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Reactive power in circuits with sampled resistive load

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The process of power transmission lines, from source to load is a well-known, if the voltages and currents vary harmonically. With the lines is transmitted active power that is dissipated in load (this power exits from the system) and reactive power (this power oscillates between the source and the load). Note that it would be more appropriate designation the external power and the internal power. Such systems are known as cyclo-dissipative. The active power is dissipated in the load and the reactive power oscillates between the source and the load. Physically, reactive power is delivered to reactive elements of load. Transmitting reactive power increases Joule's losses and voltage drops on lines. Reactive power can be compensated in a known manner. Compensation reduces the effective value of the current in the line. To the case of periodic but non-sinusoidal voltages and currents has been devoted many publications, conferences, etc. during the past 100 years. But despite much effort, this problem has not yet been fully solved. In the present article, we show that in a system with a harmonic source of voltage even in the case of a linear pure resistive load a reactive power can be generated and can be compensated. A necessary, but not sufficient, condition is that the resistive load is time-varying. The presented study deals with a periodically sampled resistive load.

K e y w o r d s: reactive power, non-sinusoidal voltages, time-varying resistive load

1 Introduction

One of the most significant current discussions in electrical engineering is the definition of the reactive power (RP) under non-sinusoidal conditions in nonlinear electric systems [1-4]. Physically, reactive power is supplied to the reactive elements (*ie* by a time-varying magnetic field energy of coils and electrical field energy of capacitors), which are in the load, [5]. Transfer of the reactive power increases Joule losses and voltage drop in the line. It can be compensate with reactive passive elements, inductor (L) or capacitor (C) connected parallel to the load [6, 7]. The compensation leads to a decrease of the root mean square (RMS) current in the line. Although the mechanism of electric energy flow for non-sinusoidal conditions is well described today, so fare is not yet available a generalized power theory, and theoretical calculations for the design of such devices as active filters or dynamic compensators. Therefore, the task of designing compensators for optimize energy transmission with nonlinear time-varying loads in non-sinusoidal regimes is, far from clear. Moreover, in the last time more and more the switching power supplies are used. In this paper will show that if the sinusoidal voltage and sampled pure resistive load are used, the load current can contain reactive part which can be compensated by means of simple compensating elements. In the present work we show that in a system with a harmonic voltage source can, even at a purely linear resistive load arise reactive power. A necessary (but not sufficient) condition is that the load is time-varying. The reactive power can then be compensated in the usual way. This will be illustrated in several examples where cyclo-dissipative approach is used [8, 9].

2 Theoretical part

2.1 Reactive power compensation principle

The simplified electrical circuit with sampled resistance load is displayed in Fig. 1. Suppose that equation of harmonic source is

$$v_s(t) = A\sin(\omega_0 t) \tag{1}$$

and the load resistor is R. Switch is sampled (switched ON/OFF) by signal

$$S(t) = \frac{1 + \mathrm{sign}[\sin(k_S\omega_0 t - \alpha)]}{2} \tag{2}$$

synchronously with signal v_S but with k_S times higher frequency, with phase shift α and width $= 0.5$. The example is shown in Fig. 2, where $\alpha = \pi$ and $k_S = 2$.

