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Combination of the Viterbi algorithm and cross-Wigner distribution for the instantaneous frequency estimation phase signals in high noise environments

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In this paper, combination of the cross-Wigner distribution (XWD) and the Viterbi algorithm (VA) for the instantaneous frequency (IF) estimation of frequency modulated (FM) signals in high noise environments is proposed. The favourable properties of the VA, the IF reconstruction based on minimization of the path penalty functions, and the XWD, iterative accuracy improvement of the IF estimation, give hybrid IF estimator with improved accuracy for high noise environments.

K e y w o r d s: instantaneous frequency, Wigner distribution, VITERBI algorithm, high noisy environments

1 Introduction

The instantaneous frequency (IF) estimation of the frequency modulated (FM) signals for high noise environments is considered [1, 2]. Two particular popular estimators are: the Viterbi algorithm (VA) [3–6], and the cross-Wigner distribution (XWD) [7–10]. Recent reviews [1, 11], confirm that these techniques outperform all existing estimators for high noise environments. The VA is based on the path penalty function optimization in the time-frequency (TF) plane taking into account two criteria: the IF is related to high values of the TF representation; the IF is slow varying function. The XWD is iterative procedure starting with the IF estimation obtained by the Wigner distribution (WD) position maxima. Then, a unit amplitude signal is reconstructed based on the IF estimate. The unit amplitude signal is 'crossed' (correlated) with a noisy signal. The XWD is the Fourier transform of this cross-correlation. The IF estimate is updated using the XWD position maxima. Several iterations of this procedure are usually performed for the IF estimation improvement. There are two limitations of the XWD-IF estimator: (a) increased bias; (b) accuracy of the phase filtering used for the signal reconstruction is limited for high noise environments. Therefore, it is interesting to combine these two tools to further improve the IF estimate.

The paper is organized as follows. Section 2 gives description of the proposed technique in a general form that can cover all algorithm ingredients. Simulations are given in Section 3. Concluding remarks are presented in Section 4.

2 Proposed algorithm

Considered signal is $x(n) = A\exp\big(j\varphi(n)\big)+v(n)$, where A is signal amplitude, $\varphi(n)$ is signal phase, $v(n)$ is Gaussian white noise with variance σ^2. Our goal is to estimate the IF, $\omega(n) = \varphi'(n)$, from noisy observations $x(n)$ under condition that the signal-to-noise ratio (SNR), $\mathrm{SNR} = A^2/\sigma^2$, is as low as possible.

The proposed algorithm can be written in the following form.

Calculate the WD

$$WD(n,\omega) = \sum_k x(n+k)x^*(n-k)\exp(-j2\omega k)\,. \quad (1)$$

Position of the WD maximum is one of the most popular IF estimators

$$\hat{\omega}^{(0,m)}(n) = \arg\max_{\omega} WD(n,\omega)\,. \quad (2)$$

The alternative tool for estimation of the WD-based IF estimation is the VA estimator. The IF estimate using the VA is

$$\hat{\omega}^{(0,v)}(n) = \arg\min_{l(n)}\Bigg[\sum_{k=n_1}^{n-1} g_{c,\Delta}\big(l(k), l(k-1)\big) + \sum_{k=n_1}^{n} f\big(WD\big(l, l(k)\big)\big)\Bigg] \quad (3)$$

$$= \mathrm{Viterbi}\{WD(n,\omega), c, \Delta)\}\,, \quad (4)$$

where c and Δ are the VA design parameters described in details in [3]. Here, function $g_{c,\Delta}(k_1,k_2)$ penalizes the IF estimate variations. It is given as $g_{c,\Delta}(k_1,k_2) = c\big(|k_1-k_2|-\Delta\big)$ for $|k-1-k_2| \geq \Delta$ and $g_{c,\Delta}(k_1,k_2) = 0$

