J. Hydrol. Hydromech., *57*, 2009, 2, 113–120
DOI: 10.2478/v10098-009-0010-4

ANALYTICAL SOLUTION FOR TRANSIENT HYDRAULIC HEAD, FLOW RATE AND VOLUMETRIC EXCHANGE IN AN AQUIFER UNDER RECHARGE CONDITION

RAJEEV KUMAR BANSAL[1)], SAMIR KUMAR DAS[2)*]

[1)]Department of Mathematics, National Defense Academy, Khadakwasla, Pune - 411023, India;
mailto: bansal_rajeev31@hotmail.com
[2)*]Corresponding Author, Department of Computational Fluid Dynamics, International Institute of Information Technology, Pune Infotech Park, Hinjewadi, Pune- 411057, India; Tel: +91-20-22933441, Fax: +91-20-22934191,
mailto:samird@isquareit.ac.in; samirkumar_d@yahoo.com

This paper presents closed form solution for unsteady flow equation corresponding to the transient hydraulic head, flow rate and volumetric exchange of a confined aquifer which is in contact with a constant piezometric head at one end and a stream whose water level is rising at a constant rate at the other end. The aquifer is also subjected to receive constant inflow due to rain infiltration. The unsteady groundwater flow equation is solved using Laplace transform to get analytical expressions for the transient hydraulic head and flow rate at the left and right interfaces and the net volumetric exchange of water at the aquifer-stream interface. The analytical results presented here show the effect of recharge due to rain infiltration on the net volumetric exchange and reveal the conditions for which net inflow in the aquifer could be positive, negative or zero. The results obtained have the capability to determine transient hydraulic head for two extreme scenarios: (i) very slow rise and (ii) very fast rise in the stream water. Analytical result show that the net volumetric exchange could be positive, zero or negative depending on the surface infiltration and stream water rise rate.

KEY WORDS: Piezometric Head, Flow Rate, Volumetric Exchange, Confined Aquifer, Analytical Solution, Laplace Transform.

Rajeev Kumar Bansal, Samir Kumar Das: HYDRAULICKÉ VÝŠKY, RÝCHLOSTI PRÚDENIA A OBJEMOVÉ TOKY VO ZVODNENOM KOLEKTORE, URČENÉ ANALYTICKÝM RIEŠENÍM ROVNICE NEUSTÁLENÉHO PRÚDENIA PODZEMNÝCH VÔD. J. Hydrol. Hydromech., *57*, 2009, 2; 13 lit., 6 obr.

Príspevok obsahuje analytické riešenie rovnice neustáleného prúdenia vzhľadom na neustálenú hydraulickú výšku, rýchlosť prúdenia a objemové toky vo zvodnenom kolektore s napätou hladinou, ktorý je v kontakte s konštantnou piezometrickou výškou na jednej strane a s tokom s konštantne sa zvyšujúcou hladinou vody na strane druhej.

Zvodnený kolektor je tiež napájaný konštantnou rýchlosťou infiltrovanou vodou zo zrážok. Rovnica neustáleného prúdenia podzemnej vody je riešená s použitím Laplaceovej transformácie, aby sme získali neustálenú tlakovú výšku na ľavej aj pravej strane a objemový prítok vody na rozhraní zvodnený kolektor – tok.

Výsledky analytického riešenia, ktoré predkladáme, ukazujú vplyv infiltrácie zrážok na doplňovanie podzemnej vody a odhaľujú podmienky, za ktorých prítok do zvodneného kolektora môže byť kladný, negatívny, alebo nulový. Získané výsledky umožňujú určiť neustálené hydraulické výšky pre dva extrémne scenáre: (i) veľmi pomalé a (ii) veľmi rýchle zvýšenie hladiny vody v toku. Analytické riešenie ukazuje, že objem vody, ktorou je zvodnený kolektor doplňovaný, môže byť kladný, záporný, alebo nulový, v závislosti na intenzite infiltrácie a rýchlosti zvyšovania sa hladiny vody v toku.

KĽÚČOVÉ SLOVÁ: piezometrická výška, rýchlosť prúdenia, prítok do podzemných vôd, ohraničený zvodnený kolektor, analytické riešenia, Laplaceova transformácia.

1. Introduction

The study of water table rise in aquifers due to surface infiltration and canal recharge has received considerable importance from hydrogeologists and environmental engineers. Water table rise due to seepage from canals, with or without vertical infiltration is investigated by a number of researchers. The change in water table under the steady state condition has been studied by *Maasland* (1959), while the unsteady cases have been examined by *Hantush* (1967), *Marino* (1974), Gill (1984), *Mustafa* (1987), *Barlow* and *Moench* (2000). Exhaustive reference on earlier works can be obtained from the monographs of *Polubarinova-Kochina* (1962) and *Huisman* (1978). Recently *Boufadel* and *Peridier* (2002) have presented the analytical expressions for hydraulic head and volumetric exchange of water between an aquifer and a constantly rising stream under no recharge condition.

