Feasible Generalized Stein-Rule Restricted Ridge Regression Estimators

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Abstract

Several versions of the Stein-rule estimators of the coefficient vector in a linear regression model are proposed in the literature. In the present paper, we propose new feasible generalized Stein-rule restricted ridge regression estimators to examine multicollinearity and autocorrelation problems simultaneously for the general linear regression model, when certain additional exact restrictions are placed on these coefficients. Moreover, a Monte Carlo simulation experiment is performed to investigate the performance of the proposed estimator over the others.

Keywords: Feasible Estimator; Stein-rule Estimator; Restricted Ridge Estimator; Multicollinearity; Autocorrelation

1. INTRODUCTION

Stein estimator is proposed by Stein (1956). The family of Stein-rule (SR) estimators proposed by James and Stein (1960) has smaller variability than the ordinary least squares (OLS) estimator under quadratic risk with a simple condition that the number of explanatory variables are more than two. For the coefficient vector in a linear regression model with spherical disturbances, SR estimators have been extensively used. The comparisons under different loss functions and several versions of the SR estimators have also received considerable attention in the literature. Chaturvedi and Shukla (1990) consider a SR estimator for estimating the regression model with autocorrelated errors and derive its asymptotic properties. Wan and Chaturvedi (2000) extend the analysis of Chaturvedi and Shukla (1990) to a class of operational variants of the minimum mean squared error estimators. Srivastava and Srivastava (1983, 1984) and Srivastava and Chandra (1991) consider families of improved restricted estimators obtained by mixing SR with restricted least squares. Chaturvedi et al. (1996) extends the analysis of Srivastava and Srivastava (1984) to models with nonspherical disturbances. Chaturvedi et al. (2001)

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propose a SR estimator for the general linear regression model with nonspherical disturbances and a set of linear restrictions binding the regression coefficients.

Many authors have studied the predicted performance of predictors obtained by using different estimators for either the actual values or the average values of the study variable at a time. Toutenburg and Shalabh (1996) analyzed the performance properties of predictors arising from the methods of restricted regression and mixed regression besides least squares according to the target function. Then, Toutenburg and Shalabh (2000) considered the family of SR estimators proposed by Srivastava and Srivastava (1983) and analyzed performance properties of this family when the objective is to predict values outside the sample and within the sample. Kumar et al. (2008) considered two families of SR estimators proposed by Srivastava and Srivastava (1984) and analyzed performance properties of these families under the target function. Shalabh, Toutenburg and Heumann (2009) introduced the extended balanced loss function (EBLF) under the target function and discussed the SR estimation. Moreover, Chaturvedi and Shalabh (2014) discussed the bayesian estimation of regression coefficients under the EBLF.

In the present paper, we extend the idea of Chaturvedi et al. (2001) to models under multicollinearity and propose new families of feasible generalized SR restricted ridge regression (FGSRR) estimator for the general linear regression model with autocorrelation, multicollinearity and a set of linear restrictions binding the regression coefficients, in Section 2. In Section 3, we indicate the prediction mean square error (PMSE) under the target function criterion. Moreover, we perform a Monte Carlo simulation experiment to compare the performance of the new estimators to the others in terms of the PMSE criterion under the target function in Section 4.

2. THE MODEL AND ESTIMATORS

Consider the following linear regression model

$$Y = X\beta + \varepsilon,\ \varepsilon \sim (0, \sigma^2 \mathrm{I}_n) \tag{1}$$

where Y is an $n \times 1$ vector of observations on the dependent variable, X is an $n \times p$ known design matrix of rank p, β is an $p \times 1$ vector of unknown parameters, and ε is an n × 1 vector of random errors with zero mean and variance $\sigma^2 I_n$, where I_n is an identity matrix of order n.

The OLS estimator of β is defined as:

$$\hat{\beta} = (X'X)^{-1} X'Y \,. \tag{2}$$

Both the OLS estimator and its covariance matrix heavily depend on the characteristics of the $X'X$ matrix. If $X'X$ is ill-conditioned, i.e. the column vectors of X are linearly dependent, the OLS estimator is sensitive to a number of errors. One of the most popular estimator dealing with multicollinearity is the ordinary ridge regression (ORR) estimator proposed by Hoerl and Kennard (1970a) and is defined as

$$\hat{\beta}_k = (X'X + k I_p)^{-1} X'Y = (I_p + k(X'X)^{-1})^{-1} \hat{\beta}\,, \tag{3}$$

where the constant $k > 0$ is known as the biasing parameter.

One may often have some exact linear restrictions binding the regression coefficients. Hence, we assume the exact linear restrictions binding the regression coefficients vector β of m linearly independent rows as follows:

$$r = R\beta \tag{4}$$

where r is an $m \times 1$ vector and R is an $m \times p$ matrix of rank $m\,(< p)$. The well-known restricted least squares (RLS) estimator is given by

$$\hat{\beta}_R = \hat{\beta} - (X'X)^{-1}R'\left[R(X'X)^{-1}R'\right]^{-1}(r - R\hat{\beta}). \quad (5)$$

Let us consider the general linear regression model

$$Y = X\beta + \varepsilon,\ \varepsilon \sim (0, \sigma^2 V^{-1}). \quad (6)$$

When V is a known positive definite (p.d.) matrix, the following estimators are proposed:

Aitken (1935) derived the generalized least squares (GLS) estimator as:

$$\hat{\beta}_{GLS} = (X'VX)^{-1}X'VY. \quad (7)$$

However, in this case the problem of multicollinearity may also arise. So, Trenkler (1984) extended the idea of ridge regression to reduce the effects of autocorrelation and multicollinearity problems and he proposed the generalized ridge (GR) estimator of β in the general linear regression model as:

$$\hat{\beta}_{GR}(k) = (X'VX + kI_p)^{-1}X'VY,\quad k > 0. \quad (8)$$

He concluded that the ridge regression estimator which takes the autocorrelation into account can perform better than some other methods.

Many authors consider some estimation methods for the general linear model in case of V is unknown. So, the feasible generalized restricted least squares (FGRLS) estimator under the prior restrictions in Equation (4) is given by Chaturvedi et al. (2001)

$$\breve{\beta}_R = \breve{\beta} + (X'\hat{V}X)^{-1}R'[R(X'\hat{V}X)^{-1}R']^{-1}(r - R\breve{\beta}), \quad (9)$$

where $\hat{V} = V(\hat{\theta})$, $\hat{\theta}$ is a consistent estimator of θ, and

$$\breve{\beta} = (X'\hat{V}X)^{-1}X'\hat{V}Y \quad (10)$$

is the feasible generalized least squares (FGLS) estimator of β .Then, the feasible generalized ridge regression (FGR) estimator is given by Eledum and Zahri (2013)

$$\breve{\beta}_{GR}(k) = (X'\hat{V}X + kI_p)^{-1}X'\hat{V}Y\,,\; k > 0\,. \tag{11}$$

Chaturvedi and Shukla (1990) consider the following SR estimator of β :

$$\breve{\beta}_S = \left[1 - \frac{a}{n}\frac{(Y - X\breve{\beta})'\hat{V}(Y - X\breve{\beta})}{\breve{\beta}'X'\hat{V}X\breve{\beta}}\right]\breve{\beta}\,, \tag{12}$$

where $a \geq 0$ is the shrinkage factor. Replacing $\breve{\beta}$ by $\breve{\beta}_R$ in Equation (12), Chaturvedi et al. (2001) considered the following estimator as a restricted SR estimator of β ,

$$\breve{\beta}_{RS} = \left[1 - \frac{a}{n}\frac{(Y - X\breve{\beta}_R)'\hat{V}(Y - X\breve{\beta}_R)}{\breve{\beta}_R'M^{*}{}'(X'\hat{V}X)M^{*}\breve{\beta}_R}\right]\breve{\beta}_R \tag{13}$$

where $M^{*} = I - (X'\hat{V}X)^{-1}R'\left[R(X'\hat{V}X)^{-1}R'\right]^{-1}R$. Moreover, if $\breve{\beta}_S$ is replaced for $\breve{\beta}$ in Equation (9), they obtain the restricted estimator,

$$\bar{\beta}_{RS} = \breve{\beta}_S + (X'\hat{V}X)^{-1}R'[R(X'\hat{V}X)^{-1}R']^{-1}(r - R\breve{\beta}_S) \tag{14}$$

which can be expressed equivalently as,

$$\bar{\beta}_{RS} = \breve{\beta}_R - \frac{a}{n}\frac{(Y - X\breve{\beta})'\hat{V}(Y - X\breve{\beta})}{\breve{\beta}'X'\hat{V}X\breve{\beta}}\hat{S}\,\breve{\beta} \tag{15}$$

where $\hat{S} = I_p - (X'\hat{V}X)^{-1}R'[R(X'\hat{V}X)^{-1}R']^{-1}R$.

More recently, Özbay et al. (2016) have considered replacing $\breve{\beta}$ by $\breve{\beta}_{GR}(k)$ in Equation (9) and extending the idea of restricted ridge estimation method (see, Gross, 2003; Kaçıranlar et al. 2011) for the case of V is unknown, they have defined the feasible generalized restricted ridge regression (FGRR) estimator as follows:

$$\breve{\beta}_{GRR}(k) = \breve{\beta}_{GR}(k) + (X'\hat{V}X + kI_p)^{-1} R'[R(X'\hat{V}X + kI_p)^{-1} R']^{-1}(r - R\breve{\beta}_{GR}(\mathrm{k})) \quad (16)$$

However, since $\breve{\beta}_S$ in Equation (12) depends on the $X'\hat{V}X$ matrix, the multicollinearity problem also arises in connection with the FGLS estimator in Equation (10). To reduce the effects of the multicollinearity for the estimators $\breve{\beta}_s$, $\breve{\beta}_{RS}$ and $\overline{\beta}_{RS}$, we now propose the following estimators:

Combining the philosophy behind the FGR and SR estimation, if $\breve{\beta}_{GR}(k)$ is replaced for $\breve{\beta}$ in Equation (12), we obtain the following feasible generalized SR ridge (FGSRR) regression estimator:

$$\breve{\beta}_{GSR}(k) = \left[1 - \frac{a}{n}\frac{(Y - X\breve{\beta}_{GR}(\mathrm{k}))'\hat{V}(Y - X\breve{\beta}_{GR}(\mathrm{k}))}{\breve{\beta}'_{GR}(\mathrm{k})(X'\hat{V}X + kI_p)\breve{\beta}_{GR}(k)}\right]\breve{\beta}_{GR}(k). \quad (17)$$

Following Srivastava and Srivastava (1983, 1984) and Chaturvedi et al. (2001), if we replace $\breve{\beta}_{GR}(k)$ by $\breve{\beta}_{GRR}(k)$ in Equation (17), we obtain the following FGSRR estimator:

$$\breve{\beta}_{GSRR}(k) = \left[1 - \frac{a}{n}\frac{(Y - X\breve{\beta}_{GRR}(\mathrm{k}))'\hat{V}(Y - X\breve{\beta}_{GRR}(\mathrm{k}))}{\breve{\beta}'_{GRR}(\mathrm{k})M^{**}{}'(X'\hat{V}X + kI_p)M^{**}\breve{\beta}_{GRR}(k)}\right]\breve{\beta}_{GRR}(k), \quad (18)$$

where $M^{**} = I - (X'\hat{V}X + kI_p)^{-1} R'\left[R(X'\hat{V}X + kI_p)^{-1} R'\right]^{-1} R$.

