond – 0.67- mm bakelite, Queen's University (*Nnadi*, 1992), \triangleleft – 1.05- mm bakelite, Queen's University (*Nnadi*, 1992), \triangleleft – 0.30- mm sand, SRC (*Pugh*, 1995), * – 0.56- mm sand, SRC (*Pugh*, 1995), + – 1.05- mm bakelite, SRC (*Pugh*, 1995).

Obr. 1. Zrnitostní rozdělení testovaných sypanin. Legenda: • – 1,36-mm písek, ÚH AV ČR (nová data), ∇ – 0,37-mm písek, DUT (*Matoušek*, 2007a), Δ – 0,35-mm písek, Queen's University (*Nnadi*, 1992), \Box – 0,56-mm písek, Queen's University (*Nnadi*, 1992), \circ – 1,1- mm písek, Queen's University (*Nnadi*, 1992), \Diamond – 0,67- mm bakelit, Queen's University (*Nnadi*, 1992), \triangleleft – 1,05- mm bakelit, Queen's University (*Nnadi*, 1992), \varkappa – 0,30- mm písek, SRC (*Pugh*, 1995), * – 0,56- mm písek, SRC (*Pugh*, 1995), + – 1,05- mm bakelit, SRC (*Pugh*, 1995).

2.1.3 Institute of Hydrodynamics and Hydraulic Engineering of Technical University of Denmark

Sumer et al. (1996) used a tilting flume, 10 m long, 0.3 m wide and 0.3 m deep with a lid rigidly attached to the top of the flume in order to create an enclosed conduit. A brief description of the circuit is given in *Sumer* et al. (1996). Tests on flows above deposits were carried out for solids (sand, acrylic, and two plastic fractions respectively) of very different densities ($\rho_s = 2650$, 1130, 1114, and 1270 kg m⁻³), particle diameters ($d_{50} = 0.13$, 0.60,

T a b l e 1. Summary of tested materials. T a b u l k a 1. Přehled testovaných sypanin.

Material	d_{18}	d_{50}	d_{84}	Source
	[mm]	[mm]	[mm]	
Sand	1.15	1.36	1.55	IAHSCR (new data)
Sand	0.30	0.37	0.46	DUT (Matoušek, 2007a)
Sand	0.29	0.35	0.48	Queen's University (Nnadi, 1992)
Sand	0.32	0.56	0.80	Queen's University (Nnadi, 1992)
Sand	0.90	1.10	1.04	Queen's University (Nnadi, 1992)
Bakelite	0.42	0.67	0.80	Queen's University (Nnadi, 1992)
Bakelite	0.60	1.05	1.04	Queen's University (Nnadi, 1992)
Nylon	3.94 (uniform size)			Queen's University (Nnadi, 1992)
Sand	0.21	0.30	0.40	SRC (Pugh, 1995)
Sand	0.40	0.56	0.80	SRC (Pugh, 1995)
Bakelite	0.63	1.05	1.50	SRC (Pugh, 1995)

2.6, and 3.0 mm), and particle shapes (natural sand, acrylic spherical grains, plastic elliptic cylinders, and plastic circular cylinders).

2.2 Own data

2.2.1 Laboratory of Dredging Engineering of Delft University of Technology (DUT)

The laboratory circuit for slurry-flow testing had pipes of the inner diameter 150 mm. A detailed description of the circuit is given in *Matoušek* (2002). One fraction of the narrow graded quartz sand of $d_{50} = 0.37$ mm and $\rho_s = 2650$ kg m⁻³ was tested (see Fig. 1 and Tab. 1); some of the results have been published in *Matoušek* (2007a and 2007b).

2.2.2 Institute of Hydrodynamics of Academy of Sciences of the Czech Republic (IHASCR)

The laboratory circuit (Fig. 2) for slurry-flow testing is composed of pipes of the inner diameter 100 mm. The total length of the circuit is 55 meter. Tested slurry of a required concentration of solids is prepared in a sump tank and let into the test loop by lifting the funnel closing the sump outlet. During the operation, intensive mixing prevents settling of solids in the tank. At the end of the operation the funnel is sunk to close the outlet again and the solids are collected within the tank leaving the test loop free of solids. Two 3-meter long measuring sections for differential pressure measurements are positioned one after another in one leg of the horizontal test loop. The first pressure tap is located 10 meter behind the U-bend leaving the straight section of pipe of the sufficient length of 100 pipe diameters for flow development between the fitting