In this study, we present analytical solution of the groundwater flow equation to obtain pertinent expressions for transient hydraulic head, flow rate and volumetric exchange in a confined aquifer. The aquifer is in contact with constant piezometric head at one end and a stream whose water level is rising at a constant rate at the other end. The aquifer also receives a constant recharge due to surface infiltration. The closed form solution establishes the dependence of hydraulic head, flow rate and volumetric exchange on the recharge rate.

2. Problem formulation and analytical solution

We consider the aquifer to be confined, homogeneous, isotropic, and incompressible. As shown in Fig. 1, the aquifer is in contact with a constant water head h_0 at one end and a stream with initial water level h_L at the other end. The water level in the stream is rising at a constant rate from its initial level h_L up to h_0 in time t_r and remains there for indefinite time. During the time t_r, the aquifer also receives percolation recharge at a constant rate N. The groundwater flow equation in the aquifer, which is a continuity equation (*Forchheimer*, 1901), can be written as

$$K\frac{\partial^2 h}{\partial x^2}+\frac{N\varepsilon(t)}{b}=S_s\frac{\partial h}{\partial t}, \tag{1}$$

where h is the hydraulic head [L], K – the saturated hydraulic conductivity [L T^{-1}], S_s – specific storativity [L^{-1}], b – the average thickness of the aquifer [L], N – constant recharge per unit area of the aquifer [L T^{-1}], L – the length of the domain and

$$\varepsilon(t) = \begin{cases} 0 & \text{if } t \le 0 \\ 1 & \text{if } 0 < t \le t_r \\ 0 & \text{if } t > t_r \end{cases} \tag{2}$$

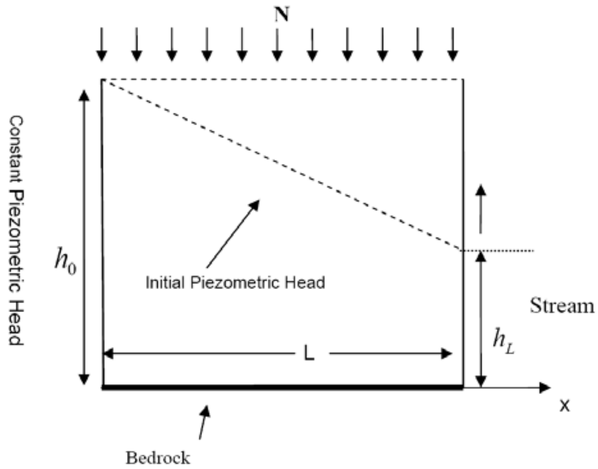

Fig. 1. Schematic diagram of a confined aquifer with recharge.
Obr. 1. Schematický diagram zvodneného kolektora s napätou hladinou s prítokom.

We prescribe the initial condition as

$$h(x,t=0)=h_0-\frac{h_0-h_L}{L}x \tag{3}$$

and the boundary conditions as

$$h(x=0,t)=h_0 \tag{4}$$

$$h(x=L,t)=h_L+\frac{h_0-h_L}{t_r}t \qquad \text{for } 0\le t\le t_r \tag{5}$$

$$h(x=L,t)=h_0 \qquad \text{for } t>t_r \tag{6}$$

Introducing the following dimensionless variables

$$\theta=\frac{h-h_0}{h_L-h_0}; \qquad X=\frac{x}{L}; \qquad \tau=\frac{K}{S_sL^2}t$$

$$\text{and } \gamma=\frac{K}{S_sL^2}t_r \tag{7}$$

we get

$$\frac{\partial^2\theta}{\partial X^2}+\frac{L^2N\varepsilon(\tau)}{Kb(h_L-h_0)}=\frac{\partial\theta}{\partial\tau} \tag{8}$$

where

$$\varepsilon(\tau)=\begin{cases}0 & \text{if}\,\tau\le 0\\ 1 & \text{if}\quad 0<\tau\le\gamma\\ 0 & \text{if}\,\tau>\gamma\end{cases} \tag{9}$$

Letting, $m=\dfrac{L^2}{Kb(h_L-h_0)}$. (10)

Eq. (8) can be written as

$$\frac{\partial^2\theta}{\partial X^2}+mN\,\varepsilon(\tau)=\frac{\partial\theta}{\partial\tau} \tag{11}$$

and accordingly the initial and boundary conditions become

$$\theta\left(X,\tau=0\right)=X \tag{12}$$

$$\theta\left(X=0,\tau\right)=0 \tag{13}$$

$$\theta(X=1,\tau)=1-\frac{\tau}{\gamma}\ \text{for}\ \tau\le\gamma \tag{14}$$

$$\theta(X=1,\tau)=0\ \text{for}\ \tau>\gamma\,. \tag{15}$$

Analytical solution of the groundwater flow Eq. (11) can be obtained by applying the Laplace transform, defined as

$$\overline{\theta}(X,p)=\int_0^\infty \mathrm{e}^{-p\tau}\theta(X,\tau)d\tau. \tag{16}$$

