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GEOMETRIC NON- LINEAR APPROACH TO STIFFNESS STATE OF SEMI –RIGID STRUCTURES

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ABSTRACT:

Present contribution intends to emphasize the contribution of geometric non-linearity to the stiffness state of semi-rigid multi –storey steel structures. Though semi-rigidity of beam – column connections involves a non- linearity at constitutive bending moment-relative rotation level, the geometric non- linearity associated to deformed conFigure uration at element level is less referred to. The main objective of the study is to express the stiffness state of geometric non-linear elements semi-rigidly connected at its ends. Stiffness state is, in its term, expressed by element level stiffness matrix considering the six degrees of freedom of the planar element. Regarding the reference system, both local and global systems are employed allowing a simple and direct transition from element level vectorial relations to their structural level forms. The three fundamental vectorial relations (static equilibrium, kinematic compatibility, material constitutivity) emphasize that the principle of virtual work holds in the case of semi-rigidly connected skeletal structures as well.

1. INTRODUCTION

Connectivity of structural elements has been, for a long time, an important topic in modelling skeletal structures aiming at their analysis. Semi-rigid connectivity has covered a long way from its starting status - as connection imperfection to its current status - intermediate connectivity between pinned and rigid connections. Regarding the semi-rigid of steel skeletal structures, it has been addressed, both experimentally and analytically (King et al.,1993), (Kishi et al., 1993), (Kishi et al., 1993), (Parfitt et al., 1976). Several mechanical models of beam- to- column connections have been proposed in order to allow a proper and realistic insertion in associated soft products (Kishi et al., 1993), (Bjorhovde et al.,1990), (Jones et al.,1983).

Laboratory results made up an extensive and documented basis for several satisfactory analytical models relating, mainly, bending moment to relative rotation (Jones et al.,1983), (Stewart et al. 1947), (Montforton et al., 1963). Both, mechanical and analytical models imply monolithical and cyclic behaviour and connections and structural levels. All these aspects allowed a large and various set of structural analyses and their static and kinematic results (Romstad et al., 1970), (Frye et al., 1975), (Moldovan, 1997), (Moldovan, 2005), (Lui et al., 1987). Nevertheless, a certain level of discrepancy may be detected in what regards the gradual transition of semi-rigidity to either one of the two more traditional pinned / rigid connections. Element level stiffness matrix, largely used in structural analyses, may be one factor that paves the way from pinned to rigid connections via semi-rigidity to fulfill such a role, stiffness matrix should take into account three sources of elasticity of element level: geometric non- linearity, post elastic (plastic) behaviour and semi-rigid connections. Present contribution addresses to geometric non-linearity and semi-rigidity at element level. Also, an important practical aspect of connection zone- its finite dimensions- has been taken into account. A very general stiffness matrix has been obtained for geometric non- linear semi-rigidly connected element into finite dimensions zone. The matrix is, further, analysed and computed by gradually neglecting finite dimensions of connecting zone, geometric non-linearity and semi-rigid connectivity at element leading in this way, to well known forms of stiffness matrices in the traditional cases of pinned-pinned, rigid-rigid and pinned-rigid elements.

2. DEFORMED SEMI-RIGIDLY CONNECTED ELEMENT

In work follows, the vectorial fundamental relations (static equilibrium, kinematic compatibility and material constitutivity) will be derived independently of each other allowing , consequently, to emphasize the principle of virtual work- in its elementary form- in the case of geometric non linear semi-rigidly connected elements.

2.1 Fundamental vectors

Mechanical state associated to deformed semi-rigidly connected elements is expressed via the set of kinematic parameters (Figure 1) and static parameters (Figure 2) collected, in their term, in the following vectors:

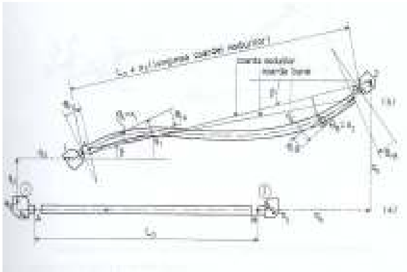

Figure 1. Kinematic parameters

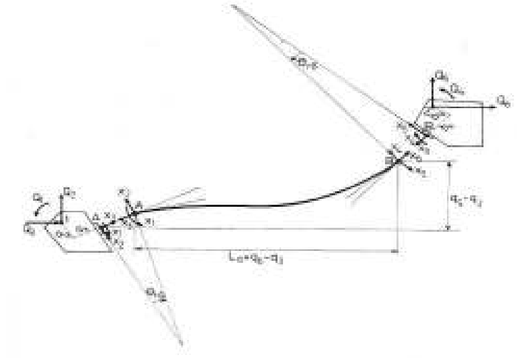

Figure 2. Static parameters

- Vector **q** of the six degrees of freedom:

$\mathbf{q}^T = [\, q_1 \; q_2 \; q_3 \; q_4 \; q_5 \; q_6 \,]$

- Vector **x** of element end deformations:

$\mathbf{x}^T = [\, \theta_A \;\; \theta_B \;\; x_3 \,]$

- Vector **Q** of nodal forces:

$\mathbf{Q}^T = [\, Q_1 \; Q_2 \; Q_3 \; Q_4 \; Q_5 \; Q_6 \,]$

-Vector $\mathbf{X}_e$ of the six stress resultants:

$\mathbf{X}_e = [M_A \;\; M_B \;\; N\,]$

Vectorial association of **Q** and **q** and **X** and **x**, respectively can be straight forwardly assessed.

2.2 Static equilibrium

Equilibrium of nodal forces $\mathbf{Q}_i$ acting on node i, $\mathbf{Q}_j$ acting on node j and element-end stress resultants **X** of deformed state (Figure 2) read from the three in- plane equilibrium equations of the element:

$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix}_{(i)} = \begin{bmatrix} 1 & a_x & 0 \\ 0 & \cos\beta & -\sin\beta \\ 0 & -\sin\beta & -\cos\beta \end{bmatrix}_{(i)} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}_{(A)} \tag{1}$$

$$\boldsymbol{Q}^{(i)} = \boldsymbol{D}_i \cdot \boldsymbol{X}_A \tag{2}$$

$$\begin{bmatrix} Q_4 \\ Q_5 \\ Q_6 \end{bmatrix}_{(j)} = \begin{bmatrix} 1 & b_x & 0 \\ 0 & -\cos\beta & \sin\beta \\ 0 & \sin\beta & \cos\beta \end{bmatrix}_{(j)} \begin{bmatrix} X_4 \\ X_5 \\ X_6 \end{bmatrix}_{(B)} \tag{3}$$

$$\boldsymbol{Q}^{(j)} = \boldsymbol{D}_j \cdot \boldsymbol{X}_B \tag{4}$$

Element equilibrium leads to the selection of the three independent stress resultants **X** out of the six elements $\mathbf{X}_e$:

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{L} & \frac{1}{L} & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ \frac{1}{L} & \frac{1}{L} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} M_A \\ M_B \\ N \end{bmatrix} \tag{5}$$

or, in a condensed form:

$$\boldsymbol{Xe} = \boldsymbol{C} \cdot \boldsymbol{X} \tag{6}$$

By substituting (5) into (4) it yields into:

$$\mathbf{Q} = \mathbf{D} \cdot \mathbf{X}_e$$

where:

$$\boldsymbol{D} = \begin{bmatrix} \boldsymbol{D_i} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{D_j} \end{bmatrix}$$

Let

$$\boldsymbol{A}^T = \boldsymbol{D} \cdot \boldsymbol{C} \tag{7}$$

Length $\mathbf{L}_d$ of deformed element can be expressed (Figure 2) in the form:

$$L_d = L_o + q_6 - q_3$$

and nodal matrix **A** reads:

$$\boldsymbol{A}^T = \begin{bmatrix} 1+\frac{a_x}{L} & \frac{a_x}{L} & 0 \\ \frac{1}{L} & \frac{1}{L} & -\frac{q_2-q_5}{L} \\ \frac{q_2-q_5+a_x q_1+b_x q_4}{L^2} & \frac{q_2-q_5+a_x q_1+b_x q_4}{L^2} & -\frac{L}{L} \\ \frac{b_x}{L} & 1+\frac{b_x}{L} & 0 \\ -\frac{1}{L} & -\frac{1}{L} & -\frac{q_2-q_5}{L} \\ -\frac{q_2-q_5+a_x q_1+b_x q_4}{L^2} & -\frac{q_2-q_5+a_x q_1+b_x q_4}{L^2} & \frac{L}{L} \end{bmatrix} \tag{8}$$

