Finite Population Mean Estimation through a Two-Parameter Ratio Estimator Using Auxiliary Information in Presence of Non-Response

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Abstract

In surveys covering human populations it is observed that information in most cases are not obtained at the first attempt even after some callbacks. Such problems come under the category of non-response. Surveys suffer with non-response in various ways. It depends on the nature of required information, either surveys is concerned with general or sensitive issues of a society. Hansen and Hurwitz (1946) have considered the problem of non-response while estimating the population mean by taking a subsample from the non-respondent group with the help of extra efforts and an estimator was suggested by combining the information available from the response and nonresponse groups. We also mention that in survey sampling auxiliary information is commonly used to improve the performance of an estimator of a quantity of interest. For estimating the population mean using auxiliary information in presence of non-response has been discussed by various authors. In this paper, we have developed estimators for estimating the population mean of the variable under interest when there is non-response error in the study as well as in the auxiliary variable. We have studied properties of the suggested estimators under large sample approximation. Comparison of the suggested estimators with usual unbiased estimator reported by Hansen and Hurwitz (1946) and the ratio estimator due to Rao (1986) have been made. The results obtained are illustrated with aid of an empirical study.

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1. INTRODUCTION

In various human surveys, information is in most cases not obtained from all the units in the survey even after call backs. An estimate derived from such incomplete data may be misleading especially when the respondents differ from the non-respondents because the estimate can be biased. To cope with this problem, survey statisticians generally consider and adopt the non-respondents sub-sampling scheme developed by Hansen and Hurwitz (1946) to a wide range of practical situations. One topic which is discussed at great length in sampling theory is the

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estimator of population mean $\bar{Y}$ of the study variable y using auxiliary information in presence of non-response. Cochran (1977) and Rao (1986) suggested the use of the ratio method of estimation for population mean $\bar{Y}$ of the study variable y with sub-sampling from amongst the non- respondents.

When the population mean $\bar{X}$ of the auxiliary information x is known, the work of Rao (1986) has been further extended by Khare and Srivastava (1997), Singh and Kumar (2008), Kumar (2012), Kumar and Vishwanathaiah (2013), Olufadi and Kumar (2014) and Chanu and Singh (2015) in presence of non-response.

For the case of non-response in sample survey, this paper addresses the problem of efficiently estimating the population mean $\bar{Y}$ of the study variable y using auxiliary information. Taking motivation from Singh and Pal (2015) a two-parameter ratio estimators for population mean $\bar{Y}$ in presence of non-response using auxiliary variable x have been proposed. The properties of these estimators have been studied in finite population approach under large sample approximation.

2. THE USUAL RATIO AND PRODUCT ESTIMATORS

Let $U = (U_1, U_2, ..., U_N)$ be a finite population of N identifiable units and (y , x) be the study and auxiliary variables respectively taking values (y_i , x_i) on the i^{th} population units U_i, $i = 1, 2, ..., .N$. Let n be the size of a sample drawn from the population of size N by using simple random sampling without replacement (*SRSWOR*) to observe the study variable y . In this approach, the population of size N is assumed to be composed of two strata of size N_1 and $N_2 = (N - N_1)$ of 'respondent' and 'non-respondents' respectively. Out of n units, n_1 respond and n_2 do not. From the n_2 non response units, $r(r = n_2 / k, k > 1)$ units are again randomly selected, hence of n selected units we have $n_1 + r$ observations on variable y. It is assumed that no non-response is observed in re-selected units.

Hansen and Hurwitz (1946) suggested the estimator of the population mean $\bar{Y}$ as

$$\bar{y}^* = (n_1/n)\bar{y}_1 + (n_1/n)\bar{y}_2', \tag{2.1}$$

where $\bar{y}_1 = \sum_{i=1}^{n_1} y_i / n_1$ and $\bar{y}_2' = \sum_{i=1}^{r} y_i / r$ are sample means based on n_1 and r units respectively. The estimator $\bar{y}^*$ is unbiased estimator with variance given by

$$\begin{aligned} V(\bar{y}^*) &= \lambda S_y^2 + \theta S_{y(2)}^2 \\ &= \bar{Y}^2[\lambda C_y^2 + \theta C_{y(2)}^2], \end{aligned} \tag{2.2}$$

where $f = n/N$, $\lambda = (1-f)/n$, $W_2 = N_2/N$; $\theta = W_2(k-1)/n$; and

$S_y^2 = \sum_{i=1}^{N}(y_i - \bar{Y})^2/(N-1)$, $S_{y(2)}^2 = \sum_{i=1}^{N_2}(y_i - \bar{Y}_2)^2/(N_2-1)$ and $\bar{Y}_2 = \sum_{i=1}^{N_2} y_i / N_2$ are the population variances for the entire population, for the non-response group of the population, and the population mean of the non-response group respectively, $C_y^2 = S_y^2/\bar{Y}^2$ and $C_{y(2)}^2 = S_{y(2)}^2/\bar{Y}^2$.

When the population mean $\bar{X}$ of the auxiliary variable x is known and information on y and x variables for the n selected units is incomplete (designated as Case I) the usual ratio estimator for the population mean $\bar{Y}$ of the study variable y is given by

$$t_{R1} = (\bar{y}^*/\bar{x}^*)\bar{X}, \tag{2.3}$$

where $\bar{x}^* = \{(n_1/n)\bar{x}_1 + (n_2/n)\bar{x}_2'\}$ is an unbiased estimator of the population mean $\bar{X}$ of the auxiliary variable x, $\bar{x}_1 = \sum_{i=1}^{n_1}(x_i/n_1)$ and $\bar{x}_2' = \sum_{i=1}^{r}(x_i/r)$.

The variance of the estimator $\bar{x}^*$ is given by

$$\begin{aligned} V(\bar{x}^*) &= \lambda S_x^2 + \theta S_{x(2)}^2 \\ &= \bar{X}^2[\lambda C_x^2 + \theta C_{x(2)}^2], \end{aligned} \tag{2.4}$$

where S_x^2 and $S_{x(2)}^2$ are the respectively population variances for the whole population and for the non-response group, $C_x^2 = S_x^2/\bar{X}^2$ and $C_{x(2)}^2 = S_{x(2)}^2/\bar{X}^2$.

If $\bar{X}$ is known and we have incomplete information on the study variable y and the complete information on the auxiliary variable x [designed as Case II], then an alternative ratio estimator is given by Rao (1986):

$$t_{R2} = (\bar{y}^* / \bar{x})\bar{X}\ , \tag{2.5}$$

where $\bar{x} = \sum_{i=1}^{n}(x_i / n)$ is the sample mean of the auxiliary variable x based on a sample of size n. In similar fashion the conventional product estimation for the population mean $\bar{Y}$ of the study variable y under Cases I and II are respectively defined by

$$t_{P1} = \bar{y}^* (\bar{x}^* / \bar{X}) \tag{2.6}$$

and

$$t_{P2} = \bar{y}^* (\bar{x} / \bar{X}) \tag{2.7}$$

To the first degree of approximation, the mean squared errors of the ratio and product estimators are respectively given by

$$MSE(t_{R1}) = \bar{Y}^2[\lambda(C_y^2 + C_x^2 - 2\rho_{yx}C_yC_x) + \theta(C_{y(2)}^2 + C_{x(2)}^2 - 2\rho_{yx(2)}C_{y(2)}C_{x(2)})], \tag{2.8}$$

$$MSE(t_{R2}) = \bar{Y}^2[\lambda(C_y^2 + C_x^2 - 2\rho_{yx}C_yC_x) + \theta C_{y(2)}^2], \tag{2.9}$$

$$MSE(t_{P1}) = \bar{Y}^2[\lambda(C_y^2 + C_x^2 + 2\rho_{yx}C_yC_x) + \theta(C_{y(2)}^2 + C_{x(2)}^2 + 2\rho_{yx(2)}C_{y(2)}C_{x(2)})], \tag{2.10}$$

$$MSE(t_{P2}) = \bar{Y}^2[\lambda(C_y^2 + C_x^2 + 2\rho_{yx}C_yC_x) + \theta C_{y(2)}^2], \tag{2.11}$$

where $\rho_{yx} = (S_{yx} / S_yS_x)$ is correlation between y and x for the entire population, and $\rho_{yx(2)} = (S_{yx(2)} / S_{y(2)}S_{x(2)})$ is the correlation coefficient between y and x for the 'non-respondent' group with

$$S_{yx} = (N-1)^{-1}\sum_{i=1}^{N}(y_i - \bar{Y})(x_i - \bar{X}) \text{ and } S_{yx(2)} = (N_2 - 1)^{-1}\sum_{i=1}^{N_2}(y_i - \bar{Y}_2)(x_i - \bar{X}_2)\,.$$

The ratio and product estimators t_{R1} and t_{P1} are better than the usual unbiased estimator $\bar{y}^*$ if

(i) $C > (1/2)$ and $C_{(2)} > (1/2)$;

and

(ii) $C < -(1/2)$ and $C_{(2)} < -(1/2)$

respectively hold good,

where $C = \rho_{yx}(C_y / C_x)$ and $C_{(2)} = \rho_{yx(2)}(C_{y(2)} / C_{x(2)})$.

