

# Asymptotic performance modelling of DCF protocol with prioritized channel access

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Recently, the modification of the DCF (Distributed Coordination Function) protocol by the prioritized channel access was proposed to resolve the problem that the DCF performance worsens exponentially as more nodes exist in IEEE 802.11 wireless LANs. In this paper, an asymptotic analytical performance model is presented to analyze the MAC performance of the DCF protocol with the prioritized channel access.

**Key words:** wireless LAN, MAC, DCF, throughput

## 1 Introduction

Since the DCF (Distributed Coordination Function) protocol was introduced for the fundamental MAC (Media Access Control) access mechanism for IEEE 802.11 wireless LANs in 1997, the DCF protocol has been the primary MAC protocol for wireless LANs [1–3]. However, because the DCF protocol depends on the exponential back-off mechanism, it suffers from the exponential MAC performance degradation as more nodes exist in a wireless LAN [4, 5]. To resolve the problem of the exponential MAC performance degradation, the efficient modification of the DCF protocol was recently proposed allowing each node in back-off stage zero to access to wireless medium without back-off by the prioritized channel access [5]. (To correct the typos in (1) and (3) in [5],  $s$  should be replaced by  $w$  in (1) and both sides of the inequality in (3) should be multiplied by  $(n - r)E[T_s]$ .) The modified DCF protocol with the prioritized channel access was shown to outperform the conventional DCF protocol and the DCF protocol with optimized CW (Contention Window) minimum value through computer simulation [5].

This paper presents an asymptotic analytical MAC performance model of the DCF protocol with the prioritized channel access by which the asymptotic saturated MAC throughput can be derived under the condition of ideal channel and no hidden node existence. Although the asymptotic saturated throughput model does not consider fully the real traffic condition, it provides the maximum capacity of wireless LAN system and helps us forecast the MAC performance of wireless LAN system as more nodes participate in the transmission procedure of the DCF protocol with the prioritized channel access.

## 2 DCF Protocol with prioritized channel access

According to the DCF protocol with the prioritized channel access in [5], each node in back-off stage zero is allowed with probability  $p$  to send the data frames after a PIFS (PCF Inter-Frame Space), which is smaller than a DIFS (DCF Inter-Frame Space), following the start of idle channel state without back-off. The channel access without back-off for the nodes belonging to back-off stage zero is called the prioritized channel access. If a node in back-off stage zero transmits its data frame successfully by the prioritized channel access, it can be continuously granted the prioritized channel access with probability  $p$  remaining in back-off stage zero. However, if the prioritized channel access fails, that is, a node belonging to back-off stage zero that transmitted its data frame by the prioritized channel access does not hear the ACK frame from the destination node, it transits to back-off stage 1 with a random CW size. The nodes belonging to other back-off stages than zero comply with the conventional DCF transmission method for their data transmissions. Besides the prioritized channel access, the nodes in back-off stage zero can use also the existing DCF protocol for their transmissions.

APs (Access Points) optimize the probability  $p$  monitoring the MAC performance of wireless LANs and updating the probability  $p$  in the direction of improving the MAC performance. By allowing the nodes in back-off stage zero to access to wireless medium by the prioritized channel access, we can differentiate the nodes belonging to back-off stage zero and the nodes belonging to other back-off stages than zero into two contention domains and drastically reduce the collisions between data transmissions.

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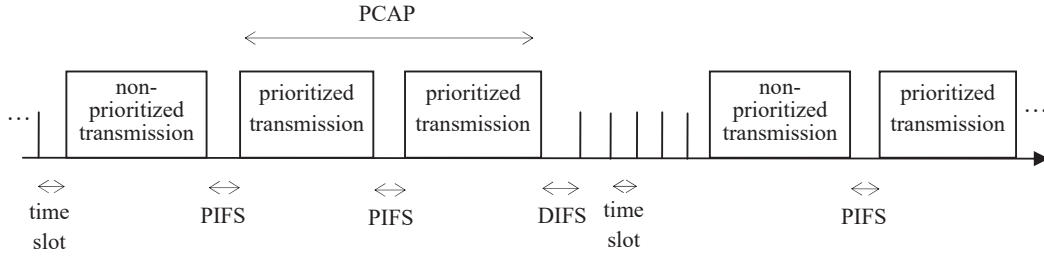


