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# ON CERTAIN SUBCLASS OF MEROMORPHIC CLOSE-TO-CONVEX FUNCTIONS

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## Abstract

In this paper we introduce and investigate a certain subclass of functions which are analytic in the punctured unit disk and meromorphically close-to-convex. The sub-ordination property, inclusion relationship, coefficient inequalities, distortion theorem and a sufficient condition for our subclass of functions are derived. The results presented here would provide extensions of those given in earlier works.


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## 1. INTRODUCTION

Let $\Sigma$ denote the class of functions of the form

$$f(z) = \frac{1}{z} + \sum_{n=1}^{\infty} a_n z^n \tag{1.1}$$

which are analytic in the punctured open unit disk

$$\mathcal{U}^* = \{z : z \in \mathbb{C}; 0 < |z| \ < 1\} =: \mathcal{U} \backslash \{0\}.$$

where $\mathcal{U}$ is an open unit disk.
Let $\mathcal{P}$ denote the class of functions $p$ given by

$$p(z) = 1 + \sum_{n=1}^{\infty} p_n z^n \quad (z \in \mathcal{U}) \tag{1.2}$$

which are analytic and convex in $\mathcal{U}$ and satisfy the condition

$$p(z) \prec \frac{1 + Az}{1 + Bz} \qquad (z \in \mathcal{U}; -1 \leq B < A \leq 1) \tag{1.3}$$

Let $f, g \in \Sigma$, where $f$ is given by (1.1) and $g$ is defined by

$$f(z) = \frac{1}{z} + \sum_{n=1}^{\infty} b_n z^n \tag{1.4}$$

A function $f \in \Sigma$ is said to be in the class $\mathcal{MS}^*(\alpha)$ of meromorphic starlike of order $\alpha$ if it satisfies the inequality

$$R\left(\frac{-zf'(z)}{f(z)}\right) > \alpha \qquad (z \in \mathcal{U}; 0 \leq \alpha < 1) \tag{1.5}$$

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Moreover, a function $f \in \Sigma$ is said to be in the class $\mathcal{MC}$ of meromorphic close-to-convex functions if it satisfies the condition

$$R\left(\frac{zf'(z)}{g(z)}\right) < 0 \qquad (z \in \mathcal{U}; g \in \mathcal{MS}^*(0) \equiv \mathcal{MS}^*) \tag{1.6}$$

Further let

$$f(z) = z + a_2 z^2 + ... \tag{1.7}$$

be analytic in $\mathcal{U}$. If there exists a function $g(z) \in \mathcal{S}^*(\frac{1}{2})$ such that

$$R\left(\frac{z^2 f'(z)}{g(z)g(-z)}\right) < 0 \qquad (z \in U)$$

then we say that $f \in \mathcal{K}_s$, where $\mathcal{S}^*\left(\frac{1}{2}\right)$ denotes the usual class of starlike functions of order $\frac{1}{2}$. The function class $\mathcal{K}_s$ was introduced and studied by Gao and Zhou [3]. Also Srivastava et al. [10] considered the class $\mathcal{MS}_s^*$ of meromorphic starlike functions with respect to symmetric points which satisfy the condition

$$R\left(\frac{zf'(z)}{f(z) - f(-z)}\right) < 0 \tag{1.8}$$

Again Kowalczyk and Bomba [5] discussed $\mathcal{K}_s(\gamma)$ of analytic functions related to starlike functions. Let $f$ is given by (1.3). Then $f \in \mathcal{K}_s(\gamma)$ if it satisfies the inequality

$$R\left(\frac{-z^2 f'(z)}{g(z)g(-z)}\right) > \gamma \qquad (z \in \mathcal{U}; 0 \leq \gamma < 1) \tag{1.9}$$

where $g(z) \in \mathcal{S}^*(\frac{1}{2})$.

