DE GRUYTER
OPEN

Journal of ELECTRICAL ENGINEERING, VOL 68 (2017), NO6, 476–481

Analysis of the transients on the multi-circuit overhead transmission line

Tomáš Nazarčík, Zdeňka Benešová *

In this paper a method for a transient analysis on the multi-circuit overhead transmission line with different voltage levels (EHV and HV) is presented. The influence of inductive and capacitive couplings between the parallel circuits placed on the same tower is studied. The transmission line model consists of mutual coupled two-port cascades which can be described by a system of ordinary differential equations (ODE). This system has been solved numerically in MATLAB, the obtained results provide the time distribution of currents and voltages in all conductors along the transmission line. The presented algorithm allows solving various types of transients which can occur during switching off- and on- operations, shortcircuits *etc.*

K e y w o r d s: transient analysis, multi-circuit line, mutual couplings, method of state variables

1 Introduction

The development of the electric grid is joined with a construction of the new transmission and distribution lines, but nowadays it is not easy to find new routes. One of the possible solutions can be an upgrade of the existing lines to the multi-circuit lines. In the case that all circuits have the same voltage level the impact of inductive and capacitive coupling could be reduced by using of the suitable phase conductors layout. But, in the case that circuits with different voltage levels are placed on the same tower the mutual electromagnetic couplings play more important role. There are two basic arrangements of the tower in such systems [1]: a vertical configuration (the EHV conductors are placed above HV system) and a horizontal one (EHV conductors are surrounding HV conductors or opposite). With regards to this fact the towers of those transmission lines are more complicated and usually the transposition which enables to compensate the parameters non-symmetry caused by geometric layout of conductors is not carried-out. The electromagnetic couplings between conductors generate additional induced voltages and currents in each circuit. Their value is affected not only by non-symmetry of parameters but also by very different values of voltages and currents in the EHV and HV system. This influence has to be taken into account by the transmission line and protection system design. For this reason a detailed analysis respecting the mentioned phenomena is needed. According to [2] the current unbalance during the standard operation generates permanent current flowing through the earth wires. The transient analysis with regard to the mutual couplings was described in [3] using the frequency domain analysis. The mutual coupled multi-circuit line modelled with the circuit with distributed parameters was studied in [4]. This paper deals with transient analysis on the multi-circuit lines in the time domain. It is based on the transmission line model consisting of cascades of mutually coupled two-ports. This approach allows respecting the effect of mutual inductive and capacitive couplings between all conductors and it enables to analyse the distribution of voltage and current along the line during the non-standard situations which can occur *eg* switching off/on processes, short-circuits on some phase, unloaded line *etc.*

2 Transmission line model

The phase conductors of two voltage levels have been modelled with the help of cascades of gamma two-ports that are mutually coupled.

2.1 Two-port model

As usual, the basic gamma two-port of the cascade involves passive parameters of the line (R, L, C and G). The voltage and current source in each two-port respects the induced voltage via mutual inductive couplings and the induced current caused by mutual capacitive couplings. The final structure of the k-th two-port of the cascade of i-th phase conductor is depicted in Fig. 1.

Considering system with m phase conductors and n earth wires then there are $(m+n)$ coupled conductors, the value of the voltage source in the k-th two-port belonging to the i-th conductor is given by the following formula

$$u_{Li,k} = \sum_{j=1, j\neq i}^{m+n} L_{ij} \frac{\mathrm{d} i_{Lj,k}}{\mathrm{d}t} \,. \tag{1}$$

In (1) L_{ij} denotes the mutual inductance between the i-th and j-th conductor, $i_{Lj,k}$ is the current of the j-th conductor in k-th two-port. The value of the current

* Department of Theory of Electrical Engineering, Faculty of Electrical Engineering, University of West Bohemia, Univerzitní 8, 306 14 Plzeň, Czech Republic, bene@kte.zcu.cz

DOI: 10.1515/jee-2017–0083, Print (till 2015) ISSN 1335-3632, On-line ISSN 1339-309X © 2017FEI STU

