Journal of ELECTRICAL ENGINEERING, VOL. 62, NO. 2, 2011, 99–103

STABLE MODEL PREDICTIVE CONTROL DESIGN: SEQUENTIAL APPROACH

Vojtech Veselý *

The paper addresses the problem of output feedback stable model predictive control design with guaranteed cost. The proposed design method pursues the idea of sequential design for N prediction horizon using one-step ahead model predictive control design approach. Numerical examples are given to illustrate the effectiveness of the proposed method.

K e y w o r d s: model predictive control, output feedback, Lyapunov function, guaranteed cost

1 INTRODUCTION

Model predictive control (MPC) has attracted notable attention in control of dynamic systems. The idea of MPC can be summarized as follows [1–6]:

– prediction of the future behavior of the process state/output over the finite time horizon;
– computation of the future input signals online at each step by minimizing a cost function under inequality constraints on the manipulated (control) and/or controlled variables;
– application on the controlled plant of the first vector control variable $u(t)$ and repetition of the previous step with new measured input/state/output variables.

Availability of the plant model is a necessary condition for the development of the predictive control [7]. The main criticism related to MPC is that due to the finite prediction horizon the algorithm in its original formulation does not guarantee closed-loop stability [8, 9, 12]. Excellent survey about stability, robustness properties and optimality of MPC is given in [10].

In this paper new design method is proposed that pursue the idea of sequential design for N prediction horizon using one-step ahead model predictive control design approach. The first modification of proposed sequential design method is based on classical LQ state feedback and solution of Diophantine equation. In the second modification of proposed method the Lyapunov function with guaranteed cost is adapted to obtain output feedback predictive control for N prediction horizon model with constraints on input variables. For both design approaches the bilinear matrix inequality (BMI) is adapted to obtain a feasible solution with respect to corresponding output feedback gain matrices.

The paper is organized as follows. The next Section gives problem formulation and some preliminaries about a predictive output/state model. In Section 3, two modification of sequential design methods for output feedback predictive control for N prediction horizon are proposed. The obtained methods are tested on three examples. At the end of the paper the conclusions are drawn.

Hereafter, the following notational conventions will be adopted: given a symmetric matrix $P = P^\top \in R^{n\times n}$, the inequality $P > 0$ denotes matrix positive definiteness. Given two symmetric matrices P, Q, the inequality $P > Q$ indicates that $P - Q > 0$. The notation $x(t+k)$ will be used to define at time t k-steps ahead prediction of a system variable x from time t onwards under specified initial state and input scenario. That is estimated predictive output at time $k = 1, 2, \ldots$ $y(t+k|t)$ will be denoted as $y(t+k)$. I denotes the identity matrix of corresponding dimensions.

2 PRELIMINARIES AND PROBLEM FORMULATION

Consider a time invariant linear discrete-time system

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $x(t) \in R^n$, $u(t) \in R^m$, $y(t) \in R^l$ are state, control and output variables of the system, respectively; A, B, C are known matrices of corresponding dimensions. The problem studied in this paper is to design model predictive control for the plant with following control algorithm

$$u(t) = F_{11}y(t) + F_{12}y(t+1) \tag{2}$$

and model predictive control for given prediction horizon N

$$u(t+k-1) = \sum_{i=1}^{k+1} F_{ki}y(t+i-1)\,,\ k = 2, 3, \ldots, N \tag{3}$$

where $F_{ki} \in R^{m\times l}$, $k = 1, 2, \ldots, N$, $i = 1, 2, \ldots, k+l$ is output (state) feedback gain matrix to be determined so that the stability of closed-loop MPC system is guaranteed and the below given cost function (6) is minimized with respect to the constraint.

* Slovak University of Technology in Bratislava, Slovakia, Faculty of Electrical Engineering and Information Technology, vojtech.vesely@stuba.sk

DOI: 10.2478/v10187-011-0016-0, ISSN 1335-3632

3 MODEL PREDICTIVE CONTROL DESIGN

Consider the system (1) with control algorithm (2). For the plant control algorithm we obtain

$$u(t) = F_{11}Cx(t) + F_{12}Cx(t+1) \rightarrow u(t) = (I - F_{12}CB)^{-1}(F_{11}C + F_{12}CA)x(t) = K_1x(t) \quad (4)$$

where $K_1 = (I - F_{12}CB)^{-1}(F_{11}C + F_{12}CA)$, I is the identity matrix of corresponding dimensions. For (4) the closed-loop system is as follows