Motivated by the class $\mathcal{K}_s(\gamma)$, Seker [8] introduced a new class $\mathcal{K}_s^{(k)}(\gamma)$ of analytic functions related to starlike functions as follows

Let $f$ be an analytic function defined by (1.3). Then $f \in \mathcal{K}_s^{(k)}(\gamma)$, if it satisfies the condition

$$R\left(\frac{z^k f'(z)}{g_k(z)}\right) > \gamma \qquad (z \in \mathcal{U}; 0 \leq \gamma < 1) \tag{1.10}$$

where $g \in \mathcal{S}^*(\frac{k-1}{k})$, $k \geq 1$ is a fixed positive integer and $g_k(z)$ is defined by the following equality

$$g_k(z) = \prod_{\nu=0}^{k-1} \in^{-\nu} g(\in^{\nu} z) \qquad \left(\in = e^{\frac{2\pi\iota}{k}}\right) \tag{1.11}$$

Recently Wang et al.[11] considered and investigated the class $\mathcal{MK}$ of meromorphic close-to-convex function, if $f \in \Sigma$ it satisfies the inequality

$$R\left(\frac{f'(z)}{g(z)g(-z)}\right) > 0 \qquad (z \in \mathcal{U}) \tag{1.12}$$

where $g \in \mathcal{MS}^*(\frac{1}{2})$.

Motivated essentially by the aforementioned function classes $\mathcal{MK}$ and $K_s^{(k)}(\gamma)$, in this paper we introduce and investigate a new class $\mathcal{MK}^{(k)}[A, B]$ of meromorphic functions.

**Definition**. *A function $f \in \Sigma$ is said to be in the class $\mathcal{MK}^{(k)}[A,B]$ if it satisfies the inequality*

$$\frac{-f'(z)}{z^{k-2}g_k(z)} \prec \frac{1+Az}{1+Bz} \qquad (z \in \mathcal{U}; -1 \leq B < A \leq 1) \tag{1.13}$$

*where $g \in \mathcal{MS}^*(\frac{k-1}{k})$, $k \geq 1$ is fixed positive integer and $g_k(z)$ is defined by the following equality*

$$g_k(z) = \prod_{\nu=0}^{k-1} \rho^\nu g(\rho^\nu z) \qquad \left(\rho = e^{\frac{2\pi\iota}{k}}\right) \tag{1.14}$$

**Remark 1.** $\mathcal{MK}^{(2)}[1,-1] = \mathcal{MK}$, where $\mathcal{MK}$ were studied by Wang et al.[11]. By simple calculations we see that the inequality (1.13) is equivalent to

$$\left|\frac{f'(z)}{z^{k-2}g_k(z)} + 1\right| < \left|\frac{Bf'(z)}{z^{k-2}g_k(z)} + A\right| \qquad (z \in \mathcal{U}; -1 \leq B < A \leq 1)$$

The class $\mathcal{MK}^{(k)}[A,B]$ is generalization of $\mathcal{MK}^{(2)}[A,B]$ which was defined by Sim and Kwon [9].

In this paper we prove that the class $\mathcal{MK}^{(k)}[A,B]$ is a subclass of meromorphic close-to-convex functions. Furthermore, we investigate coefficient inequalities, distortion theorems and inclusion relationship for functions belonging to the class $\mathcal{MK}^{(k)}[A,B]$.

**2. Results Required**

To prove our main results given in the next section, we shall require the results contained in following Lemmas:

**Lemma 1.** *Let $\varphi_i(z) \in \mathcal{MS}^*(\alpha_i)$ where $0 \leq \alpha_i < 1 \quad (i = 0,1,2,...,k-1)$. Then for $k-1 \leq \sum_{i=0}^{k-1} \alpha_i < k$, we have*

$$z^{k-1}\prod_{i=0}^{k-1} \varphi_i(z) \in \mathcal{MS}^*\left(\sum_{i=0}^{k-1}\alpha_i - (k-1)\right)$$

