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# ROBUST DECENTRALIZED CONTROLLER DESIGN: SUBSYSTEM APPROACH

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The paper addresses the problem of the robust output feedback PI controller design for complex large-scale stable systems with a state decentralized control structure. A decentralized control design procedure is proposed for static output feedback control which is based on solving robust control design problems of subsystems' size. The presented approach is based *on the Generalized Gershgorin Theorem* and uses the so-called equivalent subsystems approach to consider the interactions in the local robust controller design. The resulting decentralized control scheme has been successfully tested on two examples: a linearized model of three interconnected boiler-turbine subsystems and a linear model of four cooperating DC motors where the problem is to design four local PI controllers for a large scale system which will guarantee robust stability and performance of the closed-loop uncertain system.

K e y w o r d s: decentralized control, robust stability, bilinear matrix inequalities

## 1 INTRODUCTION

The control of complex or large-scale system is a new approach to science that studies how relationships between parts give rise to the collective behaviours of a system and forms the relationship with its environments. Nowadays, most industrial processes are naturally large-scale systems which need a control strategy using the system approach. One of the main problems of large-scale system is its high dimension which restricts the possibility to synthesize the controller using the LMI/BMI approach. Our attention now is to study a suitable control strategy to overcome this obstacle. In this paper, we focus on the control of linear large-scale stable dynamic systems using a decentralized approach. We assume that the large-scale system consists of $M$ subsystems with order as small as possible to design a robust decentralized controller on the subsystem level. The stability and robustness properties of the complex system are checked in the LMI framework.

Decentralized control has become an effective tool for control design of large scale systems during the past decades. Robustness is one of the attractive qualities of the decentralized control scheme, since such a control structure can be inherently resistant to a wide range of uncertainties both in subsystems and in the interconnections. Considerable effort has been made to consider robustness issues in the decentralized control structure and decentralized control design schemes, *eg* in [4–7] and [9]. The upper bound on nonlinear terms or/and interactions is extensively used in decentralized control design approaches based on Lyapunov stability conditions, *eg* in [5]. To receive computationally tractable results for large scale systems, LMI formulation gains a notable interest. In [9], design of a static output feedback using LMI is proposed, based on Lyapunov stability and factorization of the respective matrices to receive a linear formulation. The above approaches compute decentralized control by solving the problem of the overall system size.

To reduce the problem size in decentralized control design for large scale systems, the diagonal dominance or block diagonal dominance concept can be adopted. Recently, the so-called Equivalent Subsystems Method has been developed for decentralized control in the frequency domain by Kozákov and Veselý [2]. The main concept of the Equivalent Subsystems Method, originally developed as a Nyquist based frequency domain decentralized controller design technique, is the so-called equivalent subsystem. Equivalent subsystems are generated by shaping the Nyquist plot of each decoupled subsystem using any selected characteristic locus of the matrix of interactions. The important point in this approach is that the controllers of equivalent subsystems can be independently tuned for stability (and specified feasible performance) according to the specified stability and/or performance indices (*eg* degree of stability, overshoot ... ) so that the resulting decentralized controller guarantees the same stability/performance indices for the full system.

In this paper an analogy of equivalent subsystems' approach is proposed for decentralized control design in he state space. The major advantage is that the overall control problem is reduced to the subsystems' size; on subsystem level we adopt the robust static output feedback control design, the interaction bound is considered via the subsystem stability degree. The robust decentralized PI controller is designed using the polytopic description of the uncertain system and applying the robust optimal control design procedure with an extended cost func-

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tion for state-space subsystems generated in each vertex of the polytopic uncertainty domain. In the illustrating two examples, the proposed decentralized control design strategy is applied to design a robust decentralized PI controller for an interconnected system of boiler-turbines and four cooperating DC motors.

The controller design procedure proposed in the paper is not proved but we hope that it can give interesting results in many practical situations.