If we replace $\breve{\beta}_{GR}(k)$ by $\breve{\beta}_{GSR}(k)$ in Equation (16), we obtain the following FGSRR estimator:

$$\bar{\beta}_{GSRR}(k)=\breve{\beta}_{GSR}(k)+(X'\hat{V}X+kI_p)^{-1}R'[R(X'\hat{V}X+kI_p)^{-1}R']^{-1}(r-R\breve{\beta}_{GSR}(\mathrm{k})) \tag{19}$$

Furthermore, our new estimators have some tempting properties. Initially, it is easy to see that the new estimator $\bar{\beta}_{GSRR}(k)$ satisfy the prior restrictions in Equation (4). In fact, from the definitions of the $\breve{\beta}_{GSR}(k)$ and $\breve{\beta}_{GSRR}(k)$, we can see that these estimators are general estimators that include the $\breve{\beta}_S$ and $\bar{\beta}_{RS}$ as special cases:

(i) Since $\breve{\beta}_{GR}(0)=\breve{\beta}$, then $\breve{\beta}_{GSR}(0)=\breve{\beta}_S$

(ii) Since $\breve{\beta}_{GSR}(0)=\breve{\beta}_S$, then $\bar{\beta}_{GSRR}(0)=\bar{\beta}_{RS}$.

3. THE PMSE UNDER THE TARGET FUNCTION

In this section, we will represent the prediction performance of our new estimators under the target function. Generally predictions from a linear regression model are made either for the actual values of the study variable or for the average values at a time. However, situations may occur in which one may be required to consider the predictions of both the actual and the average values simultaneously.

If $\tilde{\beta}$ denotes an estimator of β, then the predictor for the values of study variable within the sample is generally formulated as $\hat{T}=X\tilde{\beta}$ which is used for predicting either the actual values y or the average values $E(y)=X\beta$ at a time. When the situation demands prediction of both the actual and the average values together, the target function is defined as follows,

$$T(y)=ty+(1-t)E(y) \tag{20}$$

and use $\hat{T}=X\tilde{\beta}$ for predicting it where $0\le t\le 1$ is a nonstochastic scalar specifying the weightage to be assigned to the prediction of actual and average values of the study variable, see Shalabh (1995), Toutenburg and Shalabh (1996), Toutenburg and Shalabh (2000), Kumar et al. (2008). The PMSE matrix under the target function of the predictor $\hat{T}$ is given by

$$PMSE\left(\hat{T}\right)=E\left(\hat{T}-T\right)\left(\hat{T}-T\right)'. \tag{21}$$

Then, the scalar PMSE under the target function of the predictor $\hat{T}$ is given by

$$pmse\left(\hat{T}\right)=E\left(\hat{T}-T\right)'\left(\hat{T}-T\right). \tag{22}$$

This strong criterion is used by many authors who are working on the predictive performance of the estimators. Shalabh (1995), Toutenburg and Shalabh (1996, 2000) and Kumar et al. (2008) have theoretically studied the predictive performance of some predictors obtained by using different estimators under the PMSE criterion. They derived some conditions for the superiority of the proposed estimator over the other known estimators.

However, since these conditions depend on the unknown parameters, it is difficult to know how to make use of the theoretical results in practice. Thus, we consider the prediction performance of the estimators $\breve{\beta}_R$, $\breve{\beta}_{GRR}(k)$, $\breve{\beta}_S$, $\breve{\beta}_{GSR}(k)$, $\breve{\beta}_{RS}$, $\breve{\beta}_{GSRR}(k)$ and $\bar{\beta}_{GSRR}(k)$ according to the PMSE criterion under the target function using a Monte Carlo simulation study.

4. A MONTE CARLO SIMULATION STUDY

In this section, we will discuss the simulation study to compare the performances of the $\breve{\beta}_R$, $\breve{\beta}_{GRR}(k)$, $\breve{\beta}_S$, $\breve{\beta}_{GSR}(k)$, $\breve{\beta}_{RS}$, $\breve{\beta}_{GSRR}(k)$ and $\bar{\beta}_{GSRR}(k)$ estimators. MATLAB is used for the simulation experiment. Following

McDonald and Galarneau (1975) and Kibria (2003), the explanatory variables are generated by

$$x_{ij} = (1-\gamma^2)^{1/2} z_{ij} + \gamma z_{i\,p+1} \ , \ i = 1,2,...,n, \quad j = 1,2,..., p \tag{23}$$

where z_{ij} are independent standard normal pseudo-random numbers, γ is specified so that the correlation between any two explanatory variables is given by γ^2. Following Kibria (2003), three different sets of correlation are considered, corresponding to $\gamma = 0.7, 0.8, 0.9$. Following Chaturvedi et al. (2001), data are generated using the following orthonormal model with $p = 4$ and $n = 20$:

$$y_i = \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} + \beta_4 x_{4i} + u_i; \quad i = 1,\ldots,20 \tag{24}$$

where the u_i's are assumed to be generated by either an AR(1) process ($u_i = \rho u_{i-1} + \varepsilon_i$) or a MA(1) process ($u_i = \varepsilon_i - \rho \varepsilon_{i-1}$); where ρ is taken as -0.8, -0.4, 0.0, 0.4, 0.8, and $\varepsilon_i \sim IN(0,1)$. Also, $a = p-3$ and $\alpha = 2$ are taken like this.

The estimators are compared according to scalar PMSE under target function defined as

$$pmse\left(\hat{T}\right) = \frac{1}{MCN} \sum_{mci=1}^{MCN} \left(\hat{T}_{mci} - T\right)' \left(\hat{T}_{mci} - T\right), \tag{25}$$

where MCN represents the number of Monte Carlo replications which is taken as 5000.

Following Kaçıranlar et al. (2011), two different types of restrictions are used to see how the number of restrictions affects the pmse values. The first type of restriction about the parameters is taken as $\beta_1 + \beta_2 + \cdots + \beta_p = \frac{p(p+1)}{2} \times \tau$. For this restriction, one may write $R = (1\ 1\ \dots\ 1)$, and $r = \frac{p(p+1)}{2} \times \tau$. Since the true parameter vector is $\beta = (1\ 2\ \dots\ p)'$, the sum given in the restriction is equal to $\frac{p(p+1)}{2} \times \tau$.The usage of τ allows one to control whether the restriction is true or not, and if it not true, to control the difference between the right and left hand sides of the restriction, τ is selected as 1, 1.05, or 1.10, and it is fixed constant throughout the replications. Since the left hand side of the restriction is equal to $\frac{p(p+1)}{2}$, it is obvious that the restriction $\beta_1 + \beta_2 + \cdots + \beta_p = \frac{p(p+1)}{2} \times \tau$ is true when $\tau = 1$. However, when τ is chosen as 1.05 (or 1.10), the right hand side of the restriction is equal to $\frac{p(p+1)}{2} \times 1.05$ (or $\frac{p(p+1)}{2} \times 1.10$), while the left side remains $\frac{p(p+1)}{2}$, which leads to a 5% (or 10%) deviation on the right hand side of the restriction. Since R, r, β, and τ are all defined at the beginning of the simulations and fixed through the replications, the restrictions are still nonstochastic. For the second type of restriction R and r are chosen as

$$R_{\frac{p}{2} \times p} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 1 & 1 \end{pmatrix} \text{and}\, r_{\frac{p}{2} \times 1} = \begin{pmatrix} 3 \\ 7 \\ \vdots \\ 2p-1 \end{pmatrix} \times \tau$$

where the number of restrictions is $m = p/2$. The function and values of τ are still the same as given in the first restriction.

The approaches of Hoerl et al. (1975) are used to specify the values of k. The estimates are denoted as $\hat{k}_{HK} = \frac{\hat{\sigma}^2_{GLS}}{\sum_{i=1}^{p} \breve{\beta}_i^2}$, $\hat{k}_{HKB} = \frac{p\hat{\sigma}^2_{GLS}}{\sum_{i=1}^{p} \breve{\beta}_i^2}$ and

$\hat{k}_{LW} = \dfrac{p\hat{\sigma}^2_{GLS}}{\sum_{i=1}^{p} \lambda_i \breve{\beta}_i^2}$ respectively, where $\hat{\sigma}^2_{GLS} = \dfrac{(Y - X\breve{\beta})'\hat{V}(Y - X\breve{\beta})}{n-p}$ is the FGLS estimate of error variance and λ_i 's are the eigenvalues of the matrix $X'\hat{V}X$.

The results of the simulation study are summarized in Tables 1–6.

COMMENTS:

For Tables 1 and 2, we found the pmse values of the mentioned estimators under Restriction Type 1 when it is true and AR(1) or MA(1) to study the performance of the mentioned estimators according to t=0.1, 0.5, 0.9 values.

We found the pmse values of the mentioned estimators under Restriction Type 1 and 2 when it is true or not and AR(1) or MA(1) according to t=0.1, 0.5, 0.9 values but since there are a lot of tables, we just added the results for t=0.1 in Tables 3-6 and for t=0.5 and t=0.9 are not added.

- When ρ value is increasing, pmse values for all mentioned estimators are decreasing for $t = 0.1$. Also, pmse values for all mentioned estimators are decreasing till ρ value equals 0 and then pmse values are increasing till ρ value equals 0.8 for $t = 0.5, 0.9$.
- When γ value is increasing, pmse values for $\breve{\beta}_S$, $\breve{\beta}_{RS}$ and $\breve{\beta}_R$ are increasing and decreasing for $\breve{\beta}_{GSR}(k)$, $\breve{\beta}_{GSRR}(k)$, $\breve{\beta}_{GRR}(k)$ and $\bar{\beta}_{GSRR}(k)$ of most of k 's in general.
- For the following pair-wise comparisons among the estimators, in each row the best estimator is highlighted in bold font under each restriction, AR(1) and MA(1) error term model and $t = 0.1, 0.5, 0.9$.

1- When we make a comparison between $\breve{\beta}_S$ and $\breve{\beta}_{GSR}(k)$ estimators, we have that $\breve{\beta}_{GSR}(k)$ estimator gives better results in pmse values than $\breve{\beta}_S$ estimator especially for the estimated value $\hat{k}_{HKB}$ in most cases.

2- When we make a comparison between $\breve{\beta}_{RS}$ and $\breve{\beta}_{GSRR}(k)$ estimators, we have that $\breve{\beta}_{GSRR}(k)$ estimator gives better results in pmse values than $\breve{\beta}_{RS}$ estimator especially for the estimated value $\hat{k}_{HKB}$.

3- When we make a comparison between $\breve{\beta}_R$ and $\breve{\beta}_{GRR}(k)$ estimators, we have that $\breve{\beta}_{GRR}(k)$ estimator gives better results in pmse values than $\breve{\beta}_R$ estimator especially for the estimated value $\hat{k}_{HKB}$.