$$x(t+1) = (A + BK_1)x(t) = D_1x(t)\,. \quad (5)$$

The control objective is usually to steer the state to the origin or to an equilibrium state x_r for which the output $y_r = Cx_r = w$, where w is the constant reference. A suitable change of coordinates reduces the second problem to the first one which we consider in the sequel. To determine the state feedback matrix K_r of system (5) the following cost function is defined as follows

$$J_1 = F_1\big(x(N_1)\big)+J_1(t) \quad (6)$$

where $F_1\big(x(N_1)\big)$ is a given terminal constraint at time N_1 and

$$J_1(t) = \sum_{t=t_0}^{N_1-1} x^\top(t)Q_1x(t) + u^\top(t)R_1u(t)\,,$$
$$Q_1 = Q_1^\top \geq 0 \in R^{n\times n}, \quad R_1 = R_1^\top \geq 0 \in R^{m\times m}$$

are corresponding weighting matrices.

Definition. Consider the system (1). If there exists a control law $u(t)^*$ and a positive scalar J_1^* such that the closed-loop system (5) is stable and the value of closed-loop cost function (6) satisfies $J_1 \leq J_1^*$ then J_1^* is said to be guaranteed cost and $u(t)^*$ is said to be guaranteed cost control law for the system (1).

Consider that the constraints on the maximum value of state denoted by $\max_t\big(x(t)^\top x(t)\big)$, and control input denoted by $\max_t\big(u(t)^\top u(t)\big)$ are known

$$\frac{\max_t\big(u(t)^\top u(t)\big)}{\max_t\big(x(t)^\top x(t)\big)} = \rho\,. \quad (7)$$

Then for the worst case the following linear matrix inequality (LMI) constraint can be formulated

$$\begin{bmatrix} \rho I & K_1^\top \\ K_1 & I \end{bmatrix} \geq 0\,. \quad (8)$$

Inequality (8), cost function (6) and system (1) are the basis for the calculation of the gain matrix K_1. Another approach to introduce input constraints the reader can find in [13]. There are several approaches to calculate K_1.

If (8) is missing the classical LQ design approach can be used, otherwise if $N_1 \rightarrow \infty$, $F\big(x(N_1)\big)= 0$ the bilinear matrix inequality can be formulated and solved using the linearization approach [5]; the linear matrix inequality is obtained with respect to matrix K_1. If different terminal constraints $F_1\big(x(N_1)\big)$ are chosen then different approaches for calculation of K_1 are used, for more detail see [4].

Consider that the state feedback gain matrix K_1 is known. From (4) the following Diophantine equation with respect to output feedback gain matrices F_{11}, F_{12} are obtained

$$K_1 = F_{11}C + F_{12}CD_1\,. \quad (9)$$

Note that if (A,B) is controllable then for the given matrices Q_1, R_1 a state feedback gain matrix K_1 exists guaranteeing minimal cost (6) and closed-loop stability of (5). Moreover if the solution of the Diophantine equation (9) with respect to matrices F_{11}, F_{12} exists, the proposed one-step ahead predictive control algorithm (4) guarantee for closed-loop system with output feedback the same properties as for the system obtained by state feedback with the gain matrix K_1. Note that according to the receding horizon strategy only the input variable $u(t)$ is applied to the plant as a manipulated variable. The one-step ahead design procedure is completed.

Consider the case $N = 2$. Model for prediction of state and output variable and predictive control algorithm are respectively

$$x(t+2) = Ax(+1) + Bu(t+1)\,,$$
$$u(t+1) = F_{21}y(t) + F_{22}y(t+1) + F_{23}y(t+2)\,.$$

For the manipulated variable $u(t+1)$ after some manipulation we obtain

$$u(t+1) = K_2x(t)$$

where

$$K_2 = \big(I - F_{23}CB\big)^{-1}\big(F_{21}C + F_{22}CD_1 + F_{23}CAD_1\big)\,.$$

The closed-loop system

$$x(t+2) = (AD_1 + BK_2)x(t) = D_2x(t) \quad (10)$$

for $N = 2$ is stable if and only if the matrices D_1, D_2 are stable. The matrix D_1 is already stable therefore the matrix D_2 has to be stable to guarantee closed-loop stability. The matrix K_2 can be calculated using the same approach as for the matrix K_1. The control objective for calculation of K_2 can be defined as follows

$$J_2 = F_2\big(x(N_1)\big)+J_2(t)$$

where

$$J_2(t) = \sum_{t=t_0}^{N_1-1} x^\top(t)Q_2x(t) + u^\top(t+1)R_2u(t+1)\,.$$

Note that the control input $u(t+1)$ is applied to the predictive model (10) for the calculation of the future output $y(t+2)$. It is not necessary to use the constraint (8) for K_2 calculation.