## 2 PRELIMINARIES AND PROBLEM FORMULATION

Consider the following linear large-scale stable system with polytopic uncertainty described as

$$\begin{aligned} \dot{x}(t) &= A(\xi)x(t) + B(\xi)u(t)\,, \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input, $y(t) \in R^m$ is the controlled output (measured output). Matrices $A(\xi), B(\xi) \in S$ belong to the convex hull, and $S$ is a polytope with $N$ vertices $S_1, S_2, \dots, S_N$ which can be formally defined as

$$S := \begin{cases} A(\xi) = \sum_{k=1}^{N} \xi_k A_k\,,\ B(\xi) = \sum_{k=1}^{N} \xi_k B_k\,, \\ \sum_{k=1}^{N} \xi_k = 1\,,\ \xi_k \geq 0\,. \end{cases} \tag{1}$$

Matrices $A_i, B_i$ can be divided into two parts

$$\begin{aligned} A_k &= A_{dk} + A_{mk}\,, \\ B_k &= B_{dk} + B_{mk}\,, \end{aligned} \quad k \in \{1, 2, \dots, N\} \tag{2}$$

where $A_{dk} = \mathrm{diag}\{A^k_{jj}\}$, $B_{dk} = \mathrm{diag}\{B^k_{jj}\}$, $j \in \{1, \dots, M\}$, - block diagonal matrices corresponding to subsystems and matrices $A_{mk}$, $B_{mk}$ are off-diagonal matrices, that is $A_{mk} = A_k - A_{dk}$, $B_{mk} = B_k - B_{dk}$. Assume that matrix $C = \mathrm{diag}\{C_j\}$, $j \in \{1, \dots, M\}$ is a block diagonal with dimensions corresponding to subsystems of the large-scale system. We assume that each subsystem can be stabilized by an output feedback PI controller in the following form

$$u_j = k_{P_j} y_j + k_{I_j} \int y_j \mathrm{d}t = k_{P_j} C_j x_j + k_{I_j} z_j \tag{3}$$

where
$u_j \in R^{m_j}$ is the input vector of $j$ subsystem, $\sum_{j=1}^{M} m_j = m$,
$y_j \in R^{m_j}$ is the output vector of $j$ subsystem, $\sum_{j=1}^{M} m_j = m$,
$x_j \in R^{n_j}$ is state vector of $j$ subsystem, $\sum_{j=1}^{M} n_j = n$,
$z_j \in R^{m_j}$, $\quad z_j = \int_0^t C_{I_j} x_j \mathrm{d}t$.

Consider $v_j^\top := [x_j \quad z_j]^\top$, $C_{nj} = \mathrm{diag}\{C_j, I_j\}$ and after some small deriving steps, the local law control of form (4) can be rewritten as follows

$$u_j = \begin{bmatrix} k_{P_j} C_j & k_{I_j} \end{bmatrix} v_j = F_j C_{nj} x_j\,, \tag{4}$$

$$F_j = \begin{bmatrix} k_{P_j} & k_{I_j} \end{bmatrix}\,. \tag{5}$$

For the $i$-th subsystem one obtains

$$\dot{x}_i(t) = A_{ii}(\xi)x_i(t) + B_{ii}(\xi)u_i + \sum_{\substack{j=1 \\ j\neq i}} \big(A_{ij}(\xi)x_j(t) + B_{ij}(\xi)u_j(t)\big) \tag{6}$$

with PI controller of the $i$-th subsystem is extended as follows

$$\dot{v}_i = D_{ci}(\xi)v_i + \sum_{\substack{j=1 \\ j\neq i}} D_{cji}(\xi)v_j\,, \quad i = 1, 2, \dots, M \tag{7}$$

where

$$\begin{gathered} D_{ci}(\xi) = D_{ii}(\xi) + E_{ii}(\xi)F_i C_{ni}\,, \\ D_{cij}(\xi) = D_{ij}(\xi) + E_{ij}(\xi)F_j C_{nj}\,, \\ D_{ii}(\xi) = \begin{bmatrix} A_{ii}(\xi) & 0 \\ C_{I_i} & 0 \end{bmatrix},\ E_{ii}(\xi) = \begin{bmatrix} B_i(\xi) \\ 0 \end{bmatrix}, \\ D_{ij}(\xi) = \begin{bmatrix} A_{ij}(\xi) & 0 \\ 0 & 0 \end{bmatrix},\ E_{ij}(\xi) = \begin{bmatrix} B_{ij}(\xi) \\ 0 \end{bmatrix}, \\ i, j = 1, 2, \dots, M\,,\ i \neq j \end{gathered}$$

From (7) for the complex system we obtain

$$\dot{v}(t) = \big(D_{dc}(\xi) + D_{mc}(\xi)\big)v(t) \tag{8}$$

where

$$\begin{gathered} D_{dc}(\xi) = \mathrm{diag}\{D_{ci}(\xi)\}\,, \\ D_{mc}(\xi) = \{D_{cij}(\xi)\}_{i\neq j}\,,\ i, j = 1, 2, \dots, M\,;\ i \neq j \end{gathered}$$