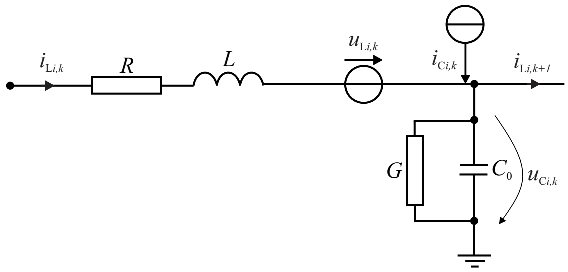

Fig. 1. Basic gamma element of k-th two-port of i-th phase conductor

source can be expressed as

$$i_{Ci,k} = \sum_{j=1,j\neq i}^{m+n} C_{ij}\Big(\frac{\mathrm{d}u_{Cj,k}}{\mathrm{d}t} - \frac{\mathrm{d}u_{Ci,k}}{\mathrm{d}t}\Big) \tag{2}$$

where C_{ij} is the mutual capacitance between the i-th and j-th conductor and the $u_{Cj,k}$ is the voltage of the j-th conductor in k-th two-port. The parameters corresponding to mutual couplings are evaluated according to the geometric layout of the conductors on the given transmission tower. Using the algorithms described in [5] and [6] it is possible to obtain the matrices of inductances and capacitances. Supposing the earth wire connected directly to the ground than its model is given by the two-port without the capacitance and the conductance. The ground is respected by a fictional conductor with the resistance R_g and inductance L_g in series according to the Rüdenberg theory [7]. The part of diagram of the transmission line model is shown in Fig. 2.

2.2. Mathematical model

To formulate the equations describing the two-port diagram the method of state variables using current of the inductance i_L and voltage on the capacitance u_C as uknown was applied. In each two-port modelling phase conductor there are two state variables, in each two-port of earth wires or ground there is only one state variable. Supposing a transmission system with m phase conductors (EHV and HV together) and n earth wires, we obtain $(m+n)$ cascades each of them consists of N two-ports. The number of state variables including the back ground way in the k-th two-port is $K = 2*m+n+1$, the total number of unknown is then $M = N*K$ so the mathematical model describing the multi-circuit transmission line is given by a system of M differential equations of the first order. Using the voltage Kirchhoffs law on the loop involving the phase conductor and ground then we receive for the i-th conductor in k-th two-port of the cascade the following equation

$$R_i i_{Li,k} + L_i\frac{\mathrm{d}i_{Li,k}}{\mathrm{d}t} + \sum_{j=1,j\neq i}^{m+n} L_{ij}\frac{\mathrm{d}i_{Lj,k}}{\mathrm{d}t} + u_{Ci,k} - \Big(R_g i_{Lg,k} + L_g\frac{\mathrm{d}i_{Lg,k}}{\mathrm{d}t}\Big) - u_{Ci,k-1} = 0\,. \tag{3}$$

In similar way the equations for the loop earth wire - ground can be written. Applying the current Kirchhoffs law on the node between the k-th and $(k+1)$-th two-port of the i-th phase conductor we obtain

$$i_{Li,k} + \sum_{j=1,j\neq i}^{m+n} C_{ij}\Big(\frac{\mathrm{d}u_{Cj,k}}{\mathrm{d}t} - \frac{\mathrm{d}u_{Ci,k}}{\mathrm{d}t}\Big) = G_i u_{Ci,k} + C_{i0}\frac{\mathrm{d}u_{Ci,k}}{\mathrm{d}t} + i_{Li,k+1}\,. \tag{4}$$

The equations for the nodes of grounding can be determined likewise (4). The load is respected in the equation of last node in cascade or in a new loop, it depends on the type of load. The obtained system of ODE can be rewritten into the matrix form. If the elements with derivatives