Using (16), Eq. (11) reduces to

$$\frac{d^2}{dX^2}\overline{\theta}(X,p)+mN\left(\frac{1-\mathrm{e}^{-\gamma p}}{p}\right)= \\ =p\overline{\theta}(X,p)-\theta(X,\tau=0). \tag{17}$$

Invoking (12) in (17), we obtain

$$\frac{d^2}{dX^2}\overline{\theta}(X,p)+mN\left(\frac{1-\mathrm{e}^{-\gamma p}}{p}\right)= \\ =p\overline{\theta}(X,p)-X. \tag{18}$$

Eq. (18) is an ordinary differential equation whose solution can be defined as

$$\overline{\theta\left(X,p\right)}=\left\{A\cosh\left(X\sqrt{p}\right)+B\sinh\left(X\sqrt{p}\right)\right\}+ \\ +\frac{X}{p}+mN\left(\frac{1-\mathrm{e}^{-\gamma p}}{p^2}\right) \tag{19}$$

where A and B are arbitrary constants and can be found out by taking Laplace transform of Eqs. (13) to (15) and using them in Eq. (19). Substituting the values of A and B in Eq. (19), we obtain

$$\overline{\theta}(X,p)=\frac{X}{p}+\frac{mN}{p^2}\left(1-\mathrm{e}^{-\gamma p}\right)-mN\frac{\sinh(1-X)\sqrt{p}}{p^2\sinh\sqrt{p}}+mN\frac{\mathrm{e}^{-\gamma p}\sinh(1-X)\sqrt{p}}{p^2\sinh\sqrt{p}}- \\ -\left(mN+\frac{1}{\gamma}\right)\frac{1}{\gamma}\frac{\sinh X\sqrt{p}}{p^2\sinh\sqrt{p}}+\left(mN+\frac{1}{\gamma}\right)\frac{1}{\gamma}\frac{\mathrm{e}^{-\gamma p}\sinh X\sqrt{p}}{p^2\sinh\sqrt{p}}. \tag{20}$$

The inverse Laplace transform of Eq. (20) can be obtained using the calculus of residues (*Sneddon*, 1972; *Brown* and *Churchill*, 1996). This leads to

$$\theta(X,\tau)=X\left(1-\frac{\tau}{\gamma}\right)+2mN\sum_{n=1}^{\infty}\frac{\sin n\pi X\left(1-\mathrm{e}^{-n^2\pi^2\tau}\right)}{n^3\pi^3}-2\left(mN+\frac{1}{\gamma}\right)\sum_{n=1}^{\infty}\frac{(-1)^n\sin n\pi X\left(1-\mathrm{e}^{-n^2\pi^2\tau}\right)}{n^3\pi^3}\quad\text{for}\ \tau\le\gamma \tag{21}$$

and

$$\theta(X,\tau)=2mN\sum_{n=1}^{\infty}\frac{\sin n\pi X\left(\mathrm{e}^{-n^2\pi^2(\tau-\gamma)}-\mathrm{e}^{-n^2\pi^2\tau}\right)}{n^3\pi^3}-2\left(mN+\frac{1}{\gamma}\right)\sum_{n=1}^{\infty}\frac{(-1)^n\sin n\pi X\left(\mathrm{e}^{-n^2\pi^2(\tau-\gamma)}-\mathrm{e}^{-n^2\pi^2\tau}\right)}{n^3\pi^3}\ \text{for}\quad\tau\ge\gamma \tag{22}$$

Eq. (21) provides the expression for the hydraulic head from the initial time up to time t_r (equivalently γ) whereas Eq. (22) provides it for the subsequent time. Eqs. (21) and (22) becomes identical for $\tau = \gamma$.