Sequentially, for $N = k$ step prediction one obtains the following closed-loop system

$$x(t+k) = AD_{k-1} + BK_k)x(t) = D_k x(t) \tag{11}$$

where $D_0 = I$, $D_k = AD_{k-1} + BK_k$, $k = 1, 2, \dots, N$.

The Diophantine equation for calculation of output feedback gain matrices (3) is

$$K_k = \sum_{j=1}^{k} F_{kj} C D_{j-1} + F_{kk+1} C D_k \,. \tag{12}$$

Cost function is given as follows

$$J_k = F_k\big(x(N_k)\big) + J_k(t)\,,$$
$$J_k(t) = \sum_{t=t_0}^{N_k-1} x(t) Q_k x(t) + u(t+k-1)^\top R_k u(t+k-1)\,. \tag{13}$$

The obtained results are summarized in the following theorem.

Theorem 1. *Consider the closed-loop system (11), control algorithm (2) and (3) and prediction horizon N. The closed-loop system is stable if the following conditions hold:*

- *the couple (AD_{k-1}, B) for $k = 1, 2, \dots, N$ is controllable, ie there exists a matrix K_k which ensures the closed-loop matrix stability and guaranteed cost*
- *there exists the solution of the Diophantine equation (12) with respect to output feedback matrices F_{kj}, $k = 1, 2, \dots, N$, $j = 1, 2, \dots, k+1$. Note that some matrices F_{kj}, $k = 1, 2, \dots, N$, $j \le k-1$ can be equal to zero.*

To avoid solution of the Diophantine equation (12) the second modification of design procedure has been proposed. Introduce the following predicted control algorithm

$$u(t+k-1) = F_{kk}(t+k-1) + F_{kk+1} y(t+k)\,, \qquad k = 1, 2, \dots, N\,. \tag{14}$$

Remarks.

1. Control algorithm for $k = N$ is $u(t+N-1) = F_{NN} y(t+N-1)$.
2. If one want to use the control horizon [1] $N_u < N$, the control algorithm $u(t+k-1) = 0$, $K_k = 0$, $F_{N_{u+1}N_{u+1}} = 0$, $F_{N_{u+1}N_{u+2}} = 0$ for $k > N_u$.

For k step prediction the closed-loop system (11) can be rewritten as follows

$$x(t+k) = (A + BF_{kk}C) D_{k-1} x(t) + BF_{kk+1} C x(t+k)\,.$$

The aim of the proposed second predictive control design procedure is to design gain matrices F_{kk}, F_{kk+1}, $k = 1, 2, \dots, N$ such that the closed-loop system (15) is stable with guaranteed cost when $F_k\big(x(N_k)\big) = 0$, $N_k \to \infty$. The closed-loop system (15) on the k step ahead model predictive control will be stable if and only if the first difference of Ljapunov function $V_k(t) = x(t)^\top P_k x(t)$, $P_k = P_k^\top > 0$ on the solution of (15) will be negative definite (semidefinite), that is

$$\Delta V_k(t) = V_k(t+k) - V_k(t) \le 0\,, \quad k = 1, 2, \dots, N\,. \tag{15}$$

Closed-loop system (15) will be stable with guaranteed cost iff the following inequality holds

$$B_{ek}(t) = \Delta V_k(t) + x^\top(t) Q_k x(t) + u(t+k-1)^\top R_k u(t+k-1) < 0\,. \tag{16}$$

The following theorem gives conditions to design the above-mentioned output feedback matrices.

Theorem 2. *The closed-loop system (15) is stable with guaranteed cost if and only if for $k = 1, 2, \dots, N$ there exist matrices $N_{k1} \in R^{n\times n}$, $N_{k2} \in R^{n\times n}$, F_{kk}, F_{kk+1} and a positive definite matrix $P_k = P_k^\top \in R^{n\times n}$ such that the following bilinear matrix inequality holds*

$$\begin{bmatrix} G_{k11} & G_{k12} \\ G_{k12}^\top & G_{k22} \end{bmatrix} \le 0 \tag{17}$$

