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EVENT-TRIGGERED CLUSTER CONSENSUS OF LEADER-FOLLOWING LINEAR MULTI-AGENT SYSTEMS

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Abstract

This paper is concerned with cluster consensus of linear multi-agent systems via a distributed event-triggered control scheme. Assume that agents can be split into several clusters and a leader is associated with each cluster. Sufficient conditions are derived to guarantee the realization of cluster consensus by a feasible event-triggered controller if the network topology of each cluster has a directed spanning tree and the couplings within each cluster are sufficiently strong. Further, positive inner-event time intervals are ensured for the proposed event-triggered strategy to avoid Zeno behaviors. Finally, a numerical example is given to illustrate the effectiveness of the theoretical results.

Keywords: cluster consensus, event-triggered scheme, leader-following consensus, multi-agent systems

1 Introduction

The past decade has witnessed the dramatic progress in the study of consensus [1-4]. The consensus problem for multi-agent systems has attracted intensive attention due to its potential advantages when compared with a single individual include greater flexibility, adaptability, and performance [5-7]. The main idea of consensus is to design some appropriate control protocols based on local information guaranteeing all agents reach an agreement on a common value. Leader-following consensus [8-10] and leaderless consensus [11, 12] have been well studied in the past decade.

For the common consensus problem, all the agents converge into the same trajectory or tracking the same leader. However, in practice, some complex networks may evolve into several clusters or groups. The agents in the same cluster or group reach consensus, while the consistent states of different clusters may not coincide [13-17]. It has many applications in civilian and military, such as surveillance, reconnaissance, and battlefield assessment. For example, the search and rescue scenario by multiple robots in a disaster site, where victims are required to be located and rescued. In this case, a diverse group of robots with different capabilities should be deployed to perform different tasks [17]. If the eigenvalues of the Laplacian matrix had nonnegative parts and the clusters were balanced and balanced coupling, a linear consensus protocol was designed to solve the group average-consensus problem in [13]. In [14], the scaled group consensus problems of first/second-order linear multi-agent systems were studied. It was revealed that under interaction topologies with acyclic partition, cluster consensus was irrelevant to the magnitude of the couplings among agents [15]. In [16], cluster synchronization of coupled network systems under pinning control was investigated. a common feature of the above-mentioned works is that each agent has the continuous access to its neighbors. In some applications, continuous communication is impossible or inefficient regarding the network and computation resources. The problem of cluster formation control for a networked multi-agent system in the simultaneous presence of aperiodic sampling and communication delays was studied in [17].

In order to reduce the communication burden caused by continuous communication between agents, some event-triggered control schemes were proposed in [18-27]. The control actuation is triggered whenever a certain error becomes large enough to violate the threshold. Refs. [18] and [19] presented the centralized and decentralized consensus protocols for first and second order multi-agent systems under event-triggered schemes respectively. In [20], three different types of event-triggered strategies, namely distributed, centralized and clustered event-triggered strategies, was studied under different network topologies. It was noted that the event-triggered schemes derived in [21, 22] were based on the combinational measurements, which needed continuous monitoring of measurement errors. In the even-triggered scheme design, it is known that the minimal time distance between two consecutive event time instants must be strictly than zero, otherwise, infinite events occur when a finite time interval named as Zeno behavior will happen. Thus, Refs. [23, 25] and [26] conducted studies on sampling-data-based event-triggered control for multi-agent systems, in which the trigger instants only happened at the sampling instants, thus Zeno behaviour can be excluded. Furthermore, an overview of multi-agent event-triggered consensus control has been provided in [27].

Recently, event-triggered control has attracted tremendous attention [28-31]. The strategies in [28] and [29] were based on the sampling data for complex networks and formation control of multi-agent systems, respectively. However, sufficient conditions of these sampling-data-based strategies were expressed in LMIs, which were not easy to be solved when the number of agents was large. Ref. [30] proposed an impulsive framework to analyse the clustered event-triggered consensus. But clusters considered in [30] had no coupling between two clusters and the whole system had only one leader, thus complete consensus was achieved for multi-agent systems. Moreover, cluster synchronization of coupling neural networks under pinning control via event-triggered mechanism was studied in [31]. However, the coupling matrix was needed to be constructed, which was not suitable for the fixed topology.

Motivated by the above discussion, cluster consensus is studied for leader-following linear multi-agent systems. The agents are split into several clusters and each cluster has a leader. A distributed event-triggered scheme is proposed in this paper, which is related to the measurement error and the relative errors between agents and their neighbours. A novel controller is designed only related to discrete states at triggered instants. Sufficient conditions are derived to guarantee the realization of cluster consensus if the network topology of each cluster has a directed spanning tree and the couplings within each cluster are sufficiently large.

The major contributions are threefold. 1) Compared with the control protocol considered in [18, 19, 21] and [22], the dynamic control protocol considered in this paper is based on the model, which could reduce the number of event-trigger times obviously. 2) The distributed event-triggered scheme designed excludes Zeno behavior and the continuous event detection is not required under the event-triggered condition. 3) An algorithm is presented to find appropriate coupling strengths.