The water head expressions for a horizontal aquifer without recharge can be obtained by setting $N = 0$ in Eqs. (21) and (22). Accordingly, we obtain

$$\theta(X,\tau) = X\left(1-\frac{\tau}{\gamma}\right) - \frac{2}{\gamma}\sum_{n=1}^{\infty}\frac{(-1)^n \sin n\pi X\left(1-\mathrm{e}^{-n^2\pi^2\tau}\right)}{n^3\pi^3} \quad \text{for } \tau \le \gamma \tag{23}$$

$$\theta(X,\tau) = -\frac{2}{\gamma}\sum_{n=1}^{\infty}\frac{(-1)^n \sin n\pi X\left(\mathrm{e}^{-(\alpha^2+n^2\pi^2)(\tau-\gamma)} - \mathrm{e}^{-(\alpha^2+n^2\pi^2)\tau}\right)}{n^3\pi^3} \quad \text{for } \tau \ge \gamma\,. \tag{24}$$

These results are in conformity with the results obtained by *Boufadel* and *Peridier* (2002). One can also derive the expressions for the asymptotic cases of very fast and very slow rise of the water in the stream by taking $\gamma \to 0$ and $\gamma \to \infty$ in Eqs. (24) and (23) respectively. The corresponding analytical expressions are

$$\theta(X,\tau) = \ -2\sum_{n=1}^{\infty}\frac{(-1)^n \sin n\pi X}{n^3\pi^3}\mathrm{e}^{-n^2\pi^2\tau}, \tag{25}$$

$$\theta(X,\tau) = X + 2mN\sum_{n=1}^{\infty}\frac{\sin n\pi X}{n^3\pi^3}\left(1-\mathrm{e}^{-n^2\pi^2\tau}\right) - $$
$$-2mN\sum_{n=1}^{\infty}\frac{(-1)^n \sin n\pi X}{n^3\pi^3}\left(1-\mathrm{e}^{-n^2\pi^2\tau}\right). \tag{26}$$

3. Flow rate and volume exchange

The flow rate through a unite cross sectional area in the aquifer is given by (*Bear* and *Verrujit*, 1987)

$$q = -K\frac{\partial h}{\partial x}. \tag{27}$$

Introducing the following dimension less flow rate Q and after using Eq. (7) in Eq. (27), we get

$$Q = \frac{q}{-K(h_L - h_0)/L} \tag{28}$$

and

$$Q = \frac{\partial \theta}{\partial X}. \tag{29}$$

The expressions for dimensionless flow rate Q at the left and right boundary of the aquifer can be obtained by substituting $X = 0$ and $X = 1$.

$$Q_{X=0} = 1-\frac{\tau}{\gamma}+\frac{mN}{2}+\frac{1}{6\gamma}-2mN\sum_{n=1}^{\infty}\frac{\mathrm{e}^{-n^2\pi^2\tau}}{n^2\pi^2}-2\left(mN+\frac{1}{\gamma}\right)\sum_{n=1}^{\infty}\frac{(-1)^n\mathrm{e}^{-n^2\pi^2\tau}}{n^2\pi^2} \quad \text{for } 0 \le \tau \le \gamma \tag{30}$$

$$Q_{X=0} = 2\mathrm{mN}\sum_{n=1}^{\infty}\frac{\left(\mathrm{e}^{-n^2\pi^2(\tau-\gamma)} - \mathrm{e}^{-n^2\pi^2\tau}\right)}{n^2\pi^2} - 2\left(\mathrm{mN}+\frac{1}{\gamma}\right)\sum_{n=1}^{\infty}\frac{(-1)^n\left(\mathrm{e}^{-n^2\pi^2(\tau-\gamma)} - \mathrm{e}^{-n^2\pi^2\tau}\right)}{n^2\pi^2} \quad \text{for } \tau \ge \gamma \tag{31}$$

$$Q_{X=1} = 1-\frac{\tau}{\gamma}-\frac{mN}{2}-\frac{1}{3\gamma}-2mN\sum_{n=1}^{\infty}\frac{(-1)^n\mathrm{e}^{-n^2\pi^2\tau}}{n^2\pi^2}+2\left(mN+\frac{1}{\gamma}\right)\sum_{n=1}^{\infty}\frac{\mathrm{e}^{-n^2\pi^2\tau}}{n^2\pi^2} \quad \text{for } 0 \le \tau \le \gamma \tag{32}$$

$$Q_{X=1} = 2mN \sum_{n=1}^{\infty} \frac{(-1)^n \left(e^{-n^2\pi^2(\tau-\gamma)} - e^{-n^2\pi^2\tau} \right)}{n^2\pi^2} - 2\left(mN + \frac{1}{\gamma} \right) \sum_{n=1}^{\infty} \frac{\left(e^{-n^2\pi^2(\tau-\gamma)} - e^{-n^2\pi^2\tau} \right)}{n^2\pi^2} \quad \text{for } \tau \geq \gamma \tag{33}$$

The net volume outflow at right interface can be obtained after integrating dimensionless flow rate Q at $X = 1$.