Fig. 1. Two transmission mechanisms

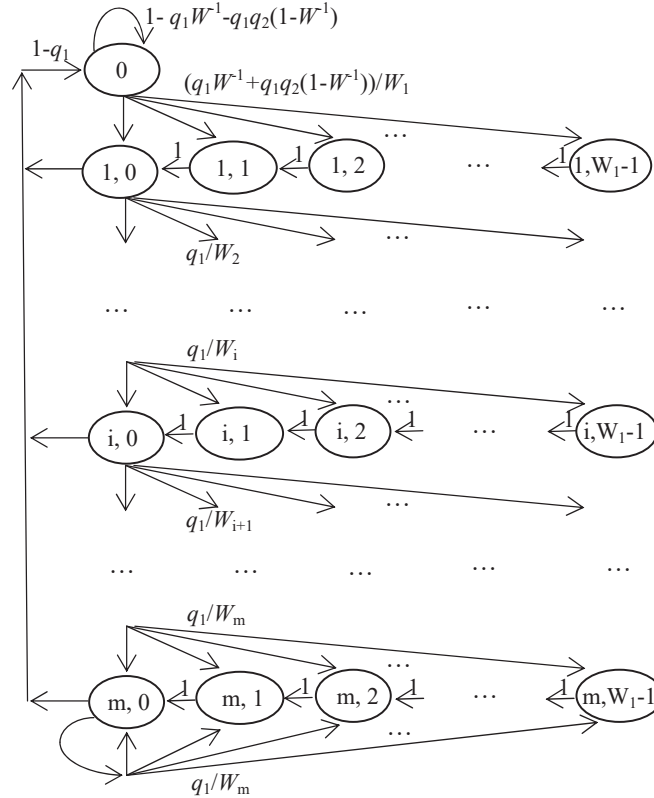


Fig. 2. Markov chain

### 3 Asymptotic MAC performance model

#### 3.1 Non-Prioritized and prioritized transmissions

We assume that each node continuously attempts to transmit its data frame according to the DCF protocol with the prioritized channel access, and the data transmissions fail only due to the collisions between transmissions. Under the assumption of no hidden node existence, the nodes approximately have a single view of the channel state (busy or idle) of a wireless LAN on a time basis like Figure 1 where the prioritized transmissions by the nodes that are granted the prioritized channel access occur after a PIFS following the end of the non-prioritized transmissions. After the non-prioritized transmissions, the periods of the prioritized transmissions called the PCAPs (Prioritized Channel Access Periods) follow.

#### 3.2 Markov chain

Let the state of a node participating in the transmission procedure of the DCF protocol with the prioritized channel access at the end of idle time slot  $t$  in Fig. 1 be characterized by back-off stage  $s(t)$  and size of the back-off widow  $b(t)$ . Similarly to [6], the stochastic process  $\{s(t), b(t)\}$  is a discrete-time Markov chain with the approximation that the probability  $q_1$  that the node about to attempt the non-prioritized channel access finds that the back-off timer of another node expires at the end of a generic idle time slot, that is, the attempted non-prioritized channel access collides and the probability  $q_2$  that the node experiences the collision of the prioritized channel access during a PCAP are independent of  $s(t)$ . Note that for the nodes in back-off stage zero,  $q_1$  is also the probability of the occurrence of the non-prioritized transmissions, after which the prioritized transmissions follow.

The nodes belonging to back-off stage zero are eligible for both the non-prioritized and prioritized channel access. A node in back-off stage zero gets the chance for the non-prioritized channel access when its back-off timer expires, and gets the chance for the prioritized channel access after the non-prioritized transmissions. As the number  $n$  of nodes in the transmission procedure becomes larger, except the case that the nodes entering back-off stage zero immediately gets the chance for the non-prioritized channel access with random back-off time of zero, the latter becomes dominant for the nodes belonging to back-off stage zero because the presence of the nodes with size of back-off window  $b(t)$  equal to zero becomes inevitable in other back-off stages than zero. Therefore, for our asymptotic MAC performance model, we do not differentiate the states of the nodes in back-off stage zero in terms of  $b(t)$ . The states of each node in back-off stage zero are consolidated into zero.

Modifying the Markov chain model in [6], we can construct the two-dimensional Markov chain in Figure 2 where  $m$  is maximum back-off stage and  $W_i = 2^i W$  maximum size of back-off stage  $i \geq 0$  for a certain constant  $W$ .