4- When we make a comparison between $\breve{\beta}_R$ and $\bar{\beta}_{GSRR}(k)$ estimators, we have that $\bar{\beta}_{GSRR}(k)$ estimator gives better results in pmse values than $\breve{\beta}_R$ estimator especially for the estimated value $\hat{k}_{HKB}$.

5- When we make a comparison between $\breve{\beta}_{GRR}(k)$ and $\bar{\beta}_{GSRR}(k)$ estimators, we have that $\bar{\beta}_{GSRR}(k)$ estimator gives better results in pmse values than $\breve{\beta}_{GRR}(k)$ estimator especially for the estimated value $\hat{k}_{HKB}$.

- Under the same restriction type, when t value is increasing, pmse values for all mentioned estimators are increasing for the AR(1) error term model rapidly than for the MA(1) error term model.

- For AR(1) and MA(1), the pmse values are so close to each other for all mentioned estimators with the same restriction type and the same value of t.

5. CONCLUSION

In this study, we introduce some feasible generalized SR restricted ridge regression estimators to examine multicollinearity and autocorrelation problems simultaneously for the general linear regression model. Then we investigate the performance of the defined estimators using a Monte Carlo simulation study in which the results show that the proposed estimators have smaller pmse values relative to the other mentioned estimators especially for the estimated value $\hat{k}_{HKB}$ in most cases under the first and the second restrictions when they are true or not and when the error term follows AR(1) or MA(1) model.

Table 1. pmse values of the mentioned estimators under Restriction Type 1 when it is true and $AR(1)$

t	ρ	γ	$\breve{\beta}_S$	$\breve{\beta}_{GSR}(k)$			$\breve{\beta}_{RS}$	$\breve{\beta}_{GSRR}(k)$			$\breve{\beta}_R$	$\breve{\beta}_{GRR}(k)$			$\bar{\beta}_{GSRR}(k)$		
			------	$\hat{k}_{HK}$	$\hat{k}_{HKB}$	$\hat{k}_{LW}$	------	$\hat{k}_{HK}$	$\hat{k}_{HKB}$	$\hat{k}_{LW}$	------	$\hat{k}_{HK}$	$\hat{k}_{HKB}$	$\hat{k}_{LW}$	$\hat{k}_{HK}$	$\hat{k}_{HKB}$	$\hat{k}_{LW}$
0.1	-0.8	0.7	18.0752	17.5445	**16.5308**	18.0529	17.5803	17.1089	**16.0429**	17.5610	17.6298	17.0476	**15.6627**	17.6062	16.8711	**15.5246**	17.4148
		0.8	18.1756	17.4352	**15.9321**	18.1415	18.0454	17.4712	**16.2882**	18.0190	17.6298	16.8586	**15.0997**	17.5950	16.7068	**14.9831**	17.4285
		0.9	18.2132	17.0279	**14.7033**	18.1520	19.3181	18.5618	**17.4360**	19.2773	17.6298	16.4047	**13.8638**	17.5671	16.2843	**13.7768**	17.4311
	-0.4	0.7	5.9627	5.9104	**5.8522**	5.9595	5.3915	5.3399	**5.2273**	5.3885	5.2872	5.2157	**5.0442**	5.2832	5.1918	**5.0276**	5.2568
		0.8	5.9161	5.8298	**5.6792**	5.9105	5.6359	5.5717	**5.4433**	5.6318	5.2872	5.1866	**4.9495**	5.2810	5.1657	**4.9346**	5.2579
		0.9	5.9253	5.7557	**5.4185**	5.9135	6.8395	**6.7917**	6.7973	6.8355	5.2872	5.1066	**4.7014**	5.2749	5.0894	**4.6891**	5.2558
	0.0	0.7	3.0574	3.0520	**3.0546**	3.0570	2.3542	2.3492	**2.3409**	2.3539	2.2883	2.2787	**2.2565**	2.2877	2.2745	**2.2537**	2.2831
		0.8	3.0462	3.0358	**3.0234**	3.0454	2.4226	2.4159	**2.4050**	2.4221	2.2883	2.2747	**2.2435**	2.2873	2.2712	**2.2412**	2.2835
		0.9	3.0330	3.0098	**2.9657**	3.0311	2.6078	2.5966	**2.5816**	2.6068	2.2883	2.2630	**2.2052**	2.2864	2.2602	**2.2033**	2.2832
	0.4	0.7	1.6191	**1.6144**	1.6161	1.6188	1.4263	1.4227	**1.4169**	1.4261	1.3738	1.3662	**1.3485**	1.3734	1.3640	**1.3472**	1.3709
		0.8	1.6580	1.6488	**1.6379**	1.6574	1.4323	1.4257	**1.4132**	1.4319	1.3738	1.3630	**1.3382**	1.3731	1.3612	**1.3370**	1.3711
		0.9	1.7016	1.6803	**1.6396**	1.7002	1.4234	1.4074	**1.3746**	1.4223	1.3738	1.3536	**1.3084**	1.3725	1.3522	**1.3074**	1.3708
	0.8	0.7	0.6778	0.6780	0.6809	**0.6778**	0.6731	0.6734	0.6747	**0.6731**	0.6658	**0.6656**	0.6657	0.6658	**0.6656**	0.6658	0.6658
		0.8	0.6789	**0.6789**	0.6815	0.6789	0.6716	**0.6716**	0.6728	0.6716	0.6658	**0.6656**	0.6657	0.6658	**0.6655**	0.6657	0.6658
		0.9	0.6805	**0.6801**	0.6822	0.6804	0.6692	**0.6690**	0.6699	0.6692	0.6658	**0.6653**	0.6655	0.6658	**0.6653**	0.6654	0.6658
0.5	-0.8	0.7	63.4563	63.0433	**62.3445**	63.4386	63.6144	63.2576	**62.5007**	63.5997	63.5406	63.0762	**62.0090**	63.5216	62.9418	**61.9119**	63.3727
		0.8	63.4838	62.9030	**61.8160**	63.4566	64.1478	63.7262	**62.9431**	64.1280	63.5406	62.9259	**61.5768**	63.5126	62.8094	**61.4939**	63.3822
		0.9	63.3291	62.3990	**60.6992**	63.2804	65.5211	65.0104	**64.4877**	65.4924	63.5406	62.5675	**60.6447**	63.4903	62.4751	**60.5832**	63.3833
	-0.4	0.7	11.9848	**11.9542**	11.9587	11.9828	12.1014	12.0720	**12.0236**	12.0997	11.9807	11.9317	**11.8256**	11.9779	11.9167	**11.8179**	11.9604
		0.8	11.9819	11.9266	**11.8648**	11.9782	12.3525	12.3192	**12.2791**	12.3502	11.9807	11.9117	**11.7651**	11.9763	11.8983	**11.7577**	11.9608
		0.9	11.9661	11.8533	**11.6726**	11.9580	13.5710	13.5783	13.7355	**13.5707**	11.9807	11.8572	**11.6100**	11.9720	11.8462	**11.6036**	11.9592
	0.0	0.7	4.7491	**4.7478**	4.7626	4.7489	4.8222	**4.8217**	4.8267	4.8222	4.7500	4.7449	**4.7361**	4.7497	4.7429	**4.7354**	4.7472
		0.8	4.7492	**4.7449**	4.7507	4.7488	4.8926	**4.8921**	4.8998	4.8925	4.7500	4.7428	**4.7305**	4.7495	4.7411	**4.7299**	4.7474
		0.9	4.7493	4.7380	**4.7285**	4.7484	5.0797	5.0802	5.0993	**5.0797**	4.7500	4.7366	**4.7137**	4.7489	4.7352	**4.7132**	4.7472
	0.4	0.7	7.3552	7.3570	7.3781	**7.3552**	7.5822	7.5854	7.5996	**7.5822**	7.5180	**7.5172**	7.5197	7.5179	**7.5172**	7.5205	7.5176
		0.8	7.3563	7.3570	7.3753	**7.3562**	7.5976	7.6006	7.6164	**7.5976**	7.5180	**7.5169**	7.5204	7.5179	**7.5168**	7.5210	7.5177
		0.9	7.3478	**7.3462**	7.3622	7.3476	7.5918	7.5938	7.6128	**7.5918**	7.5180	**7.5159**	7.5224	7.5178	**7.5158**	7.5228	7.5176
	0.8	0.7	15.0135	15.0151	15.0220	**15.0135**	15.1445	15.1459	15.1509	**15.1445**	15.1344	15.1354	15.1391	**15.1344**	15.1356	15.1394	**15.1344**
		0.8	14.9723	14.9744	14.9830	**14.9723**	15.1435	15.1452	15.1515	**15.1435**	15.1344	15.1358	15.1410	**15.1344**	15.1360	15.1412	**15.1344**
		0.9	14.9232	14.9268	14.9408	**14.9232**	15.1404	15.1433	15.1536	**15.1404**	15.1344	15.1370	15.1466	**15.1344**	15.1371	15.1467	**15.1344**
0.9	-0.8	0.7	180.4704	180.175	**179.7911**	180.4573	181.2816	181.0393	**180.5916**	181.2713	181.0844	180.7378	**179.9883**	181.0701	180.6454	**179.9322**	180.9637
		0.8	180.4249	180.0039	**179.3329**	180.4048	181.8832	181.6142	**181.2311**	181.8701	181.0844	180.6262	**179.6869**	181.0632	180.5451	**179.6378**	180.9689
		0.9	180.0779	179.4032	**178.328**	180.0418	183.3571	**183.092**	183.1724	183.3404	181.0844	180.3633	**179.0585**	181.0464	180.2989	**179.0225**	180.9686
	-0.4	0.7	31.767	**31.7581**	31.8253	31.7661	32.5715	**32.5642**	32.58	32.5709	32.4342	32.4078	**32.3671**	32.4326	32.4017	**32.3683**	32.4241
		0.8	31.8078	**31.7836**	31.8104	31.8059	32.8291	**32.8267**	32.8751	32.8287	32.4342	32.3969	**32.3408**	32.4317	32.3911	**32.3409**	32.4239
		0.9	31.767	31.7111	**31.6869**	31.7626	34.0626	34.125	34.4339	**34.0626**	32.4342	32.368	**32.2788**	32.4293	32.3631	**32.2783**	32.4227
	0.0	0.7	12.5208	12.5236	12.5506	**12.5208**	13.3703	13.3742	13.3925	**13.3703**	13.2917	**13.2911**	13.2958	13.2916	**13.2912**	13.2972	13.2912
		0.8	12.5322	12.5341	12.558	**12.5322**	13.4426	13.4483	13.4746	**13.4426**	13.2917	**13.2909**	13.2974	13.2916	**13.2909**	13.2985	13.2912
		0.9	12.5457	12.5462	12.5714	**12.5456**	13.6317	13.6438	13.697	**13.6317**	13.2917	**13.2902**	13.3021	13.2915	**13.2902**	13.303	13.2912
	0.4	0.7	23.8835	23.8919	23.9324	**23.8835**	24.5304	24.5403	24.5746	**24.5304**	24.4545	24.4605	24.483	**24.4545**	24.4626	24.486	**24.4545**
		0.8	23.8468	23.8575	23.9049	**23.8468**	24.5551	24.5677	24.6119	**24.5551**	24.4545	24.463	24.4949	**24.4545**	24.4648	24.4973	**24.4545**
		0.9	23.7862	23.8044	23.8771	**23.7862**	24.5524	24.5725	24.6432	**24.5524**	24.4545	24.4703	24.5286	**24.4545**	24.4717	24.5304	**24.4545**
	0.8	0.7	49.5926	49.5955	49.6064	**49.5926**	49.8592	49.8618	49.8704	**49.8592**	49.8462	49.8485	49.8558	**49.8462**	49.849	49.8564	**49.8462**
		0.8	49.5092	49.5132	49.5278	**49.5092**	49.8587	49.8622	49.8735	**49.8587**	49.8462	49.8494	49.8597	**49.8462**	49.8498	49.8601	**49.8462**
		0.9	49.4092	49.4169	49.4427	**49.4092**	49.8549	49.861	49.8807	**49.8549**	49.8462	49.8521	49.871	**49.8462**	49.8524	49.8712	**49.8462**

