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PEAK TO AVERAGE POWER RATIO REDUCTION IN NC–OFDM SYSTEMS

Saadia Tabassum * — **Sajjad Hussain** ** — **Abdul Ghafoor** *

Non contiguous orthogonal frequency division multiplexing (NC-OFDM) is an efficient and adaptable multicarrier modulation scheme to be used in cognitive radio communications. However like OFDM, NC-OFDM also suffers from the main drawback of high peak to average power ratio (PAPR). In this paper PAPR has been reduced by employing three different trigonometric transforms. Discrete cosine transform (DCT), discrete sine transform (DST) and fractional fourier transform (FRFT) has been combined with conventional selected level mapping (SLM) technique to reduce the PAPR of both OFDM and NC-OFDM based systems. The method combines all the transforms with SLM in different ways. Transforms DCT, DST and FRFT have been applied before the SLM block or inside the SLM block before IFFT. Simulation results show the comparative analysis of all the transforms using SLM in case of both OFDM and NC-OFDM based systems.

Keywords: peak to average power ratio, non contiguous orthogonal frequency division multiplexing, selected level mapping, discrete cosine transform, discrete sine transform, fractional Fourier transform

1 INTRODUCTION

To fulfill the increasing demand of bandwidth, efficient spectrum utilization is necessary. Dynamic spectrum access (DSA) provides efficient spectrum utilization by allowing secondary users to opportunistically use the licensed spectrum when it is not being utilized. Cognitive radio (CR) [1] is an emerging technology for the efficient use of spectrum that enables DSA. CR also requires an adaptable multicarrier modulation technique to be used at its physical layer. Non contiguous orthogonal frequency division multiplexing (NC-OFDM) [2] can be the very suitable modulation technique for DSA networks as it provides the agile use of spectrum when it is occupied by both primary and secondary users. It achieves high data rates by deactivating some sub-carriers that can interfere with the primary sub-carriers. Besides several advantages like high bandwidth efficiency, robustness against frequency selective fading and relatively simple receiver implementation NC-OFDM also suffers from a major drawback of high peak to average power ratio (PAPR). This high PAPR leads to in-band distortion and out of band radiation, requires amplifiers with wide dynamic range, increases complexity of analog-to-digital and digital-to-analog converters and reduces the efficiency of power amplifiers. Therefore it is desirable to reduce the PAPR of NC-OFDM systems.

Several techniques have been proposed to combat this problem in OFDM systems like amplitude clipping [3], coding [4], partial transmit sequence [5], selected level mapping (SLM) [6], interleaving [7] and tone reservation [8]. However incase of NC-OFDM systems as some sub-carriers are used for primary transmission and some for secondary transmission so any non linearity effect for secondary users can disturb the primary users as well. Due to this, PAPR is worse in NC-OFDM systems so PAPR reduction techniques used for OFDM need to be modified for NC-OFDM.

Different techniques have also been proposed to reduce the PAPR of NC-OFDM systems like joint sidelobe and PAPR reduction [9], adaptive mode PAPR reduction algorithm [10], PAPR reduction with trellis shaping [11] and an efficient adaptive technique with low complexity [12]. SLM is also an efficient technique to reduce PAPR in NC-OFDM systems [13]. It is one of the probabilistic PAPR reduction technique as the probability of occurrence of high PAPR is compensated by signal modification [14]. Therefore SLM gives better PAPR reduction without any data rate loss and also without any signal distortion. However SLM has high computational complexity as well. Incase of OFDM systems an efficient technique has been proposed that combines discrete cosine transform (DCT) and SLM and provides better PAPR reduction than each one can individually provide [15, 16]. This technique has not been applied in NC-OFDM systems.