The rest of this paper is organized as follows. Section 2 gives some basic theory on graphs. The mathematical model and the event-triggered scheme are also presented. Section 3 gives some sufficient conditions on reaching cluster consensus. A simulation example is given in Section 4. Finally, the conclusion of this paper is given.

Notations: $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $\|\cdot\|$ stands for either the Euclidean vector norm or its induced matrix 2-norm. For real symmetric matrix X, the notation $X > 0$ means X is positive definite, and similarly definition for $X < 0$, $X \geq 0$ and $X \leq 0$. I_n is the identity matrix of order n. If not explicitly stated, matrices are assumed to have compatible dimensions. $\lambda_{\min}(M)$ and $\lambda_{\max}(M)$ denote the smallest and largest eigenvalues of the symmetric matrix M, respectively.$\sigma_{\min}(N)$ and $\sigma_{\max}(N)$ denote respectively the minimum and maximum singular values of the matrix N.

2 Preliminaries and problem statement

The interaction between N agents can be described as a directed graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$ consisting of a set of vertices $\mathcal{V} = \{v_1, v_2, \cdots, v_N\}$, a set of directed edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and a weighted adjacency matrix $\mathcal{A} = (a_{ij})$ with nonnegative entries. An edge e_{ij} is denoted by an ordered pair of vertices (v_j, v_i), where v_j and v_i are called the parent and child vertices, respectively, and $e_{ij} \in \mathcal{E}$ if and only if $a_{ij} > 0$. A path from node v_i to node v_j is a sequence of edges (v_j, v_{i_1}), $(v_{i_1}, v_{i_2}), \ldots, (v_{i_p}, v_i)$, with distinct nodes i_k, $k = 1, 2, \ldots, p$. The Laplacian matrix $L = (l_{ij})_{N\times N}$ associated with an adjacency matrix $\mathcal{A}$ is defined as $l_{ii} = \sum_{j=1, j\neq i}^{N} a_{ij}$, $l_{ij} = -a_{ij}$ for $i \neq j$. Further, $D = diag\{d_1, d_2, \ldots, d_N\}$ is a nonnegative diagonal matrix, which is defined according to the following roles: there is an edge from the leader to the node $i \in 1, 2, \ldots, N$ if and only if $d_i > 0$; otherwise $d_i = 0$.

2.1 The model of multi-agent system

Cluster consensus of multi-agent systems is studied in this paper. Consider a multi-agent system consisting of N followers and p leaders where N followers will finally achieve p-cluster states. Let $G = \{G_1, G_2, \ldots, G_p\}$ be a disjoint partition of the agent set $\{1, 2, \ldots, N\}$, i.e. $G_l \cap G_r = \emptyset$ for $l \neq r$, $l, r = 1, 2, \ldots, p$, $\bigcup_{l=1}^{p} G_l = \{1, 2, \ldots, N\}$. For $i \in \mathcal{V}$, let $\hat{i}$ denote the cluster to which the ith agent belongs, i.e. $i \in G_{\hat{i}}$. One can say that agents i and j are in the same cluster if $\hat{i} = \hat{j}$.

Suppose that the linear dynamic of the ith agent can be described by

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t),\ i = 1, 2, \ldots, N, \tag{1}$$

and the dynamic of the lth leader is given by

$$\dot{s}_l(t) = As_l(t),\ l = 1, 2, \ldots, p, \tag{2}$$

where $x_i(t) \in \mathbb{R}^n$ and $s_l(t) \in \mathbb{R}^n$ are the states of agent i and the state of the lth leader, respectively, $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$ are constant matrices. $u_i(t) \in \mathbb{R}^m$ is the control input of agent i. One can say $\hat{i} = l$ if the ith agent belongs to the lth cluster.

Remark 1. The homogeneous system $\dot{s}_l(t) = As_l(t)$ is considered in this paper. If one requires that there are no consensus between two different clusters, i.e. $\lim_{t\to\infty} \|s_l(t) - s_k(t)\| \neq 0$ for any $l, k = 1, 2, \ldots, p$, $k \neq l$, it can be realized by choosing different initial values since $s(t) = e^{At}s(0)$ if A is not a Hurwitz matrix. Note that the states of all the linear systems approach zeros asymptotically if A is a Hurwitz matrix.

Some definitions, assumptions and useful lemmas are presented as follows.