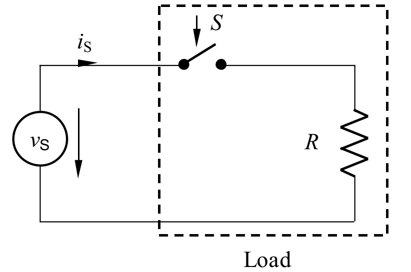

Fig. 1. Simplified circuit diagram of sampled resistive load

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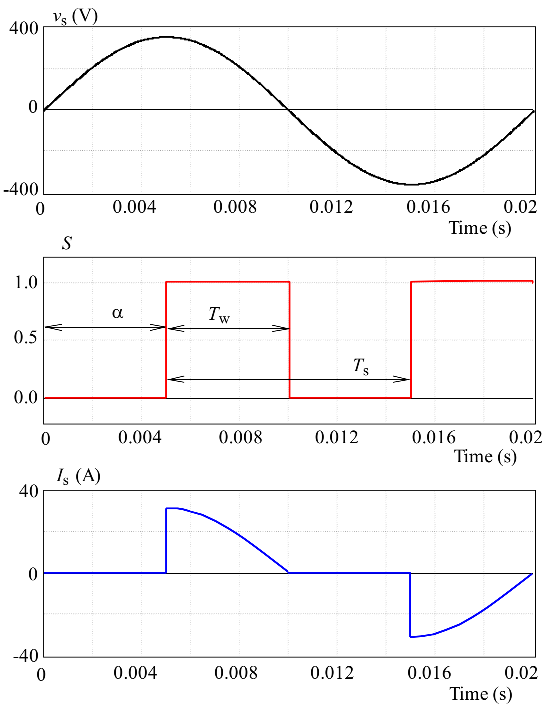

Fig. 2. Voltage source, switching signal and current concerning switched resistive load. Source v_S (top), switching signal S (middle), current from source i_S (bottom), here T_S is sampling period, T_W is sampling pulse width and α is the phase shift. For this example $T_W = T_S/2$ and $k_S = 2$.

Time evolution of signals for several values of α is presented in Fig. 3. The simplified electrical circuit with sampled resistance load and compensation admittance Y_C is shown in Fig. 4. The current from power source is i_S, compensation current i_{CO} and load current i. Principle of RP compensation is described in next part.

The RP compensation of sampling resistive load can be proved by means of spectral decomposition of current i_S, see Fig. 5. Suppose time evolution of signals according Fig. 2 (sampling with $\alpha = \pi$ and $k_S = 2$). For non-harmonic current i_S the Fourier decomposition was used. The first harmonics of i_S (designated in Fig. 6 as ${}^1i_S$) and source voltage ($v_S/10$) is shown in Fig. 6. From this figure is clear that phase shift exist between voltage and first spectral component of current, therefore RP can be compensated.

The RP compensation is possible by means of capacitor C_C, used instead of compensator Y_C in Fig. 4. The optimal value of capacitor can be find by optimization, see Fig. 7. From this figure can be seen, that for optimal compensator the effective current source decreased from 15.5 A (without compensation) to 13.9 A (with optimal compensation). Time diagram of voltage and currents after compensation are plots in Fig. 8.

2.1 Mathematical approach

It must be pointed, that is possible calculate optimal value of compensation by means of mathematical approach [8–12]. For any given periodic signal $x(t)$ and $y(t)$ with period T the inner product is defined as