Matrices $D_{dc}(\xi), D_{mc}(\xi) \in S$ belong to the convex hull with $N$ vertices $S_1, S_2, \dots, S_N$ which can be formally described as follows

$$D_{dc}(\xi) = \sum_{k=1}^{N} \xi) k D^k_{dc}\,,\ \ D_{mc}(\xi) = \sum_{k=1}^{m} D^k_{mc}\xi_k \tag{9}$$

where

$$\begin{gathered} D^k_{dc} = \mathrm{diag}\{D^k_{ci}\} = \mathrm{diag}\{D^k_{ii} + E^k_{ii} F_i C_{ni}\}\,, \\ D^k_{mc} = \{D^k_{cij}\} = \{D^k_{ij} + E^k_{ij} F_j C_{ni}\}\,, \\ i, j, = 1, 2, \dots, M\,;\ i \neq j\,. \end{gathered}$$

Simultaneously, with system (8) we consider the following complex system

$$\dot{x}(t) = \big(G_d(\xi) + G_m(\xi)\big)x(t) \tag{10}$$

where

$$\begin{gathered} G_d(\xi) = \sum_{k=1}^{N} \xi_k\, \mathrm{diag}_{j\in\{1,\dots,M\}}\{-\gamma^k_{jj}\}\,,\ \gamma^k_{jj} > 0\,, \\ G_m(\xi) = \sum_{k=1}^{N} \xi_k \{\rho^k_{ij}\}_{i,j\in\{1,\dots,M\}}\,, \\ \rho_{ij} = \begin{cases} 0\,, & i = j\,, \\ \rho^k_{ij} > 0\,, & i, j \in \{1, \dots, M\}\,,\ i \neq j\,. \end{cases} \end{gathered}$$

We introduce the following known results [3]

LEMMA 2.1 (Gershgorin Circle). *Let $A = \{a_{ij}\}_{i,j\in\{1,\dots,n\}}$ and assume that $a_{ii} > 0$ for each $i$ and $a_{ij} \le 0$, $i \ne j$. If $A$ is positive diagonally dominant, that is*

$$a_{ii} > \sum_{\substack{j=1\\ j\ne i}} |a_{ij}| \tag{11}$$

*then $A$ is an $M$-matrix. Note that if the system is negative diagonally dominant, it is stable.*

LEMMA 2.2 (Generalized Gershgorin Theorem) [12]. *Let $A = \{A_{ij}\}_{i,j\in\{1,\dots,M\}}$ if $c_1,\dots,c_M$ are positive numbers such that the following matrix*

$$\begin{bmatrix} c_1 & -\|A_{12}\| & \dots & -\|A_{1M}\} \\ -\|A_{21}\| & c_2 & \dots & -\|A_{2M}\} \\ \vdots & \vdots & \ddots & \vdots \\ -\|A_{M1}\| & -\|A_{M2}\| & \dots & c_M \end{bmatrix}$$

*is $M$-matrix, then all eigenvalues of matrix $A$ lie in the region*

$$D_i = \{\{z\colon r(A_{ii} - zI_i) \le c_i\}\,,\ i\in\{1,\dots,M\}$$

*where $r(\cdot)$ is regularity of square matrix. $r(A) = 0$ if $A$ is singular and $r(A) = \left\|A^{-1}\right\|^{-1}$ if $A$ is a non-singular matrix.*

LEMMA 2.3 [3]. *T Let $A \in R^{n\times n}$ be a square matrix with $n$ different eigenvalues: $\mu_1,\mu_2,\dots,\mu_n$ and $B \in R^{n\times n}$. Then each eigenvalue of $A+B$ lies in one of the circles*

$$|z-\mu_i| \le r\,, \quad r = \|B\|v(P) \tag{12}$$

*where $\|B\|$ denotes norm of $\|B\|$ and $v(P)$ denotes condition number of a matrix $P$ which transforms $A$ to a diagonal matrix, ie $P^{-1}AP = \operatorname{diag}\{\mu_1,\mu_2,\dots,\mu_n\}$. When $A$ is a normal matrix ($A^\top A = AA^\top$), then $v(P) = 1$.*

We consider 2-norm of matrices: $\|P\|$ = the largest singular value of $P$, and the respective condition number (the ratio of the largest singular value of $P$ to the smallest one).

