

Robust PID controller design for nonlinear systems

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In this paper the new approach to the design of robust PID controller for the case of nonlinear Lipschitz systems is proposed. The proposed method is based on the uncertain gain scheduling plant model and Bellman Lyapunov equation. The designed robust controller ensures parameter dependent quadratic stability and in the frame of H_2 performance guaranteed cost. Examples show the effectiveness of the proposed method.

Key words: robust controller; Lipschitz systems; gain scheduled controller, Bellman Lyapunov equation

1 Introduction

PID controllers have found large number of practical applications, in the more than eight decades since Ziegler and Nichols proposed their first PID controller tuning method. Tuning rules of PID controller parameters can be divided into the following groups based on:

- measured step response.
- a measure of critical (ultimate) points of closed-loop system.
- the mathematical plant model and model uncertainties analytical approach.

Due to [7], there are more than 184 tuning rules for PI and 258 tuning rules for PID controllers. Most of them are suited for a particular problem or give the specified closed-loop performance. In the paper [12] the PID tuning rules for specified performance for stable or not stable systems have been proposed. The paper [8] is devoted to design of the PID controller gains using gain margin methods to compensate the instability of closed loop system which may induced by time delays for the network control system. Robust PID controller parameters design for mechanical actuators using Kharitonovs theorem is given in [10]. In the frequency domain with uncertainties [4] linear programming is used for the design of robust PID controllers. Robust PID controller design for linear MIMO system is given [1, 3, 5, 15]. For nonlinear Lipschitz systems, there are two basic methods one could use the Lyapunov function or the linearization and multi-model approach. The latest approaches lead to the use of switched controller design [13] or linear parameter varying (LPV). In this paper, we will use the idea to transform the nonlinear system to the linear parameter varying (LPV) system [6], and for LPV system develop the new design procedure for the design of robust PID controller. In this paper two design procedures have been developed. The first method is less conservativeness than

the second ones, but the computation loads are greater for the first method.

After the problem formulation, transformation of nonlinear systems to LPV model, some preliminary results are given. A sufficient robust stability conditions in the form of BMI for proposed robust PID controller design procedures are introduced, and obtained results are illustrated on two examples. Hereafter the following notational conditions will be adopted. Given a symmetric matrix $P = P^T \in R^{n \times m}$ the inequality $P > 0$ means the matrix positive definiteness. I denotes the identity matrix.

2 Problem statement and preliminaries

Consider the uncertain nonlinear Lipschitz systems

$$\dot{z} = f(z, v, w), \bar{y} = h(z) \quad (1)$$

where $z \in R^{\bar{n}}$ is the state, the input, $v \in R^m$, the output $\bar{y} \in R^{\bar{l}}$ and exogenous input $w \in R^k$, which captures the parametric dependence of the plant model on exogenous input. Assume that the nonlinear function $f(\cdot)$ is locally Lipschitz, that is for each state vector $z_0 \in R^{\bar{n}}$ one can linearize the nonlinear function $f(\cdot)$. Let the number of linearized points of system be \bar{p} . Transform plant model (1) to the linear parameter varying system using the following steps.

- 1 - Choose the number of gain scheduled variables θ , $p = \bar{p} - 1$ and their upper ($\bar{\theta}$) and lower bound ($\underline{\theta}$) as a function of exogenous input w .
- 2 - For robust controller design choose r uncertain parameters of the plant (1) which are ranges within given two values.
- 3 - At each linearized point $\bar{j} = 1, 2, \dots, \bar{p}$ and uncertain parameters $i = 1, 2, \dots, 2^r$ one builds a linearized

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model of the plant (1)

$$\begin{aligned}\dot{\bar{x}} &= \bar{A}_{ij}\bar{x} + \bar{B}_{ij}u \\ y_s &= \bar{C}\bar{x}\end{aligned}\quad (2)$$

where, $\bar{x} = z - z_e$, $u = v - v_e$, $y_s = \bar{y} - y_e$, (z_e, v_e, \bar{y}_e) define the equilibrium points of the nonlinear system (1) at each linearized point for the case of different combinations of θ . For each i we transform (2) to the gain scheduling time varying model using simple equality.