$$\langle x, y\rangle := \frac{1}{T}\int_0^T x(t)y(t)\mathrm{d}t \qquad (3)$$

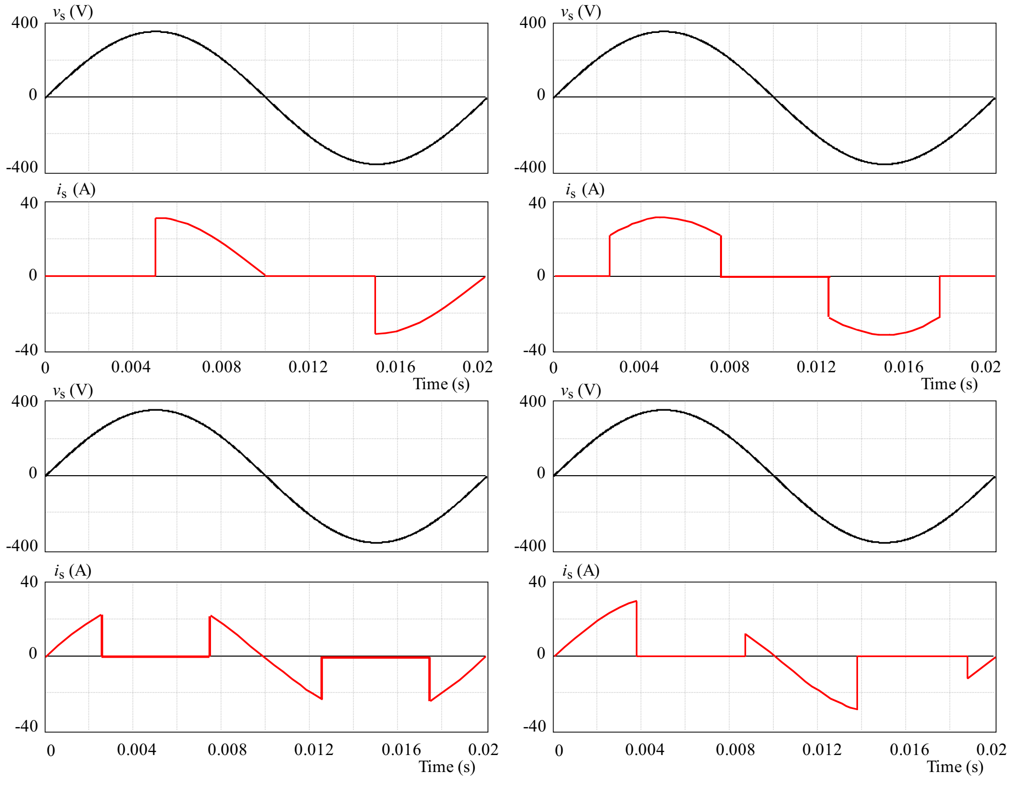

Fig. 3. Examples of time evolution of voltage source v_S and current i_S for different phase shifting of $S(t)$. Left top $\alpha = 0$, right top $\alpha = \pi/2$, left bottom $\alpha = 3\pi/2$, right bottom $\alpha = 7\pi/4$. For all examples $T_W = T_S/2$ and $k_S = 2$.

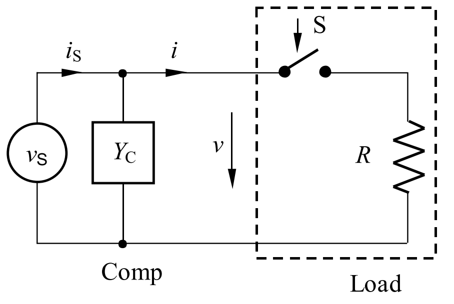

Fig. 4. Simplified circuit diagram of sampled resistive load with reactive power compensation by means of compensator Y_C

Table 1. Compensation results for different phase of sampling

α (rad)	$PF_{\rm n}$ (-)	$PF_{\rm c}$ (-)	$I_{\rm sn}$ (A)	$I_{\rm sc}$ (A)	$C_{\rm c}$ (μF)	$L_{\rm c}$ (mH)
0	0.708	0.793	15.57	13.91	0	100
0.314	0.773	0.841	17.02	15.66	0	100
0.628	0.829	0.872	18.234	17.33	0	120
0.943	0.87	0.891	19.138	18.69	0	170
1.257	0.9	0.902	19.712	19.59	0	325
1.571	0.904	0.904	19.9	19.89	0	∞
1.885	0.896	0.902	19.72	19.6	31	∞
2.199	0.87	0.891	19.14	18.69	59	∞
2.513	0.828	0.872	18.23	17.32	2	∞
2.827	0.773	0.841	17.02	15.66	96	∞
3.142	0.707	0.791	15.54	13.88	100	∞
3.456	0.633	0.721	13.93	12.24	96	∞
3.77	0.559	0.63	12.31	10.92	82	∞
4.084	0.493	0.533	10.85	10.04	59	∞
4.398	0.444	0.455	9.77	9.52	31	∞
4.712	0.426	0.426	9.38	9.38	0	∞

In Table: α – phase shift of sampling signal, PF_n – power factor for non-compensated circuit, PF_c – power factor of compensated circuit, $I_{\rm sn}$ – effective current from source for non-compensated circuit, $I_{\rm sc}$ – effective current from source for compensated circuit, C_c – value of compensation capacitor, L_c – value of compensation inductor. If the compensation by capacitor is not possible, value in table is 0. If the compensation by inductor is not possible, value in table is ∞.

and norm of variable $x(t)$ (X is rms - root mean square) is

$$\|x\| = \sqrt{\langle x,x\rangle} = \sqrt{\frac{1}{T}\int_0^T x^2(t)\mathrm{d}t} = X \tag{4}$$

Let $x(t)$ and $y(t)$ be differentiable periodic function with period T. Then

$$\left\langle \frac{\mathrm{d}}{\mathrm{d}t}x, y\right\rangle = -\left\langle x, \frac{\mathrm{d}}{\mathrm{d}t}y\right\rangle \tag{5}$$

and also

$$\left\langle \int x, y\right\rangle = -\left\langle x, \int y\right\rangle. \tag{6}$$