$$V(\tau) = \int_0^{\tau} Q(X=1,\tau')\,d\tau'. \tag{34}$$

Utilizing Eq. (34) in (32) and (33), we obtain following expressions

$$V(\tau < \gamma) = \tau\left(1 - \frac{\tau}{2\gamma}\right) - \frac{mN\tau}{2} - \frac{\tau}{3\gamma} - 2mN \sum_{n=1}^{\infty} \frac{(-1)^n (1 - e^{-n^2\pi^2\tau})}{n^4\pi^4} + \left(2mN + \frac{1}{\gamma}\right) \sum_{n=1}^{\infty} \frac{(1 - e^{-n^2\pi^2\tau})}{n^4\pi^4}, \tag{35}$$

$$V_{\gamma}(\tau = \gamma) = \frac{\gamma}{2} - \frac{1}{3} - \frac{mN\gamma}{2} - 2mN \sum_{n=1}^{\infty} \frac{(-1)^n (1 - e^{-n^2\pi^2\gamma})}{n^4\pi^4} + \left(2mN + \frac{1}{\gamma}\right) \sum_{n=1}^{\infty} \frac{(1 - e^{-n^2\pi^2\gamma})}{n^4\pi^4}, \tag{36}$$

$$V(\tau > \gamma) = V_{\gamma} + 2mN \sum_{n=1}^{\infty} \frac{(-1)^n (1 - e^{-n^2\pi^2\gamma})(1 - e^{-n^2\pi^2(\tau-\gamma)})}{n^4\pi^4} - \left(2mN + \frac{1}{\gamma}\right) \sum_{n=1}^{\infty} \frac{(1 - e^{-n^2\pi^2\gamma})(1 - e^{-n^2\pi^2(\tau-\gamma)})}{n^4\pi^4}. \tag{37}$$

The steady state net inflow or outflow of the volume is obtained from Eq. (37) by setting $\tau \to \infty$ and this yields

$$V_{\infty} = \frac{\gamma}{2}(1 - mN) - \frac{1}{3}. \tag{38}$$

From the definition of m, given by Eq. (10) one can easily infer that m is negative (as $H_L < H_0$). Since N is positive, Eq. (40) implies that V_{∞} increases linearly with recharge rate N. Based on Eq. (38), the volumetric exchange can be either positive (outflow), zero or negative (inflow) depending upon following conditions

$$\gamma > \frac{1}{3(1 - mN)} \quad \text{for } V_{\infty} > 0, \tag{39}$$

$$\gamma = \frac{1}{3(1 - mN)} \quad \text{for } V_{\infty} = 0, \tag{40}$$

$$\gamma < \frac{1}{3(1 - mN)} \quad \text{for } V_{\infty} < 0. \tag{41}$$

The Eqs. (39)–(41) show important relationship for volumetric exchange between aquifer and adjoining water body.

4. Results and discussion

In order to illustrate the applicability of this analytical solution, we consider a hypothetical aquifer of length L = 100 m, b = 4 m, K = 0.001 m s^{-1}, S_s = 0.09 m^{-1}, h_0 = 10 m, h_L = 5 m and t_r = 3 days. While computing Eqs. (21) and (22) and other series solutions, we find good convergence for all values of dimensionless time τ considered for this problem. The spatial and temporal variation in hydraulic head $(1 - \theta)$ in the aquifer during and after the rise of stream water for N = 2 mm hr^{-1} and 4 mm hr^{-1} is plotted in Fig. 2, utilizing Eqs. (21) and (22). A higher recharge rate shows overall water table rise in the aquifer; although the relative rise of water table in the middle of the aquifer is more evident than that of rest of the aquifer. Figs. 3 and 4 show the flow rates at the left and right interfaces respectively that are being plotted against time τ for N = 0, 2 and 4 mm hr^{-1}. We observe that the flow rate ($Q_{X=0}$) is higher at the beginning and decreases rapidly with time and becomes zero for large values of τ (Fig. 3). For higher recharge rate, the hydraulic gradient available to the water to at X = 0 reduces and as a result the flow rate ($Q_{X=0}$) decreases. Fig. 4 shows the flow rate at X = 1 for various recharge rate defined by $Q_{x=1}$. At the initial stage, $Q_{X=1}$ is positive (outflow) and as time passes this outflow

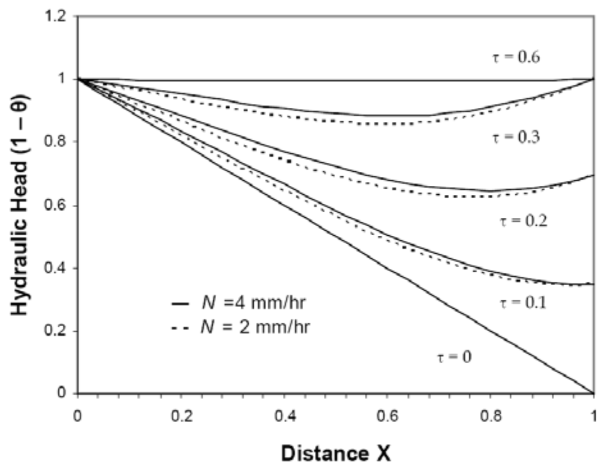

Fig. 2. Hydraulic head (1 – θ) in the aquifer for different time τ under recharge rate N = 2 and 4 [mm hr^{-1}].
Obr. 2. Hydraulická výška (1 – θ) v zvodnenom kolektore v rozdielnych časoch τ pri rýchlosti prítoku vody N = 2 a 4 [mm h^{-1}].