$$\begin{bmatrix} \bar{A}_{i1} \\ \bar{A}_{i2} \\ \vdots \\ \bar{A}_{i\bar{p}} \end{bmatrix} = \begin{bmatrix} I\bar{Q}_1 & \cdots & I\bar{Q}_p \\ \vdots & \vdots & \vdots \\ I & I\bar{Q}_1 & \cdots & I\bar{Q}_p \end{bmatrix} \begin{bmatrix} \bar{A}_{0i} \\ \bar{A}_{1i} \\ \vdots \\ \bar{A}_{pi} \end{bmatrix}, \quad (3)$$

$i = 1, 2, \dots, 2^r$

From which one obtains

$$\begin{aligned}\bar{A}_i(\theta) &= \bar{A}_{0i} + \sum_{j=1}^p \bar{A}_{ij}\theta_j \in R^{\bar{n} \times \bar{m}} \\ \bar{B}_i(\theta) &= \bar{B}_{0i} + \sum_{j=1}^p \bar{B}_{ij}\theta_j \in R^{\bar{n} \times \bar{m}}\end{aligned}$$

Due to uncertainties the obtained system matrices are in the polytope with $K = 2^r$ vertices which can be formally defined as

$$\begin{aligned}\Omega &= \{\bar{A}(\xi, \theta), \bar{B}(\xi, \theta)\} = \sum_{i=1}^K (\bar{A}_i(\theta), \bar{B}_i(\theta)) \xi_i \\ \sum_{i=1}^K \xi_i &= 1, \quad \xi_i \geq 0, \quad \xi_i \in \Omega_\xi, \quad \dot{\xi}_i \in \Omega_{t\xi}\end{aligned}\quad (4)$$

where, ξ_i , $i = 1, 2, \dots, K$ are constant or possible time varying, but unknown parameters satisfying (4), $\theta \in R^p$ is a vector of known constant or time varying gain scheduled parameters. We assume that lower, upper bounds and their rates are available, that is

$$\begin{aligned}\theta &\in \Omega_\theta = \{\theta \in R^p : \theta_j \in \langle \underline{\theta}_j, \bar{\theta}_j \rangle\} \\ \dot{\theta} &\in \Omega_{\dot{\theta}} = \{\dot{\theta} \in R^p : \dot{\theta}_j \in \langle \underline{\dot{\theta}}_j, \bar{\dot{\theta}}_j \rangle\}\end{aligned}\quad (5)$$

For more detail how to obtain the gain scheduled plant model, see the excellent surveys [6]. The following problem is studied in this paper. Design a robust static output feedback PID controller with control algorithm

$$u(t) = K_p y_s + K_i \int_{\tau=0}^t y_s(\tau) d\tau + K_d \dot{y}_s(t) \quad (6)$$

such that the controller (6) with (9) ensures closed loop parameter dependent quadratic stability and guaranteed cost in the frame of H_2 .

The plant model (4) can be rewritten as in (7)

$$\dot{\bar{x}} = \bar{A}(\xi, \theta)\bar{x} + \bar{B}(\xi, \theta)u, \quad y_s = \bar{C}\bar{x} \quad (7)$$

Due to the integral part of the PID controller, let's introduce the new system output as $e = \int_0^t y_s(\tau) d\tau \in R^{\bar{l}}$. System (7) is

$$\dot{x} = \begin{bmatrix} \dot{\bar{x}} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} \bar{A}(\xi, \theta) & 0 \\ \bar{C} & 0 \end{bmatrix} \begin{bmatrix} \bar{x} \\ e \end{bmatrix} + \begin{bmatrix} \bar{B}(\xi, \theta) \\ 0 \end{bmatrix} u \quad (8)$$

Output

$$\begin{aligned}y &= \begin{bmatrix} y_s \\ e \end{bmatrix} = Cx, \\ C &= \begin{bmatrix} \bar{C} & 0 \\ 0 & I_e \end{bmatrix} \in R^{(2\bar{l}) \times (\bar{n} + \bar{l})} \in R^{l \times n}\end{aligned}\quad (9)$$

or

$$\dot{x} = A(\xi, \theta)x + B(\xi, \theta)u, \quad y = Cx$$

Due to the augmented plant model (8) control algorithm (6) is in the form

$$u(t) = K_p C_p x + K_i C_i x + K_d C_d \dot{x}$$

where

$$\begin{aligned}C_p &= [\bar{C} \quad 0] \in R^{\bar{l} \times n}, \\ C_i &= [0 \quad I_e] \in R^{\bar{l} \times n}.\end{aligned}\quad (10)$$