R e m a r k 1. In Lemma 2.3, $A$ can be considered as a nominal matrix and $B$ as a disturbing one, then Lemma 2.3 shows how the eigenvalues of $A$ are disturbed by adding the "disturbance" matrix $B$. Concerning interconnected system, $A$ and $B$ can represent the block diagonal part $D_{dc}$ corresponding to individual subsystems and interaction part $D_{mc}$ respectively. From the observation of above Lemmas, we can conclude:

Due to Lemma 2.1 the stability of system (10) on the given vertex $k$ is guaranteed if

$$\gamma_{jj}^k \ge \sum_{\substack{i=1\\ i\ne j}} \rho_{ij}^k\,,\ j\in\{1,\dots,M\}\,,\ k\in\{1,\dots,N\}\,. \tag{13}$$

Concerning system (7) and let $G = D_{dc}(\xi)$ and $H = D_{mc}(\xi)$. If system $G$ is stable then $G+H$ will be stable if

$$0 < r < \operatorname{abs}\big(\max\{\operatorname{real}\{\operatorname{eig}(G)\}\}\big). \tag{14}$$

To reach such value of $r$ one can try to minimize the $\|H\|$, $v(G)\to 1$ and $\max\{\operatorname{real}\{\operatorname{eig}(G)\}\} < 0$.
To obtain above results for (10), due to above Lemmas

$$\rho_{ij}^k \to \min;\ i,j\in\{1,\dots,M\};\ i\ne j\,;\ k\in\{1,\dots,N\}$$
$$(14)\ \gamma_{jj}^k \to \max;\ j\in\{1,\dots,M\}\,;\ k\in\{1,\dots,N\}\,.$$

Consider the interconnected system (8) consisting of $M$ subsystems (6), where isolated subsystem is described by

$$\dot x_i = A_{ii}(\xi)x_i(t) + B_{ii}(\xi)u_i\,,\ i = 1,2,\dots,M\,. \tag{15}$$

For each isolated subsystem, an equivalent subsystem is defined by its system matrix

$$A_{ii}^{eq}(\xi) = A_{ii}(\xi) + p(\xi)I_i\,,\ i=1,2,\dots,M \tag{16}$$

where $I_i$ is identity matrix of the same size as $A_{ii}$; $p(\xi)$ represents the interaction bound and can be also used as a tuning parameter. The question how to find appropriate $p(\xi)$ is still open; here we put it equal to zero or use a norm of interconnection matrix $\max_k \|A_{mk}\|$ from (3) as a first estimate. Above Lemmas and observations give up the following results which summarizing in the next section

## 3 ROBUST DECENTRALIZED CONTROLLER DESIGN

The proposed decentralized control design strategy is based on the design of local robust controllers. The main aim is to develop the design procedure on subsystems level so that by a solution of M robust control problems of subsystems' size, the stabilizing decentralized control for the overall system is obtained. This approach has been motivated by Equivalent Subsystems Method proposed in [2] for decentralized control in the frequency domain: the crucial point is to find a suitable representation of interactions in the subsystem controller design and to use a subsystem stability degree and above observations as a stabilizing tool.

ASSERTION 3.1. *Let $\gamma_{jj}^k$ be stability degree of $j$-subsystem for $k$-vertex and*

$$\rho_{ij}^k = \|D_{ij}^k + E_{ij}^k F_j C_{nj}\|\,;\ i,j = 1,2,\dots M\,;\ \rho_{ii}^k \to \min\,. \tag{17}$$

*If for the system (8), the condition (12) holds, system is stable.*

From above observation, the following steps can give positive results to robustly stabilize the closed-loop system.

1. For a chosen $p(\xi)$ design the robust controller with gain matrix $F_j$ for $j$-subsystem (15) in such a way

that (12), (14), and (17) holds. Then the following subsystem is robustly stable with guaranteed cost

$$D^k_{cij} + \gamma^k_{jj} I\,,\ j = 1,2,\dots,M\,;\ k = 1,2,\dots,N\,.$$

2. Design a gain matrix $F_j$ so that the following condition holds

$$\begin{bmatrix} \left(p^k_{ij}\right)^2 & \left(D^k_{cij}\right)^\top \\ D^k_{cij} & I_{ij} \end{bmatrix} \geq 0\,,$$
$$i \neq j\,,\ i = 1,\dots,M\,, k = 1,2,\dots,N\,;\ i \neq j\,.$$

3. Design a gain matrix $F_j$ so that $\mathrm{trace}\left(V^k_j\right)$ is minimized, where $V^k_j = \mathrm{diag}\{\rho^k_{ij}\}\,,\ i \neq j\,.$
4. When all subsystems are robust stable with guaranteed cost, check the robust stability of complex system.
5. If the complex system is not robustly stable with performance, increase $p(\xi) > 0$ and return to first point.
6. If the complex system is not robustly stable with performance, an alternative way to get a robust stability of complex system is as follows: put $F = \{F_i\} = \alpha F\,;$ $\alpha > 0$ and using V-K iteration procedure using LMI for $\alpha = 1$ and complex system calculate matrices $P_k$, $k = 1,2,\dots,N\,;\ G, H$ (21) and then $\alpha > 0$.