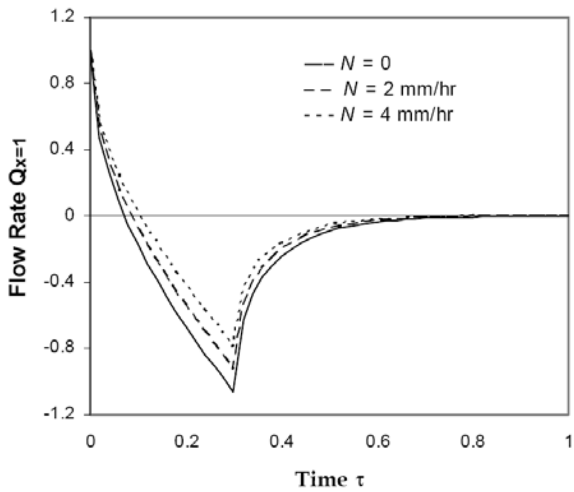

Fig. 4. Flow rate at X = 1 for N = 0, 2 and 4 [mm hr^{-1}] as a function of time τ.
Obr. 4. Rýchlosť prúdenia pri X = 1 pre N = 0, 2 a 4 [mm h^{-1}] ako funkcia času τ.

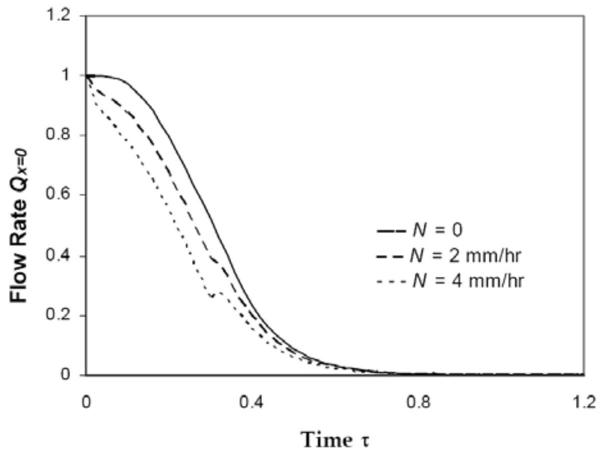

Fig. 3. Flow rate at X = 0 for N = 0, 2 and 4 [mm hr^{-1}] as a function of time τ.
Obr. 3. Rýchlosť prúdenia pri X = 0 pre N = 0, 2 a 4 [mm h^{-1}] ako funkcia času τ.

decreases and becomes zero. For higher values of γ (≥1), one can approximate the time τ_c when $Q_{X=1}$ = = 0 by neglecting the summation terms in Eq. (32), and this yields

$$\tau_c = \gamma - \frac{1}{3} - \frac{mN\gamma}{2}. \tag{42}$$

For $\tau > \tau_c$, the flow rate becomes negative (inflow) and attains its maximum value near $\tau = \gamma$. It decreases thereafter and finally becomes zero for large value of the time. Increase in recharge rate provides higher hydraulic gradient at X = 1, resulting increase in the outflow and reduction in inflow while compared with the cases for N = 0 and 2 [mm hr^{-1}]. The volumetric exchange (V) is plotted against the time τ using Eqs. (35) to (37) for different values of N and t_r (Fig. 5). Using elementary calculus, one can show that for $\gamma \geq 1$, V attains its maximum value at the same instant of time $\tau = \tau_c$ when $Q_{X=1}$ becomes zero. The corresponding value of V can be simplified from Eq. (36) as

$$V_{\max} = \frac{\gamma}{2}\left(1 - mN\right) - \frac{1}{3} + \frac{mN}{24}\left(3mN\gamma + 4\right) + \frac{1}{18\gamma}. \tag{43}$$

The volume that enters or leaves the aquifer ($V_{\gamma\infty}$) after the rise in the stream water can be expressed as

$$V_{\gamma\infty} = V_\infty - V_\gamma = 2mN \sum_{n=1}^{\infty} \frac{(-1)^n (1 - \mathrm{e}^{-n^2\pi^2\gamma})}{n^4\pi^4} - \left(2mN + \frac{1}{\gamma}\right) \sum_{n=1}^{\infty} \frac{(1 - \mathrm{e}^{-n^2\pi^2\gamma})}{n^4\pi^4}. \tag{44}$$