Definition 1. For any given initial states $x(0) = [x_1^T(0), x_1^T(0), \ldots, x_N^T(0)]^T$, cluster consensus for the interacting p clusters of agents is said to be achieved if

$$\lim_{t\to\infty} \|x_i(t) - s_{\hat{i}}(t)\| = 0,\ i = 1, 2, \ldots, N.$$

Assumption 1. The elements of the Laplacian matrix $L \in \mathbb{R}^{N\times N}$ can be divided into p clusters as defined above, then one can assume the following form holds

$$L = \begin{pmatrix} L_{11} & L_{12} & \ldots & L_{1p} \\ L_{21} & L_{22} & \ldots & L_{2p} \\ \ldots & \ldots & \ldots & \ldots \\ L_{p1} & L_{p2} & \ldots & L_{pp} \end{pmatrix},$$

with L_{lk}, $l, k = 1, 2, \ldots, d$ are zero-row-sum matrices, i.e. $\sum_{j\in G_k} a_{ij} = 0$ for all $i \in G_l$. Each L_{ll} is the Laplacian matrix if the digraph G_l, $l = 1, 2, \ldots, p$.

Remark 2. Assumption 1 indicates that the inter-cluster coupling strengths may be either positive and negative, which describe the cooperative or competitive scheme between agents. It means that a coupling balance among a cluster with other clusters are needed.

Assumption 2. The matrix pair (A, B) is stabilizable.

Lemma 1. [30] If (A, B) is stabilizable, then for any $\alpha > 0$, there exists a positive definite matrix $P > 0$ that satisfies the Riccati inequality

$$PA + A^T P - \alpha PBB^T P + \alpha I_n < 0. \tag{3}$$

Lemma 2. [16] Let L and D be the Laplacian matrix and the pinning matrix. If $\mathcal{G}$ has a directed spanning tree, then there exists a positive diagonal matrix $\Xi > 0$ such that

$$\Xi(L + D) + (L + D)^T \Xi > 0. \tag{4}$$

2.2 Distributed event-triggered control over the directed topology

In this Section, a distributed event-triggered condition is proposed.

First, the measurement error for each agent is defined as

$$e_i(t) = \hat{x}_i(t) - x_i(t),\ t \in [t_k^i, t_{k+1}^i) \tag{5}$$

where $\hat{x}_i(t) = e^{A(t-t_k^i)}x_i(t_k^i)$, t_k^i is the kth event-triggered instant for agent i. The distributed event-triggered consensus protocol is proposed as

$$\begin{aligned} u_i(t) =& Kz_i(t) \\ =& K\Big[\sum_{j\in G_{\hat{i}}} c_{\hat{i}} a_{ij}(\hat{x}_j(t) - \hat{x}_i(t)) \\ &+ \sum_{j\notin G_{\hat{i}}} a_{ij}(\hat{x}_j(t) - \hat{x}_i(t)) \\ &+ c_{\hat{i}} d_i(\hat{s}_{\hat{i}}(t) - \hat{x}_i(t))\Big],\ t \in [t_k^i, t_{k+1}^i) \end{aligned} \tag{6}$$

where $K = B^T P$ and P is a positive definite matrix to be designed later. $c_{\hat{i}} > 0$ measures the intra-cluster coupling strength in cluster G_l if $\hat{i} = l$.

The triggering time sequence t_k^i for the agent i is determined by

$$\begin{aligned} t_{k+1}^i =& t_k^i + \Delta_k^i \\ =& t_k^i + \max\{\tau_k^i, b_i\} \end{aligned} \tag{7}$$

where Δ_k^i is the triggering time interval between event-triggered instant t_k^i and t_{k+1}^i for agent i. b_i and β_i are positive constants to be designed, and τ_k^i is defined as

$$\tau_k^i = \inf_{t>t_k^i}\{t - t_k^i \mid \|e_i(t)\| > \beta_i\|z_i(t)\|\}. \tag{8}$$

Remark 3. Comparing with the consensus static control protocol in [18], [19], [21] and [22], a dynamic controller in (6) is considered. The dynamic consensus law depends on the estimating state $\hat{x}_i(t) = e^{A(t-t_k^i)}x_i(t_k^i)$ instead of $\hat{x}_i(t) = x_i(t_k^i)$ in [18], [19], [21] and [22]. In this paper, the matrix A in (1) and (2) is not Hurwitz, which even allows to have roots with positive real parts. It means that the system could be unstable or even divergent. The static control protocol may lead to a greater increase speed of $\|e_i(t)\|$ to reach the threshold value. Thus the number of event-trigger times under the control scheme (6) is smaller.

Remark 4. To reduce the communication cost and energy between different agents, a distributed event-triggered is proposed in this paper. Note that the distributed event-triggered scheme designed not only guarantees the achievement of leader-following cluster consensus, but also excludes Zeno behavior due to $b_i > 0$ in the event-triggered condition. According to (7), the event detection is not required during the time interval $(t_k^i,\ t_k^i + b_i)$, and the agent i only need to receive it's neighbors' information.

3 Main results

In this Section, some cluster consensus conditions of multi-agent systems (1) and the leader (2) under the control protocol (6) via the event-trigger strategy (7) will be derived.

