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A Comparison of Different Short-Term Macroeconomic Forecasting Models: Evidence from Armenia

Abstract: We evaluate the forecasting performance of four competing models for short-term macroeconomic forecasting: the traditional VAR, small scale Bayesian VAR, Factor Augmented VAR and Bayesian Factor Augmented VAR models. Using Armenian quarterly actual macroeconomic time series from 1996Q1 – 2014Q4, we estimate parameters of four competing models. Based on the out-of-sample recursive forecast evaluations and using root mean squared error (RMSE) criterion we conclude that small scale Bayesian VAR and Bayesian Factor Augmented VAR models are more suitable for short-term forecasting than traditional unrestricted VAR model.

Key words: vector autoregression, Bayesian estimation, principal components, recursive regression, forecast evaluation, macroeconomic indicators, Armenia.

JEL classification codes: C11, C13, C52, C53

1. Introduction

In order to conduct effective monetary policy, central bank practitioners are interested in producing accurate forecasts of the relevant macroeconomic variables. It is well known that monetary policy decisions can affect an economy with a certain lag. Therefore, monetary policy authorities must be forward-looking, that is, they should know what will happen with the key macroeconomic variables in the future. On the other hand, some important macroeconomic variables, especially real GDP growth, are available around two months after the end of reference quarter. Having accurate forecasts for the key macroeconomic variables (one or two quarters ahead) is an important ingredient for the inflation targeting model. For this reasons, in this paper we study the performance of different models for short-term macroeconomic forecasting, namely the traditional VAR, small Bayesian VAR, Factor Augmented VAR and Bayesian Factor Augmented VAR models (hereafter VAR, BVAR, FAVAR and BFAVAR).

There are some important differences between these models. For example, the unrestricted VAR can be applied, as a rule, for small dataset, the BVAR for both small and large datasets, while the FAVAR and BFAVAR models can be applied for a large dataset. The BVAR is a model with restrictions because we set priors on the parameters. The FAVAR and BFAVAR models, with the exception of the main variables also include the so-called principal components (or factors). Dynamics of static or principal components can be extracted based on the additional explanatory variables. After extracting the dynamics of principal components, the FAVAR (BFAVAR) model is estimated in the manner of the traditional unrestricted VAR (BVAR) model. Thus, one of the important questions that can arise is how we can extract the dynamics of static or dynamic principal components?

There are three factor models that are frequently used in applications (Barhoumi, Darne & Ferrara, 2009): 1) The static principal component approach (Stock & Watson, 2002), 2) The dynamic principal components estimated in the frequency domain (Forni et al., 2005), and 3) The dynamic principal components estimated in the time domain (Doz, Gianonne & Reichlin, 2011, 2012)). The Stock and Watson approach uses eigenvalues and eigenvectors of the covariance (correlation) matrix of the initial variables to extract the principal components. The Forni et al. approach, also known as the generalized dynamic factor models, uses time series spectral analysis methodology, while the Doz, Gianonne & Reichlin approach uses Kalman filter and state space modelling methodology to extract the principal components. All mentioned factor models have the same purpose, namely, given a large number of initial variables, to extract only a small number of factors which summarize the most part of information contained in the whole dataset. In this paper we use both static and dynamic approaches to estimate the dynamics of principal components. As a dynamic approach, we use an algorithm proposed (Doz, Gianonne & Reichlin, 2011, 2012).

Using Armenian quarterly macroeconomic time series from 1996Q1 – 2014Q4, we estimate the dynamics of principal components. For that we use 21 additional macroeconomic variables. A set of additional macroeconomic variables comprising information on monetary and financial variables, international price indices, the European Union and the Russian Federation business activity variables. The data set is selected from the Central Bank of Armenia and the National Statistical Agency internal databases as well as from the internet source databases. Using selected macroeconomic variables, we estimate parameters of four competing models. Based on the out-of-sample recursive forecast evaluations and using root mean squared error (RMSE) criterion, we conclude that small scale BVAR and BFAVAR models perform better for short-term forecasting purposes than the traditional unrestricted VAR model.

The remaining paper is organized as following. In section 2 we briefly present four basic forecasting models (VAR, small BVAR, FAVAR and BFAVAR), as well as detail algorithms of the extraction of principal components. In section 3 we present the actual dynamics of the key macroeconomic variables in Armenia, and also provide an explanation relating to additional explanatory variables that were used for extraction of the static and dynamic principal components. In section 4 we present the recursive regression scheme for our experimental design. In section 5 we present out-of-sample forecast evaluation results. Section 6 concludes.

2. Overview of the basic forecasting models (VAR, small BVAR, FAVAR and BFAVAR)

This part of the paper outlines the basics of the competing models, namely the unrestricted VAR, BVAR, FAVAR and BFAVAR.

It is known that the unrestricted VAR model can be presented as:

$$y_t = A_0 + A_1 y_{t-1} + A_2 y_{t-2} + \dots + A_p y_{t-p} + \nu_t \quad (t = 1,\dots,T)$$

where y_t is a $(n\times 1)$ vector of variables to be forecasted, A_0 is a $(n\times 1)$ vector of constant terms, $A_1, A_2, \dots, A_p$ is $(n\times n)$ matrix of estimated parameters for different lag length $(l = 1,2, \dots ,p)$, ν_t is $(n\times 1)$ vector of error terms. We assume that $\nu_t \sim N(0, \sigma^2 I_{n\times n})$, where $I_{n\times n}$ is $(n\times n)$ identity matrix.

It is known that the parameters of the VAR model can be consistently estimated using traditional OLS algorithm (Hamilton, 1994). But from the other side in the VAR model very often we need to estimate many parameters. This over parametrization could cause inefficient estimates and hence a large out-of-sample forecast error. An alternative approach to overcoming this over parametrization is to use a Bayesian VAR approach (Gupta & Kabundi, 2009a, 2009b).