**Proof**: Since $\varphi_i(z) \in \mathcal{MS}^*(\alpha_i)$ where $0 \leq \alpha_i < 1 \quad (i = 0,1,2,...,k-1)$, we have

$$\mathrm{Re}\left(\frac{-z\varphi_0'(z)}{\varphi_0(z)}\right) > \alpha_0, \mathrm{Re}\left(\frac{-z\varphi_1'(z)}{\varphi_1(z)}\right) > \alpha_1, ..., \mathrm{Re}\left(\frac{-z\varphi_{k-1}'(z)}{\varphi_{k-1}(z)}\right) > \alpha_{k-1} \tag{2.1}$$

We now let

$$F_k(z) = z^{k-1}\varphi_0(z)\varphi_1(z)...\varphi_{k-1}(z) \tag{2.2}$$

Differentiating (2.2) logarithmically, we have

$$\frac{zF_k'(z)}{F_k(z)} = (k-1) + \frac{z\varphi_0'(z)}{\varphi_0(z)} + \frac{z\varphi_1'(z)}{\varphi_1(z)} + ... + \frac{z\varphi_{k-1}'(z)}{\varphi_{k-1}(z)} \tag{2.3}$$

Therefore

$$\mathrm{Re}\left(\frac{-zF_k'(z)}{F_k(z)}\right) > -(k-1) + \alpha_0 + \alpha_1 + ...\alpha_{k-1}$$

$$= \sum_{i=0}^{k-1} \alpha_i - (k-1) \tag{2.4}$$

Thus, if

$$0 \leq \sum_{i=0}^{k-1} \alpha_i - (k-1) < 1$$

that is,

$$(k-1) \leq \sum_{i=0}^{k-1} \alpha_i < k$$

Then

$$F_k(z) = z^{k-1} \prod_{i=0}^{k-1} \varphi_i(z) \in \mathcal{MS}^* \left( \sum_{i=0}^{k-1} \alpha_i - (k-1) \right). \tag{2.5}$$

**Lemma 2** (see [2]). *Suppose that*

$$h(z) = \frac{1}{z} + \sum_{n=1}^{\infty} c_n z^n \in \mathcal{MS}^* \tag{2.6}$$

*Then*

$$|c_n| \leq \frac{2}{n+1} \, (n \in N) \tag{2.7}$$

*Each of these inequality is sharp, with the extremal function given by*

$$h(z) = z^{-1} \left(1 + z^{n+1}\right)^{\frac{2}{n+1}} \tag{2.8}$$

**Lemma 3** (see [1]). *Let $p \in P[A, B]$ and $p(z) = 1 + \sum\limits_{n=1}^{\infty} c_n z^n$*
*Then*

$$|c_n| \leq A - B$$

*This result is sharp.*
**Lemma 4** (see [4]). *Let $p \in P[A, B]$, then for $|z| = r < 1$*

$$\frac{1 - Ar}{1 - Br} \leq \mathrm{Re}p(z) \leq |p(z)| \leq \frac{1 + Ar}{1 + Br} \tag{2.9}$$

*These bounds are sharp.*
**Lemma 5** (see [7]) *Suppose that $g \in \mathcal{MS}^*$, then*

$$\frac{(1-r)^2}{r} \leq |g(z)| \leq \frac{(1+r)^2}{r} \qquad (|z| = r; 0 < r < 1) \tag{2.10}$$

**Lemma 6**(see [6]). *Let $-1 \leq B_2 \leq B_1 < A_1 \leq A_2 \leq 1$. Then*

$$\frac{1 + A_1 z}{1 + B_1 z} \prec \frac{1 + A_2 z}{1 + B_2 z} \tag{2.11}$$

## 3. Main Results

**Theorem 1** *Let* $g(z) = \frac{1}{z} + \sum_{n=1}^{\infty} b_n z^n \in \mathcal{MS}^*\left(\frac{k-1}{k}\right)$, *then*

$$G_k(z) = z^{k-1} g_k(z) = \frac{1}{z} + \sum_{n=1}^{\infty} B_n z^n \in \mathcal{MS}^* \tag{3.1}$$

**Proof** From (1.14), we know that

$$z^{k-1} g_k(z) = z^{k-1} \prod_{\nu=0}^{k-1} \rho^{\nu} g(\rho^{\nu} z)$$

$$= z^{k-1} \left[ \prod_{\nu=0}^{k-1} \left( \frac{1}{z} + \sum_{n=2}^{\infty} b_n \rho^{\nu(n+1)} z^n \right) \right] \tag{3.2}$$

Now since

$$g(z) = \frac{1}{z} + \sum_{n=1}^{\infty} b_n z^n \in \mathcal{MS}^* \left( \frac{k-1}{k} \right)$$

Then by above Lemma 1 and equality (3.2), we can get $G_k(z) \in \mathcal{MS}^*$.