where

$$K_{k11} = N_{k1}^\top M_{ck} + M_{ck}^\top N_{k1} + C^\top F_{kk+1}^\top R_k F_{kk+1} C + P_k\,,$$
$$G_{k22} = Q_k - P_k + D_{k-1}^\top C^\top F_{kk}^\top R_k F_{kk} C D_{k-1} + N_{k2}^\top A_{ck} D_{k-1} + D_{k-1}^\top A_{ck}^\top N_{k2}\,,$$
$$G_{k12}^\top = D_{k-1}^\top C^\top F_{kk}^\top R_k F_{kk+1} C + D_{k-1}^\top A_{ck}^\top N_{k1} + N_{k2}^\top M_{ck}$$

for $k = 1, 2, \dots, N$, where

$$M_{ck} = BF_{kk+1}C - I\,, \; A_{ck} = A + BF_{kk}C\,,$$
$$K_k = (I - F_{kk+1}CB)^{-1}(F_{kk}C + F_{kk+1}CA)D_{k-1}\,.$$

Proof. *Sufficiency*. From (15) one obtains

$$x(t+k) = -(M_{ck})^{-1} A_{ck} D_{k-1} x(t)\,.$$

Because the matrix

$$U_k^\top = \big[(-(M_{ck})^{-1} A_{ck} D_{k-1})^\top I\big]$$

has full row rank multiplying from left and right side of (17)

$$U_k^\top (\mathrm{eq}(17)) U_k$$

and obtained results multiplying from left by $x(t)^\top$ and right by $x(t)$, due to (18) the inequality (16) is obtained, which proves the sufficiency.

Necessity. Suppose that for k-step ahead model predictive control there exists such matrix $P_k = P_k^\top > 0$ that (17) holds. Necessity, there exists a scalar $\alpha > 0$ such that for the first difference of Ljapunov function

$$V_k^\top P_k V_k - P_k < -\alpha V_k^\top V_k \tag{18}$$

where $V_k = -M_{ck}^{-1} A_{ck} D_{k-1}$. Equation (18) can be rewritten as follows

$$V_k^\top (P_k + \alpha I) V_k - P_k < 0 \,.$$

Using Schur complement formula

$$\begin{bmatrix} P_k & -V_k^\top (P_k + \alpha I) \\ -(P_k + \alpha I) V_k & -(P_k + \alpha I) \end{bmatrix} < 0 \tag{19}$$

taking

$$N_{k1} = -\big(M_{ck}^{-1}\big)^\top \big(P_k + 0.5\alpha I\big) \,,$$
$$N_{k2}^\top = -D_{k-1}^\top A_{ck}^\top \big(M_{ck}^{-1}\big)^\top M_{ck}^{-1} \alpha 0.5$$

one obtains

$$-V_k(P_k + \alpha I) = D_{k-1}^\top A_{ck}^\top N_{k1} + A_{k2}^\top M_{ck}$$
$$-P_k = -P_k + N_{k2}^\top A_{k2}^\top A_{ck} D_{k-1}$$
$$+ D_{k-1}^\top A_{ck}^\top N_{k2} + \alpha V_k^\top V_k - (P_k + \alpha I) = 2 M_{ck}^\top N_{k1} + P_k \,.$$

For $\alpha \to 0$ one has got the first difference of Ljapunov function given by (17) (case $R_k = Q_k = 0$). If one substitute to second part of (16) instead of $u(t+k-1)$ (14), rewrite the obtained result to matrix form and take sum it with (19) the matrix inequality of (17) is obtained, which proves the necessity of theorem. It completes the proof.

If there exists a feasible solution to (17) with respect to F_{kk}, F_{kk+1}, N_{k1}, N_{k2}, $k = 1, 2, \dots, N$ and a positive definite matrix P_k, then the designed model predictive algorithm (14) ensures closed-loop stability and guaranteed cost.

Note that for $N = 1$, polytopic system $A = \sum_{i=1}^{S} A_i \alpha_i$ and parameter dependent Lyapunov function $P = \sum_{i=1}^{S} P_i \alpha_i$ the feasible solution of (17) guarantees the robustness properties of closed-loop onestep ahead predictive control (for more detail see [7, 10, 14]).

