A New Compound Lifetime Distribution: Model, Characterization, Estimation and Application

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Abstract

There are diverse lifetime models available to the researchers to predict the uncertain behavior of random events but at times they fail to provide adequate fit for some complex and new data sets. New probability distributions are emerging as lifetime models to meet this ever growing demand of modeling complex real world phenomena from different sciences with better efficiency. Here, in this manuscript we shall compose Ailamujia distribution with that of power series distribution. This newly developed distribution called Ailamujia power series distribution reduces to four new special lifetime models on simple specific function parametric setting. Apart from this some important mathematical properties in the form of propositions will also be discussed. Furthermore, characterization and some statistical properties that include mgf, moments, and parameter estimation have also been discussed. Finally, the potency of newly proposed model has been analyzed statistically and graphically and it has been established from the statistical analysis that newly proposed model offers a better fit when it comes to model some lifetime data set.

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1. INTRODUCTION

Lifetime distributions play an important role in almost every field of science be it engineering, industrial, medical or similar biological science. The events of interest such as death, appearance of some disease and system failure are of major concern for statisticians because of their uncertain behavior. And there are so many probability distributions such as exponential, Weibull, gamma and log normal that can be used as lifetime models to predict this uncertain behavior of random events but due to varying pattern of different data sets, these probability models can not be used adequately because of some serious limitations. To overcome these limitations researchers have developed many lifetime distributions by using different techniques such as compounding, transmutation etc. For instance, Adamidis and Loukas [1], Tahmasbi [10] and Morais and Baretto Souza [9] developed several lifetime distributions through compounding mechanisms that proved to be very effective in modeling the lifetime data having different characteristics. The efforts of Adil and

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Jan ([2], [3] [4], [5], [6]) got materialized when they obtained many compound distributions that exhibited with clinical precision to be superior in comparison to existing lifetime models.

Let us take a series system with N components, where N, the number of components is itself a discrete random variable with support N =1,2,…, The lifetime of i^{th} component in this set up can be portrayed by any suitable lifetime distribution like viz; exponential, gamma, Weibull, Lindley, Ailamujia etc. And N, the discrete random variable may have any of the ascribed distribution such as geometric, zero truncated Poisson or power series distribution in general. The lifetime for this kind of system in series combination will be denoted by $Y = \min\left\{X_i\right\}_{i=1}^{N}$. In this paper we will consider the lifetime of i^{th} component to be distributed as Ailamujia distribution and the index N itself as powers series distribution. The new lifetime distribution that is obtained by compounding Ailamujia distribution with that of powers series distribution will be known as Ailamujia power series distribution. The present paper is organized as follows: In section (2) we present the construction of the proposed lifetime distribution. Density, survival, hazard rate functions and some of the properties of the proposed family are given in section (3). Moment generating function of proposed distribution is given in section (4). Order statistics, their moments and parameter estimation are discussed in detail respectively in section (5) and (6). Special cases that include new lifetime distributions have been given in section (7). Finally, real application and conclusion about new findings are respectively given in section (8) and (9).

2. CONSTRUCTION OF THE CLASS

Let $X_i,\ i=1,2,...,N$ be independent and identically distributed (iid) random variables following Ailamujia distribution with CDF

$$G(x;\beta) = 1-(1+2\beta x)e^{-2\beta x} \tag{1}$$

Here, the index N is itself a discrete random variable following power series distribution that have been truncated at zero with probability function given by

$$P(N=n)=\frac{a_n\lambda^n}{C(\lambda)},\quad n=1,2,...$$

where a_n depends only on n, $C(\lambda)=\sum_{n=1}^{\infty}a_n\lambda^n$ and $\lambda>0$ is such that $C(\lambda)$ is finite. Table 1 is very informative and it will be helpful for obtaining the special cases of the proposed model on specific function setting.