and the pressure tap. Pressure differences over the measuring sections are measured by the Rosemount differential pressure transmitters (model 1151DP). The mean velocity in the circuit is sensed using the Krohne magnetic flow meter (model OPTIFLUX 5000 with the converter IFC 300 W) mounted to the vertical section of the circuit. The flow divider allows collecting slurry samples in the sampling tank of the calibrated volume. Weighting of slurry samples collected in the sampling tank gives information about the delivered concentration of solids in flowing slurry. The thickness of a stationary bed is determined from observed positions of the top of the bed in the glass pipe section that is 1.4-meter long and positioned 4 meters after the U-bend. So far, one sand fraction has been tested. The tested solids was the narrow graded quartz sand of d_{50} = = 1.36 mm and ρ_s = 2650 kg m⁻³ (see Fig. 1 and Tab. 1).

3. Prediction of mobile bed roughness

3.1 Analytical modeling for sheet flow

Basically, an analytical approach to the evaluation of bed friction requires knowledge of velocity and concentration distributions throughout the flow above the top of the bed. This information is available for sheet flows, i.e. for flows in which all particles are transported as a bed load (contact load) within a shear layer linked to the top of the eroded stationary bed. The theory (*Pugh* and *Wilson*, 1999) that assumes a logarithmic profile of liquid velocity linked to the profile of a certain characteristic slope at the top of the shear layer suggests that the equivalent roughness of the bed, k_s , is independent of the particle size and there is a linear relationship between k_s/d_{50} and θ_b . This can be derived from a



Fig. 2. Laboratory circuit with pipe of internal diameter 100 mm at Institute of Hydrodynamics of Academy of Sciences of the Czech Republic (IHASCR).

Obr. 2. Laboratorní trubní okruh s potrubím s vnitřím průměrem 100 mm v Ústavu pro hydrodynamiku Akademie věd České republiky (ÚH AV ČR). 1 – mísící tank, 2 – odstředivé čerpadlo, 3 – šoupě, 4 – experimentální okruh, 5 – skleněná pozorovací sekce, 6 – diferenciální tlakoměry, 7 – elektromagnetický indukční průtokoměr, 8 – rozdělovač proudu, 9 – nádrž pro odběr vzorků, 10 – sedimentační nádrž, 11 – vypouštěcí uzávěr, 12 – elektromotor.

combination of the Nikuradze's equation for a liquid-velocity profile above a rough boundary (e.g. *Schlichting*, 1979)

$$\frac{u}{u_{*b}} = 2.5 \cdot \ln\left(\frac{30 \cdot y}{k_s}\right),\tag{2}$$

where u – local velocity of liquid at vertical position y, u_{*b} – shear velocity at top of bed, k_s – equivalent roughness of bed with the experimentally determined conditions at the top of a shear layer. The vertical position of the top of the shear layer above the origin of the logarithmic profile is equal to roughly one half of the thickness of the shear layer (*Pugh* and *Wilson*, 1999), i.e.

$$y = y_{sh} = 0.44.H_{sh},$$
 (3)

where H_{sh} – thickness of shear layer and the local velocity at the top of the shear layer (*Pugh* and *Wilson*, 1999)

$$u = u_{sh} = 9.4.u_{*b}.$$
 (4)

The combination of Eqs. (2), (3), and (4) gives

$$\frac{u_{sh}}{u_{*b}} = 2.5 \cdot \ln\left(\frac{30 \cdot y_{sh}}{k_s}\right) \tag{5}$$

and thus

$$\frac{9.4 \cdot u_{*b}}{u_{*b}} = 2.5 \cdot \ln\left(\frac{30 \cdot 0.44 \cdot H_{sh}}{k_s}\right).$$
 (6)

The rearrangements of Eq. (6) give (*Pugh* and *Wilson*, 1999)

$$k_{s} = \frac{30 \cdot 0.44 \cdot H_{sh}}{\exp\left(\frac{9.4}{2.5}\right)} = 0.307 \cdot H_{sh} \approx \frac{H_{sh}}{3}.$$
 (7)