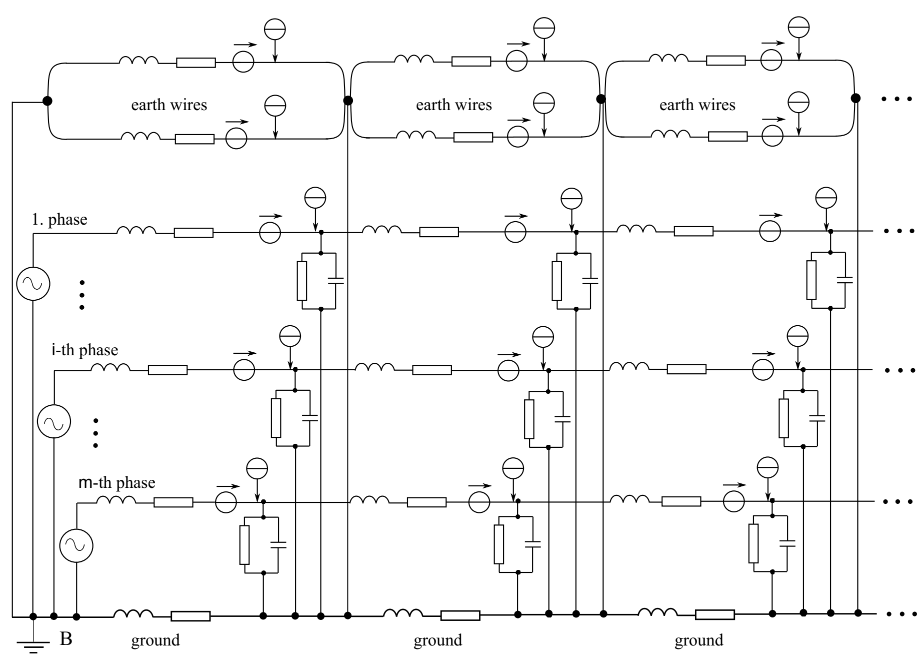

Fig. 2. Part of the two-port diagram

stay on the left side of the equation and the remaining elements on the right side we obtain

$$\mathbf{A}_1 \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x} = \mathbf{A}_2\boldsymbol{x} + \mathbf{f} \tag{5}$$

where $\boldsymbol{x}$ is a column vector full of M state variables, matrix $\mathbf{A}_1$ is full of coefficients belonging to derivatives and $\mathbf{A}_2$ is a matrix of coefficients of elements without derivatives. Multiplying (5) by the inverse of $\mathbf{A}_1$ from left we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x} = \mathbf{A}_1^{-1}(\mathbf{A}_2\boldsymbol{x} + \boldsymbol{f}) = \mathbf{A}\boldsymbol{x} + \mathbf{A}_1^{-1}\boldsymbol{f}\,. \tag{6}$$

Denoting the vector of currents in the k-th two-port of cascade as $i_L^{(k)}(K-m,1) = \{i_{L1}^{(k)}, i_{L2}^{(k)}, \dots, i_{L(K-M)}^{(k)}\}^\top$ and the vector of voltages as $u_C^{(k)}(m,1) = \{u_{C1}^{(k)}, u_{C2}^{(k)}, \dots, u_{Cm}^{(k)}\}^\top$ then the vector of state variables in the k-th two-port is defined as

$$\mathbf{x}^{(k)}(K,1) = \begin{bmatrix} \boldsymbol{i}_L^{(k)} \\ \mathbf{u}_C^{(k)} \end{bmatrix}. \tag{7}$$

The vector of all state variables is then given in form

$$\boldsymbol{x}(M,1) = \begin{bmatrix} \boldsymbol{x}^{(1)} & \mathbf{x}^{(2)} & \dots & \boldsymbol{x}^{(k)} & \dots & \boldsymbol{x}^{(N)} \end{bmatrix}^\top. \tag{8}$$

Respecting the order of state variables shown in (7) and (8) the matrices $\mathbf{A}_1$ and $\mathbf{A}_2$ consist of the repeating submatrices

$$\mathbf{A}_1 = \begin{bmatrix} \mathbf{A}_1^{(1)} & 0 & \dots & 0 \\ 0 & \mathbf{A}_1^{(2)} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \mathbf{A}_1^{(N)} \end{bmatrix} \tag{9}$$