Table 1: Useful quantities of Some Power Series Distribution

Distribution	a_n	$C(\lambda)$	$C^{'}(\lambda)$	$C^{''}(\lambda)$	$C^{-1}(\lambda)$	λ
Poisson	$n!^{-1}$	$e^{\lambda}-1$	e^{λ}	e^{λ}	$\log(\lambda+1)$	$\lambda\in(0,\infty)$
Logarithmic	n^{-1}	$-\log(1-\lambda)$	$(1-\lambda)^{-1}$	$(1-\lambda)^{-2}$	$1-e^{-\lambda}$	$\lambda\in(0,1)$
Geometric	1	$\lambda(1-\lambda)^{-1}$	$(1-\lambda)^{-2}$	$2(1-\lambda)^{-3}$	$\lambda(\lambda+1)^{-1}$	$\lambda\in(0,1)$
Binomial	$\binom{m}{n}$	$(\lambda+1)^m-1$	$m(\lambda+1)^{m-1}$	$\frac{m(m-1)}{(\lambda+1)^{2-m}}$	$(\lambda-1)^{1/m}-1$	$\lambda\in(0,\infty)$

Let $X_{(1)}=\min\{X_i\}_{i=1}^{N}$. The conditional cumulative distribution function of $X_{(1)}\mid N=n$ is given by

$G_{X_{(1)}|N=n}(x)=1-\left[\bar{G}(x)\right]^n$, where $G(x)$ is the cdf of Ailamujia distribution

$$=1-\left[(1+2\beta x)e^{-2\beta x}\right]^n$$

The joint probability function is

$$P\left(X_{(1)}\le x, N=n\right)=\frac{a_n\lambda^n}{C(\lambda)}\left\{1-\left[(1+2\beta x)e^{-2\beta x}\right]^n\right\},\ \ x>0,\ n\ge 1.$$

By using the compounding technique, the CDF of newly proposed lifetime distribution is

$$F(x)=\sum_{n=1}^{\infty}\frac{a_n\lambda^n}{C(\lambda)}\left\{1-\left[(1+2\beta x)e^{-2\beta x}\right]^n\right\}$$

$$=1-\frac{C\left[\lambda(1+2\beta x)e^{-2\beta x}\right]}{C(\lambda)},\quad x>0 \tag{2}$$

The newly proposed distribution will be called Ailamujia power series distribution and will be symbolically represented as $\text{APSD}(X;\beta,\lambda)$.

3. DENSITY, SURVIVAL AND HAZARD RATE FUNCTION

Since PDF is essentially a derivative of CDF, therefore probability density function of APSD $(X;\beta,\lambda)$ can be obtained as

$$f(x)=\frac{dF(x)}{dx}$$

$$=\frac{d}{dx}\left\{1-\frac{C\left[\lambda\left(1+2\beta x\right)e^{-2\beta x}\right]}{C(\lambda)}\right\}$$

$$f(x)=4\lambda\,\beta^2 xe^{-2\beta x}\,\frac{C'\left[\lambda\left(1+2\beta x\right)e^{-2\beta x}\right]}{C(\lambda)} \tag{3}$$

$$S(x)=1-F(x)\quad=\frac{C\left[\lambda\left(1+2\beta x\right)e^{-2\beta x}\right]}{C(\lambda)}$$

and the hazard function is

$$h(x)=\frac{f(x)}{S(x)}$$

$$h(x)=\frac{4\lambda\,\beta^2 xe^{-2\beta x}C'\left[\lambda\left(1+2\beta x\right)e^{-2\beta x}\right]}{C\left[\lambda\left(1+2\beta x\right)e^{-2\beta x}\right]}\quad ,x>0$$

Next, we will explore some important properties of $\text{APSD}(X;\beta,\lambda)$ through the following propositions.

PROPOSITION 1. The Ailamujia distribution is the limiting case of the APSD $(X;\beta,\lambda)$ whenever $\theta\to 0^+$.

PROOF. The cumulative distribution function of $\text{APSD}(X;\beta,\lambda)$

$$\lim_{\theta\to 0^+}F(x)=1-\lim_{\theta\to 0^+}\frac{C\left[\lambda\left(1+2\beta x\right)e^{-2\beta x}\right]}{C(\lambda)},\quad x>0$$

In view of the fact

$$C(\lambda)=\sum_{n=1}^{\infty}a_n\lambda^n$$

$$\lim_{\lambda\to 0^+}F\left(x\right)=1-\lim_{\lambda\to 0^+}\frac{\sum_{n=1}^{\infty}a_n\left[\lambda\left(1+2\beta x\right)e^{-2\beta x}\right]^n}{\sum_{n=1}^{\infty}a_n\lambda^n}$$

$$\lim_{\lambda\to 0^+}F\left(x\right)=1-\lim_{\lambda\to 0^+}\frac{a_1\lambda\left(1+2\beta x\right)e^{-2\beta x}+\sum_{n=2}^{\infty}a_n n\lambda^{n-1}\left[\left(1+2\beta x\right)e^{-2\beta x}\right]^n}{a_1\lambda+\sum_{n=2}^{\infty}a_n n\lambda^{n-1}}$$

$$=1-\left(1+2\beta x\right)e^{-2\beta x}$$

which is the distribution function of Ailamujia distribution.