The shear stress that the flow exerts at the bottom of the shear layer, τ_b , is balanced by the particulate shear stress from particles traveling within a shear layer (*Wilson*, 1987) and hence

$$\tau_b = \left(\rho_s - \rho_f\right) \cdot g \cdot H_{sh} \cdot C_{sh} \cdot \tan \varphi' \tag{8}$$

where $C_{\rm sh}$ – mean concentration of solids in shear layer, φ' – dynamic friction angle of solids. A combination of Eqs. (1) and (8) gives

$$H_{sh} = \frac{\theta_b \cdot d_{50}}{C_{sh} \cdot \tan \varphi'} \tag{9}$$

and an implementation of Eq. (7) produces

$$\frac{k_s}{d_{50}} = \frac{0.307}{C_{sh} \cdot \tan \varphi'} \cdot \theta_b.$$
(10)

Values of the average volumetric concentration within a shear layer, $C_{\rm sh}$, and of the coefficient of solids friction, tan φ ', can vary for different solids.

For the values of $C_{\rm sh} = 0.30$ ($C_{\rm sh} = c_{\rm b}/2$ and $c_{\rm b} = 0.6$ is the value typical for a loose-poured bed of e.g. narrow graded sands) and tanq' = 0.31, Eq. (10) simplifies to

$$\frac{k_s}{d_{50}} = 3.3 \cdot \theta_b \tag{11}$$

which is the relationship that *Wilson* (2005) proposed for sheet flows. This relationship agrees well with the new experimental data (Fig. 3) collected for the 1.36-mm sand slurry flow ($0.5 < \theta_b < 2$) in the circuit of IHASCR. Due to the large particle size of the tested sand, it is reasonable to assume that the observed flow was a sheet flow, i.e. all particles were transported as contact load at all installed flow conditions.



Fig. 3. Relative equivalent roughness of the top of the coarsesand bed, (new data). Legend: pipe of diameter 0.100 m (IHASCR), o – 1.36 -mm quartz sand, $w_t = 14.8 \text{ cm s}^{-1}$, ... – k_s/d_{50} using Eq. (11) for sheet flow.

Obr. 3. Relativní hydraulická drsnost povrchu sedliny z hrubozrnného písku, (nová data). Legenda: potrubí průměru 0,100 m (ÚH AV ČR), o – 1,36 -mm křemičitý písek, $w_t = 14,8$ cm s⁻¹, ... – k_s/d_{50} podle rov. (11).

Wilson (2005) compared Eq. (11) with his finesand results (Fig. 5 in *Wilson*, 2005) and saw that the relationship held good only for θ_b smaller than say 4.5. At higher θ_b values a change in trend was observed in the relationship between k_s/d_{50} and θ_b showing a steeper increase of k_s/d_{50} with θ_b than suggested by Eq. (11). *Wilson* attributed the observed change in the trend to the rapid increase in suspended load at the θ_b values higher than 4.5. The medium-sand flows tested by *Matoušek* (2007a) exhibit a very similar trend (Fig. 4). Also data by *Sumer* et al. (1996) exhibited the described trend. They showed a reasonable match with Eq. (11) at θ_b

< 2 for all four very different solids fractions tested (see Fig. 3 in *Sumer* et al., 1996). At the higher $\theta_{\rm h}$ values, however, the data of two plastic fractions by Sumer et al. exhibited an upward trend causing that the measured k_s/d_{50} values were higher than those predicted using Eq. (11). The sand data were confined to $\theta_{\rm b}$ < 2.2, and the acrylic data exhibited a rather anomalous trend at $\theta_{\rm b} > 2$. Perhaps, the trend was a result of combined effects of the particle properties; the acrylic particles were coarse, light, and rather broadly graded. The data by Nnadi and *Pugh*, respectively, show also higher k_s/d_{50} values than expected by Eq. (11), see Figs. 5 and 6. In summary, values of the relative equivalent roughness higher than predicted using the sheet-flow equation (Eq. 11) have been observed for different solids fractions (from fine to coarse, and from light to heavy), at θ_b values higher than say 2÷4. A steeper slope of the relation between k_s/d_{50} and θ_b indicates that the relationship is not linear at high $\theta_{\rm b}$ values.



Fig. 4. Relative equivalent roughness of the top of the bed at high shear stress, (data from *Matoušek*, 2007a). Legend: pipe of diameter 0.150 m (DUT), $\Delta - 0.37$ -mm sand, $w_t = 4.8$ cm s⁻¹, ... - k_s/d_{50} using Eq. (11) for sheet flow, - k_s/d_{50} using Eq. (13).