To assess performance quality, the following cost function is used within H_2 frame

$$J_c = \int_0^\infty J(x, u, \dot{x}) dt \quad (11)$$

where $J(x, u, \dot{x}) = x^\top Q x + u^\top R u + \dot{x}^\top S \dot{x}$ and $(Q, S) \in R^{n \times n}$ are positive definite (semidefinite) matrices, and $R \in R^{m \times m}$ is positive definite matrix. Lemma 1. Consider the uncertain plant model (9) with control algorithm (10). Control algorithm (10) is the guaranteed cost control law for the closed loop system if and only if a Lyapunov function $V(x, \theta, \xi)$ exists such that the following inequality holds

$$B_e = \frac{dV(\cdot)}{dt} + J(x, u, \dot{x}) \leq -\varepsilon x^\top x, \varepsilon \rightarrow 0, \varepsilon \geq 0 \quad (12)$$

The equation (12) is known as the Bellman-Lyapunov equation, and function V , which satisfies (12), is the Lyapunov function. For the particular structure of the Lyapunov function, the obtained robust controller design procedure may reduce from if and only if to if.

3 Robust controller design

This part of the paper is devoted to develop a new robust PID controller design procedure for the case of gain scheduled plant model and polytope uncertainty. The obtained PID controller ensures to closed loop system parameter dependent quadratic stability for the all $\theta \in \Omega_\theta$, $\xi \in \Omega_\xi$ and $\dot{\xi} \in \Omega_{t\xi}$. The two controller design procedure is obtained. Let in (12) the Lyapunov function be given as $V(x, \theta, \xi) = x^\top P(\xi, \theta)x$ where

$$P(\xi, \theta) = P_0(\xi) + \sum_{j=1}^p P_j(\xi)\theta_j = \sum_{i=1}^K (P_{i0} + \sum_{j=1}^p P_{ij}\theta_j)\xi_i \quad (13)$$

3.1 First controller design procedure

The first time derivative of the Lyapunov function (13) is as follows

$$\frac{dV}{dt} = [\dot{x}^\top x^\top u^\top] \begin{bmatrix} 0 & P(\theta, \xi) & 0 \\ P(\theta, \xi) & P(\dot{\theta}, \dot{\xi}) & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ x \\ u \end{bmatrix} \quad (14)$$

where

$$P(\dot{\xi}, \dot{\theta}) = \left(\sum_{i=1}^K \left(P_{i0} + \sum_{j=1}^p P_{ij}\theta_j \right) \dot{\xi}_i \right)' = \sum_{i=1}^K \left(P_{di0} + \sum_{j=1}^p P_{dij}\theta_j \right) \dot{\xi}_i \quad (15)$$

$$P_{di0} = \sum_{i=1}^K P_{i0}\dot{\xi}_i + \sum_{j=1}^p P_{ij}\dot{\theta}_j, \quad P_{dij} = \sum_{i=1}^K P_{ij}\dot{\xi}_i$$

To ensure less conservativeness results and split system matrices from Lyapunov matrix introduce the auxiliary matrices

$$N_i \in R^{n \times n}, \quad i = 1, 2, \quad N_3 \in R^{n \times m}, \\ N_i \in R^{m \times n}, \quad i = 4, 5, \quad N_6 \in R^{m \times m}$$

as follows

$$2(N_1\dot{x} + N_2x + N_3u)^\top (\dot{x} - A(\xi, \theta)x - B(\xi, \theta)u) = 0 \\ 2(N_4\dot{x} + N_5x + N_6u)^\top (u - K_p C_p x - K_i C_i x - K_d C_d \dot{x}) = 0 \quad (16 \text{ a,b})$$