We also note that the ratio estimator t_{R1} and the product estimator t_{P1} are also better than the usual unbiased estimator $\bar{y}^*$ respectively if

(iii) $R > (1/2)$,

(iv) $R < -(1/2)$,

where $R = \dfrac{(\lambda C C_x^2 + \theta C_{(2)} C_{x(2)}^2)}{(\lambda C_x^2 + \theta C_{x(2)}^2)}$.

The conditions (iii) and (iv) are not noticed in the literature.

Thus having the observations over the conditions (i) to (iv) the usual unbiased estimator $\bar{y}^*$ is to be preferred over t_{R1} and t_{P1} if the following conditions

(v) either $\{-(1/2) \le C \le (1/2)$ and $-(1/2) \le C_{(2)} \le (1/2)\}$

(vi) or $\{-(1/2) \le R \le (1/2)\}$

holds true.

Further the estimators t_{R2} and t_{P2} are more efficient than the usual unbiased estimator $\bar{y}^*$ if

(i) $C > (1/2)$;

and

(ii) $C < -(1/2)$;

respectively hold true.

However, the usual unbiased estimator $\bar{y}^*$ is to be preferred over ratio estimator t_{R2} and product estimator t_{P2} if the condition:

$\{-(1/2) \le C \le (1/2)\}$ holds good.

In this paper we have proposed a two-parameter ratio estimator for a finite population mean in the presence of non-response. We have obtained the bias and mean squared error (*MSE*) of the proposed class of estimators to the first degree of approximation. We have also derived the conditions for the parameter under which the proposed class of estimators has smaller *MSE* than the usual unbiased estimator $\bar{y}^*$, ratio estimator and product estimator. An empirical study is carried out in support of the present study.

3. SOME SUGGESTED RATIO-TYPE ESTIMATORS

In this section, we have suggested some ratio-type estimators for estimating the population mean $\bar{Y}$ in two different situations designated as Case I and Case II which are described below.

CASE I. When the population mean $\bar{X}$ of the auxiliary variable x is known; and there is non-response on the study variable y as well as on the auxiliary variable x. In this situation, we consider the following estimators for population mean $\bar{Y}$ as

$$t_1^* = \bar{y}^* (\bar{X}^2 / \bar{x}^{*2}), \tag{3.1}$$

$$t_2^* = \bar{y}^* (\bar{X} / \bar{x}^*)^{1/2}, \tag{3.2}$$

$$t_3^* = \bar{y}^* \exp\left\{\frac{(\bar{X} - \bar{x}^*)}{2(\bar{X} + \bar{x}^*)}\right\}, \tag{3.3}$$

$$t_4^* = \bar{y}^* \exp\left\{\frac{2(\bar{X} - \bar{x}^*)}{(\bar{X} + \bar{x}^*)}\right\}, \tag{3.4}$$

$$t_5^* = \bar{y}^* \left(\frac{\bar{X}}{\bar{x}^*}\right) \exp\left\{\frac{(\bar{X} - \bar{x}^*)}{(\bar{X} + \bar{x}^*)}\right\}, \tag{3.5}$$

$$t_6^* = \bar{y}^* \left(\frac{\bar{X}}{\bar{x}^*}\right)^{1/2} \exp\left\{\frac{(\bar{X} - \bar{x}^*)}{2(\bar{X} + \bar{x}^*)}\right\}. \tag{3.6}$$

It is to be noted that estimators t_1^*, t_2^* and (t_4^*, t_5^*) are respectively defined on the lines of Kadilar and Cingi (2003), Swain (2014) and Singh and Pal (2015) respectively.

The estimators in (3.1) to (3.6) are members of the following class of estimators of the population mean $\bar{Y}$ defined by

$$t_{(\alpha,\delta)} = \bar{y}^* \left(\frac{\bar{X}}{\bar{x}^*} \right)^{\alpha} \exp\left\{ \frac{\delta(\bar{X} - \bar{x}^*)}{(\bar{X} + \bar{x}^*)} \right\}, \tag{3.7}$$

where (α , δ) are suitable chosen constants. We note that the class of estimators:

(i) $t_{(\alpha,\delta)} \to t_1^*$ for $(\alpha,\delta) = (2,0)$,

(ii) $t_{(\alpha,\delta)} \to t_2^*$ for $(\alpha,\delta) = \left(\frac{1}{2}, 0 \right)$,

(iii) $t_{(\alpha,\delta)} \to t_3^*$ for $(\alpha,\delta) = \left(0, \frac{1}{2} \right)$,

(iv) $t_{(\alpha,\delta)} \to t_4^*$ for $(\alpha,\delta) = (0,2)$,

(v) $t_{(\alpha,\delta)} \to t_5^*$ for $(\alpha,\delta) = (1,1)$,

(vi) $t_{(\alpha,\delta)} \to t_6^*$ for $(\alpha,\delta) = \left(\frac{1}{2}, \frac{1}{2} \right)$.

In addition to t_1^* to t_6^*, many other acceptable estimators can be generated from the class of estimators $t_{(\alpha,\delta)}$. Thus to obtain the biases and *MSE*s of the estimators t_1^* to t_6^*, we will first obtained the bias and *MSE* of the generalized class of estimators $t_{(\alpha,\delta)}$.

To obtain the bias and *MSE* of the class of estimators $t_{(\alpha,\delta)}$, we write

$\bar{y}^* = \bar{Y}(1 + e_0)$, $\bar{x}^* = \bar{X}(1 + e_1)$

such that

$E(e_0) = E(e_1) = 0$

and

$E(e_0^2) = (\lambda C_y^2 + \theta C_{y(2)}^2)$, $E(e_1^2) = (\lambda C_x^2 + \theta C_{x(2)}^2)$, $E(e_0 e_1) = (\lambda C C_x^2 + \theta C_{(2)} C_{x(2)}^2)$.

Now expressing (3.7) in terms of e’s we have

$$t_{(\alpha,\delta)} = \bar{Y}(1+e_0)(1+e_1)^{-\alpha}\exp\left\{\frac{-\delta e_1}{(2+e_1)}\right\},$$

$$= \bar{Y}(1+e_0)(1+e_1)^{-\alpha}\exp\left\{\frac{-\delta e_1}{2}\left(1+\frac{e_1}{2}\right)^{-1}\right\}. \quad (3.8)$$

We assume that $|e_1|<1$ so that $(1+e_1)^{-\alpha}$ and $\left(1+\frac{e_1}{2}\right)^{-1}$ are expandable in terms of power series. Expanding and multiplying out the right hand side of (3.8) and neglecting terms of e's having power greater than two we have

$$t_{(\alpha,\delta)} \cong \bar{Y}\left[1+e_0-\frac{(\delta+2\alpha)}{2}e_1-\frac{(\delta+2\alpha)}{2}e_0e_1+\frac{(\delta+2\alpha)(\delta+2\alpha+2)}{8}e_1^2\right]$$

or

$$(t_{(\alpha,\delta)}-\bar{Y}) \cong \bar{Y}\left[e_0-\frac{(\delta+2\alpha)}{2}e_1-\frac{(\delta+2\alpha)}{2}e_0e_1+\frac{(\delta+2\alpha)(\delta+2\alpha+2)}{8}e_1^2\right]$$

(3.9)

Taking expectation of both sides of (3.9) we get the bias of the class of estimators $t_{(\alpha,\delta)}$ to the first degree of approximation as

$$B(t_{(\alpha,\delta)}) = \frac{\bar{Y}(\delta+2\alpha)}{8}(\lambda C_x^2+\theta C_{x(2)}^2)(\delta+2\alpha-4R+2) \quad (3.10)$$

which is zero, if either

$$\delta = -2\alpha \text{ (or } \alpha = -\delta/2\text{)} \quad (3.11)$$

or

$$\delta = -2(\alpha-2R+1) \text{ (or } \alpha = 2R-(\delta/2)-1\text{)}. \quad (3.12)$$

The suggested class of estimators $t_{(\alpha,\delta)}$ substituted with the values of δ (or α) from (3.11) and (3.12) becomes an (approximately) unbiased estimator for the population mean $\bar{Y}$ respectively as

$$t_{(\alpha,-2\alpha)} = \bar{y}^*\left(\frac{\bar{X}}{\bar{x}^*}\right)^{\alpha}\exp\left\{\frac{2\alpha(\bar{x}^*-\bar{X})}{(\bar{X}+\bar{x}^*)}\right\} \quad (3.13)$$

$$= t_{u(\alpha)}\ (say)$$

and

$$t_{(\delta/2,\delta)} = \bar{y}^{*}\left(\frac{\bar{x}^{*}}{\bar{X}}\right)^{\delta/2} \exp\left\{\frac{\delta(\bar{X}-\bar{x}^{*})}{(\bar{X}+\bar{x}^{*})}\right\} \tag{3.14}$$

$$= t_{u(\delta)}\,(say)$$

Here we note that the estimators $t_{u(\alpha)}$ and $t_{u(\delta)}$ are almost unbiased irrespective of the values of (α , δ).