3.1 Model transformation

Based on Assumption 1, let $D = diag\{D_1, \ldots, D_p\} = diag\{d_1, d_2, \ldots, d_N\}$, where $D_l \in \mathbb{R}^{n_l\times n_l}$, $l = 1, 2, \ldots, p$. Obviously, n_l means the number of agents in the lth cluster, and matrix D_l describes the topology between leaders and followers. Besides, let

$$\bar{L} = \begin{pmatrix} \bar{L}_{11} & L_{12} & \ldots & L_{1p} \\ L_{21} & \bar{L}_{22} & \ldots & L_{2p} \\ \ldots & \ldots & \ldots & \ldots \\ L_{p1} & L_{p2} & \ldots & \bar{L}_{pp} \end{pmatrix},$$

where $\bar{L} = L_{ll} + D_l$. Let $\bar{G}_l$, $l = 1, 2, \ldots, p$ represents the digraph consisting of the digraph of the lth cluster, its virtual leader $s_l(t)$, and the edges connecting the following agents to the leader. Define

$$\tilde{L} = \begin{pmatrix} c_1\bar{L}_{11} & L_{12} & \ldots & L_{1p} \\ L_{21} & c_2\bar{L}_{22} & \ldots & L_{2p} \\ \ldots & \ldots & \ldots & \ldots \\ L_{p1} & L_{p2} & \ldots & c_p\bar{L}_{pp} \end{pmatrix}, \tag{9}$$

where c_l is the intra-cluster coupling strength in cluster G_l defined above.

Define the error signal between the leader and the follower i, $i = 1, 2, \ldots, N$,

$$\delta_i(t) = x_i(t) - s_{\hat{i}}(t). \tag{10}$$

Substituting the controller (6) into (1), one has the closed-loop multi-agent system

$$\dot{x}_i(t) = Ax_i(t) + K\Big[\sum_{j\in G_{\hat{i}}} c_{\hat{i}} a_{ij}(\hat{x}_j(t) - \hat{x}_i(t)) + \sum_{j\notin G_{\hat{i}}} a_{ij}(\hat{x}_j(t) - \hat{x}_i(t)) + c_{\hat{i}} d_i(\hat{s}_{\hat{i}}(t) - \hat{x}_i(t))\Big]. \quad (11)$$

Then, the closed-loop multi-agent error system can be described by

$$\begin{aligned}\dot{\delta}_i(t) =\ & A\delta_i(t) + K\Big[\sum_{j\in G_{\hat{i}}} c_{\hat{i}} a_{ij}(\hat{x}_j(t) - \hat{x}_i(t)) + \sum_{j\notin G_{\hat{i}}} a_{ij}(\hat{x}_j(t) - \hat{x}_i(t)) + c_{\hat{i}} d_i(\hat{s}_{\hat{i}}(t) - \hat{x}_i(t))\Big]\\ =\ & A\delta_i(t) + K\Big[\sum_{j\in G_{\hat{i}}} c_{\hat{i}} a_{ij}(e_j(t) + x_j(t) - e_i(t) - x_i(t)) + \sum_{j\notin G_{\hat{i}}} a_{ij}(e_j(t) + x_j(t) - e_i(t) - x_i(t)) + c_{\hat{i}} d_i(s_{\hat{i}}(t) - x_i(t) - e_i(t))\Big].\end{aligned}$$

Additionally, one has

$$\begin{aligned}&\sum_{j=1}^{N} a_{ij}(x_j(t) - x_i(t))\\ =& -\sum_{j=1}^{N} l_{ij}x_j(t)\\ =& -\sum_{k=1}^{p}\sum_{j\in G_k} l_{ij}x_j(t)\\ =& -\sum_{k=1}^{p}\sum_{j\in G_k} l_{ij}[(x_j(t) - s_k(t)) + s_k(t)]\\ =& -\sum_{j=1}^{N} l_{ij}\delta_j(t) - \sum_{k=1}^{p}\Big(\sum_{j\in G_k} l_{ij}\Big)s_k(t)\end{aligned}$$

Now, according to Assumption 1, $\sum_{j\in G_c} l_{ij} = 0$. Therefore, $\sum_{j\in N_i} a_{ij}(x_j(t) - x_i(t)) = -\sum_{j=1}^{N} l_{ij}\delta_j(t)$. Consequently, the error system (10) for $t \in [t_k^i, t_{k+1}^i)$ can be rewritten as

$$\dot{\delta}_i(t) = A\delta_i(t) + K\Big[\sum_{j\in G_{\hat{i}}} c_{\hat{i}} a_{ij}(e_j(t) - e_i(t) + \delta_j(t) - \delta_i(t)) + \sum_{j\notin G_{\hat{i}}} a_{ij}(e_j(t) - e_i(t) + \delta_j(t) - \delta_i(t)) + c_{\hat{i}} d_i(s_{\hat{i}}(t) - x_i(t) - e_i(t))\Big]. \quad (12)$$

Further, denote $\delta(t) = [\delta_1^T(t), \delta_2^T(t), \ldots, \delta_N^T(t)]^T \in \mathbb{R}^{Nn}$, $e(t) = [e_1^T(t), e_2^T(t), \ldots, e_N^T(t)]^T \in \mathbb{R}^{Nn}$, then one has the following compact form

$$\dot{\delta}(t) = (I_N \otimes A)\delta(t) - (\tilde{L} \otimes BK)(\delta(t) + e(t)), \quad (13)$$

where $\tilde{L}$ is defined in (9).