Supposing only one three-phase system EHV with index 1 and one three-phase system HV with index 2 then particular submatrix $\mathbf{A}_1$ for the k-th two-port has the following structure (index 0 belongs to the earth wire, index g to the ground)

$$\mathbf{A}_1^{(k)} = \begin{bmatrix} \mathbf{L}_1 & \mathbf{L}_{12} & \mathbf{L}_{10} & -\mathbf{L}_g & & 0 \\ \mathbf{L}_{12} & \mathbf{L}_2 & \mathbf{L}_{20} & -\mathbf{L}_g & & 0 \\ \mathbf{L}_{10} & \mathbf{L}_{20} & \mathbf{L}_0 & -\mathbf{L}_g & & 0 \\ & & & 0 & & \\ & & & & \mathbf{C}_1 & \mathbf{C}_{12} \\ 0 & & \dots & & \mathbf{C}_{12} & \mathbf{C}_1 \end{bmatrix}. \tag{10}$$

The matrices $\mathbf{L}_i$ describe the self-inductances and mutual inductive couplings inside the i-th three-phase circuit. The matrices $\mathbf{L}_{ij}$ involves the mutual inductances between the systems i and j. In similar way it is possible to build the matrices of capacitances $\mathbf{C}_i$ and $\mathbf{C}_{ij}$. For example (11) expresses the matrix $\mathbf{L}_1$ of inductances in EHV system 1, $\mathbf{L}_{12}$ resp. $\mathbf{C}_{12}$ describe matrices for inductive resp. capacitive coupling between systems 1 and 2

$$\mathbf{L}_1 = \begin{bmatrix} L_{a1} & L_{a1b1} & L_{a1c1} \\ L_{a1b1} & L_{b1} & L_{b1c1} \\ L_{a1c1} & L_{b1c1} & L_{c1} \end{bmatrix} \tag{11}$$

$$\mathbf{L}_{12} = \begin{bmatrix} L_{a1a2} & L_{a1b2} & L_{a1c2} \\ L_{b1a2} & L_{b1b2} & L_{b1c2} \\ L_{c1a2} & L_{c1b2} & L_{c1c2} \end{bmatrix} \tag{12}$$

$$\mathbf{C}_{12} = \begin{bmatrix} C_{a1a2} & C_{a1b2} & C_{a1c2} \\ C_{b1a2} & C_{b1b2} & C_{b1c2} \\ C_{c1a2} & C_{c1b2} & C_{c1c2} \end{bmatrix} \tag{13}$$

The solution $\mathbf{x}(t)$ of (6) depends on the existence of the inverse matrix $\mathbf{A}_1^{-1}$, but the matrix $\mathbf{A}_1$ built according to equations (7)–(10) is singular. Application of Kirchhoffs law on the ground node B in Fig. 2., where only branches with inductive currents (state variables) are joined, results in equation without derivatives (it is a node with the redundant branch)

$$i_g + \sum_{i=1}^{n} i_{L0i} + \sum_{i=1}^{m} i_{Li} = 0\,. \tag{14}$$

For this reason the row of matrix $\mathbf{A}_1$ belonging to (14) has only zero elements. As a consequence the determinant $\mathbf{A}_1$ is equal to zero and the matrix $\mathbf{A}_1$ is singular. This problem was solved by adding a very low value capacitance ($\sim$ 1 pF) parallel to the fictional conductor that respects the ground, Fig. 3.

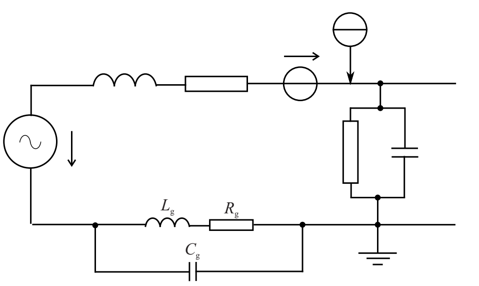

Fig. 3. The upgraded model of the ground way

The solution of the system of ODE in (6) provides the time distribution of all state variables in the considered time interval. In general, the solution $x(t)$ could be written as a sum of exponential functions

$$x(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} + \dots + C_k e^{\lambda_k t} + \dots + C_M e^{\lambda_M t}. \tag{15}$$

The C_1, C_2, C_k and C_M are integration constants, the λ_1, λ_2, λ_k, λ_M are the eigenvalues of the matrix $\mathbf{A}$. It is known that the analysed system is stable, if all eigenvalues of its state matrix $\mathbf{A}$ have negative real parts. This fact is useful for an assessment of the transients character namely for an estimation of oscillations generation. The numerical solution of the ODE has been solved in MATLAB using the Runge-Kutta method of the 4$^{\text{th}}$ order (procedure ode45).