We have to note that the above procedure does not guarantee the stability of the complex system but the above procedure gives the way how we can obtain the robust stability of the complex system.

Note that for the design of a robust controller with guaranteed cost on the subsystem level and checked robust stability of the complex system we use a robust stability notion based on the parameter dependent Lyapunov function (PDLF)

$$P(\xi) = \sum_{k=1}^{N} \xi_k P_k \text{ where } P_k = P_k^\top > 0\,. \tag{18}$$

Lemma 3.1 [4, 11]. *If there exist matrices $H, G$ and $N$ symmetric positive definite matrices $P_k$ such that for all $k = 1,\dots,N$*

$$\begin{bmatrix} S_{11}(\xi) & S_{12}(\xi) \\ S_{12}(\xi)^\top & -(G + G^\top) \end{bmatrix} < 0 \tag{19}$$

*where*

$$S_{11}(\xi) = HD_e(\xi) + D_e(\xi)^\top H^\top + Q + C_n^\top F^\top RFC_n\,,$$
$$S_{12}(\xi) = P(\xi) + D_e(\xi)^\top G - H\,,$$
$$D_e(\xi) = D_{dc}(\xi) + D_{mc}(\xi)\,,$$
$$C_n = \mathrm{diag}\{C_{nj}\}\,,\quad F = \mathrm{diag}\{F_j\}$$

*with performance*

$$J = \int_0^\infty \left(v^\top Q v + u^\top R u\right) dt\,,$$

*$Q$ and $R$ are positive definite (semidefinite) and definite weighting matrices, respectively; then the closed loop system (7) or (8) is robustly stable with guaranteed cost.*

Note that matrices $H$ and $G$ are not restricted to any special form; they were included to relax the conservatism of the sufficient condition. Note that when the robust stability is checked for complex system, matrix $F$ is known, therefore on subsystem level LMI (19) is used accordingly. Because of linearity for the $k$-th vertex (19) is read as follows

$$\begin{bmatrix} S^*_{11} & S^*_{12} \\ \left(S^*_{12}\right)^\top & -\left(G + G^\top\right) \end{bmatrix} < 0 \tag{20}$$

where

$$S^*_{11} = HD^k_c + (D^k_c)^\top H^\top + Q + C_n^\top F^\top RFC_n\,,$$
$$S^*_{12} = P_k + (D^k_c)^\top G - H\,;\ k = 1,2,\dots,N\,, \tag{21}$$
$$D^k_c = D^k_{dc} + D^k_{mc}\,,\ C_n = \mathrm{diag}\{C_{nj}\}\,,\ F = \mathrm{diag}\{F_j\}.$$

## 4 EXAMPLES

### 4.1 Boiler-turbine model

The proposed decentralized control design procedure has been used to design a PI decentralized controller for a linearized model of three interconnected subsystems: boiler-turbines. To simplify the description we consider three identical subsystems.

The boiler-turbine dynamics can be described by a nonlinear model of the 3rd order, [1, 8]. For individual subsystems we use the linearized model derived in [1], respective to one of operating points. The isolated subsystem dynamics is described by (15), subsystem matrices $A_{ii}(\xi)$, $B_{ii}(\xi)$ are from polytopic uncertainty domains given by (2), with 2 vertices

$$A^1_{ii} = \begin{bmatrix} -0.0025 & 0 & 0 \\ 0.0694 & -0.1 & 0 \\ -0.0067 & 0 & 0 \end{bmatrix},\ A^2_{ii} = \begin{bmatrix} -0.0027 & 0 & 0 \\ 0.0700 & -0.1 & 0 \\ -0.0065 & 0 & 0 \end{bmatrix},$$
$$B^1_{ii} = \begin{bmatrix} 0.9 & -0.3490 & -0.15 \\ 0 & 14.155 & 0 \\ 0 & -1.3976 & 1.6588 \end{bmatrix},\ B^2_{ii} = \begin{bmatrix} 0.95 & -0.3360 & -0.140 \\ 0 & 14.20 & 0 \\ 0 & -1.360 & 1.650 \end{bmatrix},$$
$$C_i = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ i = 1,2,3\,.$$

The subsystem input vector $u_i$ includes: the valve position for fuel flow, steam control and feedwater flow; three states of the subsystem (vector $x_i$) are: the drum pressure, electric output and fluid density. All three states can be measured (the third state variable is recalculated from the measurement of water level deviation). For more details on boiler-turbine model, see [1].