The sensitivity of recharge and corresponding volume inflow and outflow are shown in Fig. 6. We notice that $V_{\gamma\infty}$ decreases as γ increases. In the absence of recharge (N = 0), there is always an inflow

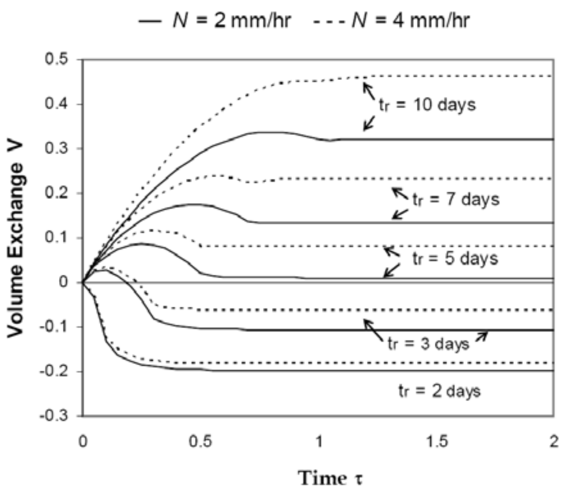

Fig. 5. Net volumetric exchange V at stream-aquifer interface as a function of time τ.
Obr. 5. Prietok cez hranicu zvodneného kolektora V na hranici tok – kolektor ako funkcia času τ.

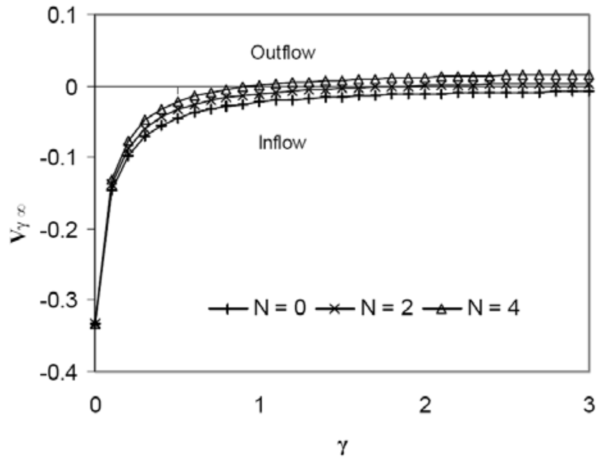

Fig. 6. Sensitivity plot with respect to the recharge parameter N.
Obr. 6. Citlivosť vzhľadom na parameter prítoku N.

($V_{\gamma\infty} < 0$). However, in the presence of recharge, one can obtain an outflow in this period by choosing an appropriate recharge rate and γ.

5. Conclusion

In this study we have obtained analytical expressions to analyze and volumetric exchange in a confined aquifer that is in contact with a constant piezometric head at one end and a constantly rising stream at the other end. The mathematical expressions presented here have the ability to quantify hydraulic head; flow rate and volume exchange and can deal with the asymptotic cases. Sensitivity of the solutions with respect to the change in the recharge rate is analyzed. It has been shown that the conditions for net volumetric exchange to be positive, zero or negative, depend on surface infiltration and stream water rise rate.

List of symbols

$h(x,t)$ – hydraulic head [L],
K – hydraulic conductivity [L T^{-1}],
S_s – specific storitivity [L^{-1}],
h_0 – piezometric head at the left boundary (Fig. 1) [L],
h_L – initial piezometric head in the stream (Fig. 1) [L],
x – horizontal x-axis [L],
t – time [T],
t_r – time in which the stream water rises from h_0 to h_L at a constant speed [T],
b – mean aquifer depth [L],
N – constant recharge rate [L T^{-1}],
q – flow rate [L T^{-1}],
L – length of the aquifer [L],
X – spatial coordinate equal to x/L [–],
Q – flow rate [–],
$Q_{X=0}$ – flow rate at the left boundary $X = 0$ [–],
$Q_{X=1}$ – flow rate at the right boundary $X = 1$ [–],
V – net volumetric exchange at the stream-aquifer interface [–],
V_∞ – steady state volumetric exchange [–],
$V_{\max}$ – maximum volume [–],
$V_{\gamma\infty}$ – volume that enters or leaves aquifer after the rise in the stream water,
$\theta(X, \tau)$ – $(h - h_0)/(h_L - h_0)$ [–],
τ – time equal to $(K\,t)/SL^2$ [–],
τ_c – time when the flow rate $Q_{X=1} = 0$ [–],
γ – time equal to $(K\,t_r)/SL^2$ corresponding to time, $t = t_r$ [–].