Squaring both sides of (3.9) and neglecting terms of e's having power greater than two we have

$$(t_{(\alpha,\delta)}\bar{Y})^{2} \cong \bar{Y}^{2}\left[e_0^2 + \frac{(\delta+2\alpha)^2}{4}e_1^2 - (\delta+2\alpha)e_0 e_1\right] \tag{3.15}$$

The mean squared error of the proposed class of estimators $t_{(\alpha,\delta)}$ to the first degree approximation is given by

$$MSE(t_{(\alpha,\delta)}) = \bar{Y}^{2}\left[(\lambda C_y^2 + \theta C_{y(2)}^2) + (\lambda C_x^2 + \theta C_{x(2)}^2)\left\{\frac{(\delta+2\alpha)^2}{4} - (\delta+2\alpha)R\right\}\right] \tag{3.16}$$

which is minimum when

$$(\delta+2\alpha) = 2R\,. \tag{3.17}$$

Putting (3.17) in (3.16) we get the minimum MSE of $t_{(\alpha,\delta)}$ as

$$\min.MSE(t_{(\alpha,\delta)}) = \bar{Y}^{2}\left[(\lambda C_y^2 + \theta C_{y(2)}^2) - \frac{(\lambda C C_x^2 + \theta C_{(2)} C_{x(2)}^2)^2}{(\lambda C_x^2 + \theta C_{x(2)}^2)}\right] \tag{3.18}$$

Thus we state that the following theorem.

THEOREM 3.1. To the first degree of approximation,

$$MSE(t_{(\alpha,\delta)}) \geq \bar{Y}^{2}(\lambda C_y^2 + \theta C_{y(2)}^2)(1-\rho^{*2})$$

with equality holding if

$$(\delta+2\alpha) = 2R\,,$$

where $\rho^* = \dfrac{Cov(\bar{y}^*, \bar{x}^*)}{\sqrt{V(\bar{y}^*)V(\bar{x}^*)}} = \dfrac{(\lambda C C_x^2 + \theta C_{(2)} C_{x(2)}^2)}{\sqrt{(\lambda C_y^2 + \theta C_{y(2)}^2)(\lambda C_x^2 + \theta C_{x(2)}^2)}}$.

Noting from Srivastava (1971, 1980) it can be shown that the minimum mean squared error of the class of estimators $t_{(\alpha,\delta)}$ in (3.18) is the minimal possible mean squared error up to first degree of approximation for a large class of estimators to which the estimators t_i^* (i=1 to 6) and the class of estimators $t_{(\alpha,\delta)}$ also belong, for example, for the estimators of the form:

$$t_g = \bar{y}^* g(\bar{x}^* / \bar{X}),$$

where $g(\bullet)$ is a function of $(\bar{x}^* / \bar{X})$ with $g(1) = 1$.

4. COMPARISON OF THE PROPOSED CLASS OF ESTIMATORS $t_{(\alpha,\delta)}$ WITH HANSEN AND HURWITZ (1946) ESTIMATOR $\bar{y}^*$, RAO (1986) RATIO ESTIMATOR t_{R1} AND PRODUCT ESTIMATOR t_{P1}

From (2.2) and (3.16) we have

$$Var(\bar{y}^*) - MSE(t_{(\alpha,\delta)}) = \bar{Y}^2(\delta + 2\alpha)\left[(\lambda C C_x^2 + \theta C_{(2)} C_{x(2)}^2) - \frac{(\delta + 2\alpha)(\lambda C_x^2 + \theta C_{x(2)}^2)}{4}\right]$$

$$= \bar{Y}^2(\delta + 2\alpha)\left[\lambda C_x^2\left\{C - \frac{(\delta + 2\alpha)}{4}\right\} + \theta C_{x(2)}^2\left\{C_{(2)} - \frac{(\delta + 2\alpha)}{4}\right\}\right] \quad (4.1)$$

which is positive if

$$\left.\begin{array}{ll} \text{either} & C > \dfrac{(\delta + 2\alpha)}{4} \text{ and } C_{(2)} > \dfrac{(\delta + 2\alpha)}{4} \text{ with } (\delta + 2\alpha) > 0 \\ \text{or} & C < \dfrac{(\delta + 2\alpha)}{4} \text{ and } C_{(2)} < \dfrac{(\delta + 2\alpha)}{4} \text{ with } (\delta + 2\alpha) < 0 \end{array}\right\}. \quad (4.2)$$

The expression in (4.1) can be re-expressed as

$$Var(\bar{y}^*) - MSE(t_{(\alpha,\delta)}) = \bar{Y}^2(\delta + 2\alpha)(\lambda C_x^2 + \theta C_{x(2)}^2)\left[R - \frac{(\delta + 2\alpha)}{4}\right] \quad (4.3)$$

which is positive if

$$\left.\begin{array}{ll}\text{either} & R > \dfrac{(\delta+2\alpha)}{4}\,\text{with}\,(\delta+2\alpha) > 0 \\ \text{or} & R < \dfrac{(\delta+2\alpha)}{4}\,\text{with}\,(\delta+2\alpha) < 0\end{array}\right\}. \tag{4.4}$$

Thus the proposed class of estimators $t_{(\alpha,\delta)}$ is more efficient than usual unbiased estimator $\bar{y}^*$ if either the condition in (4.2) or the condition (4.4) holds good. However, the condition (4.2) is sufficient for the proposed class of estimators $t_{(\alpha,\delta)}$ to be better than the usual unbiased estimator $\bar{y}^*$.

From (2.8) and (3.16) we have

$$MSE(t_{R1}) - MSE(t_{(\alpha,\delta)}) = \bar{Y}^2\left\{1-\frac{(\delta+2\alpha)}{2}\right\}\left[\lambda C_x^2\left(1+\frac{(\delta+2\alpha)}{2}-2C\right)\right.$$
$$\left.+\theta C_{x(2)}^2\left(1+\frac{(\delta+2\alpha)}{2}-2C_{(2)}\right)\right] \tag{4.5}$$

which is positive if

$$\left.\begin{array}{ll}\text{either} & C > \dfrac{(\delta+2\alpha+2)}{4}\,\text{and}\,C_{(2)} > \dfrac{(\delta+2\alpha+2)}{4}\,\text{with}\,\dfrac{(\delta+2\alpha)}{2} > 1 \\ \text{or} & C < \dfrac{(\delta+2\alpha+2)}{4}\,\text{and}\,C_{(2)} < \dfrac{(\delta+2\alpha+2)}{4}\,\text{with}\,\dfrac{(\delta+2\alpha)}{2} < 1\end{array}\right\} \tag{4.6}$$

Expression (4.5) can also be written as

$$MSE(t_{R1}) - MSE(t_{(\alpha,\delta)}) = \bar{Y}^2(\lambda C_x^2 + \theta C_{x(2)}^2)\left\{1-\frac{(\delta+2\alpha)}{2}\right\}\left(\frac{(\delta+2\alpha+2)}{2}-2R\right) \tag{4.7}$$

which is positive if

$$\left.\begin{array}{ll}\text{either} & R > \dfrac{(\delta+2\alpha+1)}{2}\,\text{with}\,\dfrac{(\delta+2\alpha)}{2} > 1 \\ \text{or} & R < \dfrac{(\delta+2\alpha+1)}{2}\,\text{with}\,\dfrac{(\delta+2\alpha)}{2} < 1\end{array}\right\} \tag{4.8}$$

Thus the proposed class of estimators $t_{(\alpha,\delta)}$ is better than the usual unbiased estimator t_{R1} if either the condition in (4.6) or the condition in (4.8) holds good. However, the condition in (4.6) is sufficient for the proposed class of estimators

$t_{(\alpha,\delta)}$ to be better than ratio estimator t_{R1}.

From (2.10) and (3.16) we have

$$MSE(t_{P1}) - MSE(t_{(\alpha,\delta)}) = \bar{Y}^2\left\{1+\frac{(\delta+2\alpha)}{2}\right\}\left[\lambda C_x^2\left(2C - \frac{(\delta+2\alpha)}{2}+1\right)\right.$$

$$\left. + \theta C_{x(2)}^2\left(2C_{(2)} - \frac{(\delta+2\alpha)}{2}+1\right)\right] \tag{4.9}$$

which is non-negative if

$$\left.\begin{array}{ll} \text{either} & C > -\frac{1}{2}+\frac{(\delta+2\alpha)}{4} \text{ and } C_{(2)} > -\frac{1}{2}+\frac{(\delta+2\alpha)}{4} \text{ with } \frac{(\delta+2\alpha)}{2} > -1 \\ \text{or} & C < -\frac{1}{2}+\frac{(\delta+2\alpha)}{4} \text{ and } C_{(2)} < -\frac{1}{2}+\frac{(\delta+2\alpha)}{4} \text{ with } \frac{(\delta+2\alpha)}{2} < -1 \end{array}\right\}. \tag{4.10}$$

Expression (4.9) be can also written as

$$MSE(t_{P1}) - MSE(t_{(\alpha,\delta)}) = \bar{Y}^2(\lambda C_x^2 + \theta C_{x(2)}^2)\left\{1+\frac{(\delta+2\alpha)}{2}\right\}\left(2R - \frac{(\delta+2\alpha)}{2}+1\right) \tag{4.11}$$

which is positive if

$$\left.\begin{array}{ll} \text{either} & R > -\frac{1}{2}+\frac{(\delta+2\alpha)}{4} \text{ with } \frac{(\delta+2\alpha)}{2} > -1 \\ \text{or} & R < -\frac{1}{2}+\frac{(\delta+2\alpha)}{4} \text{ with } \frac{(\delta+2\alpha)}{2} < -1 \end{array}\right\}. \tag{4.12}$$

Thus the proposed class of estimators $t_{(\alpha,\delta)}$ is more efficient than product estimator t_{P1} if the condition in (4.10) or the condition (4.12) is satisfied. However, the condition (4.10) is sufficient for the proposed class of estimators $t_{(\alpha,\delta)}$ to be better than the product estimator t_{P1}.