Obr. 4. Relativní hydraulická drsnost povrchu sedliny za vysokého smykového napětí (data *Matoušek*, 2007a). Legenda: potrubí průměru 0,150 m (DUT), $\Delta - 0.37$ -mm písek, $w_t = 4,8$ cm s⁻¹, ... - k_s/d_{50} podle rov. (11), - k_s/d_{50} podle rov. (13).

3.2 Semi-empirical modeling for flows different from sheet flow

For a majority of bed-friction data in the literature, vertical profiles of liquid velocity and solids concentration across flows different from a sheet flow are not available. Therefore an analytical solution of the bed roughness formula is not feasible for these flows. Instead, the Nikuradze's friction law for a hydraulically rough boundary (e.g. *Schlichting*, 1979) is adopted for the top of a bed,

$$\frac{v_a}{u_{*b}} = 2.5 \cdot \ln\left(\frac{14.8 \cdot R_{hb}}{k_s}\right),\tag{12}$$

where v_a – average velocity in discharge area above bed, R_{hb} – hydraulic radius of area associated with bed.



Fig. 5. Relative equivalent roughness of the top of the bed at high shear stress for different sand fractions tested in one circuit, (data from *Nnadi*, 1992, and *Wilson*, 2005). Legend: rectangular pipe 0.098 x 0.098 m (Queen's University), $\nabla - 0.11$ -mm sand, $w_t = 1.33$ cm s⁻¹ (*Wilson* 2005), $\Delta - 0.354$ -mm sand, $w_t = 4.63$ cm s⁻¹ (*Nnadi*, 1992), $\Box - 0.55$ -mm sand, $w_t = 6.85$ cm s⁻¹ (*Nnadi*, 1992), $\circ - 1.10$ -mm sand, $w_t = 11.76$ cm s⁻¹ (*Nnadi*, 1992), $\ldots - k_s/d_{50}$ using Eq. (11) for sheet flow, $-k_s/d_{50}$ using Eq. (13).

Obr. 5 Relativní hydraulická drsnost povrchu sedliny za vysokého smykového napětí pro různé frakce písku testované na jednom experimentálním okruhu, (data *Nnadi*, 1992, a *Wilson*, 2005). Legenda: potrubí čtvercového profilu 0,098 x 0,098 m (Queen's University), ∇ 0,11-mm písek, $w_t = 1,33$ cm s⁻¹ (*Wilson* 2005), Δ 0,354-mm písek, $w_t = 4,63$ cm s⁻¹ (*Nnadi*, 1992), \Box – 0,55-mm písek, $w_t = 6,85$ cm s⁻¹ (*Nnadi*, 1992), \circ – 1,10mm písek, $w_t = 11,76$ cm s⁻¹ (*Nnadi*, 1992), ... – k_s/d_{50} podle rov. (11), – k_s/d_{50} podle rov. (13).

Experimental tests provide measured values of v_a . The values of u_{*b} and R_{hb} are determined from the additional measured parameters (thickness of the bed, hydraulic gradient) using standard methods (e.g. *Matoušek*, 2007a). Then k_s values can be determined from Eq. (12) filled with the test results. Further, the correlation is sought among the equivalent roughness of the bed k_s and relevant parameters.



Fig. 6. Relative equivalent roughness of the top of the bed at high shear stress, (data from *Pugh*, 1995). Legend: pipe of diameter 0.105 m (SRC), $\Delta - 0.30$ -mm sand, $w_t = 5$ cm s⁻¹, $\Box - 0.56$ -mm sand, $w_t = 7$ cm s⁻¹, $\circ - 1.05$ -mm bakelite, $w_t = 6$ cm s⁻¹, $\ldots - k_s/d_{50}$ using Eq. (11) for sheet flow, $-k_s/d_{50}$ using Eq. (13).

Obr. 6 Relativní hydraulická drsnost povrchu sedliny za vysokého smykového napětí, (data *Pugh*, 1995). Legenda: potrubí průměru 0,105 m (SRC), $\Delta - 0,30$ -mm písek, $w_t = 5$ cm s⁻¹, $\Box - 0,56$ -mm písek, $w_t = 7$ cm s⁻¹, $\circ - 1,05$ -mm bakelit, $w_t = 6$ cm s⁻¹, ... - k_s/d_{50} podle rov. (11), - k_s/d_{50} podle rov. (13).