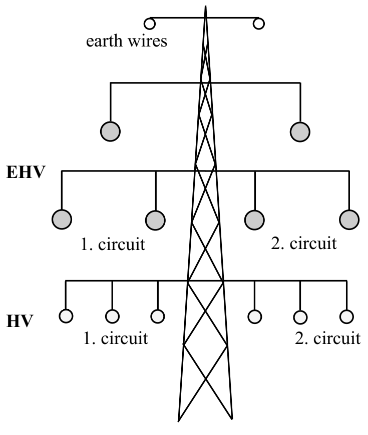

Fig. 4. Considered transmission tower

3 Illustrative examples of calculations

The above described algorithm was used to analyse a quadruple transmission line with two EHV and two HV circuits. Here, the transposition is not usually carried-out and the tower is supposed to have sufficient distance between phase conductors minimizing the phase to phase short-circuit, a sufficient it should guarantee the minimal height of conductor above ground according to hygienic standard of electromagnetic field distribution. This problem was solved in [8] where the influence of the tower type and the phase layout on electric and magnetic field distribution was investigated. Optimal results were reached for the tower Donau according to the Fig. 4 and for phase configuration that is depicted in Tab. 1. This phase configuration provides the most significant reduction of the values of electric and magnetic field strength.

Table 1. Considered phase configuration

	1. circuit	2. circuit
EHV	B A C	B A C
HV	c b a	c b a

The mutual influence between the EHV and HV has been investigated in the example of the single phase short circuit in the first EHV system on the phase C. Because this phase conductor is placed near the axis of the tower it could be expected that its impact on all HV conductors is probably the most significant. It was considered that the short circuit arises in given distance of the transmission line in the time t_{SC}. Supposing that the line length 20 km is divided into 10 two-ports the short circuit was modelled in the 7^{th} two-port via value of the G parameter. The other EHV and HV circuits have been in standard operation that is depicted in Fig. 5, the transmitted power in each EHV system was 800 MW and in the each HV system 40 MW.

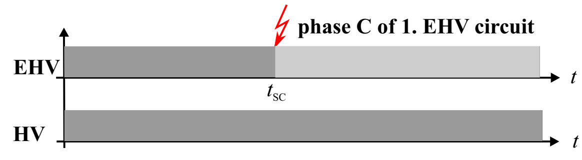

Fig. 5. Operation of the EHV and HV circuits in observed time interval

In Figures 6–9 the phase currents and voltages along the line are presented, showing some interesting featurers.

