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COMMUNICATIONS

Fast off grid compressed sensing ISAR imaging algorithm

Cheng Ping*, **Zhao Jiaqun****

To solve the off grid problem in compressed sensing (CS) based inverse synthetic aperture radar (ISAR) imaging, a fast and accurate algorithm has been proposed in the paper. By jointly estimating the off grid error and the sparse solution, off grid ISAR imaging is transformed into a joint optimization problem. Interestingly, it can be solved efficiently through two least squares problems based on first order Taylor approximation. When applied to complex sinusoids and quasi real ISAR data, the proposed algorithm has got better results than the conventional algorithm. Therefore, it is a promising off grid CS based ISAR imaging algorithm.

Keywords: inverse synthetic aperture radar (ISAR), compressed sensing (CS), off grid

1 Introduction

In compressed sensing (CS) based inverse synthetic aperture radar (ISAR) imaging, continuous parameters are discretized to grids and the strong scattering points are supposed in the grid centers. However, the strong scattering points may not be in the grid centers. So there is mismatch between the predefined dictionary and actual dictionary. It will degade the imaging performance significantly, which is named the off grid problem [1, 2].

To solve the problem, a direct method is to refine the grid, but it will increase the coherence in the sparsifying dictionary, which is adverse to reliable sparse recovery. Recently, methods based on nuclear norm optimization [3] and local optimization [4] have been proposed, however there is a requirement in the minimal distance between scatterers. Most of off grid CS methods jointly estimate the off grid error and the sparse solution. Bayesian method [5], matching pursuit algorithm [6] and alternating convex search algorithm [7] can be used with this model. However, their computation burdens are big. To decrease the heavy calculations, the first order Taylor approximation is used [5, 6]. In [6], a novel compressive sensing technique is proposed to alleviate the issues related with the reconstruction of the radar targets whose positions do not coincide with the assumed delay-Doppler grid. However, the computation burden is still very big. To reduce the computation burden further, we present a fast and accurate off grid ISAR imaging method in this work.

2 CS based ISAR imaging model

Supposing linear frequency modulated (LFM) signal is transmitted by radar, after dechirping, removing the residual video phase and range compression, the signal in a range cell can be expressed as [8]

$$\boldsymbol{z}(\tau) = \sum_{k=1}^{K} \xi_k \exp(-j2\pi f_k \tau) \tag{1}$$

where τ is azimuth time, K is the number of scattering points in the range cell, ξ_k and f_k are complex coefficient and Doppler frequency of the k-th scattering point respectively.

Then, ISAR imaging in cross-range can be transformed into a CS problem. Equation (1) can be written in matrix form

$$\boldsymbol{z} = \boldsymbol{\Phi s} \tag{2}$$

where $\boldsymbol{z} \in \boldsymbol{C}^{N\times 1}$ is signal in the range cell, N is the total number of samples in cross-range, $\boldsymbol{\Phi} \in \boldsymbol{C}^{N\times N}$ represents Fourier basis matrix, and $\boldsymbol{s} = [s_1, s_2, \ldots, s_i, \ldots, s_N] \in \boldsymbol{C}^{N\times 1}$ is K sparse coefficient vector which indicates the scattering points distribution of a target. By employing measurement matrix $\boldsymbol{\Psi} \in \boldsymbol{C}^{M\times N}$ $(M < N)$, where M is the dimension of measurements, signal is observed compressively

$$\boldsymbol{y} = \boldsymbol{\Psi z} + \boldsymbol{n} = \boldsymbol{\Theta s} + \boldsymbol{n} \tag{3}$$

where $\boldsymbol{y}$ is measurements vector, $\boldsymbol{\Theta} = \boldsymbol{\Psi\Phi}$, and $\boldsymbol{n}$ is noise. $\boldsymbol{s}$ can be recovered by a compressed sensing recovery algorithm.