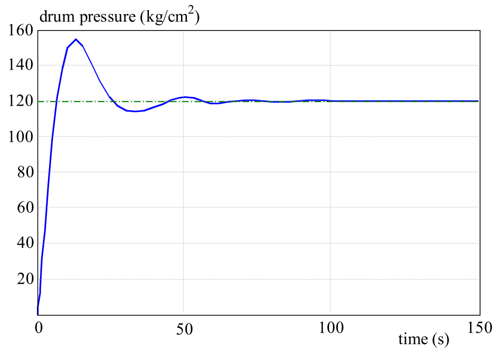


**Fig. 1.** Drum pressure step response

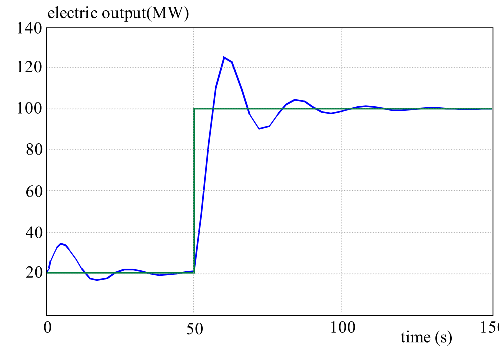


**Fig. 2.** Electric output step response

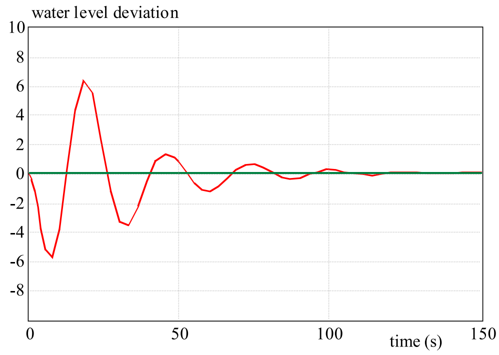


**Fig. 3.** Drum water level deviation

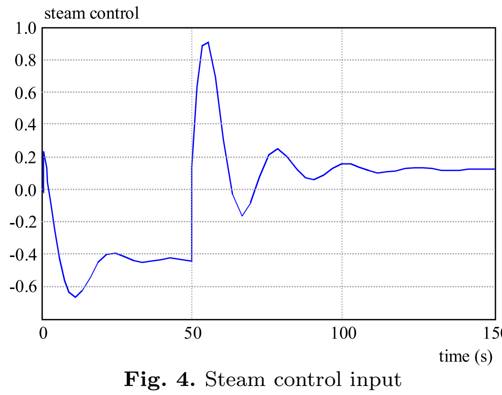


**Fig. 4.** Steam control input

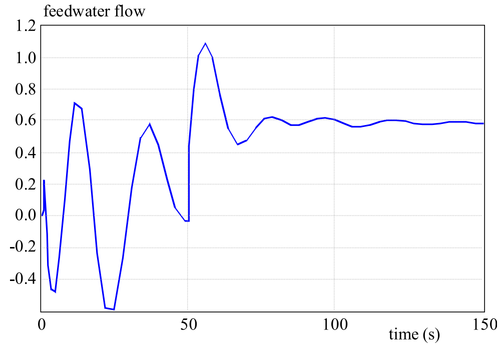


**Fig. 5.** Feedwater flow input

As stated above, we consider three identical boiler-turbine subsystems with interactions, the overall system model being in the form

$$\dot{x} = \begin{bmatrix} A_{11}(\xi) & A_{12}(\xi) & A_{13}(\xi) \\ A_{21}(\xi) & A_{22}(\xi) & A_{23}(\xi) \\ A_{31}(\xi) & A_{32}(\xi) & A_{33}(\xi) \end{bmatrix} x + \begin{bmatrix} B_{11}(\xi) & 0 & 0 \\ 0 & B_{22}(\xi) & 0 \\ 0 & 0 & B_{33}(\xi) \end{bmatrix} u \,,$$
$$y = x \,.$$

Interaction matrices $A_{ij}$ are considered to be the same for all $i,j$ (for simplicity).

$$A_{ij}^1 = \begin{bmatrix} 0.05 & 0 & 0 \\ 0 & 0.05 & 0 \\ 0 & 0 & 0.05 \end{bmatrix}, \quad A_{ij}^2 = \begin{bmatrix} 0.04 & 0 & 0 \\ 0 & 0.04 & 0 \\ 0 & 0 & 0.04 \end{bmatrix},$$
$$i,j = 1,2,3\,,\ i \neq j\,.$$

Since a tight regulation around the working point is required, a PI controller is used for all outputs (7).