From (16a) one obtains

$$[\dot{x}^\top x^\top u^\top] \begin{bmatrix} N_1^\top + N_1 & -N_1^\top A(\cdot) + N_2 \\ N_2^\top - A^\top(\cdot)N_1 & -N_2^\top A(\cdot) - A^\top(\cdot)N_2 \\ N_3^\top - B^\top(\cdot)N_1 & -N_3^\top A(\cdot) - B^\top(\cdot)N_2 \\ N_3 - N_1^\top B(\cdot) \\ -N_2^\top B(\cdot) - A^\top(\cdot)N_3 \\ -N_3^\top B(\cdot) - B^\top(\cdot)N_3 \end{bmatrix} \begin{bmatrix} \dot{x} \\ x \\ u \end{bmatrix} = 0$$

and from (16b)

$$[\dot{x}^\top x^\top u^\top] \begin{bmatrix} 2N_4^\top \\ 2N_5^\top \\ 2N_6^\top \end{bmatrix} [-K_d C_d - K_c I] \begin{bmatrix} \dot{x} \\ x \\ u \end{bmatrix} = 0 \\ K_c = K_p C_p + K_i C_i \quad (17)$$

Substituting (11), (14), (16) and (17) to (12) after manipulation we obtain the Bellman-Lyapunov function in the form

$$B_e = [\dot{x}^\top x^\top u^\top] W(\xi, \theta) [\dot{x}^\top x^\top u^\top]^\top \leq 0, \\ \text{where:} \\ W(\xi, \theta) = \{w_{ij}(\xi, \theta)\} \\ w_{11}(\xi, \theta) = N_1^\top + N_1 - N_4^\top K_d C_d - C_d^\top K_d^\top N_4 + S \\ w_{12}(\xi, \theta) = P(\xi, \theta) - N_1^\top A(\xi, \theta) + N_2 - N_4^\top K_c - C_d^\top K_d^\top N_5 \\ w_{13}(\xi, \theta) = N_3 - N_1^\top B(\xi, \theta) + N_4^\top - C_d^\top K_d^\top N_6 \\ w_{22}(\xi, \theta) = -N_2^\top A(\xi, \theta) - A(\xi, \theta)^\top - N_5^\top K_c - K_c^\top N_5 + P(\dot{\xi}, \dot{\theta}) + Q \\ w_{23}(\xi, \theta) = -N_2^\top (\xi, \theta) - A^\top(\xi, \theta)N_3 + N_5^\top - K_c^\top N_6 \\ w_{33}(\xi, \theta) = -N_3^\top B(\xi, \theta) - B(\xi, \theta)N_3 + N_6^\top + R \quad (18)$$

Correspondingly, all elements of matrix $W(\xi, \theta)$ are convex with respect to uncertainly ξ and θ , one can split the matrix $W(\xi, \theta)$ as follows.

$$W_{ke}(\xi, \theta) = \sum_{k=1}^K \left(W_{kli0} + \sum_{j=1}^p W_{keij}\theta_j \right) \xi_i \quad (19)$$

or robust stability condition is given as

$$W(\xi, \theta) = \sum_{k=1}^K (W_{i0} + \sum_{j=1}^p W_{ij}\theta_j) \xi_i \leq 0$$

where:

$$w_{11i0} = N_1^\top + N_1 - N_4^\top K_d C_d - C_d^\top K_d^\top N_4 + S, \\ w_{11ij} = 0 \\ w_{12i0} = P_{i0} - N_1^\top A_{i0} + N_2 - N_4^\top K_c - C_d^\top K_d^\top N_5 \\ w_{12ij} = P_{ij} - N_1^\top A_{ij} \\ w_{13i0} = N_3 - N_1^\top N B_{i0} + N_4^\top - C_d^\top K_d^\top N_6 \\ w_{13ij} = -N_1^\top B_{ij} \\ w_{22i0} = -N_2^\top A_{i0} - A_{i0}^\top N_2 - N_5^\top K_c - K_c^\top N_5 + Q + P_{di0}$$

$$\begin{aligned}
w_{22ij} &= -N_2^\top A_{ij} - A_{ij}^\top N_2 + P_{dij} \\
w_{23i0} &= -N_2^\top B_{i0} - A_{i0}^\top N_3 + N_5^\top - K_c^\top N_6 \\
w_{23ij} &= -N_2^\top B_{ij} - A_{ij}^\top N_3 \\
w_{33i0} &= -N_3^\top B_{i0} - B_{i0}^\top N_3 + N_6^\top + N_6 + R \\
w_{33ij} &= -N_3^\top B_{ij} - B_{ij}^\top N_3
\end{aligned} \tag{20}$$

The obtained first main results are summarized to the following theorem.