3.2.1 Flows at very high shear stress ($\theta_b > 2 \div 4$)

At very high bed shear, flows tend to exhibit high k_s/d values and their increase with the bed Shields parameter is steeper than for the linear relationship suggested by Eq. (11).

Matoušek (2007a) used Eq. (12) to determine the bed roughness values from the results of his 0.37mm sand tests. The observed flows were different from a sheet flow at very high θ_b values. As shown in *Matoušek* (2007b) particles supported by turbulent suspension occupied at least the upper half of the discharge area above the bed for θ_b larger than say 9. The proposed semi-empirical formula

$$\frac{k_s}{d_{50}} = 1.3 \cdot \theta_b^{1.65} \tag{13}$$

is a result of the best-fit procedure on the test results. The line by Eq. (13) intersects the sheet-flow line (Eq. (11)) at $\theta_b = 4.2$. Eq. (13) provides a better match to the medium-sand data than Eq. (11) at θ_b > 4.2 (Fig. 4).

The θ_b value at the intersection corresponds well with the value ($\theta_b \approx 4.5$) at which the k_s/d_{50} values for the 0.11-mm sand flow by *Wilson* (2005) start to deviate from the prediction by Eq. (11), see Fig. 5. *Wilson* suggested that a rapid increase in suspension begins at that θ_b value in the 0.11-mm sand flow. The ratio u_{*b}/w_t (w_t = terminal settling velocity of solid particle) is more appropriate to serve as a threshold between a sheet flow (bed-load flow) and a combined-load flow (bed load and suspended load together). The values of this ratio for $\theta_{\rm b} \approx 4.5$ are different for the 0.37-mm sand ($u_{\rm *b}/w_{\rm t} =$ = 3.1) and for the 0.11-mm sand $(u_{*b}/w_t = 6.5)$. Fig. 5 shows that the value of $\theta_{\rm b}$ (or the value of $u_{\rm *b}/w_{\rm t}$ associated with this $\theta_{\rm b}$ value) at the intersection of Eq. (11) with the measured data points can hardly be considered a general threshold between the bed load transport and the combined load transport. The coarser the solids fraction the lower the $\theta_{\rm b}$ value at the intersection in Fig. 5. For solids coarser than the fine sand, the values of $\theta_{\rm b}$ (and $u_{*\rm b}/w_{\rm t}$) at the intersection are far too low to be considered as limit values for sheet flows. A comparison of the results for sand fractions of very different particle sizes tested in one pipe in Fig. 5 indicates that there are effects additional to the effect of $\theta_{\rm b}$ on the $k_{\rm s}/d_{50}$ values.

3.2.2 Effect of sand-particle size

The data in Fig. 5 suggest that the finer the sand fraction the smaller the k_s/d_{50} value tends to be for a certain value of θ_b . The slope remains virtually unchanged except for the finest sand fraction that exhibits a considerably steeper relation. Together with other effects, this effect should be included in the new best-fit equation for equivalent roughness of the top of the bed.

3.2.3 Effects of discharge area and flow velocity

A comparison of different data sets in the collected database suggests that there might be an additional effect of the size and the shape of the discharge area on the relationship between k_s/d_{50} and $\theta_{\rm b}$. For instance, a rectangular shape of the discharge area tends to generate a more complex flow structure (in particular in the regions associated with the area corners) than a circular shape and hence perhaps affect bed friction. The effects of both the shape and the size are suggested to be covered by an introduction of the hydraulic radius of the discharge area above the bed to the k_s/d_{50} versus $\theta_{\rm b}$ relationship. Furthermore, the observations suggest that the effect of the mean velocity of flow through the discharge area is not fully covered by the Shields parameter for various flow conditions and should be added to the equation for the relative roughness of the bed. This is in accordance with the

proposal that *Camenen* et al. (2006) did on the basis of their analysis of a large population of flow data from predominantly open channels.