Equilibrium relation takes its final form:

$$\boldsymbol{Q} = \boldsymbol{A}^T \cdot \boldsymbol{X} \tag{9}$$

2.3 Kinematic compatibility

In the following, kinematic compatibility of nodal degrees of freedom **q** and element end-deformation **x** takes into account both, the finite dimensions of connecting zones (Jennings, 1968) and the effect of excentricities a and b (Figure 1):

$$x_1 = q_1 + arctg\frac{a_x q_1 + b_x q_4}{L_0 + q_6 - q_3} + arctg\frac{q_2 - q_5}{L_0 + q_0 - q_3} \qquad (10)$$
$$x_2 = q_4 + arctg\frac{a_x q_1 + b_x q_4}{L_0 + q_6 - q_3} + arctg\frac{q_2 - q_5}{L_0 + q_6 - q_3}$$
$$x_3 = \sqrt{(q_5 - q_2)^2 + (L_0 + q_6 - q_3)^2} - L_0$$

or, in matricial form:

$$\boldsymbol{x} = \boldsymbol{f(q)} \qquad (11)$$

2.4 Elementary form of virtual work

The two vectorial relations (9) and (11) have been independently derived, that is no constitutive relation has been imposed on the static and kinematic parameters involved. In work follows, the validity of virtual work principle is proved based on countergradience form of the two vectorial relations. Not before the kinematic relation (11) will be transformed into its elementary form .This is achieved by computing Jacobian operator to vector **f** with respect to **q**:

$$J_q^T = \nabla_q(\boldsymbol{x}^T) \qquad (12)$$

Explicitly Jacobian operator reads:

$$\nabla_q(\boldsymbol{x}^T) = \begin{bmatrix} \frac{\partial x_1}{\partial q_1} & \frac{\partial x_1}{\partial q_2} & \cdots & \frac{\partial x_1}{\partial q_6} \\ \frac{\partial x_2}{\partial q_1} & \cdots & \cdots & \frac{\partial x_2}{\partial q_6} \\ \frac{\partial x_3}{\partial q_1} & \cdots & \cdots & \frac{\partial x_3}{\partial q_6} \end{bmatrix} \qquad (13)$$

It may be easily proved (Moldovan, 1997), (Moldovan, 2005), that:

$$\boldsymbol{J}_q = \boldsymbol{A}^T \qquad (14)$$

Therefore: $$d\boldsymbol{x} = \boldsymbol{J}_q \cdot d\boldsymbol{q} \qquad (15)$$

Taking into account (14), elementary form of kinematic compatibility reads:

$$d\boldsymbol{x} = \boldsymbol{A} \cdot d\boldsymbol{q} \qquad (16)$$

Association of vectors d**x** with **X** and d**q** with **Q** emphasizes the countergradiency of the two vectorial fundamental relations (16) and (9). By simple mathematics, the two relations merge into the elementary form of virtual work principle:

$$\boldsymbol{Q}^T d\boldsymbol{q} = \boldsymbol{X}^T \cdot d\boldsymbol{x} \qquad (17)$$

It has to be underlined that the validity of this principle – in the case of geometric non linearity - only holds for elementary kinematic quantities d**x** and d**q**.

As it can be easily seen, semi-rigidity of connections does not affect the principle. A computational effect of elementary form of kinematic relation (16) is the fact that structural analysis in these (geometric non linear and semi-rigid connection) can only be performed in a step- by- step manner. For the sake of simplicity, in what follows the static (**Q** and **X**) and kinematic (**q** and **x**) parameters involved in (17) will only be considered in their elementary quantities. Therefore, kinematic compatibility (16) may formerly be put into the form:

$$\boldsymbol{x} = \boldsymbol{A} \cdot \boldsymbol{q} \qquad (16\ a)$$

By linearization of static equilibrium (9) and kinematic compatibility the relation (16a) takes the well-known forms of geometric linear analysis (Jennings, 1968), (Moldovan, 1997), (Moldovan, 2011).

2.5 Material elastic constitutivity

Intended structural analysis requires the imposing of material constitutivity between and element-end stress resultants stress resultants **X** and element –end deformations **x.** Linear elastic hypothesis leads to:

$$d\boldsymbol{X} = \boldsymbol{k} \cdot d\boldsymbol{x} \qquad (18)$$

where element stiffness matrix **k** reads:

$$\boldsymbol{k} = \begin{bmatrix} \frac{4EI}{L} & \frac{2EI}{L} & 0 \\ \frac{2EI}{L} & \frac{4EI}{L} & 0 \\ 0 & 0 & \frac{EA}{L} \end{bmatrix} \qquad (19)$$

Noticing that rotational deformation x_1 and x_2 include relative rotation θ_{rA} and θ_{rB} generated by semi-rigidity material constitutivity (18) may be expressed as:

$$dX_1 = \frac{4EI}{L}(dx_1 - d\theta_{rA}) + \frac{2EI}{L}(dx_2 - d\theta_{rB})$$
$$dX_2 = \frac{2EI}{L}(dx_1 - d\theta_{rA}) + \frac{4EI}{L}(dx_2 - d\theta_{rB}) \qquad (20)$$
$$dX_3 = \frac{EA}{L} dx_3$$