Theorem 1

The uncertain gain scheduled system (9) with controller (10) is robust PDQS with guaranteed cost, if there exists a positive definite Lyapunov matrix $P(\xi, \theta) \in R^{n \times n}$, matrices N_k , $k = 1, 2, \dots, 6$, positive definite matrix R and positive definite (semidefinite) matrices Q, S , such that inequality (20) holds for all $\theta \in \Omega_0$, $\dot{\theta} \in \Omega_t$, $\dot{\xi} \in \Omega_{\xi t}$. Proof. Due to the concrete Lyapunov function (13), the sufficient robust stability conditions follow (12), (13), (14) and (18). Note that (20) are convex with respect to the gain scheduled parameters θ and uncertainty ξ . Inequalities (20) hold if and only if they are negative for all $i = 1, 2, \dots, K$ and corners of gain scheduled parameters θ_j $j = 1, 2, \dots, p$ and their rates. As the rates of θ and ξ are in the diagonal of matrix $W(\xi, \theta)$, we can reduce the computation load using the inequalities.

$$\begin{aligned}
\sum_{i=1}^K P_{i0} \dot{\xi}_i &\leq \sum_{i=1}^K P_{i0} \bar{\xi}_i \\
\sum_{j=1}^p P_{ij} \dot{\theta}_j &\leq \sum_{j=1}^p P_{ij} \bar{\theta}_j
\end{aligned} \tag{21}$$

and $\sum_{i=1}^K P_{ij} \dot{\xi}_i \leq \sum_{i=1}^K P_{ij} \bar{\xi}_i$ assuming that: $P_{ij} > 0, i = 1, 2, \dots, K, j = 01, 2, \dots, p$

3.2 Second robust controller design

Closed loop system for uncertain plant model (9) and controller (10) one obtains as

$$\dot{x} = A(\xi, \theta)x + B(\xi, \theta)(K_c x + K_d C_d \dot{x}) \tag{22}$$

or

$$M(\xi, \theta) \dot{x} = A_c(\xi, \theta) x \tag{23}$$

where

$$\begin{aligned}
M(\xi, \theta) &= [I - B(\xi, \theta)K_d C_d] = \\
&= \sum_{i=1}^k \left(M_{i0} + \sum_{j=1}^p M_{ij} \theta_j \right) \xi_i \\
A_c(\xi, \theta) &= A(\xi, \theta) + B(\xi, \theta)K_c = \\
&= \sum_{i=1}^k \left(A_{ci0} + \sum_{j=1}^p A_{cij} \theta_j \right) \xi_i
\end{aligned}$$

$$M_{i0} = I - B_{i0}K_d C_d, \quad M_{ij} = -B_{ij}K_d C_d$$

$$A_{ci0} = A_{i0} + B_{i0}K_c, \quad A_{cij} = A_{ij} + B_{ij}K_c$$

For obtaining the convex robust stability condition with respect to gain scheduled and uncertainty parameters introduce two auxiliary matrices $M_1, M_2 \in R^{n \times n}$ as

$$[\dot{x}^\top \quad x^\top] \begin{bmatrix} 2M_1^\top \\ 2M_2^\top \end{bmatrix} [M(\xi, \theta) \quad -A_c] \begin{bmatrix} \dot{x} \\ x \end{bmatrix} = 0 \tag{24}$$

Quadratic performance (11) one can rewrite as

$$\begin{aligned}
J &= x^\top Q x + \dot{x}^\top S \dot{x} + u^\top R u \Rightarrow \\
[\dot{x}^\top \quad x^\top] &\begin{bmatrix} C_d^\top K_d^\top R K_d C_d + S & C_d^\top K_d^\top R K_c \\ K_c^\top R K_c C_d & K_c^\top R K_c + Q \end{bmatrix} \begin{bmatrix} \dot{x} \\ x \end{bmatrix}
\end{aligned} \tag{25}$$