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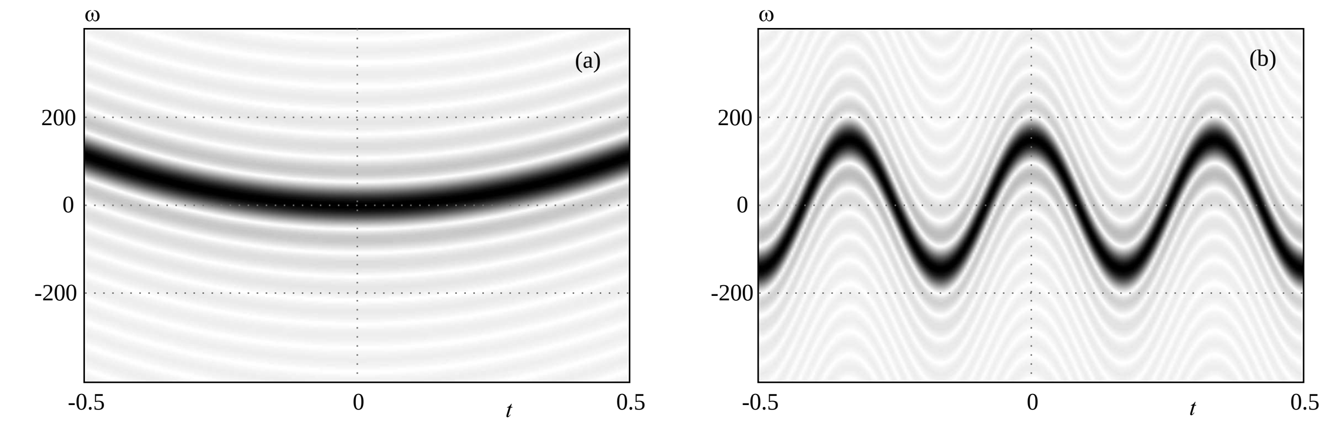

Fig. 1. WD of test signals: (a) – CPS, (b) – sinusoidal FM signal

otherwise [3]. Function $f\big(WD\big(k,l(k)\big)\big)$ penalizes more lower WD values. The highest WD value for a given instant (corresponding to the IF position maxima) is not penalized ($f(\cdot,\cdot)=0$), the next highest WD value is penalized with $f(\cdot,\cdot)=1$ with incrementing output of $f(\,)$ function for lower WD values.

Set $i=0$.

Label A. The phase reconstruction by integrating the IF estimate is performed

$$\hat{\phi}^{(i,a)}(n)=\sum_{k=n_0}^{n}\hat{\omega}^{(i,a)}(k)\,, \tag{5}$$

where exponent a can be m for the IF estimate based on the WD maxima position, or v for the VA-based IF estimate, respectively.

Then, the unit amplitude signal is formed as

$$\hat{x}^{(i,a)}(n)=\exp\big(j\hat{\phi}^{(i,a)}(k)\big). \tag{6}$$

The XWD is calculated as

$$XWD^{(i,a)}(n,\omega)=\sum_k x(n+k)\hat{x}^{(i,a)*}(n-k)\exp(-j2\omega k)\,. \tag{7}$$

The IF estimates based on the position of the XWD maxima can be denoted as

$$\hat{\omega}^{(i+1,m)}(n)=\arg\max_{\omega}|XWD^{(i,m)}(n,\omega)|\,, \tag{8}$$

while the IF estimate obtained with the VA applied to the XWD is

$$\hat{\omega}^{(i+1,v)}(n)=\text{Viterbi}\big\{|XWD^{(i,v)}(n,\omega)|,c,\Delta\big\}, \tag{9}$$

If maximal number of iterations $i_{\max}$ is reached or if the IF estimates in two consecutive estimations are close

$$\max\big|\hat{\omega}^{(i+1,a)}(n)-\hat{\omega}^{(i,a)}(n)\big|<\Delta\omega\,, \tag{10}$$

then the IF estimate is equal to $\hat{\omega}^{(i+1,a)}(n)$ and further iterations can be stopped, otherwise set $i=i+1$ and go to *Label A*.