PROPOSITION 2. The densities of $\text{APSD}\left(X;\beta,\lambda\right)$ can be expressed as an infinite linear combination of densities of 1st order statistics of Ailamujia distribution

PROOF. Since we have, $f(\text{X};\beta,\lambda)=4\lambda\,\beta^2xe^{-2\beta x}\,\dfrac{C'\left[\lambda\left(1+2\beta x\right)e^{-2\beta x}\right]}{C\left(\lambda\right)}$

It is known that

$$C'(\lambda)=\sum_{n=1}^{\infty}na_n\lambda^{n-1}$$

Therefore it follows that

$$f(\text{X};\beta,\lambda)=4\lambda\,\beta^2xe^{-2\beta x}\sum_{n=1}^{\infty}na_n\,\frac{\left[\lambda\left(1+2\beta x\right)e^{-2\beta x}\right]^{n-1}}{C\left(\lambda\right)}$$

$$f(\text{X};\beta,\lambda)=\sum_{n=1}^{\infty}\frac{a_n\lambda^n}{C\left(\lambda\right)}4n\beta^2xe^{-2\beta x}\left[\left(1+2\beta x\right)e^{-2\beta x}\right]^{n-1}$$

$$f(\text{X};\beta,\lambda)=\sum_{n=1}^{\infty}P(N=n)g_1(x,n) \tag{4}$$

Where $g_1(x,n)=4n\beta^2xe^{-2\beta x}\left[\left(1+2\beta x\right)e^{-2\beta x}\right]^{n-1}$ is the Ist order statistics of Ailamujia distribution. Therefore the densities of proposed distribution can be expressed as an infinite linear combination of the Ist order statistics of Ailamujia distribution. Hence it is obvious that properties of $\text{APSD}\left(X;\beta,\lambda\right)$ can be obtained from the Ist order statistics of Ailamujia distribution.

4. MOMENT GENERATING FUNCTION

In view of the proposition 2, the mgf APSD$(X;\beta,\lambda)$ can be obtained as

$$M_X(t)=\sum_{n=1}^{\infty}P(N=n)M_{X_{(1)}}(t)$$

where $M_{X_{(1)}}(t)$ is the moment generating function of I[st] order statistics of Ailamujia distribution

$$M_{X_{(1)}}(t)=\int_0^{\infty}e^{tx}4n\beta^2xe^{-2\beta x}\left[\left(1+2\beta x\right)e^{-2\beta x}\right]^{n-1}\ dx$$

$$M_{X_{(1)}}(t)=4n\beta^2\int_0^{\infty}e^{-(2n\beta-t)x}\left(1+2\beta x\right)^{n-1}\ dx$$

$$M_{X_{(1)}}(t)=n\left(2\beta\right)^{n-j+1}\sum_{j=1}^{n-1}\binom{n-1}{j}\int_0^{\infty}e^{-(2n\beta-t)x}x^{n-j-1}\ dx$$

$$M_{X_{(1)}}(t)=n\left(2\beta\right)^{n-j+1}\sum_{j=1}^{n-1}\binom{n-1}{j}\frac{\Gamma(n-j)}{(2n\beta-t)^{\mathrm{n-j}}}$$

Hence we get

$$M_X(t)=\sum_{n=1}^{\infty}\frac{a_n\lambda^n}{C(\lambda)}n\left(2\beta\right)^{n-j+1}\sum_{j=1}^{n-1}\binom{n-1}{j}\frac{\Gamma(n-j)}{(2n\beta-t)^{\mathrm{n-j}}}$$