Summarizing (14), (24), (25) and substitute the obtained results to (12) one obtains

$$B_e = v^\top V(\xi, \theta) v \leq 0 \tag{26}$$

where

$$V(\xi, \theta) = \{v_{ij}(\xi, \theta)\}_{2 \times 2} = \sum_{i=1}^K \left(v_{i0} + \sum_{j=1}^p v_{ij} \theta_j \right) \xi_i$$

and

$$\begin{aligned}
v_{11i0} &= M_1^\top M_{i0} + M_{i0}^\top M_1 + C_d^\top K_d^\top R K_d C_d + S \\
v_{11ij} &= M_1^\top M_{ij} + M_{ij}^\top M_1 \\
v_{12i0} &= -M_1^\top A_{ci0} + M_{i0}^\top M_2 + P_{i0} + C_d^\top K_d^\top R K_c \\
v_{12ij} &= -M_1^\top A_{cij} + M_{ij}^\top M_2 + P_{ij} \\
v_{22i0} &= -M_2^\top A_{ci0} - A_{ci0}^\top M_2 + K_c^\top R K_c + Q + P_{di0} \\
v_{22ij} &= -M_2^\top A_{cij} - A_{cij}^\top M_2 + P_{dij}
\end{aligned}$$

The obtained results are summarized in the following theorem

Theorem 2

The uncertain gain scheduled system (9) with the controller (10) is robust PDQS with guaranteed cost if there exists a positive definite matrix $P(\xi, \theta)$, matrices M_1, M_2 and positive definite R , and positive definite (semidefinite) Q, R such that inequality (26) holds for all $\theta \in \Omega_0$, $\dot{\theta} \in \Omega_t$, $\dot{\xi} \in \Omega_{\xi t}$. Note that (26) may be more conservative than (20). For the case of an uncertain linear system, the robust stability conditions (20), (26) reduces to

$$W_{i0} \leq 0, \quad V_{i0} \leq 0, \quad i = 1, 2, \dots, K \tag{27}$$

The latest condition has been obtained for the case of robust controller design by [8], and for the case of PID-PSD robust controller design with guaranteed cost in [11].

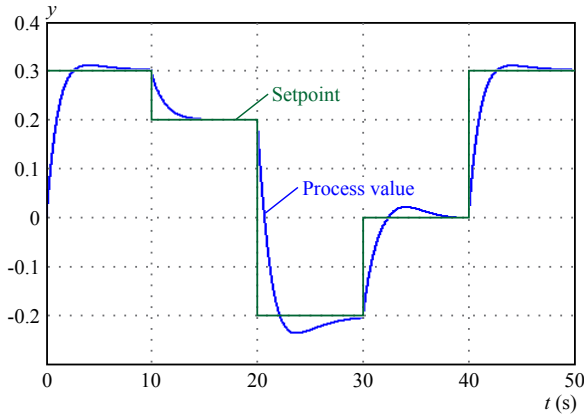


Fig. 1. Time responses of the controlled system (1. method)

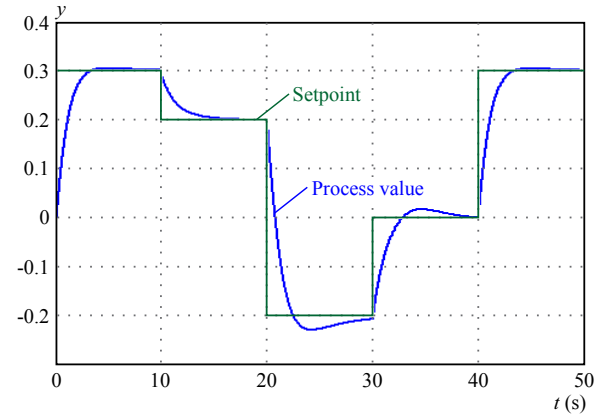


Fig. 2. Time responses of the controlled system (1. method) - zoom

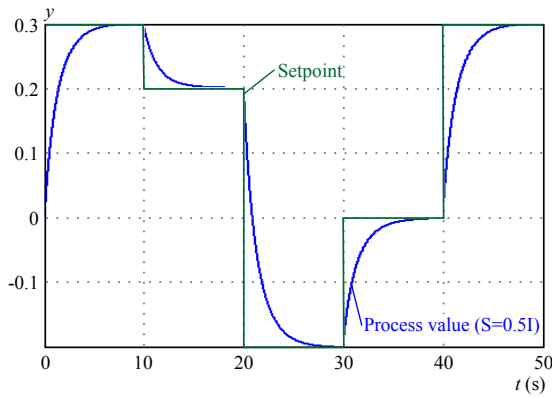


Fig. 3. Time responses of the controlled system (2nd method)