Introducing initial stiffness of the two semi-rigid connections R_{iA} and R_{iB} the following constitutivity holds (Moldovan, 1997), (Moldovan, 2005):

$$d\theta_{rA} = \frac{dX_1}{R_{iA}} \qquad d\theta_{rB} = \frac{dX_2}{R_{iB}} \tag{21}$$

Explicit matrix form of constitutivity in the presence of semi-rigidity reads:

$$\begin{bmatrix} dX_1 \\ dX_2 \\ dX_3 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & 0 \\ k_{211} & k_{22} & 0 \\ 0 & 0 & k_{33} \end{bmatrix} \begin{bmatrix} dx_1 \\ dx_2 \\ dx_3 \end{bmatrix} \tag{22 a}$$

while its condensed form:

$$d\boldsymbol{X} = \boldsymbol{k} \cdot d\boldsymbol{x} \tag{22 b}$$

The entrance of stiffness matrix **k** of (22 b) reads:

$$\begin{aligned} k_{11} &= \frac{4EIR_{iA}R_{iB}L + 12(EI)^2 R_{iA}}{(R_{iA}L + 4EI)(R_{iB}L + 4EI) - 4(EI)^2} \\ k_{12} = k_{21} &= \frac{2EIR_{iA}R_{iB}L}{(R_{iA}L + 4EI)(R_{iB}L + 4EI) - 4(EI)^2} \\ k_{22} &= \frac{4EIR_{iA}R_{iB}L + 12(EI)^2 R_{iB}}{(R_{iA}L + 4EI)(R_{iB}L + 4EI) - 4(EI)^2} \\ k_{33} &= \frac{EA}{L} \end{aligned} \tag{23}$$

Are above (23) expressions consistent with the well-known stiffness entries of rigidly connected element?

Imposing $\theta_r = 0$ and $R_i \to \infty$ it may be proved (Moldovan, 1997) that, indeed, (23) lead to the simple well-known forms of k_{ij} entries of rigidly connected members:

$$k_{11} = \frac{4EI}{L} \quad ; \quad k_{12} = k_{21} = \frac{2EI}{L} \quad ; \quad k_{22} = \frac{4EI}{L}$$

3. ELEMENT LEVEL STIFFNESS MATRICES

The three vectorial fundamental relations:

Static equilibrium:

$$\boldsymbol{Q} = \boldsymbol{A}^T \cdot \boldsymbol{X} \tag{24}$$

Kinematic compatibility:

$$\boldsymbol{x} = \boldsymbol{A} \cdot \boldsymbol{q} \tag{25}$$

Elastic constitutivity:

$$\boldsymbol{X} = \boldsymbol{k} \cdot \boldsymbol{x} \tag{26}$$

Allow the transition to force- displacement (***Q-q***) relationship at element level. By simple substitutions it reads:

$$\boldsymbol{Q} = \boldsymbol{A}^T \cdot \boldsymbol{k} \cdot \boldsymbol{A}\boldsymbol{q} \tag{27}$$

By substitution:

$$\boldsymbol{K} = \boldsymbol{A}^T \cdot \boldsymbol{k} \cdot \boldsymbol{A} \tag{28}$$

The force displacement relation becomes:

$$\boldsymbol{Q} = \boldsymbol{K} \cdot \boldsymbol{q} \tag{29}$$

where ***K*** is the stiffness matrix of geometric non linear semi-rigidly connected element.

Performing matricial computation involved in (28) one obtains:

$$\begin{aligned} &\bar{L} = L_0 + q_6 - q_3 \\ &\sqrt{(q_2 - q_5)^2 + (L_0 + q_6 - q_3)^2} = L \\ &\frac{a_x}{L_0 + q_6 - q_3} = \bar{a}_x \\ &\frac{b_x}{L_0 + q_6 - q_3} = \bar{b}_x \\ &H = \frac{1}{L^2}\left(q_2 - q_5 + a_x q_1 + b_x q_4\right) \end{aligned} \tag{30}$$