In order to obtain the moments of proposed distribution we again use proposition 2

$$E(X^r)=\sum_{n=1}^{\infty}P(N=n)\int_0^{\infty}x^r g_1(x)\,dx$$

$$=\sum_{n=1}^{\infty}P(N=n)E\left(X_{(1)}{}^{r}\right)$$

Now consider

$$E\left(X_{(1)}{}^{r}\right)=\int_0^{\infty}x^r g_1(x)\,dx=4n\beta^2\int_0^{\infty}x^{r+1}\,e^{-2\beta x}\left[\left(1+2\beta x\right)e^{-2\beta x}\right]^{n-1}dx$$

$$=4n\beta^2\sum_{j=0}^{n-1}\binom{n-1}{j}(2\beta)^{\mathrm{n-1-j}}\int_0^{\infty}x^{r+n-j}\,e^{-2n\beta x}dx$$

$$=4n\beta^2\sum_{j=0}^{n-1}\binom{n-1}{j}(2\beta)^{\mathrm{n-1-j}}\,\frac{\Gamma(r+n-j+1)}{\left(2n\beta\right)^{r+n-j+1}}$$

$$=(2\beta)^{-r}\sum_{j=0}^{n-1}\binom{n-1}{j}\frac{\Gamma(r+n-j+1)}{n^{r+n-j}}$$

Thus we have

$$E(X^r) = (2\beta)^{-r} \sum_{n=1}^{\infty} \frac{a_n \lambda^n}{C(\lambda)} \sum_{j=0}^{n-1} \binom{n-1}{j} \frac{\Gamma(r+n-j+1)}{n^{r+n-j}} \tag{5}$$

5. ORDER STATISTICS AND THEIR MOMENTS

We have developed a new lifetime distribution that can be used to model the lifetime data where order statistics plays a vital role. In this section we derive expressions for pdf and CDF of i^{th} order statistics of proposed distribution.

Let $X_1, X_2, ..., X_n$ be a random sample from APSD and $X_{1:n}, X_{2:n}, ..., X_{1:n}$ denote the corresponding order statistics. The pdf of i^{th} order statistics say $X_{i:n}$ is given by

$$f_{i:n}(x) = \frac{n!}{(n-i)!(i-1)!} \left[F(x)\right]^{i-1} \left[1-F(x)\right]^{n-i} f(x) \tag{6}$$

$$= \frac{n!\, f(x)}{(n-i)!(i-1)!} \left[1 - \frac{C\left[\lambda\left(1+2\beta x\right)e^{-2\beta x}\right]}{C(\lambda)}\right]^{i-1} \left[\frac{C\left[\lambda\left(1+2\beta x\right)e^{-2\beta x}\right]}{C(\lambda)}\right]^{n-i}$$

Expression (6) can be equivalently written as

$$f_{i:n}(x) = \frac{n!}{(n-i)!(i-1)!} \sum_{k=0}^{n-i} \binom{n-i}{k} (-1)^k f(x) \left[F(x)\right]^{k+i-1}$$

or

$$f_{i:n}(x) = \frac{n!}{(n-i)!(i-1)!} \sum_{k=0}^{i-1} \binom{i-1}{k} (-1)^k f(x) \left[1-F(x)\right]^{k+n-i}$$

In view of the fact

$$f(x)\left[F(x)\right]^{k+i-1} = \left(\frac{1}{k+i}\right) \frac{d}{dx} \left[F(x)\right]^{k+i}$$

The associated CDF of $f_{i:n}(x)$ denoted by $F_{i::n}(x)$ becomes

$$F_{i:n}(x) = \frac{n!}{(n-i)!(i-1)!} \sum_{k=0}^{n-i} \frac{\binom{n-i}{k}(-1)^k}{(k+i)} \left[1 - \frac{C\left[\lambda\left(1+2\beta x\right)e^{-2\beta x}\right]}{C(\lambda)}\right]^{k+i} \tag{7}$$

The expression for r^{th} moment of i^{th} order statistics $X_{1:n,....}X_{n:n}$ with CDF (7) can be obtained by exploiting a well-known result due to Barakat and Abdelkadir [7] as follows

$$E\left(X_{i:n}^{r}\right)=r\sum_{k=n-i+1}^{n}(-1)^{k-n+i-1}\binom{k-1}{n-i}\binom{n}{k}\int_{0}^{\infty}x^{r-1}S(x)^{k}\,dx$$

where $S(x)$ is the survival function of Ailamujia distribution. Therefore we have

$$E\left(X_{i:n}^{r}\right)=r\sum_{k=n-i+1}^{n}(-1)^{k-n+i-1}\binom{k-1}{n-i}\binom{n}{k}\int_{0}^{\infty}x^{r-1}\left(\frac{C\left[\lambda\left(1+2\beta x\right)e^{-2\beta x}\right]}{C\left(\lambda\right)}\right)^{k}dx$$

where $r=1,2,3...$ and $i=1,2,...,n$.