The main idea of the BVAR model is that this algorithm imposes restrictions on the lags. According to the BVAR we assume that parameters of the model should be closer to zero for longer lags and they should differ from zero for shorter lags. The restrictions are imposed by specifying normal prior distributions with zero mean and small standard deviation decreasing as the lag increase. The exception to this is that the coefficient on the first own lag of a variable has a mean of unity. In the econometrics such type of priors are known as the "Minnesota priors" due to its development at the University of Minnesota and the Reserve Bank of Minneapolis (Litterman, 1981). Thus according to the "Minnesota priors" rule the prior mean and standard deviation of the BVAR model parameters can be set as follows.

1. The parameters of the first lag of the dependent variables follow an AR (1) process while parameters for other lags equal to zero.

2. The variance of the priors according to the Minnesota approach can be specified as follows:

$$\left(\frac{\lambda_1}{l^{\lambda_3}}\right)^2 \textit{ if } i = j, \quad \left(\frac{\sigma_i \lambda_1 \lambda_2}{\sigma_j l^{\lambda_3}}\right)^2 \text{ if } i \neq j, \quad (\sigma_1 \lambda_4)^2 \text{ for the constant.}$$

Where, i refers to the dependent variable in the i-th equation and j to the independent variables in that equation, σ_i and σ_j are standard errors from AR regressions estimated via OLS. The ratio of σ_i and σ_j controls for the possibility that variable i and j may have different scale (l is the lag length). The λ's set by researcher, that control of the tightness of the prior. Canova (2008) reports the following values for these parameters: $\lambda_1=0.2$, $\lambda_2=0.5$, $\lambda_3=1$ or 2, $\lambda_4=10^5$.

In order to understand how we can set the priors on the parameters, let's consider the following example. Assume that we have a two-variable VAR model.

$$x_t = a_{10} + a_{11}x_{t-1} + a_{12}y_{t-1} + b_{11}x_{t-2} + b_{12}y_{t-2} + \nu_t$$

$$y_t = a_{20} + a_{21}x_{t-1} + a_{22}y_{t-1} + b_{21}x_{t-2} + b_{22}y_{t-2} + u_t$$

From this model we can see that coefficients a_{11} and a_{22} are the first order autoregression parameters. Therefore, according to the Litterman (1981) the vector of priors for the parameters equals: $\tilde{b}_0 = (0,1,0,0,0,0,0,1,0,0)'$. In other words, except the coefficients a_{11} and a_{22}, the prior mean for all other parameters equal zero.

The matrix of the prior variances is diagonal and it has the following form:

$$H=\begin{bmatrix} (\sigma_1\lambda_4)^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & (\lambda_1)^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \left(\frac{\sigma_1\lambda_1\lambda_2}{\sigma_2}\right)^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \left(\frac{\lambda_1}{2^{\lambda_3}}\right)^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \left(\frac{\sigma_1\lambda_1\lambda_2}{\sigma_2 2^{\lambda_3}}\right)^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & (\sigma_2\lambda_4)^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \left(\frac{\sigma_2\lambda_1\lambda_2}{\sigma_1}\right)^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & (\lambda_1)^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \left(\frac{\sigma_2\lambda_1\lambda_2}{\sigma_1 2^{\lambda_3}}\right)^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \left(\frac{\lambda_1}{2^{\lambda_3}}\right)^2 \end{bmatrix}$$

The diagonal elements of the matrix H are the prior variances for each corresponding coefficient. For example, the diagonal elements $(\sigma_1\lambda_4)^2$ and $(\sigma_2\lambda_4)^2$ are the prior variances for the constant parameters a_{10} and a_{20}. The elements $(\lambda_1)^2$ and $\left(\frac{\lambda_1}{2^{\lambda_3}}\right)^2$ are the prior variances for the parameters a_{11}, b_{11}, and a_{22} b_{22} respectively. The diagonal elements $\left(\frac{\sigma_1\lambda_1\lambda_2}{\sigma_2}\right)^2$ and $\left(\frac{\sigma_1\lambda_1\lambda_2}{\sigma_2 2^{\lambda_3}}\right)^2$ are the prior variances for a_{12}, b_{12}, while $\left(\frac{\sigma_2\lambda_1\lambda_2}{\sigma_1}\right)^2$ and $\left(\frac{\sigma_2\lambda_1\lambda_2}{\sigma_1 2^{\lambda_3}}\right)^2$ are the prior variances for a_{21}, b_{21}.

Thus having priors we can calculate the posterior parameters using Bayesian approach to estimation. For the VAR model, the posterior parameters can be estimated using the following formulas:

$$\beta^* = (H^{-1} + \Sigma^{-1} \otimes X_t' X_t)^{-1} (H^{-1}\tilde{b}_0 + \Sigma^{-1} \otimes X_t' X_t \hat{b})$$

$$\mathrm{var}(\beta^*) = (H^{-1} + \Sigma^{-1} \otimes X_t' X_t)^{-1}$$

Where, β^* is the vector of the posterior parameters, $\tilde{b}_0$ is the vector of the prior parameters, H is the diagonal matrix with the prior variances on the diagonal, X is the ($T\times k$) matrix of the initial time series, Σ - is the ($k\times k$) identity matrix.

In the FAVAR (BFAVAR) model, the first thing that should be solved is to estimate the dynamics of principal components. As a rule, the FAVAR (BFAVAR) model can be estimated in two steps: the first step is principal components extraction and the second step is model estimation and forecasting. Principal components are linear combinations of the initial set of variables with the property that they maximize the explained portion of the variance of the initial data set. Principal components provide the way to reduce the dimensionality of the initial set of variables.

As it was mentioned in the introduction the principal components can be extracted using three approaches:

1. The static principal components as in Stock and Watson (2002),
2. The dynamic principal component approach (frequency domain) as in Forni at al. (2005),
3. The dynamic principal component approach (time domain) as in Doz, Gianonne & Reichlin (2011, 2012).