Though the state feedback control is to be designed, the task is not trivial due to the integral character of the subsystems and required PI controller structure. (For comparison, we tried the robust SOF controller design approach proposed in [9], however, we have not received reasonable results for this example.)

We designed the decentralized PI controller using the procedure described in the previous section. To form equivalent subsystems (16), we use $p = 0.1$. The subsystem robust controllers designed for equivalent subsystems by a solution of (20) are

$$K_{P_i} = \begin{bmatrix} -0.2201 & -0.0035 & -0.0144 \\ -0.0051 & -0.0072 & 0.0001 \\ -0.0036 & -0.0059 & -0.1261 \end{bmatrix},$$

$$K_{I_i} = \begin{bmatrix} -0.0394 & 0.0044 & 0.0134 \\ 0.0004 & -0.0051 & 0.0001 \\ 0.0104 & -0.0034 & -0.0418 \end{bmatrix}, \quad i = 1,2,3\,.$$

The resulting closed loop interconnected system is stable with maximum real part of system eigenvalues $-0.0502$. It should be noted that the local subsystem controllers can be designed simultaneously.

Simulation results for the closed loop complex, interconnected system are illustrated on the responses of the first subsystem. Step responses of output variables are in Figs. 1, 2 and 3.

Control inputs respective to steam control and feedwater flow are in Figs. 4 and 5.

## 4.2 Four cooperating DC motors

In the second example we consider the linear model of four cooperating DC motors. The problem is to design