Table 2. pmse values of the mentioned estimators under Restriction Type 1 when it is true and $MA(1)$

t	ρ	γ	$\breve{\beta}_S$	$\breve{\beta}_{GSR}(k)$			$\breve{\beta}_{RS}$	$\breve{\beta}_{GSRR}(k)$			$\breve{\beta}_R$	$\breve{\beta}_{GRR}(k)$			$\overline{\beta}_{GSRR}(k)$		
			------	$\hat{k}_{HK}$	$\hat{k}_{HKB}$	$\hat{k}_{LW}$	------	$\hat{k}_{HK}$	$\hat{k}_{HKB}$	$\hat{k}_{LW}$	------	$\hat{k}_{HK}$	$\hat{k}_{HKB}$	$\hat{k}_{LW}$	$\hat{k}_{HK}$	$\hat{k}_{HKB}$	$\hat{k}_{LW}$
0.1	-0.8	0.7	26.2753	25.8226	**24.9881**	26.2642	23.9682	23.4693	**22.1583**	23.9568	24.2769	23.7634	**22.4128**	24.2652	23.5603	**22.2329**	24.0536
		0.8	26.4985	25.8557	**24.4757**	26.4808	24.0096	23.3478	**21.6430**	23.9921	24.2769	23.6009	**21.8590**	24.2590	23.4317	**21.7083**	24.0828
		0.9	26.6261	25.5850	**23.2255**	26.5932	24.0553	23.0027	**20.4215**	24.0226	24.2769	23.2095	**20.5921**	24.2437	23.0817	**20.4794**	24.1098
	-0.4	0.7	6.7027	6.6598	**6.6262**	6.6999	5.3200	5.2736	**5.1725**	5.3173	5.3043	5.2447	**5.1047**	5.3009	5.2208	**5.0885**	5.2744
		0.8	6.6740	6.6029	**6.4909**	6.6692	5.3793	5.3135	**5.1714**	5.3751	5.3043	5.2207	**5.0270**	5.2990	5.1999	**5.0123**	5.2761
		0.9	6.6803	6.5369	**6.2582**	6.6700	5.5371	5.4168	**5.1681**	5.5285	5.3043	5.1529	**4.8146**	5.2936	5.1359	**4.8022**	5.2749
	0.0	0.7	3.0574	**3.0520**	3.0546	3.0570	2.3542	2.3492	**2.3409**	2.3539	2.2883	2.2787	**2.2565**	2.2877	2.2745	**2.2537**	2.2831
		0.8	3.0462	3.0358	**3.0234**	3.0454	2.4226	2.4159	**2.4050**	2.4221	2.2883	2.2747	**2.2435**	2.2873	2.2712	**2.2412**	2.2835
		0.9	3.0330	3.0098	**2.9657**	3.0311	2.6078	2.5966	**2.5816**	2.6068	2.2883	2.2630	**2.2052**	2.2864	2.2602	**2.2033**	2.2832
	0.4	0.7	2.1772	**2.1711**	2.1742	2.1768	1.9868	1.9823	**1.9765**	1.9865	1.8956	1.8847	**1.8599**	1.8949	1.8805	**1.8574**	1.8902
		0.8	2.1997	2.1879	**2.1747**	2.1989	1.9868	1.9778	**1.9619**	1.9862	1.8956	1.8803	**1.8455**	1.8946	1.8768	**1.8434**	1.8907
		0.9	2.2328	2.2059	**2.1554**	2.2310	1.9694	1.9471	**1.9021**	1.9679	1.8956	1.8670	**1.8037**	1.8936	1.8643	**1.8021**	1.8905
	0.8	0.7	2.7900	**2.7834**	2.7905	2.7896	2.8093	2.8043	**2.7997**	2.8091	2.6545	2.6417	**2.6141**	2.6539	2.6341	**2.6096**	2.6454
		0.8	2.7963	2.7830	**2.7720**	2.7956	2.6973	2.6830	**2.6557**	2.6966	2.6545	2.6364	**2.5979**	2.6536	2.6309	**2.5945**	2.6474
		0.9	2.8183	2.7867	**2.7323**	2.8166	2.6613	2.6295	**2.5645**	2.6597	2.6545	2.6209	**2.5507**	2.6528	2.6173	**2.5483**	2.6488
0.5	-0.8	0.7	32.3871	31.9890	**31.3114**	32.3773	32.0811	31.6308	**30.4597**	32.0707	32.3593	31.8947	**30.6846**	32.3486	31.7140	**30.5269**	32.1596
		0.8	32.6195	32.0476	**30.8687**	32.6037	32.1192	31.5225	**30.0021**	32.1034	32.3593	31.7487	**30.1921**	32.3430	31.5979	**30.0595**	32.1854
		0.9	32.6959	31.7700	**29.7275**	32.6665	32.1608	31.2151	**28.9265**	32.1313	32.3593	31.3992	**29.0755**	32.3293	31.2854	**28.9762**	32.2097
	-0.4	0.7	10.3900	**10.3655**	10.3855	10.3883	10.3193	10.2906	**10.2413**	10.3176	10.2764	10.2346	**10.1469**	10.2739	10.2195	**10.1394**	10.2562
		0.8	10.3944	10.3489	**10.3112**	10.3912	10.3871	10.3462	**10.2761**	10.3844	10.2764	10.2178	**10.0967**	10.2726	10.2045	**10.0894**	10.2571
		0.9	10.4169	10.3204	**10.1736**	10.4097	10.5621	10.4873	**10.3668**	10.5566	10.2764	10.1709	**9.9620**	10.2688	10.1599	**9.9554**	10.2561
	0.0	0.7	4.7491	**4.7478**	4.7626	4.7489	4.8222	**4.8217**	4.8267	4.8222	4.7500	4.7449	**4.7361**	4.7497	4.7429	**4.7354**	4.7472
		0.8	4.7492	**4.7449**	4.7507	4.7488	4.8926	**4.8921**	4.8998	4.8925	4.7500	4.7428	**4.7305**	4.7495	4.7411	**4.7299**	4.7474
		0.9	4.7493	4.7380	**4.7285**	4.7484	5.0797	5.0802	5.0993	**5.0797**	4.7500	4.7366	**4.7137**	4.7489	4.7352	**4.7132**	4.7472
	0.4	0.7	7.2223	7.2230	7.2465	**7.2223**	7.6343	7.6371	7.6532	**7.6343**	7.5309	7.5275	**7.5248**	7.5307	7.5265	**7.5254**	7.5292
		0.8	7.2266	**7.2250**	7.2417	7.2264	7.6379	7.6394	7.6541	**7.6379**	7.5309	7.5261	**7.5222**	7.5306	7.5253	**7.5227**	7.5293
		0.9	7.2391	**7.2320**	7.2385	7.2385	7.6213	**7.6183**	7.6292	7.6210	7.5309	7.5219	**7.5148**	7.5302	7.5212	**7.5152**	7.5292
	0.8	0.7	12.9733	12.9750	13.0064	**12.9733**	13.5775	13.5811	13.6022	**13.5775**	13.4039	13.3999	**13.3982**	13.4037	**13.3970**	13.3983	13.3999
		0.8	12.9562	**12.9549**	12.9789	12.9561	13.4605	**13.4584**	13.4667	13.4603	13.4039	13.3981	**13.3955**	13.4036	13.3963	**13.3957**	13.4011
		0.9	12.9270	**12.9188**	12.9316	12.9264	13.4184	13.4091	**13.4081**	13.4179	13.4039	13.3928	**13.3870**	13.4033	13.3919	**13.3873**	13.4020
0.9	-0.8	0.7	57.2239	56.8804	**56.3597**	57.2153	58.9189	58.5173	**57.486**	58.9097	59.1666	58.7509	**57.6815**	59.157	58.5928	**57.5459**	58.9906
		0.8	57.4656	56.9644	**55.9867**	57.4516	58.9538	58.4222	**57.0863**	58.9397	59.1666	58.6214	**57.2502**	59.1521	58.4891	**57.1356**	59.0131
		0.9	57.4908	56.6799	**54.9544**	57.4648	58.9913	58.1524	**56.1565**	58.965	59.1666	58.3138	**56.284**	59.1399	58.2141	**56.1981**	59.0346
	-0.4	0.7	25.5071	**25.5008**	25.5745	25.5063	26.7483	**26.7374**	26.7399	26.7475	26.6782	26.6542	**26.6187**	26.6767	26.6479	**26.62**	26.6678
		0.8	25.5445	**25.5247**	25.5612	25.5429	26.8246	**26.8085**	26.8105	26.8234	26.6782	26.6447	**26.5961**	26.6759	26.6388	**26.5962**	26.6679
		0.9	25.5832	25.5335	**25.5187**	25.5791	27.0168	**26.9875**	26.9953	27.0143	26.6782	26.6186	**26.5391**	26.6737	26.6136	**26.5383**	26.6671
	0.0	0.7	12.5208	12.5236	12.5506	**12.5208**	13.3703	13.3742	13.3925	**13.3703**	13.2917	**13.2911**	13.2958	13.2916	**13.2912**	13.2972	13.2912
		0.8	12.5322	12.5341	12.558	**12.5322**	13.4426	13.4483	13.4746	**13.4426**	13.2917	**13.2909**	13.2974	13.2916	**13.2909**	13.2985	13.2912
		0.9	12.5457	12.5462	12.5714	**12.5456**	13.6317	13.6438	13.697	**13.6317**	13.2917	**13.2902**	13.3021	13.2915	**13.2902**	13.303	13.2912
	0.4	0.7	22.8733	22.8809	22.9246	**22.8733**	23.8876	23.8979	23.9357	**23.8876**	23.7721	23.7762	23.7955	**23.7721**	23.7783	23.7993	**23.7721**
		0.8	22.8594	22.868	22.9145	**22.8594**	23.8949	23.9068	23.9521	**23.8949**	23.7721	23.7778	23.8048	**23.7721**	23.7796	23.8079	**23.7721**
		0.9	22.8513	22.8639	22.9274	**22.8513**	23.8791	23.8954	23.9621	**23.8791**	23.7721	23.7825	23.8317	**23.7721**	23.7839	23.8341	**23.7721**
	0.8	0.7	40.4963	40.5062	40.5621	**40.4963**	41.6853	41.6976	41.7443	**41.6853**	41.4932	41.4977	41.5219	**41.4932**	41.4996	41.5267	**41.4932**
		0.8	40.4558	40.4664	40.5255	**40.4558**	41.5633	41.5734	41.6173	**41.5633**	41.4932	41.4995	41.5326	**41.4932**	41.5013	41.5365	**41.4932**
		0.9	40.3753	40.3906	40.4706	**40.3753**	41.5151	41.5282	41.5915	**41.5151**	41.4935	41.5044	41.563	**41.4935**	41.5062	41.5658	**41.4935**