The active power delivered by the source is defined as

$$P := \langle v_S, i_S\rangle = \frac{1}{T}\int_0^T v_S^\top(t) i_S(t)\mathrm{d}t \tag{7}$$

where $\langle\cdot\rangle$ denotes the inner product. From (7) and the Cauchy-Schwarz inequality

$$P \leqslant \|v_S\|\cdot\|i_S\| =: S. \tag{8}$$

where the apparent power S is defined. The compensator Y_C is placed in parallel. Also, to avoid power dissipation, Y_C is restricted to be lossless, that is

$$\langle v, i_{CO}\rangle = 0 \tag{9}$$

where i_{CO} is compensator current and notice that $v_S = v$ (see Fig. 4). From the previous it can be derived that the RP compensation problem is mathematically equivalent to the problem of minimization of $\|iS\|$ subject to the constraint (9), according

$$\|i_S\|^2 = \|i\|^2 + \|i_C\|^2 + 2\langle i_C, i\rangle. \tag{10}$$

For circuit compensated by capacitor

$$\|i_S\|^2 = \|i + i_{CO}\|^2 = \|i\|^2 + \|i_{CO}\|^2 + 2\langle i, i_{CO}\rangle \tag{11}$$

where current i_{C0} for compensation capacitor is

$$i_{CO}(t) = C_{CO}\frac{\mathrm{d}v_S(t)}{\mathrm{d}t}. \tag{12}$$

After substitution and some manipulations

$$\|i_S\|^2 = \|i\|^2 + C_{CO}^2\left\|\frac{\mathrm{d}v_S}{\mathrm{d}t}\right\|^2 + 2C_{CO}\left\langle i, \frac{\mathrm{d}v_S}{\mathrm{d}t}\right\rangle. \tag{13}$$

Minimal value of i_S (or minimum of apparent power S) can be found for

$$\frac{\mathrm{d}}{\mathrm{d}C_{CO}}\left(\|i\|^2 + C_{CO}^2\left\|\frac{\mathrm{d}v_S}{\mathrm{d}t}\right\|^2 + 2C_{CO}\left\langle i, \frac{\mathrm{d}v_S}{\mathrm{d}t}\right\rangle\right) = 0. \tag{14}$$

Optimal value of compensation capacitor is

$$C_{CO} = -\frac{\left\langle i, \frac{\mathrm{d}v_S}{\mathrm{d}t}\right\rangle}{\left\|\frac{\mathrm{d}v_S}{\mathrm{d}t}\right\|^2} \text{ or } C_{CO} = \frac{\left\langle \frac{\mathrm{d}i}{\mathrm{d}t}, v_S\right\rangle}{\left\|\frac{\mathrm{d}v_S}{\mathrm{d}t}\right\|^2} \tag{15}$$

where property (5) was used.
Optimal value of compensating inductor is

$$L_{CO} = -\frac{\left\|\int v_S\right\|^2}{\langle i, \int v_S\rangle} \text{ or } L_{CO} = \frac{\left\|\int v_S\right\|^2}{\langle \int i, v_S\rangle} \tag{16}$$

where property (6) was used. For compensating element calculation according (15) or (16) possible choice is what is "better" for calculation.

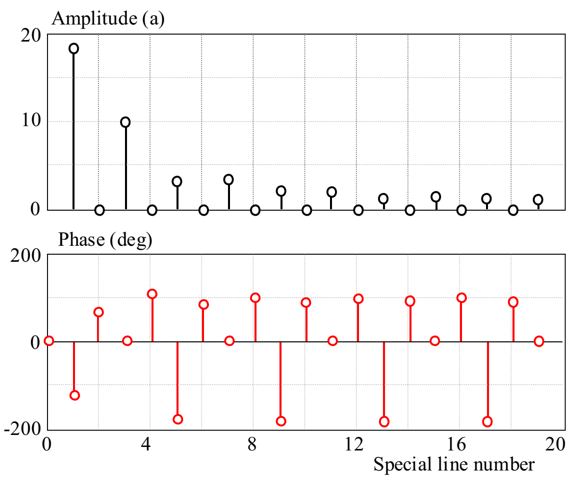

Fig. 5. Spectral decomposition of the current i_S for $\alpha = \pi$ and $k_S = 2$ (first 20 lines)