In this paper we have not only applied this technique in OFDM along with other two transforms (discrete sine transform, DST and fractional fourier transform, FRFT) but have also employed it in NC-OFDM systems. DCT, DST and FRFT based SLM has been used in this paper in the form of two different ways. *Scheme 1* reduces the PAPR of the original signal by applying the transform

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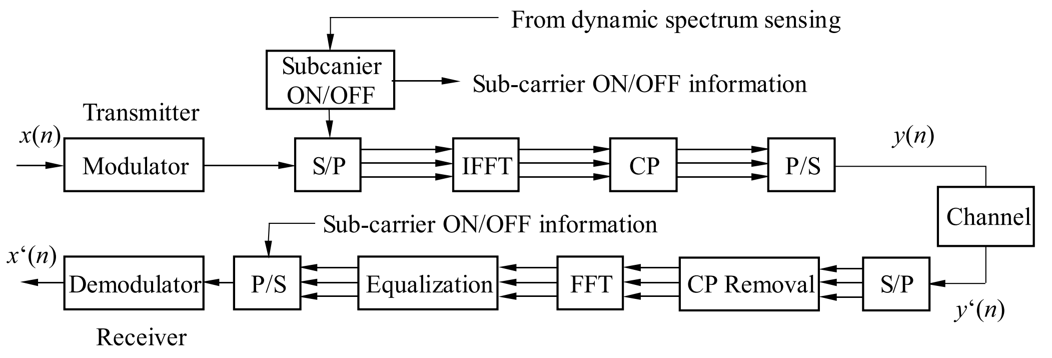

Fig. 1. Secondary NC-OFDM Transceiver with PAPR Reduction

once before applying the SLM, while *Scheme 2* applies the transforms inside SLM, before individual IFFT blocks. Results of both schemes are shown in case of OFDM and NC-OFDM systems as to give a relevant comparison.

2 NC–OFDM TRANSCEIVER

A secondary NC-OFDM transceiver is shown in Fig. 1. Its basic principle is to split a high stream serial data into slower parallel streams to be transmitted over orthogonal sub-carriers. These parallel streams are then modulated using an M-ary modulator. In case of NC-OFDM some sub-carriers are deactivated to refrain any secondary user to interfere with the primary user. The sub-carrier ON/OFF information is required to avoid this interference. This sub-carrier ON/OFF information is provided by the dynamic spectrum sensing. These modulated streams are then transformed into time domain using inverse fast fourier transform (IFFT). Cyclic prefix (CP) is then added to avoid the inter symbol interference. The parallel data is then again converted to serial using parallel to serial converter.

At the receiver side, the data are first converted to parallel streams by using serial to parallel converter. CP is removed and data are again converted to frequency domain using fast fourier transform. Distortions added to the signal through the channel are equalized. After that, the data are again converted to single stream using parallel to serial converter. Original signal is reconstructed using demodulator.

3 PAPR OF NC–OFDM

PAPR is defined as ratio between the peak power and average power, and the complex envelope of NC-OFDM signal are given by

$$PAPR = \frac{\max|s(t)^2|}{E[|s(t)^2|]}, \quad s(t) = \sum_{k=0}^{N-1} X_k e^{j2\pi kft} \qquad (1,2)$$

where $E[\cdot]$ is the expected value and $s(t)$ denotes the baseband NC-OFDM signal after IFFT, X_k is the symbol of the kth sub-carrier and N denotes the total sub-carriers. There is no complementary cumulative distribution function (CCDF) expression for the PAPR of NC-OFDM system but [17] has given an upper bound on it which is p *ie* the maximum value of PAPR for the MPSK modulated NC-OFDM signal with p active sub-carriers is equal to p, regardless of the total number of sub-carriers.

4 SELECTED LEVEL MAPPING (SLM)

In SLM firstly the source data are divided into sub-blocks all containing the same information and then each one is multiplied by different phase sequences. The most suitable sub-block which gives the minimum PAPR is then selected. Block diagram of SLM is shown in Fig. 2.

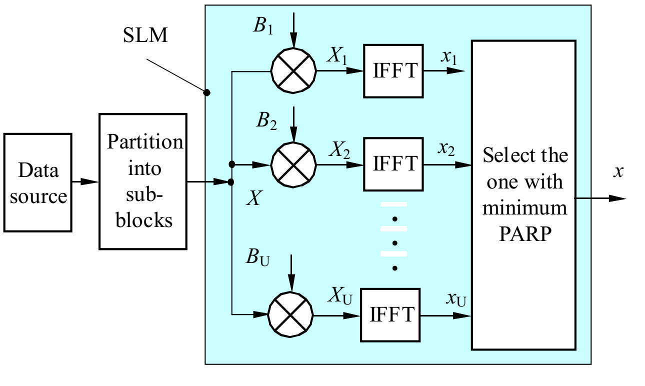

Fig. 2. Selected Mapping Block Diagram

Surce data X after partitioning are multiplied by phase vectors $\{B_1, B_2, \ldots, B_u\}$ to give the modified sub-blocks $\{X_1, X_2, \ldots, X_u\}$ to be further processed by IFFT. Out of these modified data blocks the one that gives the minimum PAPR is selected and transmitted. PAPR reduction in SLM depends on number and design of phase sequences U. Greater number of phase sequences gives more PAPR reduction. After applying SLM the