6. PARAMETER ESTIMATIONS

The log-likelihood function of the proposed model with unknown parameter vector $\Theta=(\beta,\lambda)^{T}$ is given by

$$l_n=l_n(x,\Theta)=n\log 4+2n\log\beta+n\log\lambda-2\beta\sum_{i=1}^{n}x_i+\sum_{i=1}^{n}\log x_i+\sum_{i=1}^{n}\log\left\{C'\left[\lambda(1+2\beta x)e^{-2\beta x}\right]\right\}-n\log C(\lambda)$$

The corresponding score functions are

$$\frac{\partial l_n}{\partial\beta}=\frac{2n}{\beta}-\sum_{i=1}^{n}x_i-4\lambda\beta\sum_{i=1}^{n}\frac{C''\left[\lambda\left(1+2\beta x_i\right)e^{-2\beta x_i}\right]}{C'\left[\lambda\left(1+2\beta x_i\right)e^{-2\beta x_i}\right]}x_i^{2}e^{-2\beta x_i}$$

$$\frac{\partial l_n}{\partial\lambda}=\frac{n}{\lambda}-\frac{nC'(\lambda)}{C(\lambda)}+\sum_{i=1}^{n}\frac{C''\left[\lambda\left(1+2\beta x_i\right)e^{-2\beta x_i}\right]}{C'\left[\lambda\left(1+2\beta x_i\right)e^{-2\beta x_i}\right]}\left(1+2\beta x_i\right)e^{-2\beta x_i}$$

The maximum likelihood estimate of Θ say $\hat{\Theta}$ is obtained by solving the non-linear system of equations $U_n(\Theta)=\left(\frac{\partial l_n}{\partial\beta},\frac{\partial l_n}{\partial\lambda}\right)^{T}=0$. The solution of this non-linear system of equations can be found numerically by using software such as R.

7. CONSEQUENCES OF PROPOSED MODEL

In this section we will study some important consequences of proposed model in the form of some special cases. The graphical behavior these sub models will also be discussed to show the flexibility in terms of hazard and density function.

7.1. Ailamujia Poisson Distribution (APD)

Here, we frequently exploit the use of table 1, in which it is clear that classical distributions are embodied in PSD on specific function setting. For instance, Poisson distribution is a special case of power series distribution for $C(\lambda)=e^{\lambda}-1$ and $C'(\lambda)=e^{\lambda}$. Therefore, cdf and pdf of a compound of APD is $F(\mathrm{x})=1-\frac{e^{\lambda(1+2\beta x)e^{-2\beta x}}-1}{e^{\lambda}-1}$. The associated pdf, survival and hazard rate function is

$$f(x)=\frac{4\lambda\beta^2}{e^{\lambda}-1}xe^{\lambda(1+2\beta x)e^{-2\beta x}-2\beta x}$$

$$S(x)=\frac{e^{\lambda(1+2\beta x)e^{-2\beta x}}-1}{e^{\lambda}-1} \text{ and } h(x)=\frac{4\lambda\beta^2 xe^{\lambda(1+2\beta x)e^{-2\beta x}-2\beta x}}{e^{\lambda(1+2\beta x)e^{-2\beta x}}-1}$$

for $x,\beta>0$, $0<\lambda<\infty$ respectively.

7.2. Ailamujia Logarithmic Distribution (ALD):

Again from table 1, Logarithmic distribution is a special case of PSD when $C(\lambda)=-\log(1-\lambda)$ and $C'(\lambda)=(1-\lambda)^{-1}$. The cdf of ALD is

$$F(x)=1-\frac{\log\left[1-\lambda\left(1+2\beta x\right)e^{-2\beta x}\right]}{\log\left(1-\lambda\right)},\quad x>0$$

The associated pdf, survival and hazard rate function is

$$f(x)=\frac{4\lambda\beta^2 e^{-2\beta x}}{\left[\lambda\left(1+2\beta x\right)e^{-2\beta x}-1\right]l\,\mathrm{og}(1-\lambda)}$$

$$S(x)=\frac{\log\left[1-\lambda\left(1+2\beta x\right)e^{-2\beta x}\right]}{\log\left(1-\lambda\right)}$$

$$h(x)=\frac{4\lambda\beta^2 e^{-2\beta x}}{\left[\lambda\left(1+2\beta x\right)e^{-2\beta x}-1\right]\log\left[1-\lambda\left(1+2\beta x\right)e^{-2\beta x}\right]}$$

for $x,\lambda>0$ and $0<\beta<1$ respectively.