Table 3. pmse values of the mentioned estimators under Restriction Type 1, $AR(1)$ and $t=0.1$

τ	ρ	γ	$\breve{\beta}_S$	$\breve{\beta}_{GSR}(k)$			$\breve{\beta}_{RS}$	$\breve{\beta}_{GSRR}(k)$			$\breve{\beta}_R$	$\breve{\beta}_{GRR}(k)$			$\bar{\beta}_{GSRR}(k)$		
			------	$\hat{k}_{HK}$	$\hat{k}_{HKB}$	$\hat{k}_{LW}$	------	$\hat{k}_{HK}$	$\hat{k}_{HKB}$	$\hat{k}_{LW}$	------	$\hat{k}_{HK}$	$\hat{k}_{HKB}$	$\hat{k}_{LW}$	$\hat{k}_{HK}$	$\hat{k}_{HKB}$	$\hat{k}_{LW}$
1	-0.8	0.7	18.0752	17.5445	**16.5308**	18.0529	17.5803	17.1089	**16.0429**	17.5610	17.6298	17.0476	**15.6627**	17.6062	16.8711	**15.5246**	17.4148
		0.8	18.1756	17.4352	**15.9321**	18.1415	18.0454	17.4712	**16.2882**	18.0190	17.6298	16.8586	**15.0997**	17.5950	16.7068	**14.9831**	17.4285
		0.9	18.2132	17.0279	**14.7033**	18.1520	19.3181	18.5618	**17.4360**	19.2773	17.6298	16.4047	**13.8638**	17.5671	16.2843	**13.7768**	17.4311
	-0.4	0.7	5.9627	5.9104	**5.8522**	5.9595	5.3915	5.3399	**5.2273**	5.3885	5.2872	5.2157	**5.0442**	5.2832	5.1918	**5.0276**	5.2568
		0.8	5.9161	5.8298	**5.6792**	5.9105	5.6359	5.5717	**5.4433**	5.6318	5.2872	5.1866	**4.9495**	5.2810	5.1657	**4.9346**	5.2579
		0.9	5.9253	5.7557	**5.4185**	5.9135	6.8395	**6.7917**	6.7973	6.8355	5.2872	5.1066	**4.7014**	5.2749	5.0894	**4.6891**	5.2558
	0.0	0.7	3.0574	3.0520	**3.0546**	3.0570	2.3542	2.3492	**2.3409**	2.3539	2.2883	2.2787	**2.2565**	2.2877	2.2745	**2.2537**	2.2831
		0.8	3.0462	3.0358	**3.0234**	3.0454	2.4226	2.4159	**2.4050**	2.4221	2.2883	2.2747	**2.2435**	2.2873	2.2712	**2.2412**	2.2835
		0.9	3.0330	3.0098	**2.9657**	3.0311	2.6078	2.5966	**2.5816**	2.6068	2.2883	2.2630	**2.2052**	2.2864	2.2602	**2.2033**	2.2832
	0.4	0.7	1.6191	**1.6144**	1.6161	1.6188	1.4263	1.4227	**1.4169**	1.4261	1.3738	1.3662	**1.3485**	1.3734	1.3640	**1.3472**	1.3709
		0.8	1.6580	1.6488	**1.6379**	1.6574	1.4323	1.4257	**1.4132**	1.4319	1.3738	1.3630	**1.3382**	1.3731	1.3612	**1.3370**	1.3711
		0.9	1.7016	1.6803	**1.6396**	1.7002	1.4234	1.4074	**1.3746**	1.4223	1.3738	1.3536	**1.3084**	1.3725	1.3522	**1.3074**	1.3708
	0.8	0.7	0.6778	0.6780	0.6809	**0.6778**	0.6731	0.6734	0.6747	**0.6731**	0.6658	**0.6656**	0.6657	0.6658	**0.6656**	0.6658	0.6658
		0.8	0.6789	**0.6789**	0.6815	0.6789	0.6716	**0.6716**	0.6728	0.6716	0.6658	**0.6656**	0.6657	0.6658	**0.6655**	0.6657	0.6658
		0.9	0.6805	**0.6801**	0.6822	0.6804	0.6692	**0.6690**	0.6699	0.6692	0.6658	**0.6653**	0.6655	0.6658	**0.6653**	0.6654	0.6658
1.05	-0.8	0.7	18.0752	17.5445	**16.5308**	18.0529	17.7628	17.2711	**16.1401**	17.7428	19.1129	18.5326	**17.1496**	19.0895	18.3637	**17.0179**	18.9060
		0.8	18.1756	17.4352	**15.9321**	18.1415	17.7868	17.1654	**15.8392**	17.7584	19.5421	18.7655	**16.9892**	19.5071	18.6175	**16.8757**	19.3447
		0.9	18.2132	17.0279	**14.7033**	18.1520	17.8354	16.9617	**15.4926**	17.7892	20.1516	18.9090	**16.3226**	20.0881	18.7907	**16.2373**	19.9543
	-0.4	0.7	5.9627	5.9104	**5.8522**	5.9595	5.8972	5.8395	**5.7057**	5.8940	6.7619	6.6925	**6.5255**	6.7580	6.6717	**6.5118**	6.7348
		0.8	5.9161	5.8298	**5.6792**	5.9105	5.8486	5.7648	**5.5743**	5.8434	7.1864	7.0850	**6.8436**	7.1801	7.0655	**6.8300**	7.1586
		0.9	5.9253	5.7557	**5.4185**	5.9135	5.8229	5.7084	**5.5113**	5.8148	7.7793	7.5920	**7.1663**	7.7665	7.5753	**7.1543**	7.7480
	0.0	0.7	3.0574	**3.0520**	3.0546	3.0570	3.1246	3.1172	**3.1010**	3.1241	3.7565	3.7469	**3.7243**	3.7559	3.7427	**3.7215**	3.7513
		0.8	3.0462	3.0358	**3.0234**	3.0454	3.1920	3.1796	**3.1509**	3.1911	4.1738	4.1602	**4.1284**	4.1728	4.1567	**4.1260**	4.1690
		0.9	3.0330	3.0098	**2.9657**	3.0311	3.1360	3.1073	**3.0378**	3.1338	4.7486	4.7231	**4.6639**	4.7466	4.7202	**4.6619**	4.7435
	0.4	0.7	1.6191	**1.6144**	1.6161	1.6188	2.2965	2.2885	**2.2694**	2.2960	2.8704	2.8600	**2.8342**	2.8698	2.8576	**2.8327**	2.8671
		0.8	1.6580	1.6488	**1.6379**	1.6574	2.6096	2.5961	**2.5631**	2.6088	3.3170	3.3022	**3.2657**	3.3161	3.3015	**3.2657**	3.3152
		0.9	1.7016	1.6803	**1.6396**	1.7002	3.1582	3.1290	**3.0571**	3.1563	3.9440	3.9166	**3.8513**	3.9422	3.9180	**3.8529**	3.9434
	0.8	0.7	0.6778	0.6780	0.6809	**0.6778**	1.9921	1.9900	**1.9844**	1.9920	2.2199	2.2175	**2.2111**	2.2198	2.2179	**2.2115**	2.2201
		0.8	0.6789	**0.6789**	0.6815	0.6789	2.4981	2.4940	**2.4828**	2.4979	2.7416	2.7374	**2.7261**	2.7414	2.7389	**2.7276**	2.7429
		0.9	0.6805	**0.6801**	0.6822	0.6804	3.2682	3.2588	**3.2334**	3.2678	3.4988	3.4898	**3.4658**	3.4984	3.4927	**3.4685**	3.5013
1.10	-0.8	0.7	18.0752	17.5445	**16.5308**	18.0529	20.5938	20.0793	**18.8762**	20.5729	23.5721	22.9921	**21.6073**	23.5486	22.8308	**21.4820**	23.3732
		0.8	18.1756	17.4352	**15.9321**	18.1415	20.7424	20.0718	**18.5973**	20.7120	25.2824	24.4994	**22.7034**	25.2471	24.3552	**22.5931**	25.0889
		0.9	18.2132	17.0279	**14.7033**	18.1520	20.1210	19.1349	**17.3439**	20.0695	27.7119	26.4512	**23.8173**	27.6476	26.3349	**23.7338**	27.5160
	-0.4	0.7	5.9627	5.9104	**5.8522**	5.9595	9.0285	8.9627	**8.8026**	9.0248	11.1929	11.1250	**10.9609**	11.1891	11.1073	**10.9500**	11.1691
		0.8	5.9161	5.8298	**5.6792**	5.9105	9.2089	9.1020	**8.8393**	9.2024	12.8874	12.7847	**12.5379**	12.8811	12.7668	**12.5256**	12.8612
		0.9	5.9253	5.7557	**5.4185**	5.9135	8.3821	8.1986	**7.7976**	8.3695	15.2547	15.0605	**14.6140**	15.2414	15.0444	**14.6024**	15.2235
	0.0	0.7	3.0574	**3.0520**	3.0546	3.0570	6.4419	6.4313	**6.4052**	6.4412	8.1609	8.1512	**8.1283**	8.1603	8.1471	**8.1254**	8.1557
		0.8	3.0462	3.0358	**3.0234**	3.0454	6.9975	6.9771	**6.9236**	6.9961	9.8310	9.8173	**9.7849**	9.8300	9.8138	**9.7825**	9.8261
		0.9	3.0330	3.0098	**2.9657**	3.0311	7.1034	7.0485	**6.8987**	7.0993	12.1313	12.1056	**12.0452**	12.1294	12.1028	**12.0431**	12.1262
	0.4	0.7	1.6191	**1.6144**	1.6161	1.6188	5.7153	5.7025	**5.6688**	5.7146	7.3590	7.3458	**7.3116**	7.3583	7.3432	**7.3099**	7.3554
		0.8	1.6580	1.6488	**1.6379**	1.6574	7.0295	7.0072	**6.9476**	7.0282	9.1455	9.1260	**9.0756**	9.1443	9.1264	**9.0766**	9.1446
		0.9	1.7016	1.6803	**1.6396**	1.7002	9.1852	9.1348	**9.0008**	9.1820	11.6534	11.6151	**11.5189**	11.6510	11.6192	**11.5232**	11.6550
	0.8	0.7	0.6778	0.6780	0.6809	**0.6778**	6.0082	6.0034	**5.9897**	6.0080	6.8874	6.8824	**6.8685**	6.8872	6.8832	**6.8693**	6.8879
		0.8	0.6789	**0.6789**	0.6815	0.6789	7.9726	7.9626	**7.9342**	7.9721	8.9741	8.9647	**8.9379**	8.9737	8.9676	**8.9408**	8.9766
		0.9	0.6805	**0.6801**	0.6822	0.6804	10.9874	10.9621	**10.8905**	10.9863	12.0026	11.9794	**11.9145**	12.0015	11.9851	**11.9201**	12.0074