4.1. Mean Squared Errors of the Estimators $t_i^*(i=1 \text{ to } 6)$

Putting (α , δ) = (2, 0), $\left(\frac{1}{2},0\right)$, $\left(0,\frac{1}{2}\right)$, (0, 2), (1, 1), $\left(\frac{1}{2},\frac{1}{2}\right)$ in (3.16) we get the MSEs of the estimators $t_i^*(i=1 \text{ to } 6)$ to the first degree of approximation as

$$MSE(t_1^*)=\bar{Y}^2[(\lambda C_y^2+\theta C_{y(2)}^2)+4(\lambda C_x^2+\theta C_{x(2)}^2)(1-R)], \tag{4.13}$$

$$MSE(t_2^*)=\bar{Y}^2\left[(\lambda C_y^2+\theta C_{y(2)}^2)+(\lambda C_x^2+\theta C_{x(2)}^2)\left(\frac{1}{4}-R\right)\right] \tag{4.14}$$

$$MSE(t_3^*)=\bar{Y}^2\left[(\lambda C_y^2+\theta C_{y(2)}^2)+\frac{1}{16}(\lambda C_x^2+\theta C_{x(2)}^2)(1-8R)\right], \tag{4.15}$$

$$MSE(t_4^*)=\bar{Y}^2[(\lambda C_y^2+\theta C_{y(2)}^2)+(\lambda C_x^2+\theta C_{x(2)}^2)(1-2R)], \tag{4.16}$$

$$MSE(t_5^*)=\bar{Y}^2\left[(\lambda C_y^2+\theta C_{y(2)}^2)+3(\lambda C_x^2+\theta C_{x(2)}^2)\left(\frac{3}{4}-R\right)\right], \tag{4.17}$$

$$MSE(t_6^*)=\bar{Y}^2\left[(\lambda C_y^2+\theta C_{y(2)}^2)+\frac{3}{2}(\lambda C_x^2+\theta C_{x(2)}^2)\left(\frac{3}{8}-R\right)\right]. \tag{4.18}$$

The estimators $t_1^*, t_2^*, t_3^*, t_4^*, t_5^*$ and t_6^* are respectively better than $\bar{y}^*$ if

(i) $$\left.\begin{array}{l}\text{either } C>1 \text{ and } C_{(2)}>1\\ \text{or} \quad R>1\end{array}\right\}, \tag{4.19}$$

(ii) $$\left.\begin{array}{l}\text{either } C>\frac{1}{4} \text{ and } C_{(2)}>\frac{1}{4}\\ \text{or} \quad R>\frac{1}{4}\end{array}\right\}, \tag{4.20}$$

(iii) $$\left.\begin{array}{l}\text{either } C>\frac{1}{8} \text{ and } C_{(2)}>\frac{1}{8}\\ \text{or} \quad R>\frac{1}{8}\end{array}\right\}, \tag{4.21}$$

(iv) $$\left.\begin{array}{l}\text{either } C>\frac{1}{2} \text{ and } C_{(2)}>\frac{1}{2}\\ \text{or} \quad R>\frac{1}{2}\end{array}\right\}, \tag{4.22}$$

(v) $$\left.\begin{array}{l}\text{either } C > \dfrac{3}{4} \text{ and } C_{(2)} > \dfrac{3}{4} \\ \text{or} \quad R > \dfrac{3}{4}\end{array}\right\},\tag{4.23}$$

(vi) $$\left.\begin{array}{l}\text{either } C > \dfrac{3}{8} \text{ and } C_{(2)} > \dfrac{3}{8} \\ \text{or} \quad R > \dfrac{3}{8}\end{array}\right\}.\tag{4.24}$$

The estimators $t_1^*, t_2^*, t_3^*, t_4^*, t_5^*$ and t_6^* are respectively more efficient than the ratio estimator t_{R1} if

(i) $$\left.\begin{array}{l}\text{either } C > \dfrac{3}{2} \text{ and } C_{(2)} > \dfrac{3}{2} \\ \text{or} \quad R > \dfrac{3}{2}\end{array}\right\},\tag{4.25}$$

(ii) $$\left.\begin{array}{l}\text{either } C > \dfrac{3}{4} \text{ and } C_{(2)} > \dfrac{3}{4} \\ \text{or} \quad R > 1\end{array}\right\},\tag{4.26}$$

(iii) $$\left.\begin{array}{l}\text{either } C > \dfrac{5}{8} \text{ and } C_{(2)} > \dfrac{5}{8} \\ \text{or} \quad R > \dfrac{3}{4}\end{array}\right\},\tag{4.27}$$

(iv) $$\left.\begin{array}{l}\text{either } C > 1 \text{ and } C_{(2)} > 1 \\ \text{or} \quad R > \dfrac{3}{2}\end{array}\right\},\tag{4.28}$$

(v) $$\left.\begin{array}{l}\text{either } C > \dfrac{5}{4} \text{ and } C_{(2)} > \dfrac{5}{4} \\ \text{or} \quad R > 2\end{array}\right\},\tag{4.29}$$

(vi) $$\left.\begin{array}{l}\text{either } C > \dfrac{7}{8} \text{ and } C_{(2)} > \dfrac{7}{8} \\ \text{or} \quad R > \dfrac{5}{4}\end{array}\right\}.\tag{4.30}$$

It is observed from (4.19) to (4.24) and (4.25) to (4.30) that the proposed estimator $t_1^*, t_2^*, t_3^*, t_4^*, t_5^*$ and t_6^* are more efficient than the usual unbiased estimator

$\bar{y}^*$ and the ratio estimator t_{R1} as long as the corresponding conditions given by (4.25) to (4.30) are satisfied.

4.2. Mean Squared Errors of the Almost unbiased Estimators $t_{u(\alpha)}$ and $t_{u(\delta)}$

Inserting $\delta = -2\alpha$ and $\alpha = -(\delta/2)$ in (3.16) we get the mean squared error of $t_{u(\alpha)}$ and $t_{u(\delta)}$ to the first degree of approximation as

$$MSE(t_{u(\alpha)}) = MSE(t_{u(\delta)}) = \bar{Y}^2(\lambda C_y^2 + \theta C_{y(2)}^2) \tag{4.31}$$

which equals to the variance of usual unbiased estimator $\bar{y}^*$.

REMARK 4.2.1. Following the procedure adopting in Rao (1983), the cost aspects can be easily discussed when there is non-response on both the variables y and x.

REMARK 4.2.2. One can also consider the proposed estimator for the population mean under double (or two phase) sampling in presence of non-response where the population mean $\bar{X}$ of the auxiliary variable *x* is not known. For the estimate of mean $\bar{X}$ of the auxiliary variable *x*, a large first phase sample of size n' is selected from a population of *N* units by simple random sampling without replacement (*SRSWOR*). A smaller second phase sample of size n is selected from n' by *SRSWOR* sampling scheme and the study variable *y* is measured on it.

Thus the double sampling version of the proposed estimator $t_{(\alpha,\delta)}$ at (3.7) in presence of non-response on both the variables *y* and *x*, is given by

$$t_{(\alpha,\delta)}^{(d)} = \bar{y}^*\left(\frac{\bar{x}'}{\bar{x}^*}\right)^{\alpha} \exp\left\{\frac{\delta(\bar{x}' - \bar{x}^*)}{(\bar{x}' + \bar{x}^*)}\right\}, \tag{4.32}$$

The properties of the proposed estimator $t_{(\alpha,\delta)}^{(d)}$; along with cost aspects can be studied under large sample approximation, on the line of Singh et al. (2010).

4.3. Empirical Study

In this section we compare the performance of different estimators considered in this paper using a population data set. The description of the population is given below.

POPULATION I. Source: Khare and Sinha (2004, p.53)

The data on physical growth of upper-socio-economic group of 95 school children of Varanasi under an ICMR study, Deportment of Pediatrics; BHU during 1983-1984 has under taken in this study. The first 25% (i.e. 24 children) units have been considered as non-response units. The values of the parameters related to the study variable y (the weight in Kg.) and the auxiliary variable x (the chest circumferences in cm.) are given below:

$\bar{Y} = 19.4968$, $\bar{X} = 55.8611$, $S_y = 3.0435$, $S_x = 3.2735$, $S_{y(2)} = 2.3552$,

$S_{x(2)} = 3.5137$, $\rho = 0.8460$, $\rho_2 = 0.7290$, $W_2 = 0.25$, $N = 95$, $n = 35$.