4 EXAMPLES

The first example serves as a benchmark. The continuous-time model of the double integrator has been converted to a discrete-time using the sampling period 0.1 s, the model turns to (1) with

$$A = \begin{bmatrix} 2 & -0.5 \\ 2 & 0.0 \end{bmatrix}, \; B = \begin{bmatrix} 0.125 \\ 0.0 \end{bmatrix}, \; C = [\,0.08 \quad 0.4\,] \,.$$

Eigenvalues of the matrix A are

$$\mathrm{eig}(A) = \{1, 1\} \,.$$

For prediction horizon $N = 3$ and guaranteed cost specified by the following weighting matrices

$$Q_1 = 5I \;\; R_1 = I \,, \; Q_2 = 10I \,, \; R_2 = I \,, \; Q_3 = I \,, \; R_3 = I$$

the following results are obtained:
Gain matrices for the state feedback:

$$K_1 = [\,8.2747 \quad -2.727\,] \,,$$
$$K_2 = [\,2.2454 \quad -1.0169\,] \,,$$
$$K_3 = [\,0.1934 \quad -0.3645\,] \,.$$

Solution of the Diophantine equation (9) for output feedback gain matrices are
For plant control $u(t)$

$$F_{11} = -6.49985 \,, \; F_{12} = 10.0251$$

and for output predictions $u(t+1)$

$$F_{21} = -1.8969 \,, \; F_{22} = 1.1255 \,, \; F_{23} = 1.7097 \,,$$

$$u(t+2): \quad F_{31} = -0.8204 \,, \; F_{32} = -0.0608 \,,$$
$$F_{33} = 0.2495 \,, \; F_{34} = 1.20981 \times 10^{-7} .$$

For $N = 3$ the closed-loop eigenvalues are

$$\mathrm{eig}(\mathit{CLOSED\text{–}LOOP}) = -0.035 \pm 0.3329i \,.$$

The model for the second example is given as follows

$$A = \begin{bmatrix} 0.6 & 0.0097 & 0.0143 & 0 & 0 \\ 0.012 & 0.9754 & 0.0049 & 0 & 0 \\ -0.0047 & 0.01 & 0.46 & 0 & 0 \\ 0.0488 & 0.0002 & 0.0004 & 1 & 0 \\ -0.0001 & 0.0003 & 0.0488 & 0 & 1 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.0425 & 0.0053 \\ 0.0052 & 0.01 \\ 0.0024 & 0.0474 \\ 0 & 0.0012 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

For the case of one-step ahead model predictive control $N = 1$ and guaranteed cost given by the matrices $Q_1 = 51$, $R_1 = I$ the following results are obtained.
State feedback matrix obtained by LQ method

$$K_1 = \begin{bmatrix} 0.4642 & 0.4052 & 0.0091 & 2.1971 & -0.154 \\ 0.0536 & 0.6707 & 0.3362 & 0.1361 & 2.1947 \end{bmatrix}$$

Solution of the Diophantine equation for output feedback:

$$F_{11} = \begin{bmatrix} 4.4269 & 6.514 & 1.5663 & -1.3642 \\ 6.6108 & 11.0884 & -0.9085 & 0.1941 \end{bmatrix}$$

$$F_{12} = \begin{bmatrix} -7.0219 & -14.42 & -0.1983 & -0.3201 \\ -11.6202 & -238367 & -0.3284 & -0.5298 \end{bmatrix}$$

Eigenvalues of the closed-loop system are

$$\mathrm{eig}(\mathit{CLOSED–LOOP}) = \{0.5909, 0.4541, 0.9627, 0.9905\}\,.$$

It is interesting to note that for the prediction horizon $N = 3$ the closed-loop eigenvalues have improved as follows

$$\mathrm{eig}(\mathit{CLOSED–LOOP}) = \{0.208, 0.0941, 0.91, 0.9768, 0.981\}\,.$$

In the third example we have taken the same system as in the second one. The task is to design two PS (PI) model predictive decentralized controllers for plant control input $u(t)$ for the prediction horizon $N = 5$. The cost function is given by following matrices

$$Q_1 = Q_2 = Q_3 = I\,,\ R_1 = R_2 = R_3 = I\,,$$
$$Q_4 = Q_5 = 0.5I\,,\ R_4 = R_5 = I\,.$$

The obtained decentralized output feedback gain matrices (solution of (17)) for control algorithm

$$u(t) = F_{11}y(t) + F_{12}y(t+1)$$

are

$$F_{11} = \begin{bmatrix} -0.2512 & 0 & -0.4353 & 0 \\ 0 & -0.269 & 0 & -0.3435 \end{bmatrix}$$

where proportional and integral gains of the first decentralized controller are

$$K_{1p} = 0.2512\,,\quad K_{1i} = 0.4353$$

and controller parameters for the second input are

$$K_{2p} = 0.269\,,\quad K_{2i} = 0.3435\,.$$

Because the output $y(t+1)$ is obtained from model prediction there is no need to use decentralized control structure for output feedback gain matrix F_{12}.