In this paper, we use both static and dynamic approaches. For the dynamic approach we use algorithm proposed by Doz, Gianonne & Reichlin (2011, 2012). Now let's present some details relating with using the abovementioned approaches.

1. The static factor model (Stock & Watson, 2002). To estimate the dynamics of principal components according to Stock and Watson approach we proceed as follows (Schumacher, 2007). We start with a collection a stationary ($Nx1$) time series vector $x_t = (x_{1t}, x_{2t},..., x_{Nt})'$ $t = 1,2,...,T$. Let $\hat{\Gamma}_0 = \sum_{i=1}^{T} x_t x_t^{'}$ be an estimate of the variance-covariance matrix of the initial set of variables. The aim is to find r linear combinations of the time series data $f_{i,t} = \hat{s}_i^{'} x_t$ ($i = 1,2,...,r$), that maximize the variance of the factors $max_s\,(\hat{s}_i^{'}\hat{\Gamma}_0\hat{s}_i)$. Imposing the usual restriction that $\hat{s}_i^{'}\hat{s}_i = 1$ and solving the optimization problem $L = \hat{s}_i^{'}\hat{\Gamma}_0\hat{s}_i - \hat{\mu}_i(\hat{s}_i^{'}\hat{s}_i - 1)$, we find the matrix equation $\hat{\Gamma}_0\hat{s}_i = \hat{\mu}_i\hat{s}_i$, where $\hat{\mu}_i$ denotes the i-th eigenvalue of $\hat{\Gamma}_0$ and $\hat{s}_i$ the ($Nx1$) corresponding eigenvector. Since $\hat{s}_i$ cannot be zero, the matrix equation has a non-trivial solution if and only if $\|\hat{\Gamma}_0 - \hat{\mu}_i I\| = 0$. Thus, in order to estimate the principal components we need to find the eigenvalues and eigenvectors of $\hat{\Gamma}_0$.

According to the static principal component approach, the r eigenvectors corresponding to the first largest eigenvalues are the weights of the static principal components. So according to this approach, the principal components can be calculated as $f_{i,t} = \hat{s}_i^{'} x_t$, where $\hat{s}_i$ is the corresponding eigenvectors of the matrix $\hat{\Gamma}_0$.

2. The dynamic factor model (Doz, Gianonne & Reichlin, 2011, 2012). The dynamic factor model in the state – space form can be presented as:

$$y_t = \Lambda f_t + \varepsilon_t \qquad \varepsilon_t \sim N(0, R)$$

$$f_t = A_1 f_{t-1} + A_2 f_{t-2} + \ldots + A_p f_{t-p} + u_t \qquad u_t \sim N(0, Q)$$

In this model f_t is a $(r \times 1)$vector of unobserved factors and $\varepsilon_t = [\varepsilon_{1t}, \varepsilon_{2t}, \ldots, \varepsilon_{nt}]$ is the idiosyncratic component, uncorrelated with f_t at all lags and leads (Banbura & Madugno, 2010), Λ is $(n \times r)$ matrix of factor loadings, $A_1, A_2, \ldots, A_p$ are $(r \times r)$ matrices of autoregressive coefficients. In order to estimate the dynamics of the principal components, first we have to estimate eigenvalues (λ) and eigenvectors (F) of the initial set of variables using the standard principal component analysis. Then we obtain A and Q by estimating the unrestricted VAR model on $\hat{F}$ obtained in the previous step. The elements of matrix R we estimate as follows, $y_t - \hat{\Lambda}\hat{f}_t = \hat{\varepsilon}_t$. For the estimation of the dynamics of f_t we can use the two-step Kalman filter or quasi – maximum likelihood algorithm (Doz, Gianonne & Reichlin, 2011,2012). The two-step Kalman filter algorithm assumes the following steps.

First step: Kalman filter step:

$$L = (\Lambda_t P_{t|t-1} \Lambda_t^{'} + R_t)^{-1}$$

$$f_{t|t} = f_{t|t-1} + P_{t|t-1} \Lambda' L (y_t - \Lambda f_t)$$

$$P_{t|t} = P_{t|t-1} - P_{t|t-1} \Lambda' L \Lambda P_{t|t-1}$$

$$K_t = A P_{t|t} \Lambda' L$$

$$f_{t+1|t} = A f_{t|t} + K_t (y_t - \Lambda f_t)$$

$$P_{t+1|t} = A P_{t|t-1} A' + Q$$

Second step: Smoothing step:

$$f_{t|T} = f_{t|t} + P_{t|t} A' P_{t+1|t}^{-1} (f_{t|T+1} - f_{t+1|t})$$

$$P_{t|T} = P_{t|t} + P_{t|t} A' P_{t+1|t}^{-1} (P_{t|T+1} - P_{t+1|t})(P_{t|t} A' P_{t+1|t}^{-1})'$$

Relating with the quasi maximum likelihood algorithm we can note that this algorithm include two steps Kalman filter, just the main difference is that here two steps Kalman filtering can be applied many times, while desired correctness will be achieved.

For forecasting purposes we use small scale VAR model containing variables of interest augmented by extracted factors. Following the paper by Bernanke, Boivin & Eliasz, 2005, the FAVAR (BFAVAR) model can be presented as follows:

$$\begin{vmatrix} Y_t \\ F_t \end{vmatrix} = A_1 \begin{vmatrix} Y_{t-1} \\ F_{t-1} \end{vmatrix} + A_2 \begin{vmatrix} Y_{t-2} \\ F_{t-2} \end{vmatrix} + \ldots + A_p \begin{vmatrix} Y_{t-p} \\ F_{t-p} \end{vmatrix} + \begin{vmatrix} v_t \\ u_t \end{vmatrix}$$

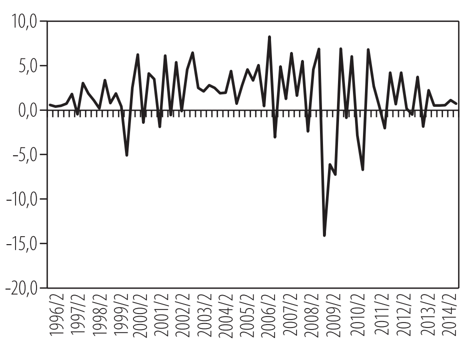
where, Y_t is the vector of observable variables, F_t is the vector of unobserved variables, which can be estimated by using static and dynamic factor models, A_1, A_2, ..., A_p are (rxr) matrices of estimated parameters. In the model above, parameters can be estimated using traditional OLS or Bayesian algorithm. ν_t and u_t is the error terms in the FAVAR (BFAVAR) model, with zero mean and diagonal variance-covariance matrices, Q and V.