We have computed the percent relative efficiency (PRE) of the proposed class of estimators $t_{(\alpha,\delta)}$ with respect to the unbiased estimator $\bar{y}^*$ by using the formula:

$$PRE(t_{(\alpha,\delta)}, \bar{y}^*) = \frac{(\lambda C_y^2 + \theta C_{y(2)}^2)}{\left[(\lambda C_y^2 + \theta C_{y(2)}^2) + (\lambda C_x^2 + \theta C_{x(2)}^2)\left(\frac{(\delta + 2\alpha)^2}{4} - (\delta + 2\alpha)R\right)\right]} \times 100$$

For $\alpha_1 = 0.0(0.25)2.0$, $\delta_1 = 0.0(0.25)2.0$, and $k = 5(1)2$; and findings are shown in Table 4.1. It is observed from Table 4.1 that

(i) for fixed (α, δ), the PRE increases as k decreases

(ii) for fixed $(\alpha \leq 1, k)$, the PRE increases as δ increases,

(iii) for fixed (δ, k), the PRE increases as α increases up to 1, beyond unity no trend is observed.

For all values of (α, δ, k), the PRE is larger than 100 percent which follows that the proposed class of estimators $t_{(\alpha,\delta)}$ is more efficient than the usual unbiased estimator $\bar{y}^*$ due to Hansen and Hurwitz (1946). For $(\alpha, \delta) = (1,0)$ in the Table 4.1

$PRE(t_{(\alpha,\delta)}, \bar{y}^*)$ gives the values of $PRE(t_{R1}, \bar{y}^*)$.

It is observed from Table 4.1 that:

(i) for $\alpha \geq 1$ and all values of (δ, k), the $PRE(t_{(\alpha,\delta)}, \bar{y}^*) \geq 181.9835 = PRE(t_{R1}, \bar{y}^*)$ which follows that the proposed class of estimators $t_{(\alpha,\delta)}$ is more efficient than the ratio estimator t_{R1} (for $\alpha \geq 1$).

(ii) (a) for $\alpha = 0$, $k = 2(1)5$ the $PRE(t_{(\alpha,\delta)}, \bar{y}^*) = PRE(t_{R1}, \bar{y}^*)$ for $\delta = 2$,

(b) for $\alpha = 0.25$, $k = 2(1)5$ the $PRE(t_{(\alpha,\delta)}, \bar{y}^*) \geq PRE(t_{R1}, \bar{y}^*)$ for $\delta \geq 1.50$,

(c) for $\alpha = 0.50$, $k = 2(1)5$ the $PRE(t_{(\alpha,\delta)}, \bar{y}^*) \geq PRE(t_{R1}, \bar{y}^*)$ for $\delta \geq 1.00$,

(d) for $\alpha = 0.75$, $k = 2(1)5$ the $PRE(t_{(\alpha,\delta)}, \bar{y}^*) \geq PRE(t_{R1}, \bar{y}^*)$ for $\delta \geq 0.50$.

Thus the proposed class of estimators $t_{(\alpha,\delta)}$ is more efficient than the ratio estimator t_{R1} as long as the conditions (a) to (d) are satisfied. Larger gain in efficiency by using the proposed class of estimators $t_{(\alpha,\delta)}$ over $\bar{y}^*$ and t_{R1} for $1 \leq (\alpha, \delta) \leq 2$ and all the values of *k*. Finally we conclude that there is enough scope of selecting the values of scalars (α, δ) involved in the class of estimators $t_{(\alpha,\delta)}$ in order to obtain estimators better than the usual unbiased estimator $\bar{y}^*$ and the ratio estimator t_{R1}. Thus the proposal of the suggested class of estimators $t_{(\alpha,\delta)}$ is justified. For the sake of convenience to the readers, we have given the percent relative efficiencies of the proposed estimators $\bar{y}^*$, t_{R1} and t_i^* (*i*=1 to 6).

Table 4.1. Percent relative efficiency of the suggested class of estimators $t_{(\alpha,\delta)}$ with respect to usual unbiased estimator $\bar{y}^*$.

δ \ α	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
					$k = 5$				
0.00	100.0000	116.1239	135.1750	157.2577	181.9835	208.0794	233.0159	253.0306	264.0285
0.25	107.7135	125.2694	145.8460	169.3440	194.9859	220.9042	243.9044	259.8803	265.2010
0.50	116.1239	135.1750	157.2577	181.9835	208.0794	233.0159	253.0306	264.0285	263.3179
0.75	125.2694	145.8460	169.3440	194.9859	220.9042	243.9044	259.8803	265.2010	258.5070
1.00	135.1750	157.2577	181.9835	208.0794	233.0159	253.0306	264.0285	263.3179	251.0826
1.25	145.8460	169.3440	194.9859	220.9042	243.9044	259.8803	265.2010	258.5070	241.4963
1.50	157.2577	181.9835	208.0794	233.0159	253.0306	264.0285	263.3179	251.0826	230.2738
1.75	169.3440	194.9859	220.9042	243.9044	259.8803	265.2010	258.5070	241.4963	217.9516
2.00	181.9835	208.0794	233.0159	253.0306	264.0285	263.3179	251.0826	230.2738	205.0272
					$k = 4$				
0.00	100.0000	116.3437	135.7686	158.4595	184.1254	211.5759	238.2755	260.2646	273.0227
0.25	107.8082	125.6523	146.7085	170.9682	197.7504	225.2379	250.1467	268.0540	274.8337
0.50	116.3437	135.7686	158.4595	184.1254	211.5759	238.2755	260.2646	273.0227	273.3595
0.75	125.6523	146.7085	170.9682	197.7504	225.2379	250.1467	268.0540	274.8337	268.7039
1.00	135.7686	158.4595	184.1254	211.5759	238.2755	260.2646	273.0227	273.3595	261.1847
1.25	146.7085	170.9682	197.7504	225.2379	250.1467	268.0540	274.8337	268.7039	251.2811
1.50	158.4595	184.1254	211.5759	238.2755	260.2646	273.0227	273.3595	261.1847	239.5633

1.75	170.9682	197.7504	225.2379	250.1467	268.0540	274.8337	268.7039	251.2811	226.6198
2.00	184.1254	211.5759	238.2755	260.2646	273.0227	273.3595	261.1847	239.5633	212.9993
$k = 3$									
0.00	100.0000	116.6355	136.5611	160.0763	187.0347	216.3813	245.6021	270.4845	285.8940
0.25	107.9337	126.1621	147.8642	173.1631	201.5265	231.2332	258.9039	279.6793	288.6922
0.50	116.6355	136.5611	160.0763	187.0347	216.3813	245.6021	270.4845	285.8940	287.8678
0.75	126.1621	147.8642	173.1631	201.5265	231.2332	258.9039	279.6793	288.6922	283.4822
1.00	136.5611	160.0763	187.0347	216.3813	245.6021	270.4845	285.8940	287.8678	275.8529
1.25	147.8642	173.1631	201.5265	231.2332	258.9039	279.6793	288.6922	283.4822	265.4981
1.50	160.0763	187.0347	216.3813	245.6021	270.4845	285.8940	287.8678	275.8529	253.0549
1.75	173.1631	201.5265	231.2332	258.9039	279.6793	288.6922	283.4822	265.4981	239.1921
2.00	187.0347	216.3813	245.6021	270.4845	285.8940	287.8678	275.8529	253.0549	224.5379
$k = 2$									
0.00	100.0000	117.0415	137.6728	162.3679	191.2138	223.3998	256.5122	286.0201	305.8380
0.25	108.1077	126.8740	149.4931	176.2939	206.9939	240.0724	272.0793	297.5281	310.3381
0.50	117.0415	137.6728	162.3679	191.2138	223.3998	256.5122	286.0201	305.8380	310.6758
0.75	126.8740	149.4931	176.2939	206.9939	240.0724	272.0793	297.5281	310.3381	306.8242
1.00	137.6728	162.3679	191.2138	223.3998	256.5122	286.0201	305.8380	310.6758	299.0867
1.25	149.4931	176.2939	206.9939	240.0724	272.0793	297.5281	310.3381	306.8242	288.0402
1.50	162.3679	191.2138	223.3998	256.5122	286.0201	305.8380	310.6758	299.0867	274.4333
1.75	176.2939	206.9939	240.0724	272.0793	297.5281	310.3381	306.8242	288.0402	259.0730
2.00	191.2138	223.3998	256.5122	286.0201	305.8380	310.6758	299.0867	274.4333	242.7261

Table 4.2. Percent relative efficiency of the suggested class of estimators $t_{(\alpha,\delta)}$ with respect to usual unbiased estimator $\bar{y}^*$.

(α, δ)	Estimator	$1/k$			
		$1/5$	$1/4$	$1/3$	$1/2$
(0,0)	$\bar{y}^*$	100.0000	100.0000	100.0000	100.0000
(1,0)	t_{R1}	181.9835	184.1254	187.0347	191.2138
(2,0)	t_1^*	**264.0285**	**273.0227**	**285.8940**	**305.8380**
(1/2,0)	t_2^*	135.1750	135.7686	136.5611	137.6728
(0,1/2)	t_3^*	116.1239	116.3437	116.6355	117.4115
(0,2)	t_4^*	181.9835	184.1254	187.0347	191.2138
(1,1)	t_5^*	233.0159	238.2755	245.6021	256.5122
(1/2,1/2)	t_6^*	157.2577	158.4595	160.0763	162.3679

It is observed from Table 4.2 that ratio estimator t_i^*'s (i=1 to 6) are more efficient than Hansen and Hurwitz (1946) estimator $\bar{y}^*$ which does not utilize auxiliary information. The proposed estimator t_4^* is at par with the ratio estimator t_{R1}. The suggested estimators t_1^* and t_5^* are more efficient than both the estimators $\bar{y}^*$ and t_{R1} with substantial gain in efficiency for all values of $(1/k)$. Largest gain in efficiency is observed by using t_1^* over $\bar{y}^*$.