Table 4. pmse values of the mentioned estimators under Restriction Type 2, $AR(1)$ and $t = 0.1$

τ	ρ	γ	$\breve{\beta}_S$	$\breve{\beta}_{GSR}(k)$			$\breve{\beta}_{RS}$	$\breve{\beta}_{GSRR}(k)$			$\breve{\beta}_R$	$\breve{\beta}_{GRR}(k)$			$\overline{\beta}_{GSRR}(k)$		
			------	$\hat{k}_{HK}$	$\hat{k}_{HKB}$	$\hat{k}_{LW}$	------	$\hat{k}_{HK}$	$\hat{k}_{HKB}$	$\hat{k}_{LW}$	------	$\hat{k}_{HK}$	$\hat{k}_{HKB}$	$\hat{k}_{LW}$	$\hat{k}_{HK}$	$\hat{k}_{HKB}$	$\hat{k}_{LW}$
1	-0.8	0.7	18.0752	17.5445	**16.5308**	18.0529	13.1534	13.0488	**12.8334**	13.1491	11.9453	11.7083	**11.0905**	11.9359	11.6041	**10.9993**	11.8269
		0.8	18.1756	17.4352	**15.9321**	18.1415	14.3349	14.2607	**14.2052**	14.3312	11.9453	11.6304	**10.8345**	11.9314	11.5415	**10.7586**	11.8374
		0.9	18.2132	17.0279	**14.7033**	18.1520	17.1246	17.2163	17.9100	**17.1246**	11.9453	11.4389	**10.2452**	11.9203	11.3679	**10.1880**	11.8433
	-0.4	0.7	5.9627	5.9104	**5.8522**	5.9595	4.4686	4.4509	**4.4079**	4.4676	3.8452	3.8060	**3.6982**	3.8431	3.7885	**3.6832**	3.8248
		0.8	5.9161	5.8298	**5.6792**	5.9105	5.0654	5.0549	**5.0418**	5.0647	3.8452	3.7900	**3.6407**	3.8419	3.7750	**3.6280**	3.8261
		0.9	5.9253	5.7557	**5.4185**	5.9135	7.1330	7.2018	7.4674	**7.1330**	3.8452	3.7454	**3.4861**	3.8386	3.7332	**3.4761**	3.8256
	0.0	0.7	3.0574	**3.0520**	3.0546	3.0570	2.0607	2.0595	**2.0572**	2.0606	1.5174	1.5102	**1.4902**	1.5170	1.5073	**1.4876**	1.5139
		0.8	3.0462	3.0358	**3.0234**	3.0454	2.1442	2.1426	**2.1403**	2.1441	1.5174	1.5073	**1.4793**	1.5167	1.5049	**1.4773**	1.5142
		0.9	3.0330	3.0098	**2.9657**	3.0311	2.2652	2.2627	**2.2608**	2.2650	1.5174	1.4987	**1.4476**	1.5160	1.4969	**1.4462**	1.5140
	0.4	0.7	1.6191	**1.6144**	1.6161	1.6188	1.3972	1.3953	**1.3908**	1.3971	1.1147	1.1086	**1.0914**	1.1144	1.1067	**1.0898**	1.1124
		0.8	1.6580	1.6488	**1.6379**	1.6574	1.2458	1.2403	**1.2258**	1.2455	1.1147	1.1061	**1.0822**	1.1142	1.1046	**1.0810**	1.1126
		0.9	1.7016	1.6803	**1.6396**	1.7002	1.1804	1.1674	**1.1334**	1.1795	1.1147	1.0988	**1.0557**	1.1137	1.0976	**1.0550**	1.1125
	0.8	0.7	0.6778	0.6780	0.6809	**0.6778**	0.6710	0.6711	0.6714	**0.6710**	0.6512	0.6510	**0.6506**	0.6512	0.6510	**0.6506**	0.6512
		0.8	0.6789	0.6789	0.6815	**0.6789**	0.6590	0.6589	**0.6589**	0.6590	0.6512	0.6510	**0.6504**	0.6512	0.6509	**0.6504**	0.6512
		0.9	0.6805	**0.6801**	0.6822	0.6804	0.6548	0.6545	**0.6539**	0.6548	0.6512	0.6507	**0.6497**	0.6512	0.6507	**0.6497**	0.6512
1.05	-0.8	0.7	18.0752	17.5445	**16.5308**	18.0529	12.5271	12.4020	**12.1257**	12.5220	13.5460	13.3110	**12.6977**	13.5367	13.2146	**12.6138**	13.4357
		0.8	18.1756	17.4352	**15.9321**	18.1415	13.0958	12.9814	**12.8083**	13.0903	13.9374	13.6204	**12.8174**	13.9234	13.5364	**12.7459**	13.8345
		0.9	18.2132	17.0279	**14.7033**	18.1520	14.5262	14.5257	14.9594	**14.5243**	14.5028	13.9873	**12.7674**	14.4773	13.9193	**12.7129**	14.4036
	-0.4	0.7	5.9627	5.9104	**5.8522**	5.9595	4.1739	4.1465	**4.0739**	4.1724	5.4346	5.3966	**5.2921**	5.4326	5.3822	**5.2799**	5.4174
		0.8	5.9161	5.8298	**5.6792**	5.9105	4.2731	4.2414	**4.1638**	4.2712	5.8212	5.7654	**5.6137**	5.8178	5.7521	**5.6026**	5.8038
		0.9	5.9253	5.7557	**5.4185**	5.9135	5.0281	5.0391	5.1336	**5.0281**	6.3727	6.2686	**5.9960**	6.3658	6.2572	**5.9867**	6.3536
	0.0	0.7	3.0574	**3.0520**	3.0546	3.0570	1.8488	1.8430	**1.8268**	1.8485	3.0959	3.0888	**3.0686**	3.0955	3.0858	**3.0660**	3.0924
		0.8	3.0462	3.0358	**3.0234**	3.0454	1.9270	1.9176	**1.8915**	1.9264	3.4739	3.4638	**3.4356**	3.4732	3.4614	**3.4336**	3.4707
		0.9	3.0330	3.0098	**2.9657**	3.0311	2.0264	2.0057	**1.9487**	2.0248	4.0082	3.9894	**3.9376**	4.0068	3.9876	**3.9361**	4.0048
	0.4	0.7	1.6191	**1.6144**	1.6161	1.6188	1.6626	1.6568	**1.6407**	1.6623	2.7379	2.7310	**2.7118**	2.7375	2.7301	**2.7110**	2.7364
		0.8	1.6580	1.6488	**1.6379**	1.6574	2.1864	2.1763	**2.1481**	2.1858	3.1457	3.1364	**3.1105**	3.1451	3.1370	**3.1114**	3.1456
		0.9	1.7016	1.6803	**1.6396**	1.7002	2.8755	2.8546	**2.7969**	2.8742	3.7253	3.7081	**3.6614**	3.7242	3.7104	**3.6639**	3.7264
	0.8	0.7	0.6778	0.6780	0.6809	**0.6778**	1.9878	1.9869	**1.9844**	1.9878	2.3680	2.3673	**2.3652**	2.3680	2.3685	**2.3664**	2.3692
		0.8	0.6789	0.6789	0.6815	**0.6789**	2.5555	2.5541	**2.5499**	2.5555	2.8414	2.8402	**2.8369**	2.8413	2.8424	**2.8391**	2.8435
		0.9	0.6805	**0.6801**	0.6822	0.6804	3.2831	3.2795	**3.2694**	3.2829	3.5173	3.5144	**3.5062**	3.5172	3.5175	**3.5093**	3.5203
1.10	-0.8	0.7	18.0752	17.5445	**16.5308**	18.0529	14.5737	14.4266	**14.0857**	14.5678	18.3581	18.1240	**17.5123**	18.3489	18.0356	**17.4356**	18.2560
		0.8	18.1756	17.4352	**15.9321**	18.1415	15.0164	14.8622	**14.5757**	15.0092	19.9203	19.6002	**18.7871**	19.9063	19.5211	**18.7200**	19.8224
		0.9	18.2132	17.0279	**14.7033**	18.1520	15.6912	**15.6148**	15.8483	15.6857	22.1773	21.6513	**20.4017**	22.1514	21.5864	**20.3498**	22.0809
	-0.4	0.7	5.9627	5.9104	**5.8522**	5.9595	6.4567	6.4177	**6.3100**	6.4546	10.2096	10.1724	**10.0697**	10.2076	10.1610	**10.0604**	10.1954
		0.8	5.9161	5.8298	**5.6792**	5.9105	6.4757	6.4204	**6.2725**	6.4724	11.7532	11.6964	**11.5410**	11.7498	11.6849	**11.5314**	11.7376
		0.9	5.9253	5.7557	**5.4185**	5.9135	6.4581	6.4210	**6.3806**	6.4553	13.9561	13.8473	**13.5601**	13.9489	13.8367	**13.5515**	13.9375
	0.0	0.7	3.0574	**3.0520**	3.0546	3.0570	3.9471	3.9356	**3.9022**	3.9464	7.8305	7.8233	**7.8030**	7.8301	7.8204	**7.8005**	7.8270
		0.8	3.0462	3.0358	**3.0234**	3.0454	4.4137	4.3942	**4.3377**	4.4124	9.3437	9.3335	**9.3050**	9.3430	9.3311	**9.3030**	9.3404
		0.9	3.0330	3.0098	**2.9657**	3.0311	4.9504	4.9053	**4.7751**	4.9471	11.4823	11.4634	**11.4109**	11.4809	11.4615	**11.4094**	11.4789
	0.4	0.7	1.6191	**1.6144**	1.6161	1.6188	4.2756	4.2647	**4.2331**	4.2750	7.6097	7.6019	**7.5794**	7.6093	7.6018	**7.5796**	7.6092
		0.8	1.6580	1.6488	**1.6379**	1.6574	6.3029	6.2852	**6.2339**	6.3019	9.2405	9.2295	**9.1983**	9.2399	9.2322	**9.2012**	9.2425
		0.9	1.7016	1.6803	**1.6396**	1.7002	8.8829	8.8453	**8.7379**	8.8805	11.5584	11.5359	**11.4738**	11.5570	11.5414	**11.4794**	11.5625
	0.8	0.7	0.6778	0.6780	0.6809	**0.6778**	6.0317	6.0288	**6.0203**	6.0315	7.5273	7.5254	**7.5199**	7.5272	7.5279	**7.5224**	7.5297
		0.8	0.6789	0.6789	0.6815	**0.6789**	8.2390	8.2339	**8.2189**	8.2388	9.4193	9.4156	**9.4049**	9.4192	9.4199	**9.4092**	9.4235
		0.9	0.6805	**0.6801**	0.6822	0.6804	11.0921	11.0787	**11.0398**	11.0915	12.1209	12.1103	**12.0795**	12.1205	12.1165	**12.0856**	12.1267