1. Data

For estimating the small-scale VAR and BVAR model we use the following three macroeconomic variables, particularly GDP growth, inflation and short-term interest rate (from 15 days to 1 year). In the process of selection of the initial variables, we closely follow the paper by (Gupta & Kabundi, 2009a, 2009b). This is because we want to keep comparability of our work with other similar works. Our data set consists of quarterly time series starting with 1996Q1 – 2014Q4, 76 observations in total for each variable. Now let's present the dynamics of the mentioned variables in more details.

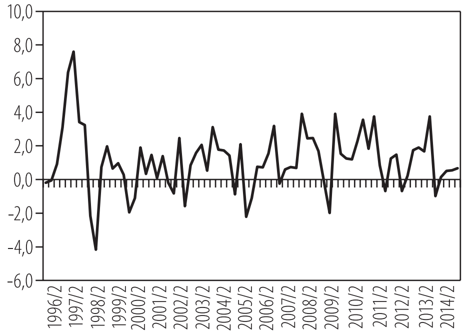
The following preliminary calculations have been done for real GDP: absolute values of real GDP were logged and then seasonally adjusted to calculate the first differences. In the result we obtain the values of GDP real growth rates (Figure 1).

Figure 1: Real GDP growth rate (in % to the previous quarter)



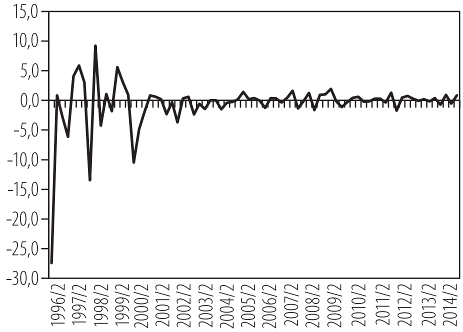
The next important macroeconomic variables that we want to include in the VAR (small scale BVAR) model is the inflation rate. As inflation we use the consumer price index (CPI). The preliminary treatment of the inflation dynamics includes the following procedures. First of all we recalculate the CPI chain indices to the base quarter (1995Q4 = 100). Then we take the logged values and apply seasonal adjustment to extract seasonality from CPI indices and after that we calculate the first differences. The result is the inflation dynamics presented in Figure 2.

Figure 2: Inflation rate (in % to the previous quarter)



The third important variable that we want to include in the small scale VAR (BVAR) model is the short-term nominal interest rate (from 15 days to 1 year) for deposits in national currency. The preliminary treatments for this variable include only first differences (in percentage points) (Figure 3).

Figure 3: Short-term nominal interest rate (difference in % points)



For the estimation of the large scale FAVAR (BFAVAR) model, we use a set of additional variables to extract the dynamics of static and dynamic principal components. This set of additional variables comprising information on monetary and financial variables, international volume and price indices, the EU and Russia business activities and growth rates. The sample periods of additional variables spans from 1996Q1 – 2014Q4. The additional data set were selected from different sources, particularly from the Central Bank of Armenia and the National Statistical Agency (NSA) internal database; international indices were selected from the following web sites: *http://www.indexmundi.com/*, and *http://stats.oecd.org/*. For some of the additional variables, the seasonal adjustment procedures have been applied. All no stationary time series are made stationary through first differencing. The name and some other important characteristics of the additional variables are presented in Appendix 1.

All calculations and forecasts have been done using the computer package developed by the author. The developed software is possible to download (for free) from the following website: *http://www.forecasting.somee.com*. Using this package, all calculations and forecasts can be done directly in the Microsoft Excel (2010, 2013) spreadsheets.

2. Experimental design

To conduct out-of-sample forecast experiments, we use recursive regression scheme. This is because our data set is relatively short and it contains structural changes. In this paper, the in-sample period spans from 1996:Q2 - 2009:Q2 (53 observations for each time series), while the out-of-sample period is 2009:Q3 - 2014:Q4 (22 observations for each time series).

The recursive simulation scheme proceeds as follows: First we estimate the model using subsample 1996:Q2-2009:Q2 (53 observations). Using the estimated model, we generate 1 to 4 steps-ahead forecasts (2009:Q3, 2009:Q4, 2010:Q1, 2010:Q2). Then we increase the sample size by one, that is 1996:Q2 - 2009:Q3 (54 observations) and generate again 1 to 4 steps-ahead forecasts (2009:Q4, 2010:Q1, 2010:Q2, 2010:Q3). We continue increasing the sample size by one and generating 1 to 4 steps-ahead forecasts until the sample spans from 1996:Q2 - 2013:Q4. Then we increase the sample size by one, that is 1996:Q2 - 2014:Q1 (72 observations) but only generate 1 to 3 steps-ahead forecasts (2014:Q2, 2014:Q3, 2014:Q4). Then we increase the sample size by one and generate 1 to 2 steps - ahead forecasts (2014:Q3, 2014:Q4). Continuing in such manner we will have 22 points for 1 step-ahead, 21 points for 2 steps-ahead, 20 points for 3 steps-ahead, and 19 point for 4 steps-ahead forecasts.