7.3. Ailamujia Geometric Distribution (AGD)

We observe from the table 1 that Geometric distribution is a particular case of PSD when $C(\lambda)=\lambda(1-\lambda)^{-1}$ and $C'(\lambda)=(1-\lambda)^{-2}$. Therefore, cdf of AGD is

$$F(x)=\frac{1-\left(1+2\beta x\right)e^{-2\beta x}}{1-\lambda\left(1+2\beta x\right)e^{-2\beta x}},x>0$$

The associated pdf, survival and hazard rate function is

$$f(x)=\frac{4\beta^2 xe^{-2\beta x}(1-\lambda)}{\left[1-\lambda(1+2\beta x)e^{-2\beta x}\right]^2}$$

$$S(x)=\frac{(1+2\beta x)e^{-2\beta x}(1-\lambda)}{1-\lambda(1+2\beta x)e^{-2\beta x}}$$

$$h(x)=\frac{4\beta^2 xe^{-2\beta x}(1-\lambda)}{\left[1-\lambda(1+2\beta x)e^{-2\beta x}\right](1+2\beta x)e^{-2\beta x}},$$

for $x,\beta,0<\lambda<1$ respectively.

7.4. Ailamujia Binomial Distribution (ABD)

Binomial distribution is a particular case of PSD for $C(\lambda)=(\lambda+1)^m-1$ and a compound of Ailamujia binomial distribution (ABD) is followed from (2) by using $C(\lambda)=(\lambda+1)^m-1$. This may be noted here that these sub models are new lifetime distributions that have been obtained on specific parameter setting in ALPSD.

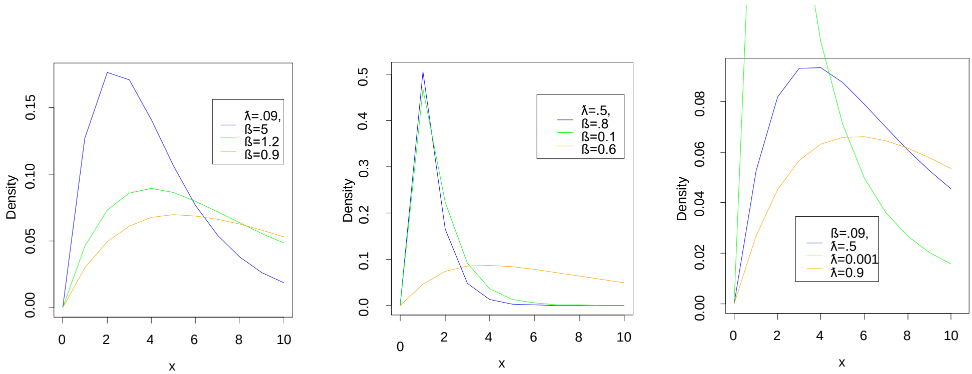

Fig. 1-3. These graphs show the flexibility of APD, ALD and AGD models for randomly selected values of parameters

8. APPLICATION

In this section, we will expose and compare the potentiality of proposed model on a real life data set based on Lifetime of fatigue fracture of Kevlar 373/epoxy [8], that are subject to constant pressure at the 90% stress level until all had failed. The data set is

0.0251	0.0886	0.0891	0.2501	0.3113	0.3451	0.4763	0.565	0.5671	0.6566	0.6748
0.6751	0.6753	0.7696	0.8375	0.8391	0.8425	0.8645	0.8851	0.9113	0.912	0.9836
1.0483	1.0596	1.0773	1.1733	1.257	1.2766	1.2985	1.3211	1.3503	1.3551	1.4595
1.488	1.5728	1.5733	1.7083	1.7263	1.746	1.763	1.7746	1.8275	1.8375	1.8503
1.8808	1.8878	1.8881	1.9316	1.9558	2.0048	2.0408	2.0903	2.1093	2.133	2.21
2.246	2.2878	2.3203	2.347	2.3513	2.4951	2.526	2.9911	3.0256	3.2678	3.4045
3.4846	3.7433	3.7455	3.9143	4.8073	5.4005	5.4435	5.5295	6.5541	9.096	