Proposition 1. Suppose Assumption 1 and Assumption 2 hold. Under the event-triggering condition (7), leader-following multi-agent system (1) and (2) with control protocol (6) can realize p-cluster consensus, if and only if the multi-agent error system (13) is asymptotically stable.

Proposition 1 implies that the p-cluster consensus of leader-follower multi-agent system (1) and (2) can be now converted into the stability of the corresponding multi-agent error system (13).

3.2 Stability condition

Theorem 1. Suppose Assumptions 1 and 2 are satisfied. If there exists a positive diagonal matrix Ξ, $\gamma_1 > 0$, $\gamma_2 > 0$, ρ, η, β_i, $b_i > 0$ such that

$$\Xi\tilde{L} + \tilde{L}^T\Xi > 0 \quad (14)$$

$$0 < \gamma_1 + \gamma_2 < 1 \quad (15)$$

$$\beta_i^2 \le \frac{\omega_2 - \omega_4/\rho}{\omega_3 + \omega_4\rho} < \gamma_1 \quad (16)$$

$$\rho > \omega_4/\omega_2 > 0 \quad (17)$$

$$b_i < b, \quad (18)$$

where

$$b = \frac{1}{q_1}\ln\Big(\frac{q_1}{q_2}\sqrt{\frac{\gamma_2}{N}} + 1\Big), \quad (19)$$

$\omega_1 = \eta\lambda_{\min}(\Xi \otimes I_n) = \omega_1^1 + \omega_1^2$, $\omega_2 = \omega_1^2\sigma_{\min}(H^TH)$, $\omega_3 = \lambda_{\min}(\Omega \otimes PBB^TP) - \omega_1^2$, $H = \tilde{L}^{-1} \otimes I_n$, $\omega_4 = \sigma_{\max}(\frac{1}{2}(\tilde{L}^{-1^T}\Omega) \otimes PBB^TP - \omega_1^2H^T)$,

$q_1 = 2\|A\|$ and $q_2 = \|BK\|$. Then the leader-following multi-agent system (1) and (2) under the the control protocol (6) and the event-triggered strategy (7) achieve cluster consensus.

Proof. Consider the time interval $[t_k^i, t_{k+1}^i)$ over which the event-triggered condition $\|e_i(t)\| \le \beta_i \|z_i(t)\|$ holds. Choose the following nonnegative Lyapunov function

$$V(t) = \delta^T(t)(\Xi \otimes P)\delta(t),$$

where P is a positive definite matrix, satisfying (3) in Lemma 1, and Ξ satisfying (14). Calculating the time derivative of $V(t)$ along with (13) yields

$$\begin{aligned}\dot{V}(t) =& 2\delta^T(t)(\Xi \otimes P)\dot{\delta}(t)\\ =& \delta^T(t)[\Xi \otimes (A^T P + PA) - \Omega \otimes PBK]\delta(t)\\ & - \delta^T(t)[(\Xi\tilde{L} + \tilde{L}^T\Xi) \otimes PBK]e(t),\end{aligned} \tag{20}$$

where $K = B^T P$. According to (14), $\Xi\tilde{L} + \tilde{L}^T\Xi > 0$. Then there exists a small positive constant $\eta > 0$ such as

$$\Xi\tilde{L} + \tilde{L}^T\Xi \ge \eta\Xi. \tag{21}$$

Then applying Lemma 1 and inequality (21) into (20) yields

$$\begin{aligned}\dot{V}(t) \le& \delta^T(t)\Big[\Xi \otimes [(A^T P + PA) - \eta PBB^T P]\Big]\delta(t)\\ & - \delta^T(t)[(\Xi\tilde{L} + \tilde{L}^T\Xi) \otimes PBB^T P]e(t)\\ \le& -\eta\delta^T(t)(\Xi \otimes I_n)\delta(t) - \delta^T(t)[\Omega \otimes PBB^T P]e(t)\\ \le& \omega_1\delta^T(t)\delta(t) - \delta^T(t)[\Omega \otimes PBB^T P]e(t).\end{aligned} \tag{22}$$