Next, we use the out of sample forecasts from recursive regression to compute the corresponding root mean squared errors (RMSE) for each forecasting horizons separately. More specifically, let us denote the out of sample period T^* by (in our case, $T^*= 22$, namely 2009:Q3-2014:Q4), and the forecast horizon by h (h=1,2,3,4). Then the RMSE is calculated:

$$RMSE_{ih} = \sqrt{\frac{1}{T^* - (h-1)} \sum_{t=1}^{T^*-(h-1)} (\hat{y}_{it} - y_{it})^2}$$

where y_{it} denotes the actual value of the i-th dependent variable (in our case we have three dependent variables i=1,2,3), $\hat{y}_{it}$ is the forecasted value of the i-th dependent variable, and $RMSE_{it}$ is the root mean squared error calculated for the i-th dependent variable and the h-th forecast horizon.

3. Forecast evaluation results

In this section we estimate four competing models, namely the traditional VAR, the small-scale BVAR, FAVAR and BFAVAR over the period 1996:Q2-2009:Q2. Using the Armenian quarterly macroeconomic time series, we compute the out-of-sample 1 to 4 steps ahead forecasts with using recursive regression. Based on the out-of-sample forecast experiments and using root mean squared error (RMSE) criterion we conclude on the most relevant model. Before presenting the results of forecast evaluation, we need to decide about the number of lags and the number of static and dynamic principal components.

We use different lag lengths to estimate parameters of the models, particularly from one lag up to four lags. There is no reason to use more than four lags since our purpose is to generate short-term forecasts up to one year. Thus we estimate the models separately for one, two, three, and four lags. From the other side, such approach will give possibilities to check the robustness of models under different lag lengths.

To estimate the number of principal components, the formal statistical tests can be used, as a rule. But in this paper we use visual graphical inspection approach to estimate the number of static principal components. The idea of this approach is to compute the eigenvalues using covariance (or correlation) matrix of the initial explanatory variables. Then we construct the graphics of the computed eigenvalues. Thus, based on the actual initial data series the eigenvalues were computed and result is presented in Figure 4.

Figure 4: The dynamics of eigenvalues of the initial variables correlation matrix

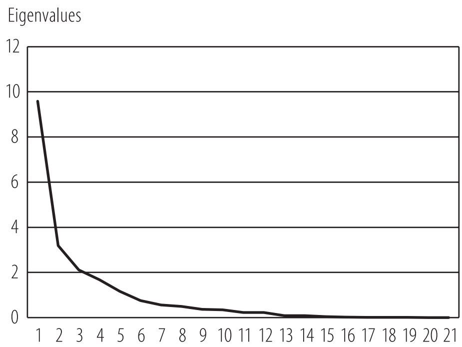


Figure 4 indicates that the difference between two successive eigenvalues significantly weakened after the fourth eigenvalue. Therefore, for the FAVAR (BFAVAR) model we can choose first four principal components, which explain about 79% percent of variability of the initial time series. Thus, the number of dynamic factors in our case can fluctuate between 1 and 4 because, as it is known, the number of dynamic factors cannot be more than the number of static factors (see MATLAB files from *http://homepages.ulb.ac.be/~dgiannon/*). Thus, to estimate the FAVAR model, we can use the following possible combinations of the dynamic and static factors: 1) one dynamic and four static factors, 2) two dynamic and four static factors, 3) three dynamic and four static factors, and 4) four dynamic and four static factors. Thus we conduct estimation experiments for all possible combinations of dynamic and static factors, which also allow us to check the robustness of the different FAVAR (BFAVAR) models to extracted principal components.

Thus, after deciding the number of lags and appropriate number of static and dynamic principal components we can conduct estimation and forecasting experiments. For that we use the following four competing models:

1. Unrestricted VAR model, where we use three endogenous variables (GDP growth, inflation, and short-term nominal interest rate),
2. Small-scale Bayesian VAR model, where we use three endogenous variables (GDP growth, inflation, and short-term nominal interest rate),
3. Factor Augmented VAR where, in addition to the three main endogenous variables, we use static and dynamic principal components.
4. Bayesian FAVAR models, where in addition to the three main endogenous variables we also use static and dynamic principal components.

Also we need to note that for small BVAR model we do grid search over all possible combinations of hyper parameters and lag lengths. As it was mentioned, we allow from 1 to 4 lags. Overall tightness is set to range from 0.1 to 0.3, with increments of 0.1 (as in Gupta and Kabundi (2009a, 2009b)). The decay factor takes values of 1 and 2. We select the hyper parameters and lag lengtha by looking at the pseudo out-of-sample forecast performances, the model having the minimum RMSE is selected as the chosen model for forecasting at all horizons.

The results of the RMSE for the recursive regression scheme are presented in tables 1 to 3.

Table 1: RMSE for the real growth of GDP[1]