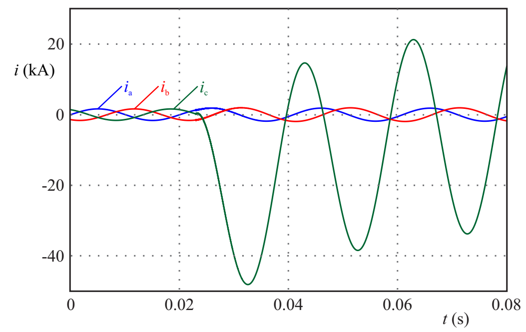

Fig. 6. Phase currents of the 1^{st} EHV circuit in the first two-port

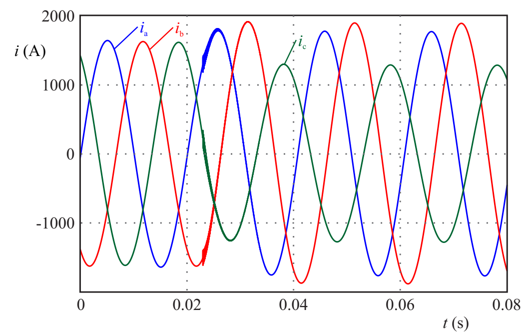

Fig. 7. Phase currents of the 2^{nd} EHV circuit in the first two-port

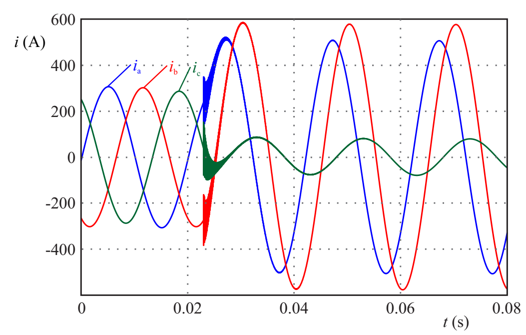

Fig. 8. Phase currents of the 1^{st} HV circuit in the first two-port

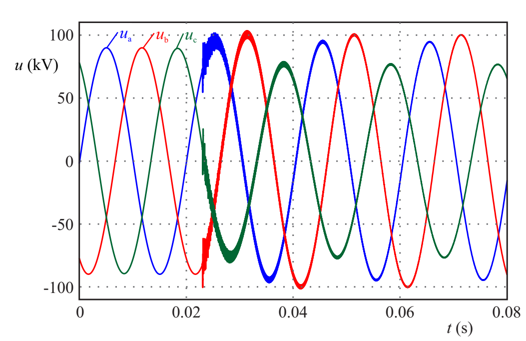

Fig. 9. Phase voltages of the 1^{st} HV circuit in the first two-port

Table 2. Voltage of the phase-C of the 1st EHV and 1st HV circuit along the transmission line

u_{Ci} (kV)	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$
HV	76.98	63.47	50.46	37.57	25.42	16.23	15.49
EHV	178.1	149.8	121.4	93.22	65.31	37.61	10.64

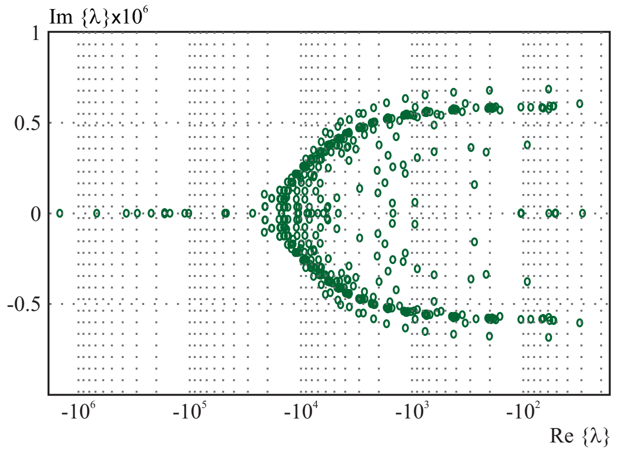

Fig. 10. The eigenvalues in the complex plane

The single phase short circuit in the EHV causes a phase current asymmetry in others circuits. This phenomenon is more significant in the HV circuits where the quickly oscillating voltage and current with high peak-value arise. The frequency of oscillations could be analysed with the help of state matrix eigenvalues λ_i. Their position in complex plane is shown in the Fig. 10. With regards to (15) the oscillation are caused by eigenvalues with non-zero imaginary part which also determine their frequency. According to Fig. 10 oscillations with the frequency about 50 kHz ($T \approx 20\,\mu$s) could be expected. The damping of arising oscillations is given by their real part value. The result impact of considered eigenvalue depends also on the integration constant C_i in (15).

The significant advantage of the presented transmission line model consisting of the cascades of the mutually coupled two-ports is the possibility to analyse the voltage and current distribution along the transmission line. Based on the results presented in Figs. 8 and 9, the short circuit in the phase-C of the 1st EHV circuit strongly influences the phase c of the 1st HV circuit. In despite of the position of phase-c conductor that is located farther than the conductors of phases-(b, a) in Fig. 11. It could be explained by the fact that the short circuit current has mostly reactive character and the induced voltage from this conductor to the conductor of the phase-c of the 1st HV circuit has a counter-phase direction.