Further, one obtains that $\delta(t) = x(t) - s(t) = \hat{x}(t) - s(t) - e(t)$, and $z(t) = -(\tilde{L} \otimes I_n)(\delta(t) + e(t)) = (\tilde{L} \otimes I_n)(s(t) - \hat{x}(t))$, then

$$\delta(t) = -Hz(t) - e(t), \tag{23}$$

where $H = \tilde{L}^{-1} \otimes I_n$. Substituting (23) into (22), and let $\omega_1 = \omega_1^1 + \omega_1^2$, by Young's inequality: $x^T y \le \frac{\rho}{2}x^2 + \frac{1}{2\rho}y^2$, it is obtained that

$$\begin{aligned}\dot{V}(t) \le& -\omega_1^1\delta^T(t)\delta(t) - \omega_1^2\delta^T(t)\delta(t)\\ & - \delta^T(t)[\Omega \otimes PBB^T P]e(t)\\ \le& -\omega_1^1\delta^T(t)\delta(t)\\ & - \omega_1^2(Hz(t) + e(t))^T(Hz(t) + e(t))\\ & + (Hz(t) + e(t))^T[\Omega \otimes PBB^T P]e(t)\\ \le& -\omega_1^1\delta^T(t)\delta(t) - \omega_1^2 z^T(t)(H^T H)z(t)\\ & - 2\omega_1^2 z^T(t)H^T e(t) + e^T(t)(\Omega \otimes PBB^T P)e(t)\\ & + z^T(t)(\tilde{L}^{-1^T}\Omega \otimes PBB^T P)e(t) - \omega_1^2 e^T(t)e(t)\\ \le& -\omega_1^1\delta^T(t)\delta(t) + \omega_3 e^T(t)e(t) - \omega_2 z^T(t)e(t)\\ & + \omega_4[\rho e^T(t)e(t) + \frac{1}{\rho}z^T(t)z(t)]\\ \le& -\omega_1^1\delta^T(t)\delta(t) - \sum_{i=1}^{N}[(\omega_2 - \frac{\omega_4}{\rho})\\ & - \beta_i^2(\omega_3 + \omega_4\rho)]z_i^T(t)z_i(t)\\ \le& -\omega_1^1\delta^T(t)\delta(t)\\ <& 0,\end{aligned} \tag{24}$$

where ω_2, ω_3, ω_4 are defined in Theorem 1. Next, the minimal inter-event interval of each agent will be presented in the rest part of this subsection.

Firstly, denote the agent sets $M_1(t)$ and $M_2(t)$ consisting of agents which are triggered by τ_k^i or b_i in the last event instant, respectively. Then, $M_1(t) \cup M_2(t) = \{1, 2, \ldots, N\}$ and $M_1(t) \cap M_2(t) = \emptyset$. To ensure the asymptotically stability of the p-cluster multi-agent systems, one can choose that

$$\sum_{i \in M_1}\|e_i(t)\|^2 \le \gamma_1 \sum_{i \in M_1}\|z_i(t)\|^2 \le \gamma_1 \sum_{i=1}^{N}\|z_i(t)\|^2, \tag{25}$$

$$\sum_{i \in M_2}\|e_i(t)\|^2 \le \gamma_2 \sum_{i \in M_2}\|z_i(t)\|^2 \le \gamma_2 \sum_{i=1}^{N}\|z_i(t)\|^2, \tag{26}$$

for $\gamma_1 + \gamma_2 = \gamma < 1$. For the agents in $M_1(t)$, a sufficient condition for (25) is given by $\|e_i(t)\| \le \beta_i\|z_i(t)\|$ with $\beta_i^2 \le \gamma_1$. Then, a sufficient condition for (26) is $\|e_i(t)\|^2 \le \frac{\gamma_2}{N}\|z(t)\|^2$. The evolution time of $\|e_i(t)\|/\|z(t)\|$ from 0 to $\sqrt{\frac{\gamma_2}{N}}$ is lower bounded by that of $\|e(t)\|/\|z(t)\|$ from 0 to $\sqrt{\frac{\gamma_2}{N}}$. The one has

$$\frac{d}{dt}\frac{\|e(t)\|}{\|z(t)\|} = \frac{e^T(t)\dot{e}(t)}{\|e(t)\|\|z(t)\|} - \frac{\|e(t)\|z^T(t)\dot{z}(t)}{\|z(t)\|^3}$$
$$\leq \frac{\dot{e}(t)}{\|z(t)\|} + \frac{e(t)}{\|z(t)\|}\frac{\dot{z}(t)}{\|z(t)\|}. \tag{27}$$

Since $\dot{e}(t) = (I_N \otimes A)e(t) - (I_N \otimes BK)z(t)$ and $\dot{z}(t) = (I_N \otimes A)z(t)$, thus (27) can be expressed as

$$\frac{d}{dt}\frac{\|e(t)\|}{\|z(t)\|} \leq q_1\frac{e(t)}{\|z(t)\|} + q_2, \tag{28}$$

where $q_1 = 2\|A\|$ and $q_2 = \|BK\|$. Hence, $\|e(t)\|/\|z(t)\| \leq \psi(t,\varphi_0)$ which is the solution of the following equation

$$\dot{\psi}(t) = q_1\psi(t) + q_2, \ \psi(0) = 0.$$

The evolution time of $\|e(t)\|/\|z(t)\|$ from 0 to $\sqrt{\frac{\gamma_2}{N}}$ is lower bounded by $b = \frac{1}{q_1}\ln(\frac{q_1}{q_2}\sqrt{\frac{\gamma_2}{N}} + 1)$. Thus, for the agents in $M_2(t)$, one can select the inter-event time $b_i \leq b$ to guarantee (26).