### 3.3 Analysis of Markov chain

Letting  $b_0 = \lim_{t \rightarrow \infty} Pr\{s(t) = 0\}$  and  $b_{i,k} = \lim_{t \rightarrow \infty} Pr\{s(t) = i, b(t) = k\}$ ,  $i \in (0, m)$ ,  $k \in (0, W_i - 1)$ , from the balance equations of the Markov chain in Fig. 2, we can derive

$$b_{i,0} = (q_1)^i \left( \frac{1}{W} + q_2 \left( 1 - \frac{1}{W} \right) \right) b_0, \quad i \in (1, m-1), \quad (1)$$

$$b_{m,0} = \frac{(q_1)^m (1 + Wq_2 - q_2)q_2}{W(1 - q_1)} b_0, \quad (2)$$

$$b_{i,k} = \frac{W_i - k}{W_i} b_{i,0}, \quad i \in (1, m), \quad k \in (0, W_i - 1) \quad (3)$$

where  $1/W$  is the probability that the node entering back-off stage zero immediately gets the chance for the non-prioritized channel access with random back-off time of zero. From the normalization condition that

$$\begin{aligned} b_0 + \sum_{i=1}^m \sum_{k=0}^{W_i-1} b_{i,k} &= b_0 + \sum_{i=1}^m \frac{W_i + 1}{2} b_{i,0} \\ &= b_0 + \frac{b_0}{2} \left( (1 + Wq_2 - q_2) \left( \sum_{i=1}^{m-1} 2^i (q_1)^i + \frac{2^m (q_1)^m}{1 - q_1} \right) + \right. \\ &\quad \left. \left( \frac{1}{W} + q_2 - \frac{q_2}{W} \right) \left( \sum_{i=1}^{m-1} (q_1)^i + \frac{(q_1)^m}{1 - q_1} \right) \right) = b_0 + \\ &\quad \frac{b_0}{2} q_1 \left( \frac{1}{W} + q_2 - \frac{q_2}{W} \right) \frac{2W(1 - q_1 - 2^{m-1}(q_1)^m) + 1 - 2q_1}{(1 - 2q_1)(1 - q_1)} \\ &= 1. \quad (4) \end{aligned}$$

We can derive  $b_0$  in terms of  $q_1$  and  $q_2$  as

$$\begin{aligned} b_0 &= 2W(1 - 2q_1)(1 - q_1) / [2W(1 - 2q_1)(1 - q_1) + \\ &\quad 2Wq_1(1 + Wq_2 - q_2)(1 - q_1 - 2^{m-1}(q_1)^m) + \\ &\quad q_1(1 + Wq_2 - q_2)(1 - 2q_1)]. \quad (5) \end{aligned}$$

Similarly to [6], we can derive the probability  $\tau$  that a node sends a data frame at the end of a generic idle time slot by the non-prioritized channel access as

$$\tau = \frac{1}{W} b_0 + \sum_{i=1}^m b_{i,0} = \left( \frac{1}{W} + \frac{q_1 q_2}{1 - q_1} \right) b_0. \quad (6)$$

The probability  $q_1$  that a data frame transmitted by the non-prioritized channel access collides with other non-prioritized transmissions can be expressed as

$$q_1 = 1 - (1 - \tau)^{n-1} = 1 - \left( 1 - \left( \frac{1}{W} + \frac{q_1 q_2}{1 - q_1} \right) b_0 \right)^{n-1}. \quad (7)$$

Assume that in addition to a node in back-off stage zero,  $r = 1$  more node belonging to back-off stage zero simultaneously attempts to send its data frames with probability  $p$  in a PCAP. Then, the probability that the node experiences the collision of the prioritized channel access during the PCAP can be expressed as

$$q_2^{r=1} = 2p(1 - p)q_2^{r=1} + p^2 \quad (8)$$

where the first term in the right hand side considers the case that only one node transmits and both nodes continuously attempt to transmit their data frames with probability  $p$  in the PCAP, and the second term the case that the collision occurs due to the simultaneous transmission of both nodes. Therefore,  $q_2^{r=1}$  can be obtained as

$$q_2^{r=1} = \frac{p^2}{1 - 2p(1 - p)}. \quad (9)$$

When in addition to a node in back-off stage zero,  $r \in (2, n1)$  more nodes in back-off stage zero simultaneously attempt to transmit their data frames with probability  $p$ , the probability  $q_2^r$  that the node experiences the collision of the prioritized channel access during a PCAP can be expressed as

$$\begin{aligned} q_2^r &= p(1 - p)^r q_2^r + p(1 - (1 - p)^r) + (1 - p)rp(1 - p)^{r-1} q_2^r \\ &\quad + (1 - p) \sum_{j=2}^{r-1} \frac{r!}{j!(r-j)!} p^j (1 - p)^{r-j} q_2^{r-j}, \quad (10) \end{aligned}$$