3 Off grid CS ISAR imaging algorithm

As the frequency components of $\boldsymbol{z}$ are not known in advance, a predefined Fourier dictionary is employed, which can not guarantee all the frequency components of $\boldsymbol{z}$ exactly lie on the grid points. As a result, the CS based

* College of Computer and Information, Hohai University, 211100 Nanjing, China, chengping1219@163.com, **College of Science, Hohai University, 211100 Nanjing, China, zhaojq@hhu.edu.cn

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imaging performance degrades significantly, which is the off grid problem.

To solve the problem, the following will give a fast off grid CS ISAR imaging algorithm. ISAR imaging requires identification of frequency grid nodes at which scatterers are present, *ie* the index set of "non-zero" coefficient, which can be accomplished by a CS recovery algorithm such as OMP algorithm. Then, the selected frequency grids are perturbed to get a better fit to the measurements, *ie* the new optimization problem is [6]

$$\arg\min_{s_i,\delta f_i}\Big\|\boldsymbol{y}-\sum_{i=1}^{K}s_i\boldsymbol{\Theta}(f_i+\delta f_i)\Big\|_2 \quad \text{s.t. } |\delta f_i|\le\Delta/2 \tag{4}$$

where i is scatterer index, s_i is coefficient of the i-th scatterer, δf_i is unknown perturbation of the i-th scatterer from its closest grid node f_i, Δ is the size of grid, and $\boldsymbol{\Theta}(f_i+\delta f_i)$ is the column of $\boldsymbol{\Theta}$ with parameters $f_i+\delta f_i$. To get the joint minimization, coefficients and frequency perturbation can be updated sequentially in the following way.

First initialize $f_i^{(1)}$ to grid centers f_i and get an initial coefficient vector estimation $\boldsymbol{s}^{(1)}=\arg\min_{\boldsymbol{s}}\Big\|\boldsymbol{y}-\sum_{i=1}^{K}s_i\boldsymbol{\Theta}\big(f_i^{(1)}\big)\Big\|_2$.

Update from $n=1$ until convergence

$$[\delta f_1^{(n)}\dots\delta f_K^{(n)}]=\arg\min_{|\delta f_i|\le\Delta/2}\Big\|\boldsymbol{y}-\sum_{i=1}^{K}s_i^{(n)}\boldsymbol{\Theta}(f_i^{(n)}+\delta f_i)\Big\|_2, \tag{5}$$

$$\boldsymbol{s}^{(n+1)}=\arg\min_{\boldsymbol{s}}\Big\|\boldsymbol{y}-\sum_{i=1}^{K}s_i\boldsymbol{\Theta}\big(f_i^{(n+1)}\big)\Big\|_2 \tag{6}$$

where n and $n+1$ denote iteration index, and $f_i^{(n+1)}=f_i^{(n)}+\delta f_i^{(n)}$. Equation (6) is a standard least squares problem, while obtaining solution of the constrained nonlinear optimization problem (5) is not easy. But linearization of the cost function in (5) around $f_i^{(n)}$ will reduce the optimization complexity greatly. $\boldsymbol{\Theta}\big(f_i^{(n)}+\delta f_i\big)$ is approximated by its first order Taylor series as $\boldsymbol{\Theta}\big(f_i^{(n)}+\delta f_i\big)\approx\boldsymbol{\Theta}\big(f_i^{(n)}\big)+\boldsymbol{\Theta}'\big(f_i^{(n)}\big)\delta f_i$, then (5) can be re-written as

$$[\delta f_1^{(n)}\dots\delta f_K^{(n)}]=\arg\min_{\boldsymbol{\gamma}}\big\|\boldsymbol{y}^{(n)}-\mathbf{A}^{(n)}\boldsymbol{\gamma}\big\|_2 \tag{7}$$

where $\boldsymbol{\gamma}=[\delta f_1\dots\delta f_K]$, $\boldsymbol{y}^{(n)}=\boldsymbol{y}-\sum\limits_{i=1}^{K}s_i^{(n)}\boldsymbol{\Theta}\big(f_i^{(n)}\big)$, $\mathbf{A}^{(n)}\in\boldsymbol{C}^{M\times K}$ is defined as $\mathbf{A}^{(n)}=\Big[s_1^{(n)}\frac{\partial\boldsymbol{\Theta}}{\partial f_1^{(n)}},\dots s_K^{(n)}\frac{\partial\boldsymbol{\Theta}}{\partial f_K^{(n)}}\Big]$ and can be easily calculated. Now (7) is a least squares problem, so its closed-form solution is

$$[\delta f_1^{(n)}\dots\delta f_K^{(n)}]=\big((\mathbf{A}^{(n)})^H\mathbf{A}^{(n)}\big)^{-1}\big(\mathbf{A}^{(n)}\big)^H\boldsymbol{y}^{(n)}. \tag{8}$$