Table 5. pmse values of the mentioned estimators under Restriction Type 1, $MA(1)$ and $t = 0.1$

τ	ρ	γ	$\breve{\beta}_S$	$\breve{\beta}_{GSR}(k)$			$\breve{\beta}_{RS}$	$\breve{\beta}_{GSRR}(k)$			$\breve{\beta}_R$	$\breve{\beta}_{GRR}(k)$			$\overline{\beta}_{GSRR}(k)$		
			------	$\hat{k}_{HK}$	$\hat{k}_{HKB}$	$\hat{k}_{LW}$	------	$\hat{k}_{HK}$	$\hat{k}_{HKB}$	$\hat{k}_{LW}$	------	$\hat{k}_{HK}$	$\hat{k}_{HKB}$	$\hat{k}_{LW}$	$\hat{k}_{HK}$	$\hat{k}_{HKB}$	$\hat{k}_{LW}$
1	-0.8	0.7	26.2753	25.8226	**24.9881**	26.2642	23.9682	23.4693	**22.1583**	23.9568	24.2769	23.7634	**22.4128**	24.2652	23.5603	**22.2329**	24.0536
		0.8	26.4985	25.8557	**24.4757**	26.4808	24.0096	23.3478	**21.6430**	23.9921	24.2769	23.6009	**21.8590**	24.2590	23.4317	**21.7083**	24.0828
		0.9	26.6261	25.5850	**23.2255**	26.5932	24.0553	23.0027	**20.4215**	24.0226	24.2769	23.2095	**20.5921**	24.2437	23.0817	**20.4794**	24.1098
	-0.4	0.7	6.7027	6.6598	**6.6262**	6.6999	5.3200	5.2736	**5.1725**	5.3173	5.3043	5.2447	**5.1047**	5.3009	5.2208	**5.0885**	5.2744
		0.8	6.6740	6.6029	**6.4909**	6.6692	5.3793	5.3135	**5.1714**	5.3751	5.3043	5.2207	**5.0270**	5.2990	5.1999	**5.0123**	5.2761
		0.9	6.6803	6.5369	**6.2582**	6.6700	5.5371	5.4168	**5.1681**	5.5285	5.3043	5.1529	**4.8146**	5.2936	5.1359	**4.8022**	5.2749
	0.0	0.7	3.0574	**3.0520**	3.0546	3.0570	2.3542	2.3492	**2.3409**	2.3539	2.2883	2.2787	**2.2565**	2.2877	2.2745	**2.2537**	2.2831
		0.8	3.0462	3.0358	**3.0234**	3.0454	2.4226	2.4159	**2.4050**	2.4221	2.2883	2.2747	**2.2435**	2.2873	2.2712	**2.2412**	2.2835
		0.9	3.0330	3.0098	**2.9657**	3.0311	2.6078	2.5966	**2.5816**	2.6068	2.2883	2.2630	**2.2052**	2.2864	2.2602	**2.2033**	2.2832
	0.4	0.7	2.1772	**2.1711**	2.1742	2.1768	1.9868	1.9823	**1.9765**	1.9865	1.8956	1.8847	**1.8599**	1.8949	1.8805	**1.8574**	1.8902
		0.8	2.1997	2.1879	**2.1747**	2.1989	1.9868	1.9778	**1.9619**	1.9862	1.8956	1.8803	**1.8455**	1.8946	1.8768	**1.8434**	1.8907
		0.9	2.2328	2.2059	**2.1554**	2.2310	1.9694	1.9471	**1.9021**	1.9679	1.8956	1.8670	**1.8037**	1.8936	1.8643	**1.8021**	1.8905
	0.8	0.7	2.7900	**2.7834**	2.7905	2.7896	2.8093	2.8043	**2.7997**	2.8091	2.6545	2.6417	**2.6141**	2.6539	2.6341	**2.6096**	2.6454
		0.8	2.7963	2.7830	**2.7720**	2.7956	2.6973	2.6830	**2.6557**	2.6966	2.6545	2.6364	**2.5979**	2.6536	2.6309	**2.5945**	2.6474
		0.9	2.8183	2.7867	**2.7323**	2.8166	2.6613	2.6295	**2.5645**	2.6597	2.6545	2.6209	**2.5507**	2.6528	2.6173	**2.5483**	2.6488
1.05	-0.8	0.7	26.2753	25.8226	**24.9881**	26.2642	25.1791	24.6717	**23.3370**	25.1675	25.8778	25.3572	**23.9878**	25.8659	25.1879	**23.8407**	25.6884
		0.8	26.4985	25.8557	**24.4757**	26.4808	25.6992	25.0147	**23.2492**	25.6810	26.4048	25.7074	**23.9092**	26.3864	25.5685	**23.7887**	26.2406
		0.9	26.6261	25.5850	**23.2255**	26.5932	26.5195	25.4184	**22.7161**	26.4853	27.2211	26.1063	**23.3735**	27.1865	26.0134	**23.2956**	27.0876
	-0.4	0.7	6.7027	6.6598	**6.6262**	6.6999	6.0296	5.9803	**5.8679**	6.0268	6.8068	6.7495	**6.6148**	6.8035	6.7338	**6.6064**	6.7853
		0.8	6.6740	6.6029	**6.4909**	6.6692	6.1081	6.0310	**5.8520**	6.1032	7.2468	7.1618	**6.9621**	7.2415	7.1474	**6.9535**	7.2251
		0.9	6.6803	6.5369	**6.2582**	6.6700	6.0761	5.9221	**5.5690**	6.0654	7.8645	7.7041	**7.3385**	7.8533	7.6923	**7.3310**	7.8400
	0.0	0.7	3.0574	**3.0520**	3.0546	3.0570	3.1246	3.1172	**3.1010**	3.1241	3.7565	3.7469	**3.7243**	3.7559	3.7427	**3.7215**	3.7513
		0.8	3.0462	3.0358	**3.0234**	3.0454	3.1920	3.1796	**3.1509**	3.1911	4.1738	4.1602	**4.1284**	4.1728	4.1567	**4.1260**	4.1690
		0.9	3.0330	3.0098	**2.9657**	3.0311	3.1360	3.1073	**3.0378**	3.1338	4.7486	4.7231	**4.6639**	4.7466	4.7202	**4.6619**	4.7435
	0.4	0.7	2.1772	**2.1711**	2.1742	2.1768	2.6155	2.6071	**2.5890**	2.6150	3.3675	3.3555	**3.3273**	3.3668	3.3507	**3.3241**	3.3615
		0.8	2.1997	2.1879	**2.1747**	2.1989	2.8876	2.8730	**2.8398**	2.8866	3.7923	3.7760	**3.7383**	3.7912	3.7732	**3.7368**	3.7880
		0.9	2.2328	2.2059	**2.1554**	2.2310	3.3919	3.3593	**3.2825**	3.3897	4.3947	4.3650	**4.2978**	4.3927	4.3648	**4.2986**	4.3922
	0.8	0.7	2.7900	**2.7834**	2.7905	2.7896	3.2862	3.2807	**3.2741**	3.2859	4.1387	4.1284	**4.1080**	4.1382	4.1209	**4.1035**	4.1298
		0.8	2.7963	2.7830	**2.7720**	2.7956	3.8911	3.8760	**3.8456**	3.8903	4.5482	4.5314	**4.4963**	4.5474	4.5280	**4.4949**	4.5433
		0.9	2.8183	2.7867	**2.7323**	2.8166	4.7618	4.7247	**4.6444**	4.7599	5.2216	5.1849	**5.1067**	5.2197	5.1905	**5.1131**	5.2250
1.10	-0.8	0.7	26.2753	25.8226	**24.9881**	26.2642	29.5244	29.0049	**27.6364**	29.5125	30.7040	30.1726	**28.7741**	30.6918	30.0371	**28.6599**	30.5486
		0.8	26.4985	25.8557	**24.4757**	26.4808	31.5208	30.8113	**28.9781**	31.5020	32.7814	32.0598	**30.1981**	32.7623	31.9513	**30.1076**	32.6470
		0.9	26.6261	25.5850	**23.2255**	26.5932	34.6697	33.5144	**30.6751**	34.6338	36.0078	34.8395	**31.9749**	35.9715	34.7816	**31.9319**	35.9078
	-0.4	0.7	6.7027	6.6598	**6.6262**	6.6999	9.4912	9.4370	**9.3077**	9.4881	11.3224	11.2662	**11.1332**	11.3192	11.2586	**11.1325**	11.3092
		0.8	6.6740	6.6029	**6.4909**	6.6692	10.2568	10.1650	**9.9394**	10.2511	13.0793	12.9916	**12.7820**	13.0738	12.9836	**12.7795**	13.0640
		0.9	6.6803	6.5369	**6.2582**	6.6700	10.8042	10.6076	**10.1248**	10.7907	15.5461	15.3745	**14.9759**	15.5341	15.3680	**14.9733**	15.5262
	0.0	0.7	3.0574	**3.0520**	3.0546	3.0570	6.4419	6.4313	**6.4052**	6.4412	8.1609	8.1512	**8.1283**	8.1603	8.1471	**8.1254**	8.1557
		0.8	3.0462	3.0358	**3.0234**	3.0454	6.9975	6.9771	**6.9236**	6.9961	9.8310	9.8173	**9.7849**	9.8300	9.8138	**9.7825**	9.8261
		0.9	3.0330	3.0098	**2.9657**	3.0311	7.1034	7.0485	**6.8987**	7.0993	12.1313	12.1056	**12.0452**	12.1294	12.1028	**12.0431**	12.1262
	0.4	0.7	2.1772	**2.1711**	2.1742	2.1768	5.7086	5.6957	**5.6638**	5.7079	7.7817	7.7687	**7.7374**	7.7810	7.7633	**7.7336**	7.7750
		0.8	2.1997	2.1879	**2.1747**	2.1989	6.9086	6.8865	**6.8299**	6.9072	9.4832	9.4658	**9.4244**	9.4821	9.4636	**9.4236**	9.4796
		0.9	2.2328	2.2059	**2.1554**	2.2310	8.9248	8.8745	**8.7437**	8.9215	11.8956	11.8626	**11.7858**	11.8934	11.8650	**11.7889**	11.8955
	0.8	0.7	2.7900	**2.7834**	2.7905	2.7896	6.2311	6.2247	**6.2143**	6.2308	8.5908	8.5830	**8.5694**	8.5905	8.5756	**8.5649**	8.5821
		0.8	2.7963	2.7830	**2.7720**	2.7956	8.4414	8.4247	**8.3887**	8.4406	10.2275	10.2117	**10.1794**	10.2267	10.2104	**10.1801**	10.2248
		0.9	2.8183	2.7867	**2.7323**	2.8166	11.6385	11.5910	**11.4807**	11.6361	12.9193	12.8756	**12.7784**	12.9171	12.8903	**12.7935**	12.9315