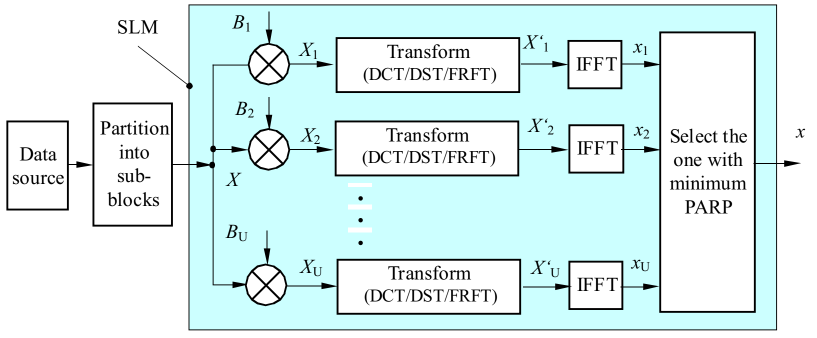

Fig. 3. Scheme-2 Transform applied before each IFFT block

baseband NC-OFDM signal consisting of N sub-carriers becomes

$$x_u(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} A_k B_{u,k} e^{j\frac{2\pi kt}{T}} \tag{3}$$

where $x(t)$ is the modified data block. $u = 1, 2, \ldots, U$ is the number of sub-block and A_k is the symbol of the k^{th} sub-carrier, while T is the symbol duration. At the receiver side an reverse operation is performed to recover the original information.

Multicarrier signals can be constructed through the complex exponential functions but this is not the only basis. DCT or DST both give a set of cosinusoidal or sinusoidal functions that can be used to implement multicarrier modulation schemes such as OFDM. They linearly transform the time domain data into frequency domain by representing the data as a sum of cosines or sines. Mathematically they can be represented as

$$X(t) = \sqrt{\frac{2}{N}} \sum_{n=0}^{N_s-1} a_n B_n \frac{\cos}{\sin} \left(\frac{n\pi t}{T_s}\right). \tag{4}$$

respectively.

DCT is like discrete Fourier transform (DFT) of real and even functions that reduces peak to average power ratio by reducing autocorrelation of the input sequence. The main advantage of DCT is that it does not require any side information for the receiver. The computational complexity of DCT is also less than that of Fourier transform.

DST is similar to DFT but of real functions only. It has the property of data compression that is viable for PAPR reduction. It decorrelates the original data and compacts the signal energy in lesser number of transform coefficients. This compaction helps to reduce the PAPR of original signal.

SLM obtains better PAPR but at a cost of increased complexity. So a better PAPR reduction is achieved by combining the features of both techniques. A DCT based SLM technique has been used in the form of two mentioned schemes and results have been compared in case of both OFDM and NC-OFDM systems. When combined with DST, SLM provides a better PAPR reduction.

FRFT is a generalized fourier transform that computes the mixed time and frequency components of the signal. Transformation is defined as

$$T^\alpha\{f(x)\} = F_\alpha(u)$$

where T^α is the α order fractional T transform and α is the fractional order. Such type of transform is called fractional transform. FRFT depends on parameter α and is interpreted by the rotation of angle *ie* if $\alpha = \pi/2$ it corresponds to conventional fourier transform and when $\alpha = 0$ it corresponds to identity operator. FRFT enhances the performance of multicarrier systems by employing orthogonal signal basis of the chirp type. In this paper FRFT has been combined with SLM to reduce the PAPR of NC-OFDM systems.

5 PAPR REDUCTION SCHEMES

PAPR has been reduced by applying the three transforms DCT, DST and FRFT.