Our algorithm allows watching changes of voltage and current distribution along the line. The voltage distribution at three two-ports in the phase-C of the 1st EHV circuit is depicted in Fig. 12. Similarly, Fig. 13 shows the voltage distribution in the phase-c of the 1st HV circuit.

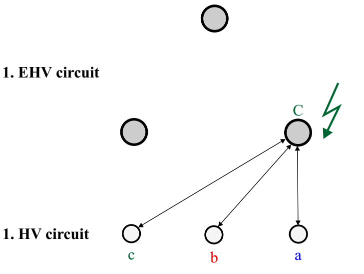

Fig. 11. Distances between the HV conductors and the conductor with short circuit

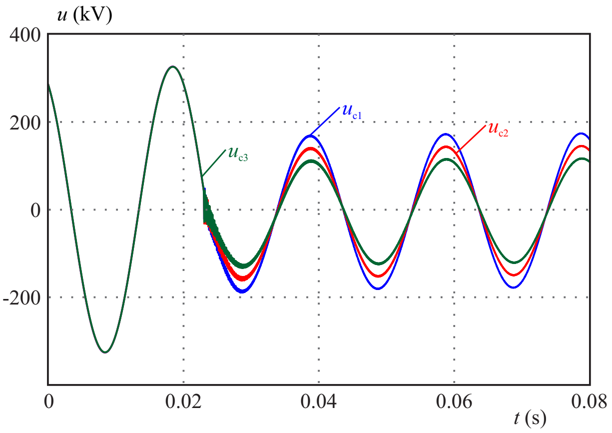

Fig. 12. Voltage of the phase-C of the 1st EHV circuit in 1st, 2nd and 3rd two-port

After the oscillation damping follows the short-circuit steady state and the decreasing voltage amplitude along the line could be watched. The phase voltages of the phase-C of the 1st EHV circuit and the phase-c of the 1st HV circuit on the particular two-ports along the line are shown in Tab. 2.

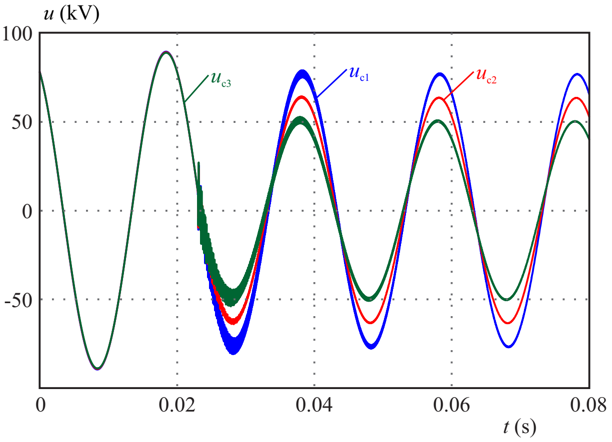

Fig. 13. Voltage of the phase-c of the 1st HV circuit in 1st, 2nd and 3rd two-port

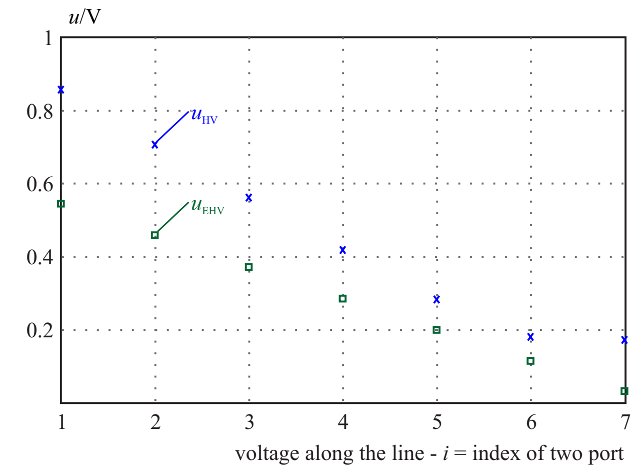

Fig. 14. Voltage decrease along the transmission line

The graph of unit voltage values is displayed in Fig. 14. While the voltage in the short circuited phase-C decreases uniformly to zero, the voltage in the phase-c of the 1st HV circuit decreases from the higher initial value quicker and does not reach the zero value.