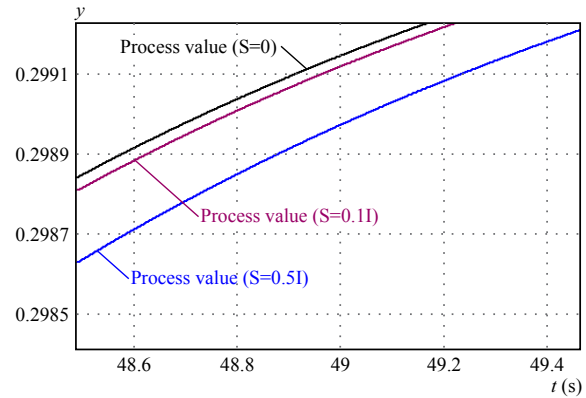


Fig. 4. Time responses of the controlled system (2nd method) - zoom

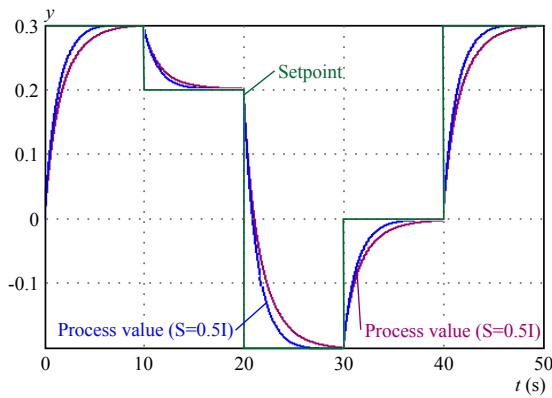


Fig. 5. Time responses of the controlled system (2nd method) detailed zoom

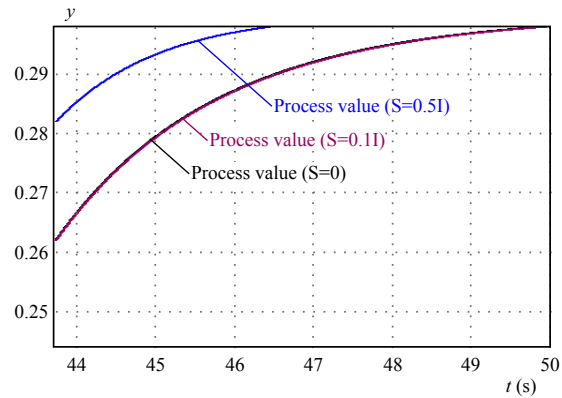


Fig. 6. Time responses of the controlled system (2nd method) - zoom

Example 1 — The first example is borrowed from [2]. It is a simple nonlinear system which has an unstable zero equilibrium point.

$$\dot{x} = -x|x| + u$$

$$y = x$$

$$-0.5 \leq u \leq 0.5$$

$$-0.5 \leq y \leq 0.5$$

One can transformed above system it into the following form

$$\dot{x} = -(a_0 + a_1\theta)x + bu, \quad y = Cx$$

where $b \in \langle 1, 1.5 \rangle$, $C = 1$, $a_1 = -0.25$, $a_0 = 0.25$ and $\theta \in \langle -1, 1 \rangle$. For PI controller introduce the new state $e = \int_0^t y dt$, we obtain

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} a_0 + a_1\theta & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} + \begin{bmatrix} b \\ 0 \end{bmatrix} u = \begin{bmatrix} a_0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} + \begin{bmatrix} a_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} \theta + \begin{bmatrix} b \\ u \end{bmatrix}$$

The aim is to design a robust PID controller which will guarantee PDQS, using in the paper the proposed meth-