Scheme 1:

Source data are first partitioned into sub-blocks and then transformed by using DCT, DST and FRFT. This operation is inserted between Partitioning sub-blocks and the SLM block in Fig. 2. The transformed data are then further processed as before. The applied transforms reduces the PAPR by decorrelating the input sequence and thus constraining the signal energy in few coefficients.

Scheme 2:

In modified SLM, data after the partitioning into sub-blocks, are again transformed by DCT/DFT/FRFT but separately "inside" the SLM block before applying IFFT on every sub-block $\{X'_1, X'_2, \ldots, X'_u\}$ to give the modified sub-blocks $\{x_1, x_2, \ldots, x_u\}$. These transforms decorrelates the data in a sequence by compacting large amount of signal energy into few transform coefficients so this gives a better reduction when a high PAPR signal is applied at the input. Block diagram of the proposed scheme is shown in Fig. 3. As every sub-block is transformed after IFFT, complexity of this scheme increases to $O(UN \log N)$ where $N \log N$ is the complexity of transform and U represents the number of sub-blocks.

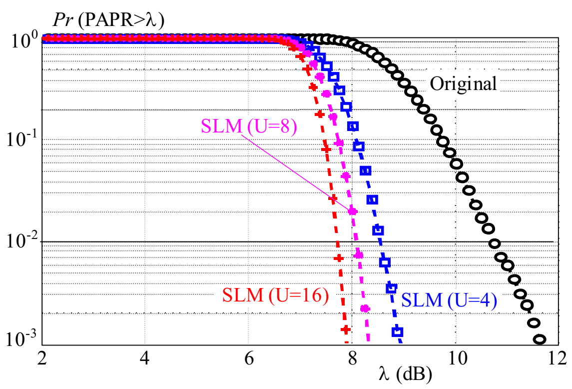

Fig. 4. CCDF curves for SLM in case of NC-OFDM

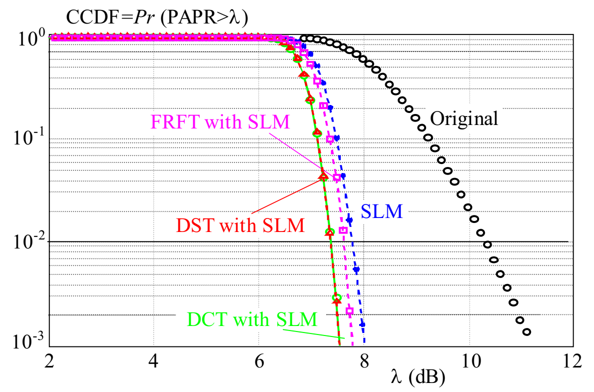

Fig. 5. CCDF curves of *Scheme 1* for OFDM

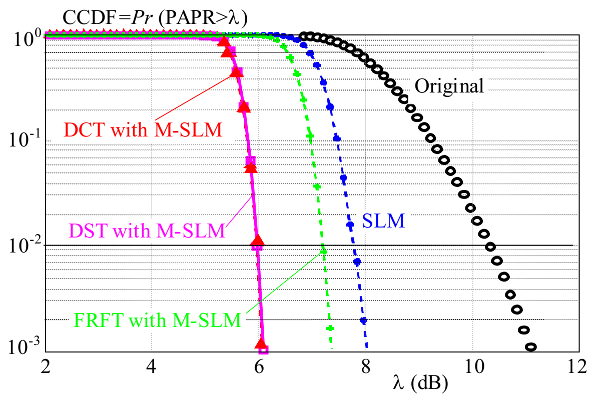

Fig. 6. CCDF curves for *Scheme 2* for OFDM

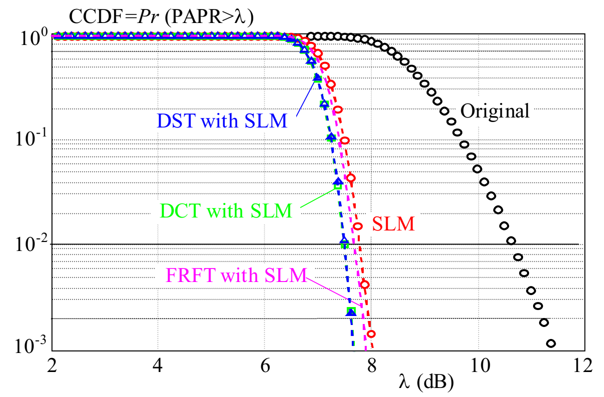

Fig. 7. CCDF curves for *Scheme 1* for NC-OFDM

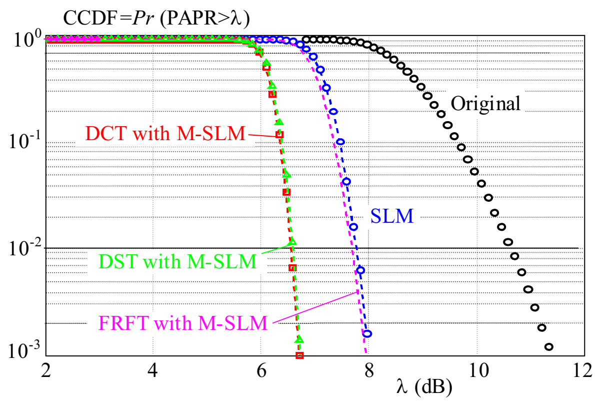

Fig. 8. CCDF curves for *Scheme 2* for NC-OFDM

6 SIMULATION RESULTS

Results of the performed simulations have been analyzed to evaluate and compare PAPR reduction performance of the two schemes in case of both OFDM and NC-OFDM systems. CCDF curves have been plotted using $N = 512$ QPSK modulated sub-carriers for 10^5 iterations. The results of applying only SLM technique in case of NC-OFDM systems are shown in Fig. 4. We see that PAPR reduces as number of blocks (U) increases. CCDF curves have been plotted for $U = 4, 8, 16$. Figure 5 shows the simulation results of *Scheme 1* in case of OFDM. Original PAPR curve of OFDM is also plotted with PAPR of 11.2 dB. All the three transforms *ie* DCT, DST and FRFT combined with SLM technique are shown to lower this value. It is seen that SLM combined with DCT reduces the PAPR to about 7.4 dB. SLM based DST also gives the same reduction as DCT transform however the performance of FRFT in case of *Scheme 1* is slightly less than that of DCT and DST. In *Scheme 2*, where data are transformed by DCT, DST and FRFT before each IFFT block in SLM. The results in Fig. 6 show a better reduction if compared with *Scheme 1* in cases of all transforms.