4 Conclusion

The model of multi-circuit transmission line with two voltage levels respecting all mutual couplings between conductors was introduced. It is based on two-port approach and it enables to study the time voltage and current distribution along the transmission line and to judge the mutual influence between various voltage level systems. For the formulation of ODE describing the considered line model the method of state variable was applied. It also allows studying the stability condition for the investigated system. The proposal algorithm can be used for evaluation of the transients such a line switching on/off, short circuits, automatic reclose cycle etc. In this article the analysis of the mutual influence in the system consisting of two EHV and two HV systems was carried-out. The dominant impact of EHV system on HV circuits was observed during the single phase short circuit arising in the EHV circuit. It results in a significant phase current asymmetry in the HV circuits and in the generation of oscillation with high frequency and high voltage and current peak value which are quickly damped. The voltage unbalance of the HV circuits is increasing along the line towards the place of the short circuit. These facts should be taken into account during the setup of the protection systems.

Acknowledgment

This article has been supported by a project SGS-2015-035.

References

[1] W. Liu, M. Pan, K. Liu and G. Xu, "Research on Electromagnetic Character of 500/220 kV Mixed-voltage Quadruple-circuit Transmission Line", *12th International Conference on Environmental and Electrical Engineering* EEEIC 2013.

[2] A. Novitskiy and D. Westermann, "Interaction of Multi-Circuit Overhead Transmission Lines of Different Voltages Located on the Same Pylon", *Electric Power Quality and Supply Reliability Conference (PQ)*, 2012.

[3] B. Gustavsen, "Modal Domain-Based Modeling of Parallel Transmission Lines With Emphasis on Accurate Representation of Mutual Coupling Effects", *IEEE Transaction on Power Delivery*, 2012.

[4] M. Kollar, "Active Homogenous Line with Arbitrary Distributed Sources", *Journal of Electrical Engineering*, vol.52, no.9-10, 2001.

[5] Z. Benesova and D. Mayer, "Algorithm for Computation of Inductances of Three-Phase Overhead Lines", , *AMTEE'03 Plzeň*: ZČU v Plzni 2003.

[6] Z. Benesova and L. Sroubova, "Capacitive Coupling Double-Circuit Transmission Lines", *Advances Electrical and Electronic Engineering*, Žilina 2004.

[7] R. Rudenberg, *Transient Performance of Electric Power Systems* first edition, McGraw-Hill Book Company, New York, pp. 393–408, 1950.

[8] T. Nazarcik and Z. Benesova, "Analysis of Electromagnetic Field a Vicinity of Multi-Circuit Overhead Lines with Different Voltage Level", *19th International Symposium on High Voltage Engineering*, Pilsen, Czech Republic, 2015.

Received 26 October 2017

Tomáš Nazarčík (Ing), 1989, graduated from the Faculty of Electrical Engineering (University of West Bohemia Pilsen, Czech Republic) in the Electrical Power Engineering in 2014. Now, he is PhD student at the Department of Theory of Electrical Engineering. His research is namely focused on the problems joined with the combined multi-circuit transmission lines.

Zdeňka Benešová (Prof, Ing, CSc), 1947, graduated from the Faculty of Electrical Engineering (Technical University in Pilsen) in 1970, in 1985 she received her CSc degree (PhD) in Theory of Electrical Engineering from the Academy of Science in Prague. In 1994 she became Assoc Professor and the Professor in 2002 from Unversity of West Bohemia in Pilsen. In 1995 -2008 she was head of Department of Theory of Electrical Engineering at FEL UWB. Her research concerns with numerical methods for electromagnetic field analysis and in theoretical problems of electric power systems (transmission line parameters, numerical methods for analysis of transient phenomena on transmission lines and EMC), she published more than 90 scientific papers.