**Corollary** For $k = 2$ in Theorem 1, we get the result of Theorem 2 obtained by Wang et al.[11].

**Remark 2** By Theorem 1 we see that $G_k(z)$ given by (3.1) belongs to $\mathcal{MS}^*$. Thus by (1.13), we find that our class $\mathcal{MK}^{(k)}[A, B]$ is a subclass of the class $\mathcal{MK}$ of meromorphic close-to-convex functions.

**Theorem 2.** *Let* $f(z)$ *given by* (1.1) *and* $-1 \le B < A \le 1$. *if*

$$\sum_{n=1}^{\infty} \left\{ (1 + |B|)\, n\, |a_n| + (1 + |A|) \frac{2}{n+1} \right\} \le A - B \tag{3.3}$$

*then* $f \in \mathcal{MK}^{(k)}[A, B]$ .

**Proof.** Let the function $f(z)$ and $g_k(z)$ be given by (1.1) and (1.14) respectively. Furthermore, let $g(z) \in \mathcal{MS}^*\left(\frac{k-1}{k}\right)$.

Then by Theorem 1 and Lemma 2, we have

$$G_k(z) = z^{k-1} g_k(z) = \frac{1}{z} + \sum_{n=1}^{\infty} B_n z^n \in \mathcal{MS}^*,$$

where $|B_n| \le \frac{2}{n+1}$.

Now we obtain

$$\Delta = |z f'(z) + G_k(z)| - |B z f'(z) + A G_k(z)|$$

$$= \left| \sum_{n=1}^{\infty} n a_n z^n + \sum_{n=1}^{\infty} B_n z^n \right| - \left| (A - B) \frac{1}{z} + A \sum_{n=1}^{\infty} B_n z^n + B \sum_{n=1}^{\infty} n a_n z^n \right|$$

Thus for $|z| = r\,(0 \le r < 1)$, we have

$$\Delta \le \sum_{n=1}^{\infty} n\, |a_n|\, r^n + \sum_{n=1}^{\infty} |B_n|\, r^n - \left( (A - B) \frac{1}{r} - |A| \sum_{n=1}^{\infty} |B_n|\, r^n - |B| \sum_{n=1}^{\infty} n\, |a_n|\, r^n \right)$$

$$= -\left(A-B\right)\frac{1}{r}+\left(1+|A|\right)\sum_{n=1}^{\infty}|B_n|\,r^n+\left(1+|B|\right)\sum_{n=1}^{\infty}n\,|a_n|\,r^n$$

$$\leq -\left(A-B\right)\frac{1}{r}+\left(1+|A|\right)\sum_{n=1}^{\infty}\frac{2}{n+1}r^n+\left(1+|B|\right)\sum_{n=1}^{\infty}n\,|a_n|\,r^n$$

$\leq 0.$ (By the given condition)

Thus we have

$$|zf'(z)+G_k(z)|<|Bzf'(z)+AG_k(z)|$$

which is equivalent to

$$\left|\frac{f'(z)}{z^{k-2}g_k(z)}+1\right|<\left|\frac{Bf'(z)}{z^{k-2}g_k(z)}+A\right| \qquad (z\in U)$$

which implies that $f\in\mathcal{MK}^{(k)}[A,B]$ .

Next, we give the coefficient estimates of functions belonging to the class $\mathcal{MK}^{(k)}[A,B]$ .