In case of NC-OFDM simulations were performed using 320 active sub-carriers out of the total 512 sub-carriers. CCDF curves of NC-OFDM were plotted for both schemes as shown in Fig. 7 and Fig. 8. Results show that by applying DCT transform PAPR of the symbol reduces to about 1.5 dB. PAPR reduction increases as we increase the number of sub-carriers. Comparison of PAPR reduction performance using both schemas for OFDM and NC-OFDM is in Tab. 1. PAPR values have been computed for $N = 512$ and $U = 16$. This shows that in *Scheme 2* better PAPR reduction is achieved than in *Scheme 1* in both OFDM and NC-OFDM systems.

Table 1. Comparison of both the schemes in case of OFDM and NC-OFDM

	DCT based SLM ΔPAPR (dB)	DST based SLM ΔPAPR (dB)	FRFT based SLM ΔPAPR (dB)
		OFDM	
Scheme-1	3.6	3.6	3.4
Scheme-2	5.1	5.1	3.8
		NC-OFDM	
Scheme-1	3.7	3.7	3.5
Scheme-2	4.6	4.6	3.4

6 CONCLUSION

High PAPR is one of the main drawbacks of NC-OFDM based communication systems. Several methods have been used to reduce it. In this paper transformed based SLM technique has been used that combines conventional SLM with different trigonometric transforms. *Scheme 1* combines DCT/DST/FRFT with SLM by applying these transforms before the SLM block. However in *Scheme 2* (modified SLM) these transforms are applied after each IFFT block in SLM. The proposed *ieScheme 2* gives better PAPR reduction as applying transform at the output of each IFFT block reduces the autocorrelation of the signal and also compacts the signal energy in fewer transform coefficients. The schemes combines the features of both SLM and all the transforms and provides better reduction in the PAPR of the signal than they can provide when used alone. By simply combining DCT, DST and FRFT with SLM PAPR of NC-OFDM symbol achieves a considerable reduction. Therefore from the results it is evident that the scheme that has been employed in this paper gives a better reduction though at the cost of a bit increased complexity.

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