It can be seen that the XWD-VA algorithm with the IF estimate $\hat{\omega}_3(n)=\hat{\omega}^{(i+1,v)}(n)$ produces also intermediate results: the VA-IF estimate $\hat{\omega}_1(n)=\hat{\omega}^{(0,v)}(n)$, the XWD-IF estimate $\hat{\omega}_2(n)=\hat{\omega}^{(i+1,m)}(n)$, and the IF estimate based on the position of the WD maximum $\hat{\omega}_0(n)=\hat{\omega}^{(0,m)}(n)$. These estimators have some advantages and disadvantages with respect to the bias and noise influence. The WD-based estimator is sensitive to the high noise influence, the XWD estimator has increased bias [8], the VA estimator can oversmooth the IF estimate [11]. Therefore, it can be expected there is no single estimator offering the best performance under all circumstances. The following procedure can be used for determination of the final output from these four estimates

$$J(q)=\Big|\sum_n x(n)\exp\Big(-j\sum_{k=n_0}^{n}\hat{\omega}_q(k)\Big)\Big|, \tag{11}$$

$$\hat{q}=\arg\max_q J(q)\,,\ q\in[0,3]\,, \tag{12}$$

$$\hat{\omega}(n)=\hat{\omega}_{\hat{q}}(n)\,. \tag{13}$$

3 Simulation

The considered techniques are tested on two test signals: cubic phase signal (CPS), $x(t)=\exp(j48\pi t^3)$, with the IF $\omega(t)=144\pi t$, and sinusoidal FM, $x(t)=\exp(j8\sin(6\pi t)$, with the IF $\omega(t)=48\pi\cos(6\pi t)$. Signals are recorded within the interval $t\in[-1,1)$ with the sampling time $\Delta t=1/256$. The WDs of these test signals are given in Fig. 1 for window width of 16 samples. Maximal number of iterations is $i_{\max}=10$, while $\Delta\omega$ is set to three frequency bins. The mean squared error (MSE) in the IF estimation for these signals and the WD and XWD evaluated with 16 samples wide window is depicted in Figs. 2(a), (c), respectively. For the CPS all considered techniques for the SNR higher than