3.2.4 New formula for bed roughness

The best-fit function for the relative equivalent roughness at the top of the bed in enclosed conduits at high bed shear is found by correlating the data from *Matoušek* (2007a), *Nnadi* (1992), and *Pugh* (1995). The experimental results for the bed roughness from these data sets could not be successfully matched by Eq. (11) for sheet flows. The data by *Sumer* et al. (1996) have not been used for the correlation because they would bring additional effects of anomalous particle shapes or broader particle size distribution into the relationship. The found formula reads

$$\frac{k_s}{d_{50}} = 260 \cdot \frac{R_{ha}}{d_{50}} \cdot \left(\frac{w_t}{v_a}\right)^{2.5} \cdot \theta_b^{1.7} \,. \tag{14}$$

The values of the coefficients in Eq. (14) are subject to modification in the light of increasing number of experimental data available for the formula calibration in future. The temporary values suggest that perhaps the equivalent roughness k_s should be normalized using R_{ha} instead of d_{50} . However, a more extensive database is required to generalize this statement. The value of the θ_b exponent is in good agreement with the earlier proposals (*Camenen* et al., 2006; *Matoušek*, 2007a).

4. Conclusions

New tests carried out in the circular 100-mm pipe for a coarse-sand slurry flow above a stationary bed confirmed a linear relationship between the relative equivalent roughness of the eroded bed and bed Shields parameter in sheet flows (Eqs. (10) and (11)).

The analysis of the database collected from the literature showed that the relationship is more complex for flows different from sheet flows (i.e. for combined-load flows). At high values of Shields parameter (higher than say $2\div4$) the roughness tends to increase more with the Shields parameter than the linear relationship predicts.

A new semi-empirical formula is proposed for the equivalent roughness of a plane bed for combined-load flows in the upper-plane-bed regime (Eq. (14)). The formula recognizes the bed Shields parameter, the hydraulic radius of the discharge area, and the ratio of the flow velocity and the particle-settling velocity as major parameters influencing the k_s value.

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List of symbols

- $c_{\rm b}$ volumetric concentration of loose-poured bed [–],
- $C_{\rm sh}$ mean volumetric concentration of solids in shear layer [–],
- d_{50} mass-median diameter of solid particle [m],
- g gravitational acceleration [m s⁻²],
- $H_{\rm sh}$ thickness of shear layer [m],
- $k_{\rm s}$ equivalent roughness of top of bed [m],
- \vec{R}_{ha} hydraulic radius of flow area [m],
- $R_{\rm hb}$ hydraulic radius of area associated with bed [m],
- u^{mod} local velocity of liquid [m s⁻¹],
- u_{sh} local velocity at top of shear layer [m s⁻¹],
- u_{*b} shear velocity at top of bed [m s⁻¹],
- v_a average velocity in discharge area above bed [m s⁻¹],
- $w_{\rm t}$ terminal settling velocity of solid particle [m s⁻¹],
- y vertical position above bed [m],
- y_{sh} vertical position of top of shear layer above origin of logarithmic profile [m],
- $\theta_{\rm b}$ bed Shields parameter [–],
- $\rho_{\rm s}$ density of solids [kg m⁻³],
- $\rho_{\rm f}$ density of liquid [kg m⁻³],
- $\tau_{\rm b}$ bed shear stress [Pa],
- φ' dynamic friction angle of solids [°].

REFERENCES

- CAMENEN B., BAYRAM A., LARSON M., 2006: Equivalent roughness height for plane bed under steady flow. J. Hydraul. Eng., 132, 11, 1146–1158.
- MATOUŠEK V., 2002: Pressure drops and flow patterns in sand-mixture pipes. Experim. Therm. Fluid Sci., 26, 693– -702.
- MATOUŠEK V., 2005: Research developments in pipeline transport of settling slurries. Powder Tech., *156*, 1, 43–51.
- MATOUŠEK V., 2007a: Interaction of slurry pipe flow with a stationary bed. J. South Afr. Inst. Min. Metal., *107*, 6, 365–-372.
- MATOUŠEK V., 2007b: Distribution of medium-sand particles in flow above erodible bed. J. Hydrol. Hydromech., 55, 4, 274–281.
- NNADI F.N., 1992: Bed-Load Transport at High Shear Stress: With Application to Friction in Rivers and Sand Waves. PhD Thesis, Queen's University at Kingston.
- NNADI F.N., WILSON K.C., 1992: Motion of contact-load particles at high shear stress. J. Hydraul. Eng., *118*, 12, 1670–1684.
- NNADI F.N., WILSON K.C., 1995: Bed-load motion at high shear stress: dune washout and plane-bed flow. J. Hydraul. Eng., *121*, 3, 267–273.
- PUGH F.J., 1995: Bed-Load Velocity and Concentration Profiles in High Shear Stress Flows. PhD Thesis, Queen's University at Kingston.
- PUGH F.J., WILSON K.C., 1999: Velocity and concentration distributions in sheet flow above plane beds. J. Hydraul. Eng., 125, 2, 117–125.