$$F_{12} \begin{bmatrix} -0.2253 & -0.1198 & -0.5957 & 0.1095 \\ -0.2024 & -0.2268 & -0.0547 & -0.6476 \end{bmatrix}$$

and finally for input of model prediction (assuming that $y(t+5)$ is available)

$$u(t+4) = F_{55}y(t+4) + F_{56}y(t+5)$$

the following gain matrices are obtained

$$F_{55} = \begin{bmatrix} -0.0046 & -0.0046 & -0.0874 & 0.0099 \\ -0.0102 & -0.0054 & -0.0044 & -0.0841 \end{bmatrix}$$
$$F_{56} = \begin{bmatrix} -0.00437 & -0.006 & -0.1067 & 0.0024 \\ -0.0134 & -0.0479 & -0.0141 & -0.1051 \end{bmatrix}.$$

Eigenvalues of the closed-loop system for model predictive control with $N = 5$ are as follows

$$\mathrm{eig}(\mathit{CLOSED–LOOP}) = \{0.074, 0.0196, 0.8864, 0.9838, 0.9883\}\,.$$

The above examples show that the proposed sequential design model predictive control procedure guarantees closed-loop stability and guaranteed cost.

5 CONCLUSION

The paper addresses the problem to develop original approach to sequential design of model predictive control. Sequential design consists of N steps. At first, one-step ahead model predictive control is designed using any of the two modified design techniques proposed in the paper. Repeating the one-step ahead design procedure N times, one obtains the N step ahead prediction horizon. The proposed design techniques guarantee closed-loop stability and guaranteed cost by minimizing in each step the given cost function. The results of the design procedure are the output feedback gain matrices for model predictive control and for real plant control with input $u(t)$.

Acknowledgment

The work has been supported by Grant N 01/0544/09 of the Slovak Scientific Grant Agency.

References

[1] CAMACHO, E. F.—BORDONS, C.: Model Predictive Control, Springer-Verlag, London, 2004.

[2] MACIEJOWSKI, J. M.: Predictive Control with Constraints, Prentice Hall, 2002.

[3] ROSITER, J. A.: Model Based Predictive Control: A Practical Approach, Control Series, 2003.

[4] CLARKE, D. W.—MOHTADI, C.: Properties of Generalized Predictive Control, Automatica **25** No. 6 (1989), 859–875.

[5] CLARKE, D. W.—SCATTOLINI: Constrained Receding-Horizon Predictive Control, Proceedings IEE **138** No. 4 (1991), 347–354.

[6] DERMICIOGLU, H.—CLARKE, D. W.: Generalized Predictive Control with End-Point Weighting, Proceedings of IEE **140**, part D No. 4 (1993), 275–282.

[7] KOTHARE, M. V.—BALAKRISHNAN, V.—MORARI, M.: Robust Constrained Model Predictive Control using Linear Matrix Inequalities, Automatica **32** No. 10 (1996), 1361–1379.

[8] RAWLINGS, J.—MUSKE, K.: The Stability of Constrained Receding Horizon Control, IEEE Trans. on Automatic Control **38** (1993), 1512–1516.

[9] ROSSITER, J. A.—KOUVARITAKIS, B.: Advances in Model Based Predictive Control. Chapter Advances in Generalized and Constrained Predictive Control, Oxford University Press, 1994.

[10] MAYNE, D. Q.—RAWLINGS, J. B.—RAO, C. V.—SCOKAERT, P. O. M.: Constrained Model Predictive Control: Stability and Optimality, Automatica **36** (2000), 789–814.

[11] de OLIVEIRA, M. C.—CAMINO, J. F.—SKELTON, R. E.: A Convexifying Algorithm for the Design of Structured Linear Controllers, Proceedings 39th IEEEE Conference on Decision and Control, Sydney, 2000, pp. 2781–2786.

[12] VESELÝ, V.—BARS, R.: Stable Output Feedback Model Predictive Control Design: LMI Approach, Archives of Control Sciences **18** No. 3 (2008), 277–286.

[13] AYD, H.—MESQUINE, F.—AITRAMI, M.: Robust Control for Uncertain Linear Systems with State and Control Constraints, In: Proc. of the 17th World Congress IFAC, Seoul, Korea, 2008, pp. 1153–1158, CD-ROM.

[14] VESELÝ, V.—ROSINOVÁ, D.: Robust Output Model Predictive Control Design: BMI Approach, International Journal of Innovative Computing, Information and Control ICIC International **5** No. 4 (Apr 2009), 1–9.

Received 27 April 2009