REFERENCES

BARLOW P., MOENCH A.F., 2000: Aquifer response to stream-stage and recharge variations. 1. Analytical step-response functions. J. Hydrology, *230*, 192–210.
BEAR J., VERRUIJT A., 1987: Modelling groundwater flow and pollution. D. Reidel Publishing Company, Dordrecht-Boston-Lancaster-Tokyo.
BOUFADEL M.C., PERIDIER V., 2002: Exact analytical expression for the piezometric profile and water exchange between the stream water and groundwater during and after a uniform rise of the stream level. Water Resource Research, *38*, 7, 1122, DOI: 10.1029/2001WR000780.
BROWN J.W., CHURCHILL R.V., 1996: Complex Variables and Application. 6th edition. McGraw-Hill, New York, 1996.
FORCHHEIMER P., 1901: Wasserbewegung durch Boden. Z. Ver. Deutsche Ing., *45*, 1782–1788.
GILL, M. A., 1984: Water table rise due to infiltration from canals. J. Hydrology, *70*, 337–352.
HANTUSH M. S., 1967: Growth and decay of groundwater mounds in response to uniform percolation. Water Resource Research, *3*, 1, 227–234.
HUISMAN L., 1978: Groundwater Recovery. The Macmillan Press Ltd., London.

MARINO M.A., 1974: Rise and fall of water table induced by vertical infiltration recharge. J. Hydrology, *23*, 289–298.
MASSLAND M., 1959: Water table fluctuation induced by intermittent recharge. J. Geophysical Research, *64*, 549–559.
MUSTAFA S., 1987: Water table rise in a semiconfined aquifer due to surface infiltration and canal recharge. J. Hydrology, *95*, 269–276.
POLUBARINOVA-KOCHINA P.Ya., 1962: Theory of groundwater movement. Princeton University Press, Princeton, New Jersey.
SNEDDON I.N., 1972: The Use of Integral Transform. McGraw-Hill, Zentralblatt-MATH.

Received 7. February 2008
Scientific paper accepted 19. February 2009

HYDRAULICKÉ VÝŠKY, RÝCHLOSTI PRÚDENIA A OBJEMOVÉ TOKY VO ZVODNENOM KOLEKTORE, URČENÉ ANALYTICKÝM RIEŠENÍM ROVNICE NEUSTÁLENÉHO PRÚDENIA PODZEMNÝCH VÔD

Rajeev Kumar Bansal, Samir Kumar Das

Štúdia obsahuje analytické výrazy na vyjadrenie objemov prítoku do zvodneného kolektora s napätou hladinou, ktorý je v kontakte s konštantnou piezometrickou výškou na jednej strane a konštantne sa zvyšujúcou hladinou vody v toku na strane druhej. Matematické výrazy, ktoré uvádzame, umožňujú kvantifikovať hydraulickú výšku, rýchlosť prúdenia a objemy prítoku do zvodneného kolektora a umožňujú zhodnotiť asymptotické prípady. Práca analyzuje citlivosť riešenia voči zmenám rýchlosti prítoku do zvodneného kolektora. Štúdia ukazuje, že rýchlosť prítoku do zvodneného kolektora môže byť pozitívna, nulová, alebo negatívna, v závislosti od rýchlostí infiltrácie a od zmien úrovne hladiny vody v toku.

Zoznam symbolov

$h(x,t)$ – hydraulická výška [L],
K – hydraulická vodivosť [LT^{-1}],
S_s – merná kapacita [L^{-1}],
h_0 – piezometrická výška na ľavej hranici (obr. 1) [L],
h_L – počiatočná piezometrická výška v toku (obr. 1) [L],
x – horizontálna os x [L],
t – čas [T],
t_r – čas, počas ktorého voda v toku stúpne konštantnou rýchlosťou z h_{0p} po h_L [T],
b – priemerná hrúbka zvodneného kolektora [L],
N – konštantná rýchlosť prítoku do zvodneného kolektora [LT^{-1}],
q – rýchlosť prúdenia [LT^{-1}],
L – dĺžka zvodneného kolektora [L],
X – priestorová súradnica, rovnajúca sa x/L [–],
Q – rýchlosť prúdenia [–] ,
$Q_{X=0}$ – rýchlosť prúdenia pri pravej hranici $X = 0$ [–],
$Q_{X=1}$ – rýchlosť prúdenia pri ľavej hranici $X = 1$ [–],
V – prietok na hranici tok – zvodnený kolektor [–],
V_∞ – ustálený prietok na hranici [–],
$V_{\max}$ – maximálny objem [–],
$V_{\gamma\infty}$ – objem, ktorý vstupuje, alebo vyteká zo zvodneného kolektora po zvýšení hladiny vody v toku [–],
τ – čas $(K\,t)/SL^2$ [–],
τ_c – čas, ak je rýchlosť prúdenia $Q_{X=1} = 0$ [–],
γ – čas rovnajúci sa $(K\,t_r)/SL^2$, zodpovedajúci času $t = t_r$[–].