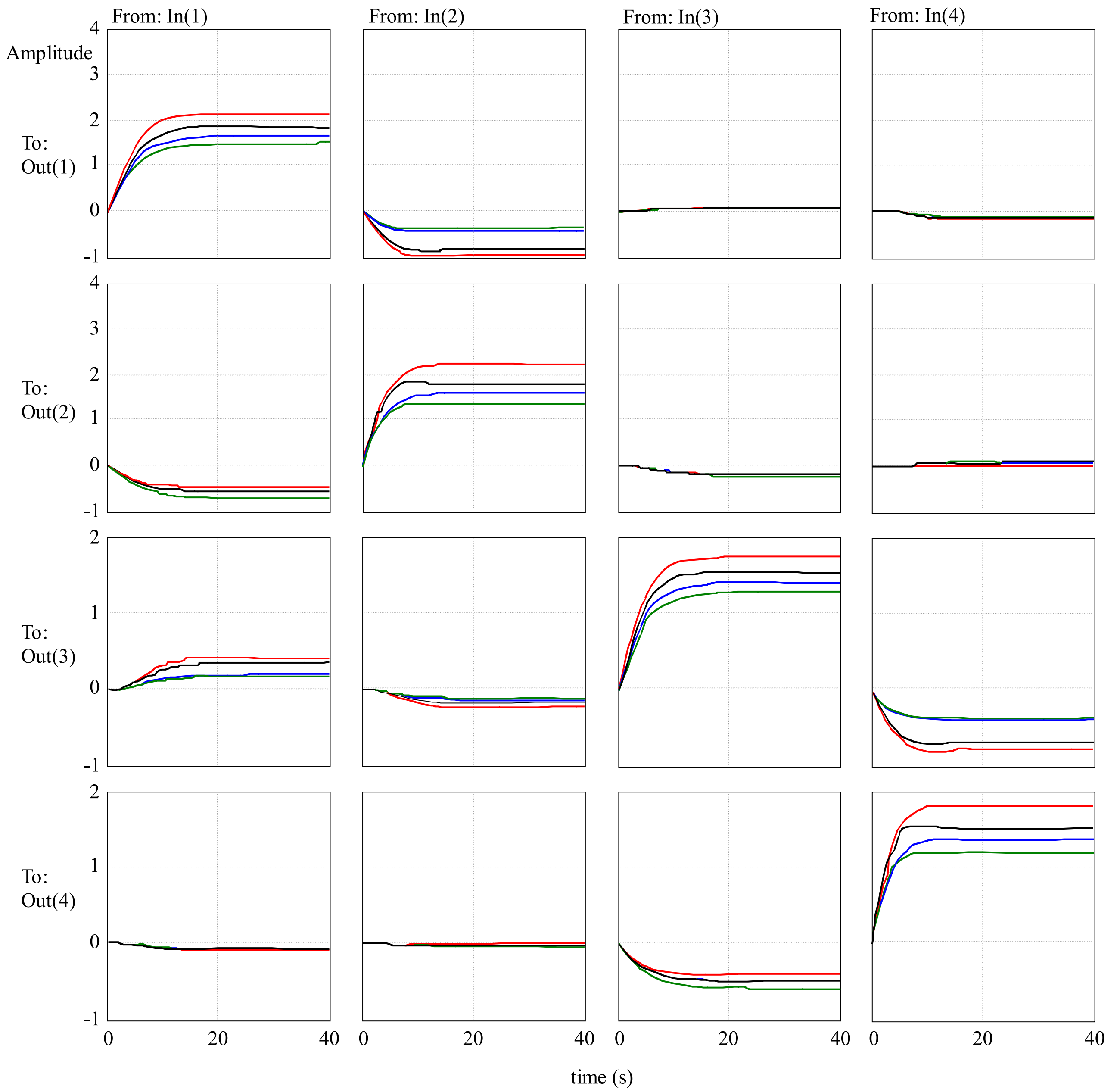


**Fig. 6.** Step response of system

four local PI controllers for a large scale DC system which will guarantee robust stability and performance of the closed-loop uncertain system. The system model is given by (1) and (2) with a time invariant matrix of 16 order type affine uncertain structure with 4 input and 4 output variables. The goal of design procedure is to design 4 PI controllers which guarantee the robustness properties and performance for closed-loop system.

Decentralized PI controller has the following parameters

$$K_P = \mathrm{diag}\{-0.5619, -0.2429, -0.7033, -0.3685\}\,,$$
$$K_I = \mathrm{diag}\{-0.1929, -0.1665, -0.2758, -0.2336\}\,.$$

V-K iteration for $\alpha = 1$ in LMI has $t_{\min} = -0.06122 < 0$, it shows that closed-loop system with above PI controller is robustly stable with the guaranteed performance.

## 6 CONCLUSIONS

In this paper, a new approach to design a robust output feedback PI controller for complex large-scale systems with a state decentralized structure is developed. The proposed design method is based on the Generalized Gershgorin Theorem and the LMI method to design robust PI controller guaranteeing feasible performance achieved in subsystems for the full system and therefore the proposed method excludes limit of system order in BMI solution. A robust decentralized PI controller has been designed using the polytopic description of the uncertain system and applying the robust optimal control design procedure including cost function to state-space subsystems generated in each vertex of the polytopic uncertainty domain.

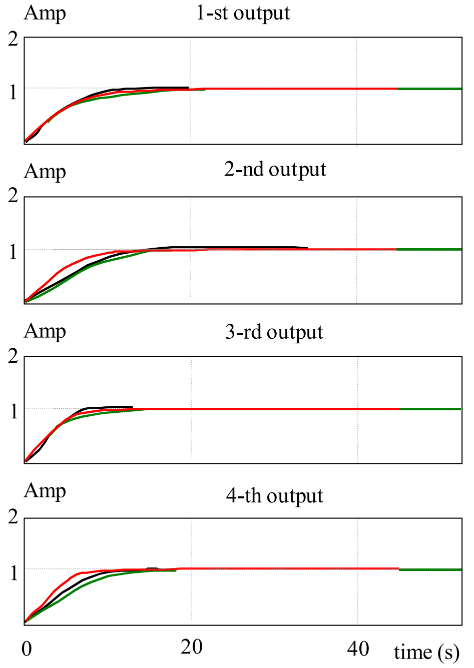


**Fig. 7.** Output signals of closed-loop feedback system

The main advantage of the proposed approach is that the order of the PI design procedure reduces to the order of the particular subsystem. The design strategy has been tested on various examples; the two presented in this article deal with three boiler-turbine subsystems and four cooperating DC motors with integral action, for which a robust PI controller has been designed. There is still many open questions starting with appropriate evaluation of interaction framework in the equivalent subsystem, feasibility or convergence, nevertheless, based on testing examples, the proposed control design scheme is believed to indicate the alternative in decentralized control, which can bring useful results.

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