where the first term in the right hand side considers the case that the node transmits and other  $r$  nodes do not transmit, the second term the case that the transmissions between the node and at least one out of other  $r$  nodes collide, the third term the case that the node does not transmit and only one out of other  $r$  nodes transmits, and the last term the case that the node does not transmit and  $1 < j < r$  out of other  $r$  nodes transmits. When for the last case all  $r$  nodes transmit, the node will not

experience the collision. Sequentially solving (9) and (10), we can derive  $q_2^r$ ,  $r \in (1, n-1)$ .

Before the prioritized channel access is granted, the event of the non-prioritized transmission should occur. As the number  $n$  of nodes of wireless LANs becomes larger, the probability  $s$  that the event of the non-prioritized transmission occurs at the end of a generic idle time slot

$$s = 1 - (1 - \tau)^n \quad (11)$$

converges to 1. Therefore, a node in back-off stage zero asymptotically observes the unconditional distribution of the number  $r$  of other nodes in back-off stage zero at the end of idle time slots where the non-prioritized transmissions start. The non-prioritized transmission only change  $r$  by at most 1, which can be ignored asymptotically. Finally, using the unconditional probability that  $1 \leq r \leq n-1$  out of  $n-1$  nodes are in back-off stage zero, which is

$$u_r = \frac{(n-1)!}{r!(n-1-r)!} (b_0)^r (1-b_0)^{n-1-r}. \quad (12)$$

We can obtain  $q_2$  as

$$q_2 = \sum_{r=1}^{n-1} u_r q_2^r. \quad (13)$$

Now, we have two equations (7) and (13) where  $b_0$  in (7) can be expressed in terms of unknown  $q_1$  and  $q_2$  using (5), and  $u_r$  and  $q_2^r$  in (13) can be expressed in terms of unknown  $q_1$  and  $q_2$  using (5), (9), (10) and (12). Utilizing the property that for a given value of  $q_1$ , the right hand side of (13) is the monotonically decreasing function of  $q_2$ , we can easily find the value of  $q_2$  satisfying (13) for a given value of  $q_1$ . By finding the values of  $q_2$  satisfying (13) for various values of  $q_1$  and checking the errors of (7) with the found values of  $q_1$  and  $q_2$ , we can numerically compute the values of  $q_1$  and  $q_2$  satisfying (7) and (13). By this, we can complete the analysis of the Markov chain in Fig. 2.

### 3.4 Throughput analysis

Using the probability  $s$  in (11), the average time length  $T_{\text{idle}}$  of consecutive idle time slots between non-prioritized transmissions can be obtained as

$$T_{\text{idle}} = T_s \left( \frac{1}{s} - 1 \right), \quad (14)$$

where  $T_s$  is the length of a time slot. Similarly to [6], the probability  $P_s$  that a non-prioritized transmission is successful is

$$P_s = \frac{n\tau(1-\tau)^{n-1}}{s}. \quad (15)$$

Therefore, the average amount  $T_{\text{non}}$  of time taken to process a simultaneous non-prioritized transmission from one or more nodes, and the average amount  $D_{\text{non}}$  of

payloads successfully transmitted by a simultaneous non-prioritized transmission can be obtained as

$$T_{\text{non}} = P_s T_{\text{success}} + (1 + P_s) T_{\text{fail}} \quad (16)$$

$$D_{\text{non}} = P_s L_{\text{payload}} \quad (17)$$

where  $T_{\text{success}}$  is the sum of the fixed amount of time taken to send a data frame, the length of a SIFS period, the amount of time taken to send the ACK frame, and the length of a DIFS period,  $T_{\text{fail}}$  the sum of the fixed amount of time taken to send a data frame, and the length of a DIFS period, and  $L_{\text{payload}}$  the fixed length of payloads in data frames.