We also note from Table 4.2 that the performance of the estimator

$$t_7^* = \bar{y}^*\left(\frac{\bar{X}}{\bar{x}^*}\right)\exp\left\{\frac{2(\bar{X}-\bar{x}^*)}{(\bar{X}+\bar{x}^*)}\right\},$$

$$t_8^* = \bar{y}^*\left(\frac{\bar{x}^*}{\bar{X}}\right)^{3/2}\exp\left\{\frac{\bar{X}-\bar{x}^*}{\bar{X}+\bar{x}^*}\right\};$$

are at par with the estimator t_1^*. It is also noted that the PREs of the estimators

t_{R1} and t_1^* to t_6^* increase as k decreases. It follows that the proposed estimator t_1^* can used in practice (which do not involve any unknown constant or population parameter) in place of $AOE(t_{(\alpha,\delta)}^{(0)})$.

It is observed from Tables 4.2 and 4.5 that the PRE of the proposed estimator t_1^* is near to the asymptotically optimum estimator (AOE) $t_{(\alpha,\delta)}^{(0)}$. With the aid of this empirical study we conclude that the estimator that the estimators t_1^*, t_7^* and t_8^* appear to be appropriate choices for use in practice.

Table 4.3. Values of δ_{opt} for different values of (α, k).

k \ α	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
5	4.2208	3.7208	3.2208	2.7208	2.2208	1.7208	1.2208	0.7208	0.2208
4	4.2629	3.7629	3.2629	2.7629	2.2629	1.7629	1.2629	0.7629	0.2629
3	4.3184	3.8184	3.3184	2.8184	2.3184	1.8184	1.3184	0.8184	0.3184
2	4.3949	3.8949	3.3949	2.8949	2.3949	1.8949	1.3949	0.8949	0.3949

Table 4.4. Values of α_{opt} for different values of (δ, k).

k \ δ	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
5	2.1104	1.9854	1.8604	1.7354	1.6104	1.4854	1.3604	1.2354	1.1104
4	2.1314	2.0064	1.8814	1.7564	1.6314	1.5064	1.3814	1.2564	1.1314
3	2.1592	2.0342	1.9092	1.7842	1.6592	1.5342	1.4092	1.2842	1.1592
2	2.1975	2.0725	1.9475	1.8225	1.6975	1.5725	1.4475	1.3225	1.1975

Table 4.5. PRE of $t_{(\alpha,\delta)}$ at optimum $\left(\frac{\delta+2\alpha}{2}\right)$ with respect to $\bar{y}^*$ for different values of *k*.

k	5	4	3	2
$\left(\frac{\delta+2\alpha}{2}\right)$	2.1104	2.1314	2.1592	2.1975
$PRE(t^{(0)}_{(\alpha,\delta)}, \bar{y}^*)$	265.2220	274.8381	288.8290	311.0505

It is to be mentioned that from equation (3.17), one can calculate the optimum values of either of the constants (α, δ) for different values of k by fixing one of them. For the readers convenience we have given the optimum values of (α, δ) in Tables 4.3 and 4.4. It is observed from Table 4.5 that the PRE of $t_{(\alpha,\delta)}$ [at optimum $\left(\frac{\delta+2\alpha}{2}\right)$] with respect to $\bar{y}^*$ [i.e. $PRE(t^{(0)}_{(\alpha,\delta)}, \bar{y}^*)$] increases as k increases.

5. CASE II: NON-RESPONSE OCCURS ONLY ON THE STUDY VARIABLE y WITH KNOWN POPULATION MEAN $\bar{X}$ OF THE AUXILIARY VARIABLE x

Let the population mean $\bar{X}$ of the auxiliary variable *x* be known. We also assume that the information on auxiliary variable *x* is available for the complete sample size *n*. Thus in this situation we define the following class of estimators for the population mean $\bar{Y}$ as

$$t_{(\alpha_1,\delta_1)} = \bar{y}^*\left(\frac{\bar{X}}{\bar{x}}\right)^{\alpha_1} \exp\left\{\frac{\delta_1(\bar{x}-\bar{X})}{(\bar{x}+\bar{X})}\right\}, \tag{5.1}$$

where (α_1, δ_1) are suitable chosen constants.

To the first degree of approximation, the bias and mean squared error (MSE) of the proposed class of estimators $t_{(\alpha_1,\delta_1)}$ are respectively given by

$$B(t_{(\alpha_1,\delta_1)}) = \frac{\bar{Y}(\delta_1 + 2\alpha_1)}{8}\lambda C_x^2(\delta_1 + 2\alpha_1 - 4C + 2), \tag{5.2}$$

$$MSE(t_{(\alpha_1,\delta_1)}) = \bar{Y}^2\left[(\lambda C_y^2 + \theta C_{y(2)}^2) + \lambda C_x^2\left\{\frac{(\delta_1 + 2\alpha_1)^2}{4} - (\delta_1 + 2\alpha_1)C\right\}\right]. \tag{5.3}$$

Equating (5.2) to zero, we have

$$\delta_1 = -2\alpha_1 \text{(or } \alpha_1 = -(\delta_1/2)) \tag{5.4}$$

or

$$\delta_1 = 2(2C - \alpha_1 - 1) \text{(or } \alpha_1 = 2 - (\delta_1/2) - 1) \tag{5.5}$$

The proposed class of estimators $t_{(\alpha_1,\delta_1)}$ substituted with the values of δ_1 from (5.4) and (5.5), becomes an (approximately) unbiased estimator for the population mean $\bar{Y}$.

Furthermore, if the sample size *n* is sufficiently large, the bias of the proposed class of estimators $t_{(\alpha_1,\delta_1)}$ becomes negligible.

The MSE of $t_{(\alpha_1,\delta_1)}$ at (5.3) is minimized when

$$\frac{(\delta_1 + 2\alpha_1)}{2} = C, \tag{5.6}$$

$$\Rightarrow (\delta_1 + 2\alpha_1) = 2C. \tag{5.7}$$

By substituting (5.6) in (5.3) we get the minimum MSE of the proposed class of estimators $t_{(\alpha_1,\delta_1)}$ as

$$\min.MSE(t_{(\alpha_1,\delta_1)}) = [\lambda S_y^2(1-\rho^2) + \theta S_{y(2)}^2)] \tag{5.8}$$

which is the same as approximate variance of the linear regression estimator $\bar{y}_{lr} = \bar{y}^* + b(\bar{X} - \bar{x})$, where *b* is the sample regression coefficient of *y* on *x*.

Thus, we established following theorem.

THEOREM 5.1. To the first degree of approximation,

$$MSE(t_{(\alpha_1,\delta_1)}) \geq [\lambda S_y^2(1-\rho^2) + \theta S_{y(2)}^2)]$$

with equality holding if

$$\frac{(\delta_1 + 2\alpha_1)}{2} = C \cdot$$

In fact Singh and Kumar (2009) showed that the quantity $[\lambda S_y^2(1-\rho^2) + \theta S_{y(2)}^2)]$ is the minimal possible mean squared error up to first degree of approximation for large class of estimators to which the estimator $t_{(\alpha_1,\delta_1)}$ in (5.1) also belongs, for example, for estimators of the form

$$\bar{y}_h = \bar{y}^* h(\bar{x} / \bar{X}), \tag{5.8}$$

where $h(\bullet)$ is a C^2-function with $h(1) = 1$. Further Singh and Kumar (2009) have shown that incorporating sample and population variance of the auxiliary variable x might yield an estimator that has smaller mean squared error than $[\lambda S_y^2(1-\rho^2) + \theta S_{y(2)}^2)]$ especially when the relationship between the study variable y and the auxiliary variable x is markedly non-linear. Thus whatever value C has, we are always able to choose an approximately optimum estimator (AOE) say $t_{(\alpha_1,\delta_1)}^{(0)}$ from the two parameter family of estimators $t_{(\alpha_1,\delta_1)}$ in (5.1).

Some members of the proposed class of estimators $t_{(\alpha_1,\delta_1)}$ of the population mean $\bar{Y}$ are given in the Table 5.1.

Table 5.1. Some members of the proposed class of estimators $t_{(\alpha_1,\delta_1)}$.