Table 2: Analysis of model fitting

MODEL	*MLE*	*AIC*	*BIC*
LG	$\hat{\beta} = 0.41,\ \hat{\lambda} = 0.38$	249.52	252.58
LP	$\hat{\beta} = 0.25,\ \hat{\lambda} = 3.58$	248.68	251.73
LL	$\hat{\beta} = 0.44,\ \hat{\lambda} = 0.51$	249.70	252.75

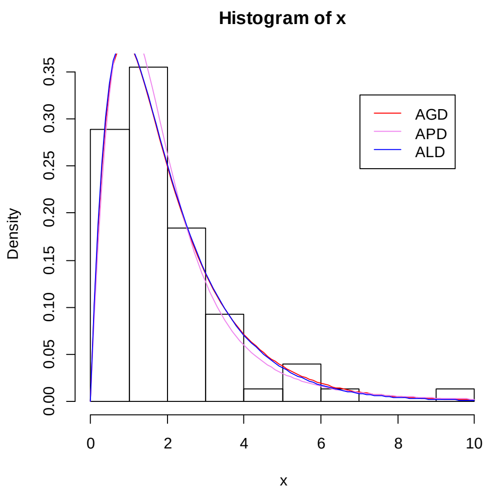

Fig 4. Fitting of AGD, APD, ALD to the fatigue lifetime data

9. CONCLUSION

We have developed a new class of compound lifetime distributions that has been named as Ailamujia power series distribution. Furthermore, we also discussed some special cases of this class of distributions that are very flexible in terms of density and hazard rate functions. Mathematical properties such as moments, order statistics and parameter estimation through MLE of the proposed class has also been discussed. Finally the potentiality of proposed model has been explored in lifetime data analysis. It is very clear from statistical analysis that proposed model performs well which is also corroborated by graphical analysis. So, we strongly recommend practitioners to use one of our models in order to get effective results when it comes to fit lifetime data.

The future course of work will be on a generalized version of Ailamujia power series distribution.

REFERENCES

[1] Adamidis, K., and Loukas, S. (1998). A lifetime distribution with decreasing failure rate. *Journal* of *Statistics & Probability Letters,* 39, 35-42.

[2] Adil, R., Zahoor, A., and Jan, T.R. (2016). A new count data model with application in genetics and ecology. *Electronic Journal of Applied Statistical Analysis,* 9(1), 213-226

[3] Adil, R., Zahoor, A., and Jan, T.R. (2017). Complementary compound Lindley power series distribution with application. *Journal of Reliability and Statistical Studies,* 10(2), 143-158.

[4] Adil, R., Zahoor, A., and Jan, T.R. (2018a). A new lifetime distribution for series system: Model, properties and application. *Journal of Modern Applied and Statistical Methods,* 17(1), 1-25.

[5] Adil, R., Zahoor, A., and Jan, T.R. (2018b). Generalized version of complementary Lindley power series distribution with application. *Pakistan Journal of Statistics and Operation Research,* 10(2), 139-155.

[6] Adil, R., Zahoor, A., and Jan, T.R. (2018c). An advanced count data model with applications in genetics. *Journal of Applied Math. Inf. Sci. Lett. Statistical Analysis*, 6(3), 113-121

[7] Barakat, H.M., and Abdelkader, Y.H. (2004). Computing the moments of order statistics from nonidentical random variables, *Statistical Methods and Applications*, 13, 15-26

[8] Barlow, R. E. Toland, R. H. and Freeman, T (1984). A Bayesian Analysis of Stress Rupture Life of Kevlar 49/epoxy Sphere-cal Pressure Vessels, in 'Proc. Conference on Applications of Statistics', Marcel Dekker, New York.

[9] Morais, A.L., and Barreto-Souza, W. (2011). A compound class of Weibull and power series distributions. Computational Statistics and Data Analysis 55, 1410-1425

[10] Tahmasbi, R., and Rezaei, S. (2008). A two-parameter lifetime distribution with decreasing failure rate, *Computational Statistics & Data Analysis*, 52, 3889-3901.

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