Let us denote by  $T_{pri}^v$  the average amount of time of a PCAP, conditioned that at the beginning of the PCAP  $v$  nodes are in back-off stage zero.  $D_{pri}^{v=1}$  can be obtained as

$$T_{pri}^{v=1} = \left( \frac{1}{1-p} - 1 \right) U_{\text{success}} \quad (18)$$

where  $U_{\text{success}}$  is the sum of the fixed amount of time taken to send a data frame, the length of a SIFS period, the amount of time taken to send the ACK frame, and the length of a PIFS period. Using the recurrence equation approach similar to that used for (10), we can derive the following equations for  $T_{pri}^v$ ,  $v \in (2, n)$

$$T_{pri}^v = vp(1-p)^{v-1}(U_{\text{success}} + T_{pri}^v) + \left( \sum_{j=2}^{v-1} \frac{v!}{j!(v-j)!} p^j (1-p)^{v-j} (U_{\text{fail}} + T_{pri}^{v-j}) \right) + p^v U_{\text{fail}} \quad (19)$$

where  $U_{\text{fail}}$  is the sum of the fixed amount of time taken to send a data frame, and the length of a PIFS period. The first term in the right hand side in (19) considers the case that only one out of  $v$  nodes transmits, and the second and the last terms the case of the collision between prioritized transmissions. When we denote by  $D_{pri}^v$  the average amount of payloads successfully transmitted during a PCAP, conditioned that at the beginning of the PCAP  $v$  nodes are in back-off stage zero, similarly to (18) and (19) we can derive the following equations for  $D_{pri}^v$ ,  $v \in (1, n)$

$$D_{pri}^1 = \left( \frac{1}{1-p} - 1 \right) L_{\text{payload}} \quad (20)$$

$$D_{pri}^v = vp(1-p)^{v-1}(L_{\text{payload}} + D_{pri}^v) + \sum_{j=2}^{v-1} \frac{v!}{j!(v-j)!} p^j (1-p)^{v-j} D_{pri}^{v-j} \quad (21)$$

Sequentially solving (18), (19), (20) and (21), we can derive  $T_{pri}^v$  and  $D_{pri}^v$ ,  $v \in (1, n)$ , and similarly to (13) the average amount  $T_{pri}$  of time of a PCAP and the average

**Table 1.** Simulation and analytical results of MAC throughput (Mbps)

Evaluation Method	Payload Size (bits)	$n$						
		10	50	100	150	200	250	300
Simulation	10,000	94.33	90.98	84.68	84.16	79.46	75.37	74.95
Analytical		91.24	88.351	82.961	82.276	78.216	74.721	74.154
Simulation	20,000	160.26	154.5	143.91	135.69	135	128.06	122.14
Analytical		157.192	151.785	142.162	134.764	133.623	127.407	122.078
Simulation	30,000	208.94	201.37	187.66	176.97	168.16	160.49	159.25
Analytical		207.089	199.542	186.531	176.534	167.983	160.809	159.379

amount  $D_{pri}$  of payloads successfully transmitted during a PCAP can be obtained as

$$T_{pri} = \sum_{v=1}^n w_v T_{pri}^v, \quad D_{pri} = \sum_{v=1}^n w_v D_{pri}^v, \quad (22)$$

where

$$w_v = \frac{n!}{v!(n-v)!} (b_0)^v (1-b_0)^{n-v}. \quad (23)$$

Finally, by the renewal cycle approach we can obtain the saturation throughput as follows

$$T_{\text{throughput}} = \frac{D_{\text{non}} + D_{\text{pri}}}{T_{\text{idle}} + T_{\text{non}} + T_{\text{pri}}}. \quad (24)$$

## 4 Numerical Results

We want to validate the analytical MAC performance model proposed in the previous section by comparing the analytical MAC throughput results derived by the performance model to the simulation results of MAC throughputs of IEEE 802.11n wireless LANs in [5]. In an IEEE 802.11n wireless LAN,  $n = 10, 50$ , or  $300$  nodes exist, data frames with  $10\,000$  bit,  $20\,000$  bit, or  $30\,000$  bit payloads are transmitted through the DCF protocol with the prioritized channel access, and the data and ACK transmission rates are set to  $600$  Mbps and  $24$  Mbps, respectively.  $24$  Mbps is one of the basic rates with which ACK frames can be transmitted [7]. The other traffic parameters can be found in Table 1 in [5]. (The ACK transfer rate should be changed to  $24$  Mbps in Table 1 in [5].)

In Table 1, we compare the analytical MAC throughput results to the simulation results. Even though due to the asymptotic nature of the performance model the analytical throughputs are smaller than the simulation results in most cases, the analytical and simulation results are closely matched with average relative error of about  $1.15\%$ .

## 5 Conclusions

We proposed the analytical MAC performance model based on the Markov chain and renewal theory approach

for the DCF protocol with the prioritized channel access. The analytical MAC performance model was validated by comparing the analytical throughput results to the simulation results.

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