- SCHLICHTING H., 1979: Boundary-Layer Theory. McGraw-Hill, New York.
- SUMER B.M., KOZAKIEWITZ A., FREDSØE J., DEI-GAARD R., 1996: Velocity and concentration profiles in sheet-flow layer of movable bed. J. Hydraul. Eng., *122*, 10, 549–558.
- VAN RIJN L.C., 2007: Unified view of sediment transport by currents and Waves. I: Initiation of motion, bed roughness, and bed-load transport. J. Hydraul. Eng., 133, 6, 649–667.
- WILSON K.C., 1987: Analysis of bed-load motion at high shear stress. J. Hydraul. Eng., ASCE, 113, 1, 97–103.
- WILSON K.C., 1989: Mobile-bed friction at high shear stress. J. Hydraul. Eng., ASCE, 115, 6, 825–830.
- WILSON K.C., 2005: Rapid increase in suspended load at high bed shear. J. Hydraul. Eng., 131, 1, 46–51.

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HYDRAULICKÁ DRSNOST POHYBLIVÉHO DNA ZA VYSOKÝCH SMYKOVÝCH NAPĚTÍ

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Nová experimentální data získaná měřeními na tlakovém kruhovém potrubí průměru 100 mm při proudění splavenin nad sedlinou z hrubozrnného písku potvrdila lineární závislost hydraulické drsnosti povrchu dna na Shiedsově parametru za předpokladu, že se splaveniny pohybují výhradně ve smykové vrstvě tvořené erodovanou částí sedliny (rov. (10) a (11)). Analýza dat sebraných z literatury ukázala, že pokud jsou splaveniny kromě smykové vrstvy transportovány i v suspenzi, je vztah pro hydraulickou drsnost složitější. Pro hodnoty Shieldsova parametru vyšší než přibližně $2 \div 4$ roste hydraulická drsnost rychleji, než předpokládá lineární závislost pro smykovou vrstvu.

V příspěvku je navržen nový semi-empirický vztah (rov. (14)) pro hydraulickou drsnost povrchu erodovatelného dna za podmínek horního režimu pohybu splavenin nad plochým dnem a kombinované podpory splaveninových částic vzájemnými mezičásticovými kontakty a interakcí částic s turbulentními víry proudící kapaliny. V novém vztahu vystupují Shieldsův parametr, hydraulický poloměr průtočné plochy a poměr rychlosti proudění k usazovací rychlosti částic jako hlavní parametry ovlivňující hodnoty hydraulické drsnosti povrchu pohyblivého dna.

Seznam použitých symbolů

g

- c_b objemová koncentrace částic v nekonsolidované sedlině [–],
- $C_{\rm sh}$ průměrná objemová koncentrace částic ve smykové vrstvě [–],
- d₅₀ střední průměr částic [m],
 - gravitační zrychlení [m s⁻²],
- $H_{\rm sh}$ tloušťka smykové vrstvy [m],
- $k_{\rm s}$ hydraulická drsnost povrchu sedliny [m],
- R_{ha} hydraulický poloměr průtočné plochy [m],

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- hydraulický poloměr části průtočné plochy, v níž $R_{\rm hb}$ proudění vyvozuje smykové napětí na povrch sedliny [m],
- místní rychlost kapaliny [m s⁻¹], и
- místní rychlost na horní hranici smykové vrstvy [m s⁻¹], u_{sh}
- třecí rychlost na povrchu sedliny [m s⁻¹], u_{*b}
- průměrná rychlost proudění směsi v průtočném profilu v_{a} $[m s^{-1}],$
- konečná usazovací rychlost částice [m s⁻¹], wt

- vertikální poloha nad povrchem sedliny [m],
- у y_{sh} - vertikální poloha horní hranice smykové vrstvy nad polohou počátku logaritmického rychlostního profilu [m],
- Shieldsův parametr na povrchu sedliny [-], $\theta_{\rm b}$
- $ho_{
 m s}$
- objemová hmotnost částic [kg m⁻³],
 objemová hmotnost kapaliny [kg m⁻³], $ho_{
 m f}$
- smykové napětí na povrchu sedliny [Pa],
 dynamický úhel vnitřního tření částic [°]. $\tau_{\rm b} \ \varphi'$