As perturbations should be real, (8) is modified to $[\delta f_1^{(n)}\dots\delta f_K^{(n)}]=\mathrm{Re}\big\{\big((\mathbf{A}^{(n)})^H\mathbf{A}^{(n)}\big)^{-1}(\mathbf{A}^{(n)})^H\boldsymbol{y}^{(n)}\big\}$. So the solution to the main problem (4) can be written as

$$\boldsymbol{f}^{(n+1)}=\boldsymbol{f}^{(n)}+\mathrm{Re}\big\{\big((\mathbf{A}^{(n)})^H\mathbf{A}^{(n)}\big)^{-1}\big(\mathbf{A}^{(n)}\big)^H\boldsymbol{y}^{(n)}\big\}, \tag{9}$$

$$\boldsymbol{s}^{(n+1)}=\big(\mathbf{B}^H\mathbf{B}\big)^{-1}\mathbf{B}^H\boldsymbol{y} \tag{10}$$

where $\mathbf{B}=\big[\boldsymbol{\Theta}(f_1^{(n+1)}),\dots\boldsymbol{\Theta}(f_K^{(n+1)}\big]$. In a word, the optimal estimation of $\boldsymbol{s}$ and $\boldsymbol{f}$ is calculated through (9) and (10) iteratively until convergence.

As the proposed algorithm can directly get the solutions of (5) and (6) during iterations, it is much faster than the algorithm proposed in [6]. Experiments show that the proposed algorithm only needs about 1 % time of the algorithm in [6] for typical step value. Moreover, the algorithm in [6] need choose step size, which is difficult in real application.

4 Experiment results and discussion

The effectiveness of the proposed algorithm is demonstrated by the following experiments.

First, the proposed algorithm is applied to simulated complex sinusoids. Signal $\boldsymbol{z}$ with length $N=320$ contains $K=5$ complex sinusoids. Their normalized frequencies are distributed randomly over $[0,1)$ and amplitudes are complex unit circle. Measurement matrix $\boldsymbol{\Phi}$ is random Gaussian matrix with $M=64$. Gaussian white noise is added to the signal, which makes SNR to vary from 0 to 40 dB. 500 trials are run for a fixed SNR. Average frequency estimation errors are shown in Fig. 1. It can be seen that the proposed algorithm can supply more accurate frequency estimation than OMP algorithm.

Then, the proposed algorithm is applied quasi real Mig-25 aircraft ISAR data. The data is provided by the US Naval Research Laboratory. The stepped frequency radar operates at 9 GHz and has a bandwidth of 512 MHz. For each pulse, 64 complex range samples are saved. Sparse basis is 128×128 discrete Fourier matrix and measurement matrix is 64×128 Gaussian random matrix. Figures 2 and 3 are the imaging results. It can be seen that the proposed algorithm has got a better image than OMP algorithm, as it can solve off grid problem in ISAR imaging.

5 Conclusion

A novel fast and accurate off grid CS ISAR imaging algorithm has been proposed, which transforms the joint optimization problem into two least squares problems. Its effectiveness has been verified by experiments of simulated complex sinusoids frequency estimation and quasi real ISAR data imaging.