Table 6. pmse values of the mentioned estimators under Restriction Type 2, $MA(1)$ and $t = 0.1$

τ	ρ	γ	$\breve{\beta}_S$	$\breve{\beta}_{GSR}(k)$			$\breve{\beta}_{RS}$	$\breve{\beta}_{GSRR}(k)$			$\breve{\beta}_R$	$\breve{\beta}_{GRR}(k)$			$\overline{\beta}_{GSRR}(k)$		
			------	$\hat{k}_{HK}$	$\hat{k}_{HKB}$	$\hat{k}_{LW}$	------	$\hat{k}_{HK}$	$\hat{k}_{HKB}$	$\hat{k}_{LW}$	------	$\hat{k}_{HK}$	$\hat{k}_{HKB}$	$\hat{k}_{LW}$	$\hat{k}_{HK}$	$\hat{k}_{HKB}$	$\hat{k}_{LW}$
1	-0.8	0.7	26.2753	25.8226	**24.9881**	26.2642	19.7786	19.3766	**18.2701**	19.7695	20.1053	19.6926	**18.5566**	20.0959	19.5293	**18.4103**	19.9264
		0.8	26.4985	25.8557	**24.4757**	26.4808	19.8339	19.3044	**17.8774**	19.8200	20.1053	19.5645	**18.1072**	20.0911	19.4261	**17.9838**	19.9470
		0.9	26.6261	25.5850	**23.2255**	26.5932	19.8852	19.0548	**16.9269**	19.8597	20.1053	19.2617	**17.1001**	20.0793	19.1562	**17.0083**	19.9683
	-0.4	0.7	6.7027	6.6598	**6.6262**	6.6999	3.6917	3.6651	**3.5946**	3.6902	3.5174	3.4825	**3.3875**	3.5154	3.4669	**3.3748**	3.4988
		0.8	6.6740	6.6029	**6.4909**	6.6692	3.7782	3.7413	**3.6454**	3.7759	3.5174	3.4684	**3.3364**	3.5144	3.4548	**3.3255**	3.4999
		0.9	6.6803	6.5369	**6.2582**	6.6700	3.9630	3.8983	**3.7398**	3.9586	3.5174	3.4282	**3.1971**	3.5113	3.4171	**3.1885**	3.4993
	0.0	0.7	3.0574	**3.0520**	3.0546	3.0570	2.0607	2.0595	**2.0572**	2.0606	1.5174	1.5102	**1.4902**	1.5170	1.5073	**1.4876**	1.5139
		0.8	3.0462	3.0358	**3.0234**	3.0454	2.1442	2.1426	**2.1403**	2.1441	1.5174	1.5073	**1.4793**	1.5167	1.5049	**1.4773**	1.5142
		0.9	3.0330	3.0098	**2.9657**	3.0311	2.2652	2.2627	**2.2608**	2.2650	1.5174	1.4987	**1.4476**	1.5160	1.4969	**1.4462**	1.5140
	0.4	0.7	2.1772	**2.1711**	2.1742	2.1768	1.9397	1.9388	**1.9378**	1.9396	1.3575	1.3494	**1.3268**	1.3570	1.3463	**1.3242**	1.3538
		0.8	2.1997	2.1879	**2.1747**	2.1989	1.6111	1.6048	**1.5884**	1.6107	1.3575	1.3461	**1.3145**	1.3568	1.3437	**1.3126**	1.3541
		0.9	2.2328	2.2059	**2.1554**	2.2310	1.4762	1.4597	**1.4166**	1.4751	1.3575	1.3364	**1.2792**	1.3561	1.3345	**1.2781**	1.3541
	0.8	0.7	2.7900	**2.7834**	2.7905	2.7896	3.0993	3.0968	**3.0901**	3.0992	2.0384	2.0310	**2.0094**	2.0381	2.0250	**2.0037**	2.0320
		0.8	2.7963	2.7830	**2.7720**	2.7956	2.1305	2.1214	**2.0951**	2.1301	2.0384	2.0280	**1.9979**	2.0379	2.0241	**1.9946**	2.0339
		0.9	2.8183	2.7867	**2.7323**	2.8166	2.0539	2.0353	**1.9831**	2.0530	2.0384	2.0190	**1.9645**	2.0375	2.0170	**1.9634**	2.0351
1.05	-0.8	0.7	26.2753	25.8226	**24.9881**	26.2642	21.0643	20.6728	**19.5935**	21.0555	21.8849	21.4849	**20.3824**	21.8759	21.3743	**20.2875**	21.7596
		0.8	26.4985	25.8557	**24.4757**	26.4808	21.5405	21.0127	**19.5873**	21.5267	22.3283	21.7910	**20.3407**	22.3142	21.6926	**20.2568**	22.2103
		0.9	26.6261	25.5850	**23.2255**	26.5932	22.2962	21.4494	**19.2733**	22.2702	23.0606	22.2023	**19.9983**	23.0343	22.1340	**19.9437**	22.9606
	-0.4	0.7	6.7027	6.6598	**6.6262**	6.6999	4.0522	4.0226	**3.9418**	4.0505	5.1460	5.1145	**5.0284**	5.1443	5.1087	**5.0253**	5.1376
		0.8	6.6740	6.6029	**6.4909**	6.6692	4.1305	4.0842	**3.9585**	4.1276	5.5457	5.4979	**5.3682**	5.5428	5.4922	**5.3649**	5.5363
		0.9	6.6803	6.5369	**6.2582**	6.6700	4.1583	4.0657	**3.8218**	4.1520	6.1183	6.0258	**5.7830**	6.1120	6.0209	**5.7802**	6.1064
	0.0	0.7	3.0574	**3.0520**	3.0546	3.0570	1.8488	1.8430	**1.8268**	1.8485	3.0959	3.0888	**3.0686**	3.0955	3.0858	**3.0660**	3.0924
		0.8	3.0462	3.0358	**3.0234**	3.0454	1.9270	1.9176	**1.8915**	1.9264	3.4739	3.4638	**3.4356**	3.4732	3.4614	**3.4336**	3.4707
		0.9	3.0330	3.0098	**2.9657**	3.0311	2.0264	2.0057	**1.9487**	2.0248	4.0082	3.9894	**3.9376**	4.0068	3.9876	**3.9361**	4.0048
	0.4	0.7	2.1772	**2.1711**	2.1742	2.1768	1.6086	1.6027	**1.5866**	1.6082	2.9448	2.9366	**2.9134**	2.9443	2.9335	**2.9108**	2.9411
		0.8	2.1997	2.1879	**2.1747**	2.1989	2.0692	2.0579	**2.0261**	2.0685	3.3357	3.3251	**3.2955**	3.3350	3.3241	**3.2951**	3.3339
		0.9	2.2328	2.2059	**2.1554**	2.2310	2.7661	2.7423	**2.6760**	2.7646	3.8987	3.8797	**3.8280**	3.8975	3.8810	**3.8299**	3.8987
	0.8	0.7	2.7900	**2.7834**	2.7905	2.7896	2.4836	2.4783	**2.4629**	2.4834	3.6237	3.6161	**3.5938**	3.6234	3.6090	**3.5872**	3.6163
		0.8	2.7963	2.7830	**2.7720**	2.7956	3.2184	3.2087	**3.1808**	3.2180	4.0264	4.0174	**3.9916**	4.0260	4.0175	**3.9921**	4.0259
		0.9	2.8183	2.7867	**2.7323**	2.8166	4.1887	4.1709	**4.1205**	4.1878	4.6762	4.6599	**4.6142**	4.6754	4.6692	**4.6239**	4.6846
1.10	-0.8	0.7	26.2753	25.8226	**24.9881**	26.2642	25.8610	25.4744	**24.4065**	25.8523	27.2907	26.8976	**25.8132**	27.2818	26.8398	**25.7696**	27.2189
		0.8	26.4985	25.8557	**24.4757**	26.4808	27.5968	27.0674	**25.6344**	27.5829	29.0311	28.4940	**27.0416**	29.0170	28.4355	**26.9970**	28.9533
		0.9	26.6261	25.5850	**23.2255**	26.5932	30.4461	29.5796	**27.3452**	30.4195	31.9172	31.0408	**28.7843**	31.8903	31.0098	**28.7669**	31.8540
	-0.4	0.7	6.7027	6.6598	**6.6262**	6.6999	7.2722	7.2370	**7.1386**	7.2702	10.0406	10.0112	**9.9306**	10.0390	10.0153	**9.9371**	10.0423
		0.8	6.6740	6.6029	**6.4909**	6.6692	7.9290	7.8692	**7.7019**	7.9253	11.6371	11.5890	**11.4571**	11.6342	11.5912	**11.4614**	11.6357
		0.9	6.6803	6.5369	**6.2582**	6.6700	8.5177	8.3880	**8.0320**	8.5090	13.9242	13.8258	**13.5642**	13.9176	13.8272	**13.5674**	13.9183
	0.0	0.7	3.0574	**3.0520**	3.0546	3.0570	3.9471	3.9356	**3.9022**	3.9464	7.8305	7.8233	**7.8030**	7.8301	7.8204	**7.8005**	7.8270
		0.8	3.0462	3.0358	**3.0234**	3.0454	4.4137	4.3942	**4.3377**	4.4124	9.3437	9.3335	**9.3050**	9.3430	9.3311	**9.3030**	9.3404
		0.9	3.0330	3.0098	**2.9657**	3.0311	4.9504	4.9053	**4.7751**	4.9471	11.4823	11.4634	**11.4109**	11.4809	11.4615	**11.4094**	11.4789
	0.4	0.7	2.1772	**2.1711**	2.1742	2.1768	3.4959	3.4841	**3.4501**	3.4952	7.7081	7.6996	**7.6758**	7.7076	7.6967	**7.6733**	7.7045
		0.8	2.1997	2.1879	**2.1747**	2.1989	5.5138	5.4946	**5.4391**	5.5126	9.2733	9.2630	**9.2342**	9.2727	9.2636	**9.2353**	9.2731
		0.9	2.2328	2.2059	**2.1554**	2.2310	8.1417	8.1020	**7.9880**	8.1391	11.5276	11.5081	**11.4547**	11.5263	11.5127	**11.4596**	11.5307
	0.8	0.7	2.7900	**2.7834**	2.7905	2.7896	4.1904	4.1821	**4.1577**	4.1901	8.3853	8.3775	**8.3546**	8.3850	8.3695	**8.3469**	8.3769
		0.8	2.7963	2.7830	**2.7720**	2.7956	7.7396	7.7278	**7.6933**	7.7391	9.9941	9.9859	**9.9620**	9.9938	9.9898	**9.9664**	9.9976
		0.9	2.8183	2.7867	**2.7323**	2.8166	11.2229	11.1990	**11.1305**	11.2218	12.5904	12.5717	**12.5189**	12.5895	12.5924	**12.5395**	12.6102

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