Hence, it can be concluded that $\dot{V}(t) < 0$, which implies all followers $x_i(t) \to s_{\hat{i}}(t)$, $i = 1,2,\ldots,N$ as t goes to infinity. Therefore, the cluster consensus can be achieved. The proof is completed.

To make Theorem 1 more applicable, the intra-cluster coupling strength construction method is given in Algorithm 1. It gives a convenient method to choose appropriate values of coupling strengths to ensure $\Xi\tilde{L} + \tilde{L}^T\Xi > 0$.

Algorithm 1.

Step 1: Construct the matrix $\tilde{L}$ defined in (9);
Step 2: According to Lemma 2, if the topology of each cluster $\bar{G}_l$ has a directed spanning tree, then there exists a positive diagonal matrix $\Xi_l > 0$ such that $\Xi_l\bar{L}_{ll} + \bar{L}_{ll}^T\Xi_l > 0$, $l = 1,2,\ldots,p$. Let $L_0 = L - diag\{L_{11}, L_{22},\ldots,L_{pp}\}$, and $\Xi = diag\{\Xi_1,\Xi_2,\ldots,\Xi_p\}$;
Step 3:
If $\lambda_{\min}(\Xi L_0 + L_0^T\Xi) \geq 0$
then $c_l > 0$ is allowed;
else one can choose

$$c_l > \frac{-\lambda_{\min}(\Xi L_0 + L_0^T\Xi)}{\lambda_{\min}(\Xi_l\bar{L}_{ll} + \bar{L}_{ll}^T\Xi_l)}, \ l = 1,2,\ldots,p. \tag{29}$$

Then, the constructed coupling strengths satisfy the requirement in Theorem 1.

It is obvious that $\Xi\tilde{L} + \tilde{L}^T\Xi > 0$ can be ensured if $\lambda_{\min}(\Xi L_0 + L_0^T\Xi) \geq 0$. If $\lambda_{\min}(\Xi L_0 + L_0^T\Xi) < 0$, it suffices to choose appropriate $c_1, c_2, \ldots, c_p$ such that $\lambda_{\min}(L_d) + \lambda_{\min}(\Xi L_0 + L_0^T\Xi) > 0$, where $L_d = diag\{c_1(\Xi_1\bar{L}_{11} + \bar{L}_{11}^T\Xi_1),\ldots,c_p(\Xi_p\bar{L}_{pp} + \bar{L}_{pp}^T\Xi_p)\}$. This can be guaranteed if for each $l = 1,\ldots,p$, the (29) holds.

Remark 5. According to Algorithm 1, it is obvious that $\Xi\tilde{L} + \tilde{L}^T\Xi > 0$ will be more possible to achieved if the coupling strengths of the intra-cluster are stronger. If the interaction topology G is a graph with acyclic partition, which was studied in [15]. Algorithm 1 could be omitted due to the fact that the Laplacian matrix is a lower triangular matrix, there exists a positive matrix Ξ satisfying $\Xi\tilde{L} + \tilde{L}^T\Xi > 0$.

4 Numerical simulations

In order to show the effectiveness of the derived results, we consider a network of $N = 7$ agents with 3 clusters $G_1 = 1,2$, $G_2 = 3,4,5$ and $G_3 = 6,7$. The topology of each cluster with a leader is shown in Figure 1.

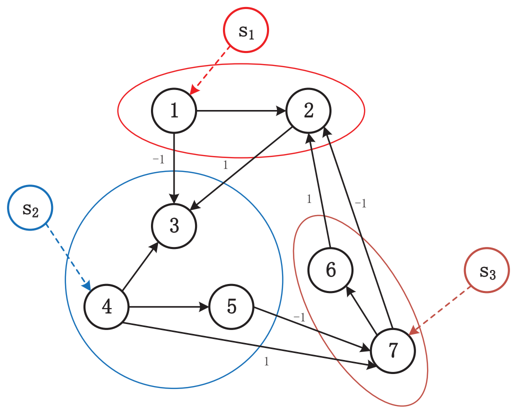

Figure 1. Network topology of the clustered multi-agent systems.

Let $A = [-2,1,1;1,-1,0;0,1,-1]$ and $B = [0;1;0]$. Thus (A,B) is stabilizable. The initial states of seven agents are chosen from $[-50,50]^3 \subset \mathbb{R}^3$ randomly. Initial states of $s_1(t)$, $s_2(t)$ and $s_3(t)$ are chosen as $(-25,15,0)^T$, $(0,-10,-30)^T$ and $(25,0,30)^T$ to guarantee that $\lim_{t\to\infty}\|s_l(t) - s_k(t)\| \neq 0$ for any $l,k = 1,2,\ldots,p, k \neq l$.