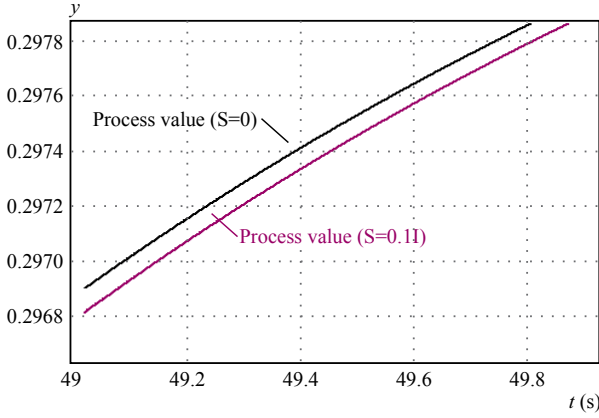


Fig. 7. Time responses of the controlled system 2nd method detailed zoom

ods. For the following parameters

$$0 < P_{ij} < r_0 I, \quad r_0 = 100, \quad \bar{\theta} = 0.5/s, \quad \bar{\xi} = 0.2/s, \\ R = I, \quad Q = 0.001I, \quad S = 0$$

one obtains PID controllers $R(s) = \alpha + \frac{\beta}{s} + \gamma s$, as shown in Tab. 1

Table 1. Example 1 controller parameters and eigenvalues

Method	α	β	γ	Max. eigen. cl.
1	-1.0473	-0.4713	0.1577	-0.4733
2	-0.9876	-0.3765	0.1532	0.4355*

* a more conservative result

The simulation results for different S are shown in Fig. 3 till Fig. 7. The results for the first model are almost similar, there is only a small influence of the S on the time response (detailed zoom is in Fig. 4). Higher impact on the dynamics regarding of the value of S can be seen in Fig. 5 to Fig. 7. The higher value of S increases the rate of the model output which leads to faster dynamics of the process.

Example 2 — In the second example we have used a single nonlinear system

$$\dot{x} = -a \sin x + bu, \quad \text{where: } a \in \langle 0.8, 1 \rangle, \\ \text{when } a = 0.8, \text{ then } b = 1 \\ \text{and when } a = 1, \text{ } b = -0.5$$

One can linearize the above model in the three working points $x_0 = \{0, \pi/4, \pi/2\}$. For PI robust controller design the system state space needs to be augmented, finally for matrices $A(\xi, \theta), B(\xi, \theta)$, one obtains:

$$A(\xi, \theta) = \left\{ \begin{bmatrix} -0.4 & 0 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 0.117 & 0 \\ 0 & 0 \end{bmatrix} \theta_1 + \right. \\ \left. \begin{bmatrix} -0.28 & 0 \\ 0 & 0 \end{bmatrix} \theta_2 \right\} \xi_1 + \left\{ \begin{bmatrix} -0.5 & 0 \\ 1 & 0 \end{bmatrix} + \right. \\ \left. \begin{bmatrix} 0.14645 & 0 \\ 0 & 0 \end{bmatrix} \theta_1 + \begin{bmatrix} -0.35355 & 0 \\ 0 & 0 \end{bmatrix} \theta_2 \right\} \xi_2$$

The aim is to design a robust PID controller, and in an experimental way show how the matrix S influences the dynamic behavior of the closed loop system. Under the parameters $0 < P_{ij} < r_0 I$, $r_0 = 100$, $\bar{\theta}_i = 0.5$, $i = 1, 2$, $\bar{\xi} = 0.1$, $R = I$, $Q = 0.0001I$, $S = 0$.

Obtained PID controllers are in Tab. 2

Table 2. Example 2 controller parameters and eigenvalues

Method	α	β	γ	Max. eigen. cl.
1	-0.8633	-0.5823	0.0870	-0.487
2	-0.7114	-0.4545	0.0738	-0.4443
Case $S = 0.1I$				
1	-0.7146	-0.5822	0.0873	-0.4879
2	-0.7114	-0.4538	0.0738	0.074
Case $S = 0.5I$				
1	-0.8753	0.5787	0.0860	-0.4899
2	-0.8351	-0.5999	0.01031	-0.4837

4 Conclusion

The paper devoted to the robust controller design for the nonlinear Lipschitz system. Designed controller guarantees closed loop robust stability and PDQS. A new method of PID controller design procedure based on the gain scheduled plant LPV model. The obtained results are in the form of BMI. In the paper two controller design procedures were proposed, where the first proposed method is less conservative than the second. The obtained results were summarized in the theorems and verified using two examples, which show the effectiveness of the proposed method. Numerical solution has been carried out by MATLAB 7.5 using YALMIP with solver PENBMI21.

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