Number of lags	Forecasting model	Forecast horizon 1	2	3	4	Average value of the RMSE
1 lag	VAR	3.26	3.17	3.19	3.11	**3.19**
	BVAR (w = 0.3; d = 1)[2]	3.43	3.16	3.20	3.11	3.23
	FAVAR_SW (4)[3]	3.52	3.35	3.26	3.05	3.29
	FAVAR_TS (2, 4)[4]	3.11	3.44	3.11	3.27	3.23
	FAVAR_QML (2, 4)	3.03	3.39	3.11	3.21	**3.19**
	BFAVAR_SW (4) (w = 0.3; d = 1)	3.47	3.20	3.28	3.03	3.25
	BFAVAR_TS (1, 4) (w = 0.3; d = 1)	3.27	3.21	3.16	3.17	3.20
	BFAVAR_QML (1, 4) (w = 0.3; d = 1)	3.22	3.19	3.21	3.16	**3.19**
2 lags	VAR	3.64	3.32	3.29	3.14	3.35
	BVAR (w = 0.3; d = 2)	3.51	3.21	3.22	3.12	3.27
	FAVAR_SW (4)	4.27	3.51	3.04	3.50	3.58
	FAVAR_TS (2, 4)	3.94	3.58	3.24	2.87	3.41
	FAVAR_QML (2, 4)	3.93	3.64	3.35	3.21	3.53
	BFAVAR_SW (4) (w = 0.3, d = 2)	3.59	3.21	3.22	3.18	3.30
	BFAVAR_TS (1, 4) (w = 0.3; d = 2)	3.33	3.30	3.20	3.10	**3.23**
	BFAVAR_QML (1, 4) (w = 0.3; d = 2)	3.30	3.30	3.24	3.22	3.27
3 lags	VAR	4.24	3.39	3.64	2.77	3.51
	BVAR (w = 0.3; d = 2)	3.76	3.19	3.37	3.07	**3.35**
	FAVAR_SW (4)	5.25	4.00	3.30	3.34	3.97
	FAVAR_TS (1, 4)	4.30	3.97	3.23	2.95	3.61
	FAVAR_QML (1, 4)	4.28	4.02	3.65	3.48	3.86
	BFAVAR_SW (4) (w = 0.3; d = 1)	4.08	3.23	3.24	3.23	3.44
	BFAVAR_TS (3, 4) (w = 0.3; d = 1)	4.03	3.18	3.19	3.19	3.40
	BFAVAR_QML (3, 4) (w = 0.3; d = 1)	4.13	3.21	3.31	3.29	3.48
4 lags	VAR	4.27	3.30	3.57	3.08	3.55
	BVAR (w = 0.2; d = 1)	3.75	3.22	3.32	3.10	**3.35**
	FAVAR_SW (4)	6.33	4.79	3.84	3.03	4.50
	FAVAR_TS (3, 4)	5.51	4.06	4.48	3.05	4.28
	FAVAR_QML (3, 4)	5.49	4.40	4.55	4.02	4.62
	BFAVAR_SW (4) (w = 0.3; d = 1)	4.91	3.66	3.50	3.24	3.83
	BFAVAR_TS (1, 4) (w = 0.3; d = 1)	4.40	3.04	3.05	3.01	3.37
	BFAVAR_QML (1, 4) (w = 0.3; d = 1)	3.95	3.28	3.15	3.25	3.41

1 FAVAR_SW is a FAVAR model with static principal components (Stock and Watson, 2002), FAVAR_TS is a FAVAR model with dynamic principal components (Two steps Kalman filter) as in (Doz et al, 2011), FAVAR_QML is a FAVAR model with dynamic principal components (Quazi Maximum Likelihood) as in (Doz et al, 2012).

2 w = 0.3 and d = 1, the coefficients that we use for BVAR model parameters estimation. The first coefficient (so called overall tightness) is implementing to the diagonal matrix of standard errors, while the second coefficient (decay) is implemented to the lags. In this paper we set the overall tightness (w) equal to 0.1, 0.2 and 0.3, and lag decay (d) equal to 1 and 2. This parameter values are chosen so that they are consistent with the ones used by R. Gupta and A. Kabundi (2009a, 2009b).

3 The figure in the bracket is the number of static principal components that have been used for estimation of the FAVAR model.

4 The numbers in the bracket shows how many dynamic and static principal components have been used for estimation of FAVAR model. For this particular case 2 dynamic and 4 static principal components have been used.

Table 2: RMSE for CPI inflation

Number of lags	Forecasting model	Forecast horizon 1	2	3	4	Average value of the RMSE
1 lag	VAR	1.36	1.40	1.41	1.44	**1.40**
	BVAR (w = 0.3; d = 1)	1.41	1.44	1.41	1.46	1.43
	FAVAR_SW (4)	1.63	1.54	1.45	1.43	1.51
	FAVAR_TS (1, 4)	1.43	1.38	1.44	1.45	1.42
	FAVAR_QML (1, 4)	1.43	1.41	1.44	1.43	1.43
	BFAVAR_SW (4) (w = 0.3; d = 1)	1.49	1.51	1.45	1.42	1.47
	BFAVAR_TS (4, 4) (w = 0.3; d = 1)	1.48	1.50	1.43	1.40	1.45
	BFAVAR_QML (4, 4) (w = 0.3; d = 1)	1.48	1.49	1.42	1.40	1.45
2 lags	VAR	1.54	1.46	1.47	1.48	1.49
	BVAR (w = 0.3; d = 1)	1.43	1.43	1.41	1.45	1.43
	FAVAR_SW (4)	2.38	2.37	1.87	1.44	2.02
	FAVAR_TS (1, 4)	1.56	1.32	1.31	1.29	**1.37**
	FAVAR_QML (1, 4)	1.52	1.37	1.42	1.41	1.43
	BFAVAR_SW (4) (w = 0.3; d = 2)	1.75	1.72	1.62	1.53	1.66
	BFAVAR_TS (1, 4) (w = 0.3; d = 2)	1.63	1.64	1.56	1.55	1.59
	BFAVAR_QML (1, 4) (w = 0.3; d = 2)	1.68	1.66	1.55	1.53	1.60
3 lags	VAR	1.69	1.62	1.63	1.50	1.61
	BVAR (w = 0.3; d = 2)	1.41	1.35	1.52	1.53	**1.45**
	FAVAR_SW (4)	2.55	1.86	2.10	1.65	2.04
	FAVAR_TS (3, 4)	2.41	1.57	1.69	1.79	1.86
	FAVAR_QML (3, 4)	2.96	2.31	1.57	1.79	2.16
	BFAVAR_SW (4) (w = 0.3; d = 1)	1.79	1.62	1.78	1.69	1.72
	BFAVAR_TS (1, 4) (w = 0.3; d = 1)	1.62	1.36	1.49	1.54	1.50
	BFAVAR_QML (1, 4) (w = 0.3; d = 1)	1.60	1.41	1.51	1.48	1.50
4 lags	VAR	1.46	1.45	1.56	1.42	1.47
	BVAR (w = 0.3; d = 1)	1.58	1.54	1.55	1.51	1.54
	FAVAR_SW (4)	2.70	1.94	2.18	1.81	2.16
	FAVAR_TS (3, 4)	2.39	2.16	1.97	2.13	2.16
	FAVAR_QML (3, 4)	2.35	2.24	2.50	2.01	2.28
	BFAVAR_SW () (w = 0.3; d = 1)	1.86	1.52	1.64	1.75	1.69
	BFAVAR_TS (1, 4) (w = 0.3; d = 1)	1.74	1.36	1.45	1.43	1.49
	BFAVAR_QML (1, 4) (w = 0.3; d = 1)	1.52	1.26	1.35	1.25	**1.34**