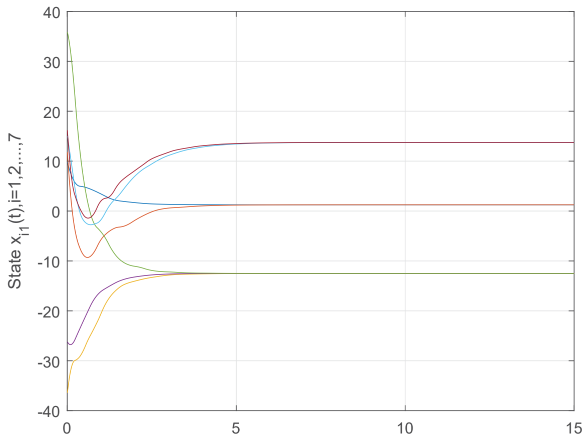

Figure 2. States trajectory of 7 followers under the distributed event-triggered scheme.

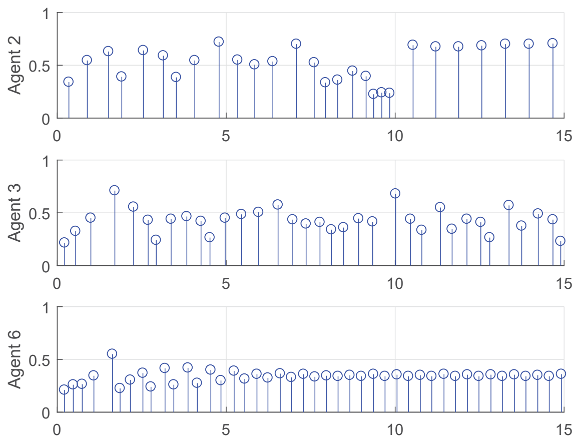

Figure 3. Event-triggered time instants and release intervals for the ith agent $(i = 2,3,6)$.

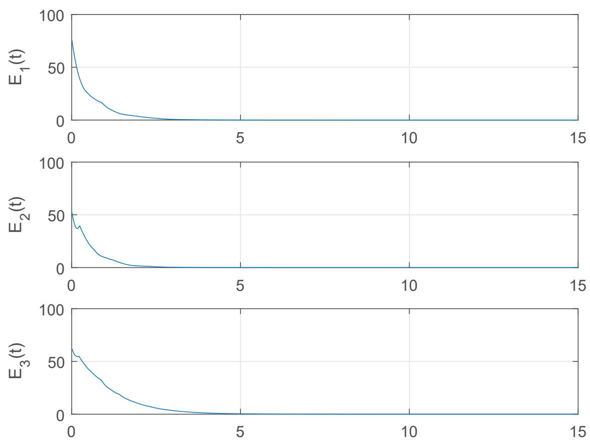

Figure 4. Evolution of $E_1(t)$, $E_2(t)$ and $E_3(t)$.

According to Algorithm 1, one can choose $c_1 = 3$, $c_2 = 4$, $c_3 = 3$ ensuring (29) holds. Solving Riccati inequality (3) with $\alpha = \eta = 0.8$, one has

$$P = \begin{pmatrix} 0.3492 & 0.4488 & 0.2378 \\ 0.4488 & 0.9789 & 0.4697 \\ 0.2378 & 0.4697 & 0.4827 \end{pmatrix}$$

and $K = B^T P = [0.4488, 0.9789, 0.4697]$. According to Theorem 1, choose $\beta_i = 0.2$ and $b_i = 0.2$ since $b = 0.2603$ with $\gamma_2 = 0.9$. From Figure 2, the trajectories of 7 followers under the distributed event-triggered scheme (8) achieves cluster consensus with 3 clusters. Figure 3 shows event-triggered time instants and release intervals for the ith agent $(i = 2,3,6)$.blue The trigger times of agent $2,3,6$ are $28,35,44$ during $[0,15]s$, respectively. It is obvious that the numbers of the trigger times under the event-triggered schemes considered in this paper are smaller, which indicates that the communication cost could be saved. It Let $E_1(t) = \frac{1}{2}\sum_{i=1}^{2} \|x_i(t) - s_1(t)\|$, $E_2(t) = \frac{1}{3}\sum_{i=3}^{5} \|x_i(t) - s_2(t)\|$ and $E_3(t) = \frac{1}{2}\sum_{i=6}^{7} \|x_i(t) - s_1(t)\|$. It can be seen from Figure 4 that the cluster consensus is achieved.

Conclusion

Cluster consensus of linear multi-agent systems via a distributed event-triggered control scheme is considered in this paper. By associating each agent within a cluster and a leader, a novel event-triggered condition is proposed and the corresponding control protocol is designed. If the network topology of each cluster has a directed spanning tree and the couplings within each cluster are sufficiently strong, sufficient conditions are derived to ensure cluster consensus. As Assumption 1 is a little bit conservative for general topologies, it is possible to relax the condition on the inter-cluster couplings to achieve cluster consensus, which is our future work.

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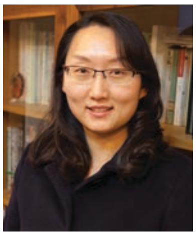

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