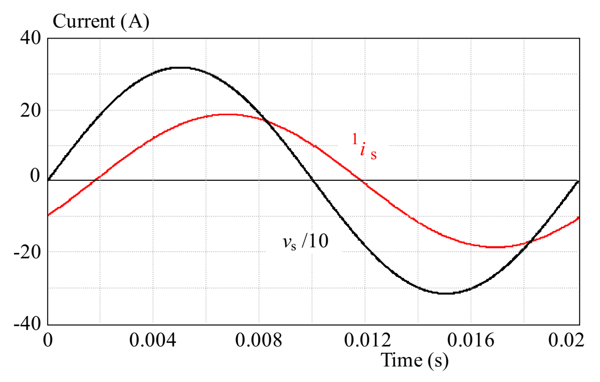

Fig. 6. Phase shift between the first spectral component of non-harmonic current i_S (according Fig. 2, $\alpha = 2$ and $k_S = 2$ and spectral decomposition Fig. 5) and voltage source $v_S/10$

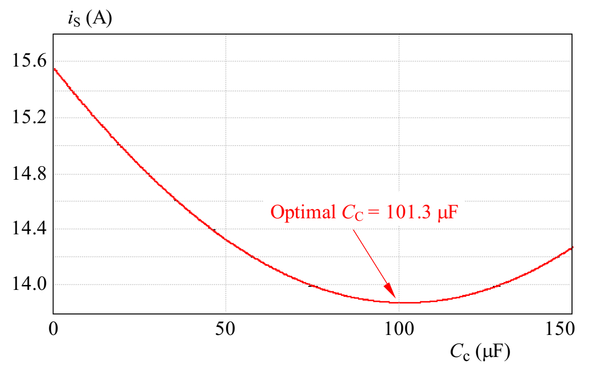

Fig. 7. Effective current from source i_S versus value of compensating capacitor (for example according Fig. 2)

3 Experimental part

In this part the results of RP compensation are presented [13–16]. It must be pointed that calculation of compensation by means of inductor, according to (16) is also possible, but the result is negative (for $\alpha = \pi$), which means that the compensation by inductor is not real. The types of compensation (capacitive, reactive) depends on phase shift α, circuits diagrams are in Fig. 10.

In present example is less complicated use left part of equation (15) to avoid current derivation. Compensation capacitor calculation, according (15) is $C_{CO} = 101.27 \times 10^{-6}$ F The results for different values of α are presented in Table 1 (for $k_S = 2$). The power factor (PF) is defined as

$$PF := \frac{P}{S} \tag{17}$$

PF for non-compensated and compensated circuit versus sampling phase shift α is shown in Fig. 9.

For real-time automatic compensation of RP the digital control system was designed [17–20]. The blocks diagrams of electronic control systems for RP compensation are displayed in Fig. 11. Implementation of the control part of system is possible by means of microcontroller. The examples of time diagrams of automatic RP compensation are presented in Figs. 12 and 13.

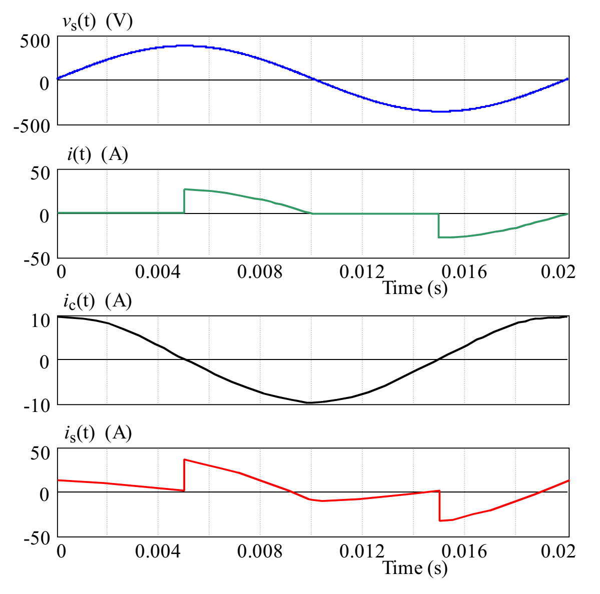

Fig. 8. Signals after compensation. From top to bottom: voltage source $v_S(t)$, current through the load $i(t)$, current through the compensation capacitor $i_{CO}(t)$, current from the source $i_S(t)$