Table 3: RMSE for nominal short-term interest rate

Number of lags	Forecasting model	Forecast horizon 1	2	3	4	Average value of the RMSE
1 lag	VAR	1.10	0.80	0.79	0.84	0.88
	BVAR (w = 0.1; d = 1)	0.84	0.77	0.73	0.81	**0.79**
	FAVAR_SW (4)	1.35	0.81	0.78	0.82	0.94
	FAVAR_TS (4, 4)	1.29	0.84	0.79	0.82	0.93
	FAVAR_QML (4, 4)	1.23	0.85	0.80	0.83	0.93
	BFAVAR_SW (4) (w = 0.1; d = 1)	0.97	0.81	0.75	0.81	0.83
	BFAVAR_TS (4, 4) (w = 0.1; d = 1)	0.94	0.80	0.77	0.81	0.83
	BFAVAR_QML (4, 4) (w = 0.1; d = 1)	0.92	0.80	0.78	0.81	0.83
2 lags	VAR	1.28	0.96	0.93	0.82	1.00
	BVAR (w = 0.1; d = 1)	0.87	0.79	0.76	0.83	**0.81**
	FAVAR_SW (4)	2.30	1.64	1.57	1.35	1.72
	FAVAR_TS (1, 4)	1.94	1.38	0.88	0.91	1.28
	FAVAR_QML (1, 4)	1.68	1.54	1.03	0.98	1.31
	BFAVAR_SW (4) (w = 0.1; d = 2)	1.11	0.94	0.85	0.85	0.94
	BFAVAR_TS (1, 4) (w = 0.3; d = 2)	0.96	0.82	0.76	0.79	0.83
	BFAVAR_QML (1, 4) (w = 0.3; d = 2)	1.11	0.95	0.74	0.79	0.90
3 lags	VAR	1.38	0.93	0.80	0.84	0.99
	BVAR (w = 0.1; d = 2)	0.87	0.79	0.79	0.84	**0.82**
	FAVAR_SW (4)	3.06	2.40	2.39	2.10	2.49
	FAVAR_TS (2, 4)	2.39	1.94	1.83	3.05	2.30
	FAVAR_QML (2, 4)	2.06	1.79	1.66	2.75	2.06
	BFAVAR_SW () (w = 0.1; d = 1)	1.60	1.20	0.97	0.96	1.19
	BFAVAR_TS (2, 4) (w = 0.1; d = 1)	1.48	1.15	0.90	0.89	1.11
	BFAVAR_QML (2, 4) (w = 0.1; d = 1)	1.31	1.07	0.87	0.89	1.03
4 lags	VAR	1.78	1.40	0.76	1.10	1.26
	BVAR (w = 0.1; d = 1)	0.92	0.85	0.84	0.81	**0.86**
	FAVAR_SW (4)	3.57	3.62	1.55	2.23	2.74
	FAVAR_TS (2, 4)	4.31	1.89	1.55	2.27	2.50
	FAVAR_QML (2, 4)	4.25	2.26	1.65	4.13	3.07
	BFAVAR_SW (4) (w = 0.3; d = 2)	1.95	1.21	1.23	1.33	1.43
	BFAVAR_TS (2, 4) (w = 0.1; d = 1)	1.93	1.14	1.04	1.17	1.32
	BFAVAR_QML (2, 4) (w = 0.1; d = 1)	1.51	1.08	0.92	1.24	1.19

We can conclude from the tables above that the Bayesian approach to estimating and forecasting is more suitable than the more traditional VAR approach. As we can see from Tables 1 to 3, in most cases small BVAR of BFAVAR models outperform more traditional unrestricted VAR models in terms of forecast accuracy of the key macroeconomic variables.

1. GDP growth: In the one lag model, as we can see from Table 1, there is no separate model that outperforms all other models. In the two lags models, the BFAVAR_TS model outperforms all other models producing the minimum average RMSEs. In the two lags model, the 'optimal' BFAVAR_TS is followed by the BFAVAR_QML and small BVAR. In the three lags model, the BVAR outperforms all other models producing the smallest value of RMSE's. The 'optimal' BVAR model is followed by the BFAVAR_TS, BFAVAR_SW and BFAVAR_QML. In the four lags model, the BVAR outperforms all other models producing the smallest value of RMSE's. In the four lags model, the 'optimal' BVAR is followed by the BFAVAR_TS and BFAVAR_QML. Thus, it is more appropriate to use Bayesian approach to estimating and forecasting GDP growth, particularly the small BVAR or BFAVAR approach.
2. Inflation: In the one lag model, the unrestricted VAR model outperforms all other models producing the lowest minimum average RMSEs. In the two lags models, the unrestricted FAVAR_TS model outperforms all other models producing the minimum average RMSEs. In the three lags model, the BVAR outperforms all other models producing the smallest value of RMSE's. In the three lags model, the 'optimal' BVAR is followed by the BFAVAR_TS and BFAVAR_QML. In the four lags model, the BFAVAR_QML outperforms all other models producing the smallest value of RMSE's. Thus, it is more appropriate to use large scale modelling approach for inflation dynamics, particularly the unrestricted FAVAR or BFAVAR.
3. Nominal short-term interest rate: In the one lag model, the BVAR model outperforms all other models producing the lowest minimum average RMSEs. As we can see from table 3, the 'optimal' BVAR model is followed by the BFAVAR_SW, BFAVAR_TS and BFAVAR_QML. In two, three and four lags, the BVAR model outperforms all other models producing the minimum average RMSEs. Thus, it is more appropriate to use Bayesian approach to estimating and forecasting the nominal interest rate, particularly the small BVAR or BFAVAR approach.