S. No.	**Estimator**	**Values of Constants**	
		α_1	δ_1
1.	$t_{(0,0)} = \bar{y}^*$	0	0
2.	$t_{(1,0)} = \bar{y}^* (\bar{X} / \bar{x}) = t_{R2}$	1	0
3.	$t_{(0,1)} = \bar{y}^* (\bar{x} / \bar{X}) = t_{P2}$	-1	0

4.	$t_{(2,0)} = \bar{y}^* (\bar{X}^2 / \bar{x}^2) = t_1$	2	0
5.	$t_{(1/2,0)} = \bar{y}^* (\bar{X} / \bar{x})^{1/2} = t_2$	1/2	0
6.	$t_{(0,1/2)} = \bar{y}^* \exp\left\{\frac{(\bar{X}-\bar{x})}{2(\bar{X}+\bar{x})}\right\} = t_3$	0	1/2
7.	$t_{(0,2)} = \bar{y}^* \exp\left\{\frac{2(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})}\right\} = t_4$	0	2
8.	$t_{(1,1)} = \bar{y}^* \left(\frac{\bar{X}}{\bar{x}}\right) \exp\left\{\frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})}\right\} = t_5$	1	1
9.	$t_{(1/2,1/2)} = \bar{y}^* \left(\frac{\bar{X}}{\bar{x}}\right)^{1/2} \exp\left\{\frac{1}{2}\frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})}\right\} = t_6$	1/2	1/2

To the first degree of approximation, the mean squared errors of the estimators t_1 to t_6 (listed in Table 5.1) are respectively given by

$$MSE(t_1) = \bar{Y}^2[(\lambda C_y^2 + \theta C_{y(2)}^2) + 4C_x^2(1-C)], \tag{5.10}$$

$$MSE(t_2) = \bar{Y}^2\left[(\lambda C_y^2 + \theta C_{y(2)}^2) + \frac{C_x^2}{4}(1-C)\right], \tag{5.11}$$

$$MSE(t_3) = \bar{Y}^2\left[(\lambda C_y^2 + \theta C_{y(2)}^2) + \frac{\lambda C_x^2}{16}(1-8C)\right], \tag{5.12}$$

$$MSE(t_4) = \bar{Y}^2[(\lambda C_y^2 + \theta C_{y(2)}^2) + \lambda C_x^2(1-2C)], \tag{5.13}$$

$$MSE(t_5) = \bar{Y}^2\left[(\lambda C_y^2 + \theta C_{y(2)}^2) + 3\lambda C_x^2\left(\frac{3}{4} - C\right)\right], \tag{5.14}$$

$$MSE(t_6) = \bar{Y}^2\left[(\lambda C_y^2 + \theta C_{y(2)}^2) + \frac{3\lambda C_x^2}{2}\left(\frac{3}{8} - C\right)\right]. \tag{5.15}$$

5.1. Comparison of Mean Squared Error of the Suggested Class of Estimators $t_{(\alpha_1,\delta_1)}$ with $\bar{y}^*, t_{R2}$ and t_{P2}

From (2.2) and (5.3) we have

$$Var(\bar{y}^*) - MSE(t_{(\alpha_1,\delta_1)}) = \lambda C_x^2 \bar{Y}^2 \frac{(\delta_1 + 2\alpha_1)}{4}\left[4C - (\delta_1 + 2\alpha_1)\right]$$

which is positive if

$\left[4C - (\delta_1 + 2\alpha_1)\right] > 0$, $(\delta_1 + 2\alpha_1) > 0$

i.e. if

$$\left.\begin{array}{ll} \text{either} & C > \dfrac{(\delta_1 + 2\alpha_1)}{4}, (\delta_1 + 2\alpha_1) > 0 \\ \text{or} & C < \dfrac{(\delta_1 + 2\alpha_1)}{4}, (\delta_1 + 2\alpha_1) < 0 \end{array}\right\}. \tag{5.16}$$

From (2.9) and (5.3) we have

$$MSE(t_{R2}) - MSE(t_{(\alpha_1,\delta_1)}) = \lambda C_x^2 \bar{Y}^2 \left\{1 - \frac{(\delta_1 + 2\alpha_1)}{2}\right\}\left[1 + \frac{(\delta_1 + 2\alpha_1)}{2} - 2C\right]$$

which is non-negative if

$$\left.\begin{array}{ll} \text{either} & C > \dfrac{(\delta_1 + 2\alpha_1 + 2)}{4}, \dfrac{(\delta_1 + 2\alpha_1)}{2} > 1 \\ \text{or} & C < \dfrac{(\delta_1 + 2\alpha_1 + 2)}{4}, \dfrac{(\delta_1 + 2\alpha_1)}{2} < 1 \end{array}\right\}. \tag{5.17}$$

Further, from (2.11) and (5.3) we have

$$MSE(t_{P2}) - MSE(t_{(\alpha_1,\delta_1)}) = \lambda C_x^2 \bar{Y}^2 \left\{1 + \frac{(\delta_1 + 2\alpha_1)}{2}\right\}\left[2C - \frac{(\delta_1 + 2\alpha_1)}{2} + 1\right]$$

which is greater than zero if

$$\left.\begin{array}{ll} \text{either} & C > -\dfrac{1}{2} + \dfrac{(\delta_1 + 2\alpha_1)}{4}, \dfrac{(\delta_1 + 2\alpha_1)}{2} > -1 \\ \text{or} & C < -\dfrac{1}{2} + \dfrac{(\delta_1 + 2\alpha_1)}{4}, \dfrac{(\delta_1 + 2\alpha_1)}{2} < -1 \end{array}\right\}. \tag{5.18}$$

Thus it follows that the proposed class of estimators $t_{(\alpha_1,\delta_1)}$ is more efficient than the usual unbiased estimator $\bar{y}^*$, ratio estimator t_{R2} and product estimator t_{P2} as long as the conditions in (5.16), (5.17) and (5.18) respectively hold true.

5.2. Comparison of the proposed estimator $t_j\,(j=1 \text{ to } 6)$ with respect to usual unbiased estimator $\bar{y}^*$ and the ratio estimator t_{R2}

It can be shown that the suggested class of estimators:

(i) t_1 is more efficient than $\bar{y}^*$ and t_{R2} respectively if

$$C > 1 \tag{5.19}$$

and

$$C > \frac{3}{2}\cdot \tag{5.20}$$

(ii) t_2 is more efficient than $\bar{y}^*$ and t_{R2} respectively if

$$C > \frac{1}{4} \tag{5.21}$$

and

$$C > \frac{3}{4}\cdot \tag{5.22}$$

(iii) t_3 is more efficient than $\bar{y}^*$ and t_{R2} respectively if

$$C > \frac{1}{8} \tag{5.23}$$

and

$$C > \frac{5}{8}\cdot \tag{5.24}$$

(iv) t_4 is more efficient than $\bar{y}^*$ and t_{R2} respectively if

$$C > \frac{1}{2} \tag{5.25}$$

and

$$C > 1. \tag{5.26}$$

(v) t_5 is more efficient than $\bar{y}^*$ and t_{R2} respectively if

$$C > \frac{3}{4} \tag{5.27}$$

and

$$C > \frac{5}{4} \cdot \tag{5.28}$$

(vi) t_6 is more efficient than $\bar{y}^*$ and t_{R2} respectively if

$$C > \frac{3}{8} \tag{5.29}$$

and

$$C > \frac{7}{8} \cdot \tag{5.30}$$

REMARK 5.2.1. Following the same procedure as adopted by Rao (1983), the cost aspects can be also studied when there is non-response only on the variable *y*.

REMARK 5.2.2. The double sampling version of suggested class of estimators $t_{(\alpha_1,\delta_1)}$ given by (5.1) can be given when the population mean $\bar{X}$ is not known. Suppose that complete information on the auxiliary variable *x* is available for both the first and second samples, and that incomplete information on the study variable *y* is available.

So, in this case, we use information on the $(n_1 + r)$ responding units on the study variable *y*, and complete information on the auxiliary variable *x* from the sample of size *n*. Thus one can suggest a double sampling version of the class of estimators $t_{(\alpha_1,\delta_1)}$ defined at (5.1) for population mean $\bar{Y}$ when the non-response occurs only on the study variable y as

$$t^{(d)}_{(\alpha_1,\delta_1)} = \bar{y}^* \left(\frac{\bar{x}'}{\bar{x}}\right)^{\alpha_1} \exp\left\{\frac{\delta_1(\bar{x} - \bar{x}')}{(\bar{x} + \bar{x}')}\right\}, \tag{5.31}$$

The properties of the suggested class of estimators $t^{(d)}_{(\alpha_1,\delta_1)}$ along with cost aspects can be studied under large sample approximation, on the line of Tabasum and Khan (2006) and Singh et al.(2011).

5.3. Empirical Study

In this section, we consider the same population data set which is given in Section 4.3. We have computed the percent relative efficiency (PRE) of the proposed class of estimators $t_{(\alpha_1,\delta_1)}$ with respect the usual unbiased estimator $\bar{y}^*$ by using the formula:

$$PRE(t_{(\alpha_1,\delta_1)},\bar{y}^*)=\frac{(\lambda C_y^2+\theta C_{y(2)}^2)}{\left[(\lambda C_y^2+\theta C_{y(2)}^2)+\lambda C_x^2\left(\frac{(\delta_1+2\alpha_1)^2}{4}-(\delta_1+2\alpha_1)C\right)\right]}\times 100\,.$$

For $\alpha_1 = 0.0(0.25)2.0$, $\delta_1 = 0.0(0.25)2.0$, and $k = 5(1)2$; and findings are shown in Table 5.2.

Table 5.2 exhibits that the values of PREs are larger than 100 percent. It follows that the proposed class of estimators $t_{(\alpha_1,\delta_1)}$ is better than usual unbiased estimator $\bar{y}^*$ for the values of the constants (α_1,δ_1) and k considered here. It is further observed that the proposed class of estimators $t_{(\alpha_1,\delta_1)}$ is better than the ratio estimator t_{R2} for $1 \le \alpha_1, \delta_1 \le 2$ and $k = 5(1)2$. Comparing the Tables 4.1 and 5.2 we find that the $PRE(t_{(\alpha,\delta)},\bar{y}^*)$ [i.e. when there is non-response present in both the variables y and x] is larger than the class of estimators $t_{(\alpha_1,\delta_1)}$ [i.e. when there is non-response occurs only on the study variables y and information on the auxiliary variable x is available for complete sample size n].