Our scientific contribution is that we propose a simple and fast off grid CS algorithm. The off grid problem in CS has attracted much attention recently. However, the existing off grid algorithms are very complex. Interestingly,

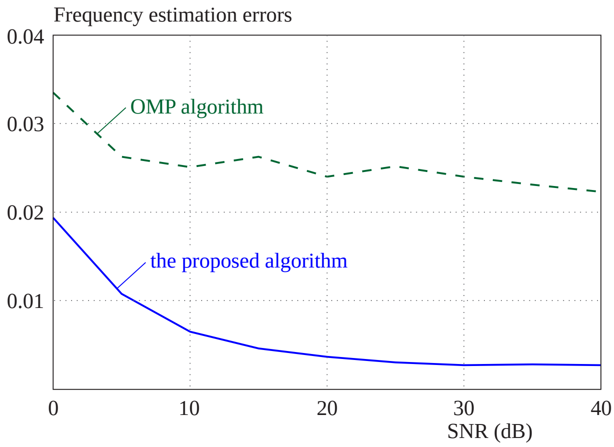

Fig. 1. Frequency estimation errors in different SNRs

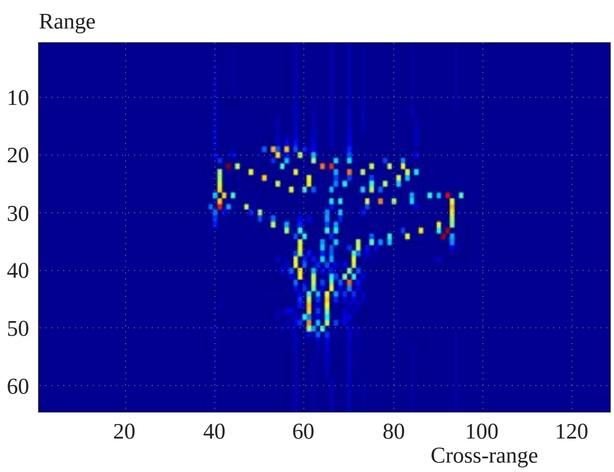

Fig. 2. Imaging result by OMP algorithm

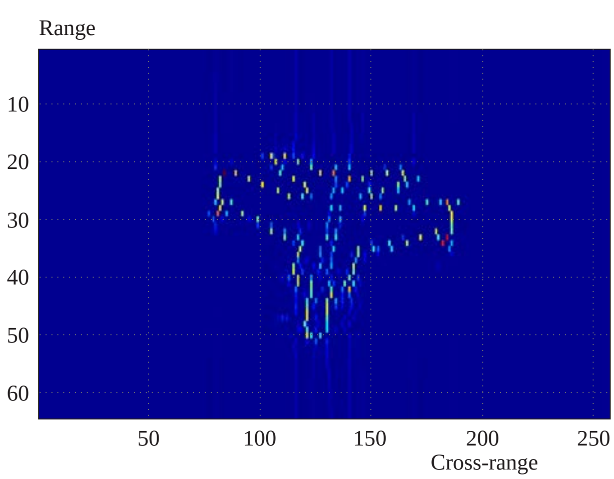

Fig. 3. Imaging result by the proposed algorithm

the proposed off grid algorithm is much simpler and faster than the existing algorithms, as it transforms the joint optimization problem into two least squares problems.

The off grid problem becomes one of the major constraints in promoting the application of CS. The proposed algorithm can solve off grid problem very well and is very simple and fast. Moreover, the novel algorithm can be used not only for ISAR imaging but also for other CS based applications.

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Cheng Ping (lecturer) was born in Jiangsu, China, on July 2, 1976. She received Bachelor Degree and Master Degree in electronic engineering from Harbin Institute of Technology in 2000 and 2002 respectively, and PhD degree in the field of communication and information system in Harbin Institute of Technology in 2007. She worked in Harbin Engineering University from 2007 to 2016, and has been working in Hohai University since 2016. Her research interest is compressed sensing, ISAR (inverse synthetic aperture radar) imaging. She has published nearly 30 journals and conference papers.

Zhao Jiaqun (associate professor) was born in Heilongjiang, China, in 1976. He received Master Degree in optics and PhD degree in physical electronics from Harbin Institute of Technology in 2003 and 2011, respectively. His research is mainly focused on solid state laser and nonlinear optical frequency conversion.