5. Conclusion

In this paper we evaluate the forecasting performance of the four competing models for short-term forecasting of the key macroeconomic variables: the traditional VAR, Bayesian VAR, Factor Augmented VAR and Bayesian Factor Augmented VAR models. Using quarterly Armenian macroeconomic variables from 1996Q1-2014Q4 we estimate parameters of the above mentioned models. Then based on the out-of-sample root mean squared error (RMSE) criterion, we conclude that the Bayesian approach to estimating and forecasting is more appropriate to use for short-term forecasting than the more traditional unrestricted VAR approach. Therefore, we suggest that it is more appropriate to develop and use non-traditional forecasting models such as small scale BVAR or BFAVAR models for the short-term forecasting of the key macroeconomic variables in the Central Bank of Armenia.

Appendix 1

Data description and transformation

	Variable name	Transformation	Seasonal adjustment
1	Broad money (without accrued interest), mln. AMD	Δ and Ln	Yes
2	Cash money outside of the banking system, mln. AMD	Δ and Ln	Yes
3	Deposits in the banking system (without accrued interest), mln. AMD	Δ and Ln	No
4	Monetary base, mln. AMD	Δ and Ln	Yes
5	Cash money outside of the Central bank, mln. AMD	Δ and Ln	Yes
6	Commodity Agricultural Raw Materials Index, 2005 = 100, includes Timber, Cotton, Wool, Rubber, and Hides Price Indices	Δ and Ln	No
7	Commodity Fuel (energy) Index, 2005 = 100, includes Crude oil (petroleum), Natural Gas, and Coal Price Indices	Δ and Ln	No
8	Commodity Food Price Index, 2005 = 100, includes Cereal, Vegetable Oils, Meat, Seafood, Sugar, Bananas, and Oranges Price Indices	Δ and Ln	No
9	Commodity Industrial Inputs Price Index, 2005 = 100, includes Agricultural Raw Materials and Metals Price Indices	Δ and Ln	No
10	Commodity Metals Price Index, 2005 = 100, includes Copper, Aluminum, Iron Ore, Tin, Nickel, Zinc, Lead, and Uranium Price Indices	Δ and Ln	No
11	Crude Oil (petroleum), Price index, 2005 = 100, simple average of three spot prices; Dated Brent, West Texas Intermediate, and the Dubai Fateh	Δ and Ln	No
12	Crude Oil (petroleum), Dated Brent, light blend 38 API, fob U.K., US Dollars per Barrel	Δ	No
13	Crude Oil (petroleum), Dubai Fateh 32 API, fob Dubai, US Dollars per Barrel	Δ	No
14	Crude Oil (petroleum), West Texas Intermediate 40 API, Midland Texas, US Dollars per Barrel	Δ	No
15	New York Harbor Conventional Gasoline Regular Spot Price FOB, US$ per gallon	Δ	No
16	EU(28) industrial production index, % change to the previous period	Δ and Ln	Yes
17	EU(28) Gross domestic product, % change to the previous period	Δ and Ln	Yes
18	EU(19 countries) business activity index, % change to the previous period	Δ and Ln	No
19	Russia construction sector activity, balance, %	--	--
20	Russia employment in construction sector, balance, %	--	--
21	Russia manufacturing sector activity, balance, %	--	--

References

1. Banbura, M., & M. Madugno (2010). Maximum likelihood estimation of factor models on data sets with arbitrary pattern of missing data. Working Paper Series 1189, European Central Bank.
2. Barhoumi, K., Darne, O. & L. Ferrara (2009). Are disaggregated data useful for factor analysis in forecasting French GDP?. Documents de Travail 232, Banque de France.
3. Bernanke, B., J. Boivin & P. Eliasz (2005). Measuring the effects of monetary policy: A factor-augmented vector autoregressive (FAVAR) approach. Quarterly Journal of Economics 120, 387-422.
4. Canova, F. (2007). Methods for Applied Macroeconomic Research. Princeton University Press.
5. Doz, C., Gianonne D. & L. Reichlin (2011). A two-step estimator for large approximate dynamic factor models. Review of Economics and Statistics 164, 188-205.
6. Doz, C., Gianonne D. & L. Reichlin (2012). A quasi-maximum likelihood approach for large approximate dynamic factor models. Review of Economics and Statistics 94, 1014-1024.
7. Forni, M., Hallin, M., Lippi, M. & L. Reichlin (2005). The generalized dynamic factor model: one-sided estimation and forecasting. Journal of American Statistical Association 100, 830-840.
8. Gupta, R. & A. Kabundi (2009a). A large factor model for forecasting macroeconomic variables in South Africa. Working paper 137, University of Cape Town.
9. Gupta, R. & A. Kabundi (2009b). Forecasting macroeconomic variables using large scale datasets: Dynamic factor model versus large-scale BVAR's. Working paper 143, University of Cape Town.
10. Hamilton, J. (1994). Time series analysis. Princeton University Press.
11. Litterman, R. (1981). A Bayesian procedure for forecasting with vector autoregressions. Working Paper, Federal Bank of Minneapolis.
12. Schumacher, C. (2007). Forecasting German GDP using alternative factor models based on large datasets. Journal of Forecasting 26, 271-302.
13. Stock, J. & M. Watson (2002). Macroeconomic forecasting using diffusion indexes. Journal of Business & Economic Statistics 20, 147-162.