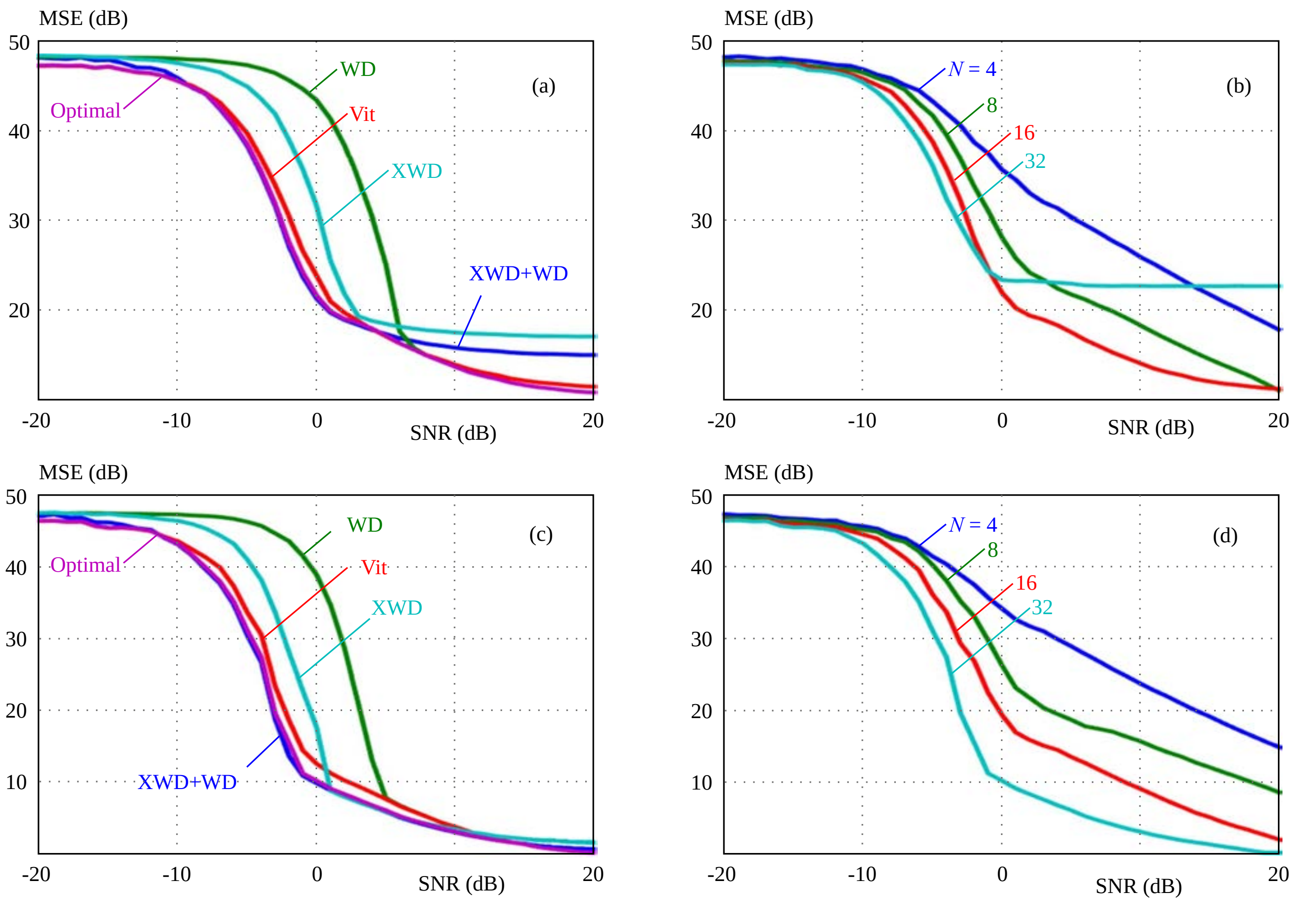

Fig. 2. MSE in IF estimation: (a) – CPS for fixed window width and various algorithms; (b) – CPS and the proposed procedure for various window widths; (c) – sinusoidal FM signal for fixed window width and various algorithms; (d) – sinusoidal FM signal and the proposed procedure for various window widths

6 dB achieves similar results. The breakdown point of the WD-based estimator is the highest, the XWD estimator achieves the SNR threshold about 2 dB, the VA is better with the SNR threshold about 1 dB while the best results are obtained with the proposed XWD-VA algorithm. Final output (13) follows the XWD-VA in the entire SNR range. The IF estimators for the sinusoidal FM signal exhibit higher bias that is additionally emphasized by the XWD iterative procedure. It means that for high SNR the WD-and VA-based estimators, are better than the XWD-counterparts. However, for low SNR (the VA, XWD, and XWD-VA estimators are primarily developed to address high noise influence) the XWD-VA estimator gives the best results. Final algorithm output (13) follows the best results for the entire considered SNR range. In addition, it is obvious that it is enough to consider combination of only two estimators in the algorithm: the WD-based (good for high SNR), and the XWD-VA (appropriate for low SNR). Figs. 2(b), (d), demonstrate results of the proposed algorithm as function of the window width. For the CPS it can be noticed that as window width increases the MSE decreases due to robustness of the considered approaches to the noise influence. However, for the sinusoidal FM signal the best results are achieved with window of 16 samples since enlarging window causes increased bias. Therefore, our recommendation is to keep size of the window between 12 and 20 samples. This is counter intuitive with respect to the other IF estimators where wider windows are recommended for suppressing high noise influence. Namely, the VA and XWD algorithms are powerful enough to reduce high noise influence with narrow windows but they are unable to remove bias from initial estimates so narrower windows are preferable here.

4 Conclusion

The XWD-VA IF estimator is proposed to further improve the IF estimation accuracy for high noise environments. The proposed estimator enhances robustness to the noise influence with respect to both of ingredients. The additional algorithm step is proposed for selecting the best IF estimator from four considered: WD-based, XWD-based, VA-IF estimator, and combined XWD-VA IF-estimator, giving excellent results for the entire the SNR range. Simulations confirm that the proposed strategy is robust to high noise influence and that excellent results can be obtained with windows narrower than in other IF estimators for high noise environment. In addition, it is also shown that final output can be selected by switching between only WD-based and XWD-VA IF estimators.

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