**Theorem 3.** *Let $f\in\mathcal{MK}^{(k)}[A,B]$ $(-1\leq B<A\leq 1)$ and $g_k(z)$ is given by* (1.1) *and* (1.14) *respectively. Then for $k\geq 1$, we have*

$$\sum_{k=1}^{n}|ka_k+B_k|^2-\sum_{k=1}^{n-1}|A.B_k+kBa_k|^2<(A-B)^2 \tag{3.4}$$

**Proof** Let $f\in\mathcal{MK}^{(k)}[A,B]$. Then we have

$$\frac{-zf'(z)}{G_k(z)}=\frac{1+Aw(z)}{1+Bw(z)}$$

where $w$ is an analytic function in $\mathcal{U}$, $|w(z)|<1$ for $z\in\mathcal{U}$ and $G_k(z)=z^{k-1}g_k(z)$. Then,

$$-zf'(z)-G_k(z)=\left(A.G_k(z)+Bzf'(z)\right)w(z).$$

Thus, putting

$$w(z)=\sum_{n=1}^{\infty}t_nz^n,$$

we obtain

$$-\sum_{n=1}^{\infty}na_nz^n-\sum_{n=1}^{\infty}B_nz^n=\left\{(A-B)\frac{1}{z}+\sum_{n=1}^{\infty}A.B_nz^n+\sum_{n=1}^{\infty}nBa_nz^n\right\}\left(\sum_{n=1}^{\infty}t_nz^n\right) \tag{3.5}$$

Now equating the coefficient of $z^n$ , we get

$$-na_n-B_n=(A-B)\,t_{n+1}+(A.B_1+Ba_1)\,t_{n-1}+...+\{A.B_{n-1}+(n-1)Ba_{n-1}\}t_1$$

and thus the coefficient combination on the R.H.S. of (3.5) depends only upon the coefficient combinations

$$(A.B_1+Ba_1)\,,...,\{A.B_{n-1}+(n-1)Ba_{n-1}\}$$

Hence for $n \geq 1$

$$\left[(A-B)\frac{1}{z}+\sum_{k=1}^{n-1}(A.B_k+kBa_k)\,z^k\right]w(z)=\sum_{k=1}^{n}(-ka_k-B_k)\,z^k+\sum_{k=n+1}^{\infty}d_kz^k \tag{3.6}$$

Then squaring the modulus of both sides of the above equality and integrating along $|z|=r$ and using the fact that $|w(z)|<1$, we obtain

$$\sum_{k=1}^{n}|ka_k+B_k|^2\,r^{2k}+\sum_{k=n+1}^{\infty}|d_k|^2\,r^{2k}<(A-B)^2\,\frac{1}{r^2}+\sum_{k=1}^{n-1}|A.B_k+kBa_k|^2\,r^{2k} \tag{3.7}$$

Letting $r \to 1$ on both sides of (3.7), we obtain

$$\sum_{k=1}^{n}|ka_k+B_k|^2<(A-B)^2+\sum_{k=1}^{n-1}|A.B_k+kBa_k|^2$$

Hence we have

$$\sum_{k=1}^{n}|ka_k+B_k|^2-\sum_{k=1}^{n-1}|A.B_k+kBa_k|^2<(A-B)^2$$

which implies the required inequality.

**Theorem 4.** *Suppose that*

$$f(z)=\frac{1}{z}+\sum_{n=1}^{\infty}a_nz^n\in\mathcal{MK}^{(k)}\,[A,B]$$

*Then*

$$|a_n|\leq\frac{(A-B)}{n}\left[-1+2\sum_{m=1}^{n}\frac{1}{m}\right]+\frac{2}{n\,(n+1)} \tag{3.8}$$