Table 5.2. PREs of the proposed class of estimators $t_{(\alpha_1,\delta_1)}$ with respect to $\bar{y}^*$

$k=5$									
δ_1 \ α_1	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
0.00	100.0000	108.3410	116.9502	125.6028	133.9946	141.7477	148.4338	153.6180	156.9189
0.25	104.1259	112.6243	121.2882	129.8533	137.9770	145.2518	151.2395	155.5238	157.7746
0.50	108.3410	116.9502	125.6028	133.9946	141.7477	148.4338	153.6180	156.9189	158.0727
0.75	112.6243	121.2882	129.8533	137.9770	145.2518	151.2395	155.5238	157.7746	157.8070
1.00	116.9502	125.6028	133.9946	141.7477	148.4338	153.6180	156.9189	158.0727	156.9831
1.25	121.2882	129.8533	137.9770	145.2518	151.2395	155.5238	157.7746	157.8070	155.6185
1.50	125.6028	133.9946	141.7477	148.4338	153.6180	156.9189	158.0727	156.9831	153.7412
1.75	129.8533	137.9770	145.2518	151.2395	155.5238	157.7746	157.8070	155.6185	151.3887
2.00	133.9946	141.7477	148.4338	153.6180	156.9189	158.0727	156.9831	153.7412	148.6063
$k=4$									
0.00	100.0000	109.6075	119.7623	130.2214	140.6166	150.4486	159.1092	165.9438	170.3510
0.25	104.7245	114.6289	124.9737	135.4552	145.6389	154.9662	162.7950	168.4830	171.5005
0.50	109.6075	119.7623	130.2214	140.6166	150.4486	159.1092	165.9438	170.3510	171.9018
0.75	114.6289	124.9737	135.4552	145.6389	154.9662	162.7950	168.4830	171.5005	171.5442
1.00	119.7623	130.2214	140.6166	150.4486	159.1092	165.9438	170.3510	171.9018	170.4372
1.25	124.9737	135.4552	145.6389	154.9662	162.7950	168.4830	171.5005	171.5442	168.6095
1.50	130.2214	140.6166	150.4486	159.1092	165.9438	170.3510	171.9018	170.4372	166.1074

1.75	135.4552	145.6389	154.9662	162.7950	168.4830	171.5005	171.5442	168.6095	162.9918
2.00	140.6166	150.4486	159.1092	165.9438	170.3510	171.9018	170.4372	166.1074	159.3349
$k = 3$									
0.00	100.0000	111.3274	123.6931	136.8731	150.4425	163.7311	175.8212	185.6280	192.0807
0.25	105.5262	117.3905	130.2025	143.6449	157.1739	169.9915	181.0800	189.3332	193.7808
0.50	111.3274	123.6931	136.8731	150.4425	163.7311	175.8212	185.6280	192.0807	194.3759
0.75	117.3905	130.2025	143.6449	157.1739	169.9915	181.0800	189.3332	193.7808	193.8455
1.00	123.6931	136.8731	150.4425	163.7311	175.8212	185.6280	192.0807	194.3759	192.2079
1.25	130.2025	143.6449	157.1739	169.9915	181.0800	189.3332	193.7808	193.8455	189.5186
1.50	136.8731	150.4425	163.7311	175.8212	185.6280	192.0807	194.3759	192.2079	185.8657
1.75	143.6449	157.1739	169.9915	181.0800	189.3332	193.7808	193.8455	189.5186	181.3628
2.00	150.4425	163.7311	175.8212	185.6280	192.0807	194.3759	192.2079	185.8657	176.1412
$k = 2$									
0.00	100.0000	113.7975	129.5759	147.2792	166.5398	186.5075	205.7080	222.0640	233.2333
0.25	106.6556	121.4371	138.2007	156.7546	176.5118	196.3214	214.3883	228.4368	236.2320
0.50	113.7975	129.5759	147.2792	166.5398	186.5075	205.7080	222.0640	233.2333	237.2873
0.75	121.4371	138.2007	156.7546	176.5118	196.3214	214.3883	228.4368	236.2320	236.3466
1.00	129.5759	147.2792	166.5398	186.5075	205.7080	222.0640	233.2333	237.2873	233.4568
1.25	138.2007	156.7546	176.5118	196.3214	214.3883	228.4368	236.2320	236.3466	228.7585
1.50	147.2792	166.5398	186.5075	205.7080	222.0640	233.2333	237.2873	233.4568	222.4696
1.75	156.7546	176.5118	196.3214	214.3883	228.4368	236.2320	236.3466	228.7585	214.8610
2.00	166.5398	186.5075	205.7080	222.0640	233.2333	237.2873	233.4568	222.4696	206.2304

We have also given the percent relative efficiency of different estimators t_{R2} and t_1 to t_6 with respect to usual unbiased estimator $\bar{y}^*$ for different values of $k = 5(1)2$ in Table 5.3.

Table 5.3. Percent relative efficiency of the suggested class of estimators $t_{(\alpha,\delta)}$ with respect to usual unbiased estimator $\bar{y}^*$.

α_1	δ_1	Estimator	$1/k$			
			$1/5$	$1/4$	$1/3$	$1/2$
0	0	$\bar{y}^*$	100.0000	100.0000	100.0000	100.0000
1	0	t_{R2}	133.9946	140.6166	150.4425	166.5398
2	**0**	t_1	**156.9189**	**170.3510**	**192.6280**	**233.2333**
1/2	0	t_2	116.9502	119.7623	123.6931	129.5759
0	1/2	t_3	108.3410	109.6075	111.3274	113.7975
0	2	t_4	133.9946	140.6166	150.4425	166.5398
1	1	t_5	148.4338	159.1092	175.8212	205.7080
1/2	1/2	t_6	125.6028	130.2214	136.8731	147.2792

We have further computed the optimum values of α_1 for given values of δ_1, and optimum values of δ_1 for given of α_1 respectively tabulated in Tables 5.4 and 5.5.

Table 5.4. Values of δ_{1opt} for different values of (α_1, k).

k \ α_1	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
5	4.5072	4.2572	4.0072	3.7572	3.5072	3.2572	3.0072	2.7572	2.5072
4	4.5072	4.2572	4.0072	3.7572	3.5072	3.2572	3.0072	2.7572	2.5072
3	4.5072	4.2572	4.0072	3.7572	3.5072	3.2572	3.0072	2.7572	2.5072
2	4.5072	4.2572	4.0072	3.7572	3.5072	3.2572	3.0072	2.7572	2.5072

Table 5.5. Values of α_{1opt} for different values of (δ_1, k).

δ_1 \ k	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
5	4.5072	4.0072	3.5072	3.0072	2.5072	2.0072	1.5072	1.0072	0.5072
4	4.5072	4.0072	3.5072	3.0072	2.5072	2.0072	1.5072	1.0072	0.5072
3	4.5072	4.0072	3.5072	3.0072	2.5072	2.0072	1.5072	1.0072	0.5072
2	4.5072	4.0072	3.5072	3.0072	2.5072	2.0072	1.5072	1.0072	0.5072

Table 5.6. PRE of $t_{(\alpha_1,\delta_1)}$ at optimum $\left(\frac{\delta_1 + 2\alpha_1}{2}\right)$ with respect to $\bar{y}^*$ for different values of *k*.

k	5	4	3	2
$\left(\frac{\delta_1 + 2\alpha_1}{2}\right)_{opt}$	2.2536	2.2536	2.2536	2.2536
$PRE(t^{(0)}_{(\alpha_1,\delta_1)}, \bar{y}^*)$	158.0729	171.9021	194.3764	237.2882

Tables 5.3 and 5.6 demonstrate that the proposed estimator t_1 gives the PRE closer to the asymptotically optimum estimator (AOE) $t^{(0)}_{(\alpha_1,\delta_1)}$. Thus in practice t_1 is to be preferred as an alternative to AOE. In general, there is increasing trend in PRE as k decreases. The PRE of t_5 with respect to $\bar{y}^*$ is larger than that of t_{R2}, t_2, t_3, t_6. Also $PRE(t_4, \bar{y}^*)$ is at par with $PRE(t_{R2}, \bar{y}^*)$. It follows that the estimator t_1 and t_5 are appropriate choice among the estimators $\bar{y}^*, t_{R2}, t_1, t_2, t_3, t_4$ t_5 and t_6. Comparing Table 5.2 and Table 5.3 we conclude that there is enough scope of selecting the values of scalars (α_1, δ_1) in order to obtain estimators better than usual unbiased estimator $\bar{y}^*$ and the ratio estimator t_{R2} from the proposed class of estimators $t_{(\alpha_1,\delta_1)}$. Thus our recommendation is in the favor of proposed class of

estimators $t_{(\alpha_1,\delta_1)}$ regarding its use in practice.

Finally, it is observed that the estimator formulated in Case I [i.e. when there is non-response present on both the variables y and x] is more efficient than the corresponding estimators in Case II [i.e. when there is non-response occurs only on the study variables y].

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