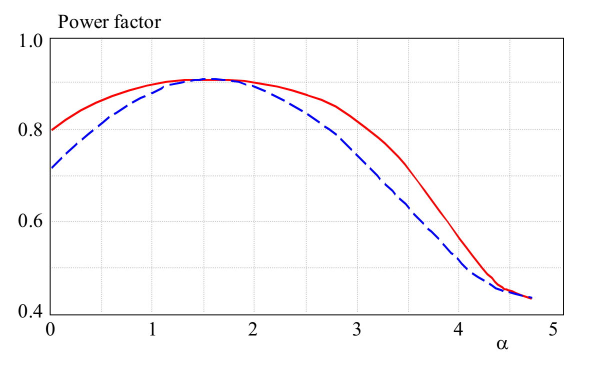

Fig. 9. PF for non-compensated circuit (dash – blue) and PF for compensated circuit (solid – red) versus sampling phase α

The electronic control system for RP compensation can be used also for circuits with nonlinear load. In next example the resistor with nonlinear inductor in series is used as load, see Fig. 14. The value of nonlinear inductance versus current is

$$L(i) = \frac{0.55}{1 + 0.1 \cdot i^2} \qquad \text{(H, A)} \tag{18}$$

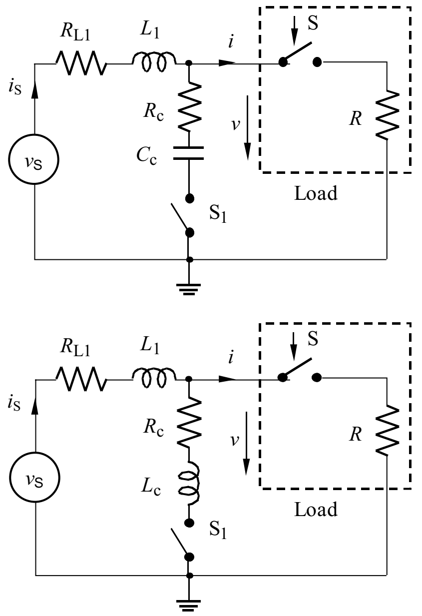

Fig. 10. Simplified block diagram of electronic control systems for RP compensation – inductive (up) or capacitive (down). This system is suitable for different types of loads (linear, nonlinear or switched load).

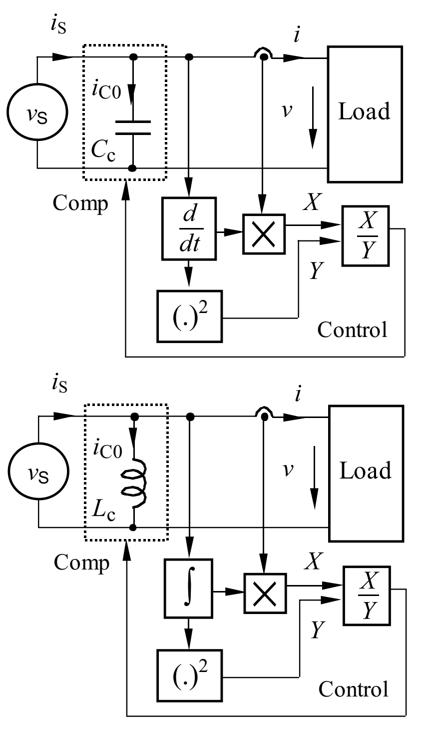

Fig. 11. Simplified block diagram of electronic control systems for RP compensation – inductive (up) or capacitive (down). This system is suitable for different types of loads (linear, nonlinear or switched load)

Graph of nonlinear inductance is displayed in Fig. 15 and time evolution of signals in circuit is presented in Fig. 16. RP compensation start in $t = 0.6$ s when compensation capacitor is connected in circuit. Effective value of current from source, without compensation is 0.94 A and with compensation is 0.624 A, therefore source current decreasing is 34 %.