**Proof** Suppose that $f\in\mathcal{MK}^{(k)}[A,B]$. Then we know that

$$\frac{-f'(z)}{z^{k-2}g_k(z)}\prec\frac{1+Az}{1+Bz}$$

this implies that

$$\frac{-zf'(z)}{G_k(z)}\prec\frac{1+Az}{1+Bz}$$

If we set

$$q(z)=\frac{-zf'(z)}{G_k(z)} \tag{3.9}$$

it follows that

$$q(z)=1+d_1z+d_2z^2+....\in\mathcal{P}$$

In view of Lemma 3, we know that

$$|d_n|\leq A-B\quad(n\in N)$$

By substituting the series expressions of functions $f$, $G_k$ and $q$ in (3.9), we obtain

$$\left(1+d_1z+d_2z^2+...+d_nz^n+...\right)\left(\frac{1}{z}+B_1z+B_2z^2+...+B_nz^n+...\right) \tag{3.10}$$

$$=\frac{1}{z}-a_1z-2a_2z^2-...-na_nz^n-...$$

Comparing like coefficients of $z^n$ in (3.10), we get

$$-na_n=B_1d_{n-1}+B_2d_{n-2}+...+B_{n-2}d_2+B_{n-1}d_1+B_n+d_{n+1} \tag{3.11}$$

By using Lemma 2 and 3, we get

$$n\left|a_n\right|\leq 2\left(A-B\right)\left[1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n}\right]+\frac{2}{n+1}-\left(A-B\right)$$

Thus

$$\left|a_n\right|\leq\frac{(A-B)}{n}\left[-1+2\sum_{m=1}^{n}\frac{1}{m}\right]+\frac{2}{n\left(n+1\right)}$$

which implies the required inequality. The proof of Theorem 4 is completed.

**Theorem 5**. *Let $f\in\mathcal{MK}^{(k)}[A,B]$. Then*

$$\frac{\left(1-r\right)^2}{r^2}\left(\frac{1-Ar}{1-Br}\right)\leq\left|f'(z)\right|\leq\frac{\left(1+r\right)^2}{r^2}\left(\frac{1+Ar}{1+Br}\right)\qquad(|z|=r,0<r<1) \tag{3.12}$$

**Proof.** Suppose $f\in\mathcal{MK}^{(k)}[A,B]$. By definition we know that

$$\frac{-zf'(z)}{G_k(z)}\prec\frac{1+Az}{1+Bz}$$

and since $G_k(z)\in\mathcal{MS}^*$, thus by Lemma 5, we have

$$\frac{\left(1-r\right)^2}{r}\leq\left|G_k(z)\right|\leq\frac{\left(1+r\right)^2}{r}$$

and also by Lemma 4, we have

$$\frac{(1-Ar)}{(1-Br)}\leq\left|q(z)\right|\leq\frac{(1+Ar)}{(1+Br)}$$

Thus by virtue of (3.9) and Lemma 5, we obtain

$$\frac{\left(1-r\right)^2}{r^2}\left(\frac{1-Ar}{1-Br}\right)\leq\left|f'(z)\right|\leq\frac{\left(1+r\right)^2}{r^2}\left(\frac{1+Ar}{1+Br}\right)$$

Thus the proof is complete.

**Remark** For $A=1$ and $B=-1$ in Theorem 5, we obtain result of Theorem 7 by Wang et al.[11]

**Theorem 6**. *Let $-1\leq B_2\leq B_1<A_1\leq A_2\leq 1$. Then*

$$\mathcal{MK}^{(k)}\left(A_1,B_1\right)\subset\mathcal{MK}^{(k)}\left(A_2,B_2\right) \tag{3.13}$$

**Proof**. Suppose that $f \in \mathcal{MK}^{(k)}[A_1, B_1]$, we have

$$\frac{-f'(z)}{z^{k-2}g_k(z)} \prec \frac{1+A_1z}{1+B_1z}$$

Since $-1 \leq B_2 \leq B_1 < A_1 \leq A_2 \leq 1$, by lemma 6, we get

$$\frac{-f'(z)}{z^{k-2}g_k(z)} \prec \frac{1+A_1z}{1+B_1z} \prec \frac{1+A_2z}{1+B_2z}$$

Hence $f \in \mathcal{MK}^{(k)}[A_2, B_2]$.
This means that $\mathcal{MK}^{(k)}(A_1, B_1) \subset \mathcal{MK}^{(k)}(A_2, B_2)$.
Hence the proof of Theorem 6 is complete.

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