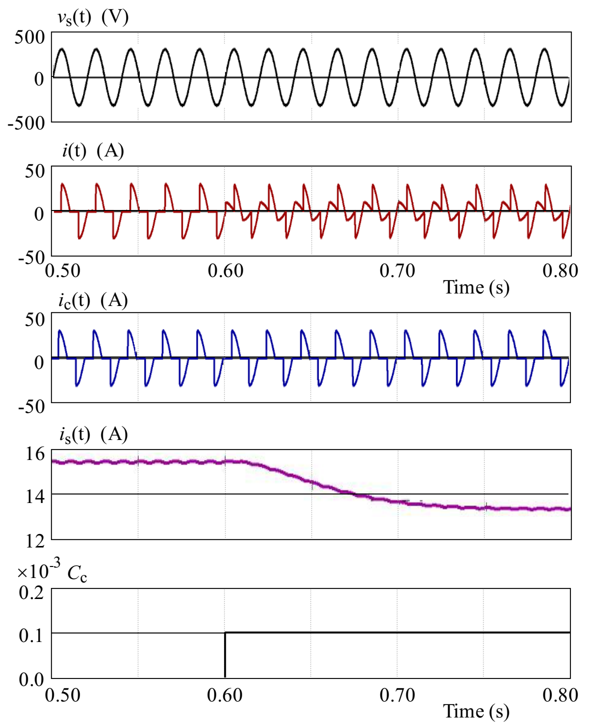

Fig. 12. Time diagram – before RP compensation (S_1 = OFF) and after RP compensation (S_1 = ON) for ($\alpha = \pi$ (and $k_S = 2$). The switch S_1 (see Fig. 8) is switched ON for time > 0.6 s. From top to bottom: v_S, i_S, i, $i_{S\text{eff}}$ – effective current from source, C_c – compensation capacitor

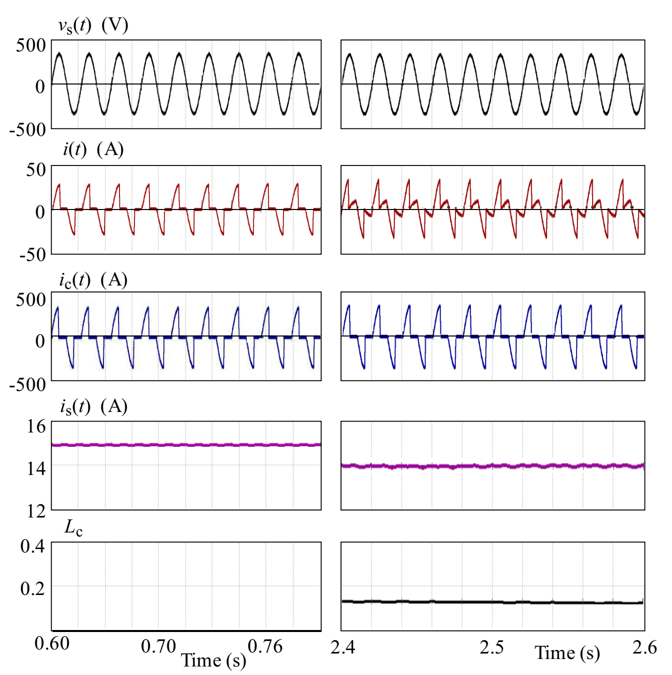

Fig. 13. Time diagram – before RP compensation (S_1 = OFF) and after RP compensation (S_1 = ON) for ($\alpha = \pi$ and $k_S = 2$).The switch S_1 (see Fig. 8) is switched ON for time > 0.8 s. From top to bottom: v_S, i_S, i, $i_{S\text{eff}}$ – effective current from source, L_c – compensation inductor. The figure is separated on left and right parts because of long transient response after L_c is connected in circuit

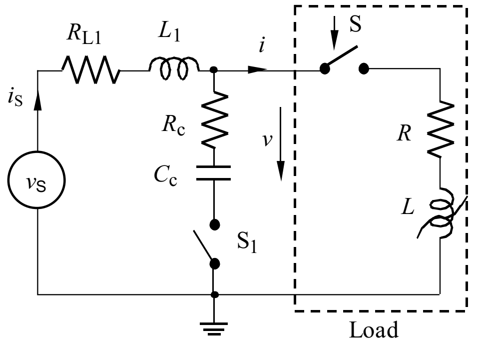

Fig. 14. Circuit diagram of circuit with nonlinear sampling load, $L_1 = 0.3\,\mathrm{mH}$, $R_{L1} = 0.1\,\Omega$, $R_C = 0.05\,\Omega$ $R = 10\,\Omega$, l see (18) and Fig. 15, C_c see Fig. 16

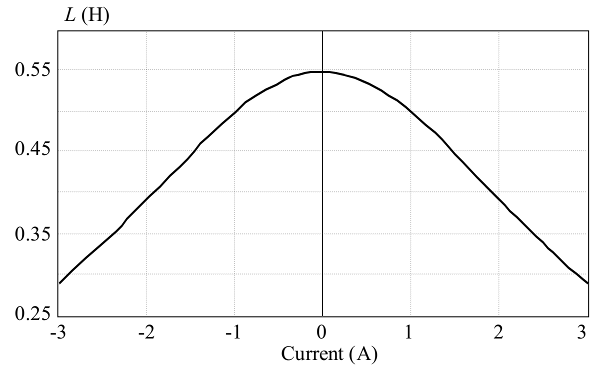

Fig. 15. Graph of nonlinear inductance versus current used in nonlinear load. Inductance versus current

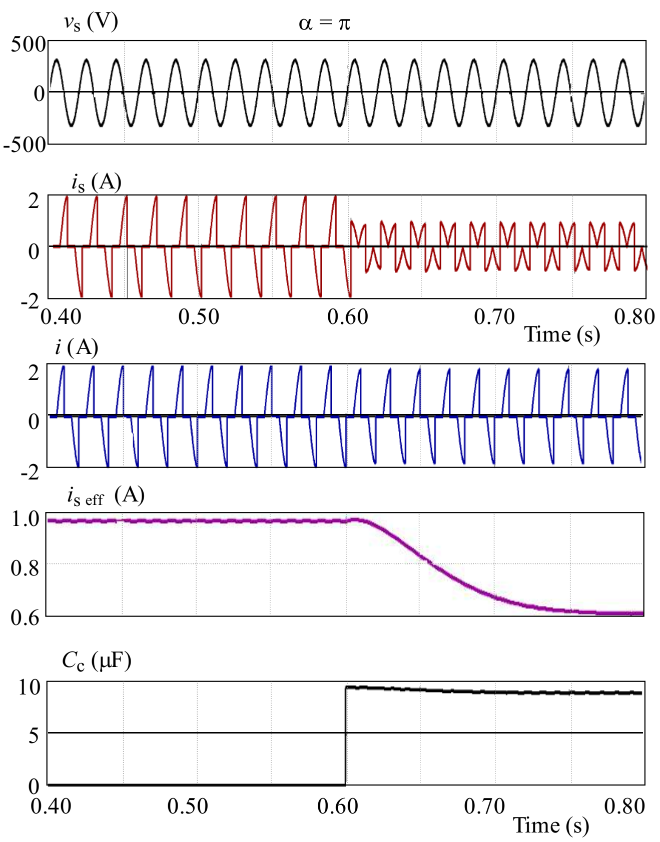

Fig. 16. Time diagram – before RP compensation ($S_1 = \mathrm{OFF}$) and after RP compensation ($S_1 = \mathrm{ON}$) for ($\alpha = \pi$ and $k_S = 2$).The switch S_1 (see Fig. 14) is switched ON for time > 0.6 s. From top to bottom: v_S, i_S, i, $i_{S\mathrm{eff}}$ – effective current from source, C_c – compensation capacitor $C_c = 9.5\,\mu\mathrm{F}$, $i_{S\mathrm{eff}} = 0.94$ A (without compensation), $i_{S\mathrm{eff}} = 0.624$ A (with compensation)

4 Discussion

The presented examples were devoted to the analysis of single phase systems. The extension of the above theory to poly-phase systems is simple for 4 wire Y connection, but for 3 wire network with unbalanced terms, delta connection and compensation of reactive power opens several questions [21–23], but the poly-phase extension is not described in this paper.

5 Conclusion

In this paper the reactive power compensation for sinusoidal and nonsinusoidal situations, where nonlinear circuit voltages and currents contain harmonics and also the control algorithms of automatic compensators were presented. The main aim of the reactive power compensation was based on the dissipative systems and cyclodissipativity theories for calculation of compensation elements for minimizing line losses and power factor increasing. The digital control system for automatic compensation was also presented with simulation results.

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