$$\begin{aligned} &K_{11} = \left(1 + \bar{a}_x\right)^2 k_{11} + 2\bar{a}_x\left(1 + \bar{a}_x\right)k_{12} + \bar{a}_x^2 k_{22} \\ &K_{12} = \frac{1}{L}\left[\left(1 + \bar{a}_x\right)k_{11} + \left(1 + 2\bar{a}_x\right)k_{12} + \bar{a}_x k_{22}\right] \\ &K_{13} = H\left[\left(1 + \bar{a}_x\right)k_{11} + \left(1 + 2\bar{a}_x\right)k_{12} + \bar{a}_x k_{22}\right] \\ &K_{14} = \left(1 + \bar{a}_x\right)\bar{b}_x K_{11} + \left(1 + \bar{a}_x + \bar{b}_x 2\bar{a}_x \bar{b}_x\right)k_{12} + \\ &\left(1 + \bar{b}_x\right)\bar{a}_x k_{22} \\ &K_{15} = -K_{12} \\ &K_{16} = -K_{13} \\ &K_{22} = \frac{1}{L^2}\left(k_{11} + 2_{12} + k_{22}\right) + \frac{\left(q_2 - q_5\right)^2}{L^2} k_{33} \\ &K_{23} = \frac{H}{L}\left(k_{11} + 2_{12} + k_{22}\right) - \frac{\left(q_2 - q_5\right)}{L^2} k_{33} \\ &K_{24} = \frac{1}{L}\left[\bar{b}_x k_{11} + \left(1 + 2\bar{b}_x\right)k_{12} + \left(1 + \bar{b}_x\right)k_{22}\right] \\ &K_{25} = -K_{22} \\ &K_{26} = -K_{23} \end{aligned} \tag{31}$$

$$K_{33} = H^2\left(k_{11} + 2k_{12} + k_{22}\right) + \left(\frac{\bar{L}^2}{L}\right)^2 k_{33}$$

$$K_{34} = H\left[\bar{b}_x k_{11} + \left(1 + 2\bar{b}_x\right)k_{12} + \left(1 + \bar{b}_x\right)k_{22}\right]$$

$$K_{35} = -K_{23}$$

$$K_{36} = -K_{33}$$

$$K_{44} = \bar{b}_x^2 k_{11} + 2\bar{b}_x\left(1 + \bar{b}_x\right)k_{12} + \left(1 + \bar{b}_x\right)^2 k_{22}$$

$$K_{45} = -K_{24}$$

$$K_{46} = -K_{34}$$

$$K_{55} = K_{22}$$

$$K_{56} = K_{23}$$

$$K_{66} = K_{33}$$

A first step in adapting stiffness matrix **K** may consist of neglecting the finite dimension a_x and b_x (Figure 1) of connection zone. By imposing a_x and b_x in (30) and (31) it leads to the entries of the stiffness matrix of geometric non linear semi-rigdly connected into non-dimension zone element:

$$K_{11}^q = k_{11}$$

$$K_{12}^q = \frac{1}{\bar{L}}\left(k_{11} + k_{12}\right)$$

$$K_{13}^q = \frac{\left(q_2 - q_5\right)}{\bar{L}^2} k_{11} + \frac{\left(q_2 - q_5\right)}{\bar{L}^2} k_{12}$$

$$K_{14}^q = k_{12}$$

$$K_{15}^q = -K_{12}^q$$

$$K_{16}^q = -K_{13}^q$$

$$K_{22}^q = \frac{1}{\bar{L}^2}\left(k_{11} + 2k_{12} + k_{22}\right) + \frac{(q_2 - q_5)^2}{L^2} k_{33}$$

$$K_{23}^q = \frac{q_2 - q_5}{\bar{L}^3}\left(k_{11} + 2k_{12} + k_{22}\right) - \frac{\bar{L}\left(q_2 - q_5\right)}{L^2} k_{33}$$

$$K_{24}^q = \frac{1}{\bar{L}}\left(k_{12} + k_{22}\right) \tag{32}$$

$$K_{25}^q = -K_{22}^q$$

$$K_{26}^q = -K_{23}^q$$

$$K_{33}^q = \frac{\left(q_2 - q_5\right)^2}{\bar{L}^4}\left(k_{11} + 2k_{12} + k_{22}\right) + \left(\frac{\bar{L}^2}{L}\right) k_{33}$$

$$K_{34}^q = \frac{\left(q_2 - q_5\right)^2}{\bar{L}^2}\left(k_{12} + k_{22}\right)$$

$$K_{35}^q = -K_{33}^q$$

$$K_{36}^q = -K_{33}^q$$

$$K_{44}^q = k_{22}$$

$$K_{44}^q = -K_{24}^q$$

$$K_{46}^q = -K_{34}^q$$

$$K_{55}^q = K_{22}^q$$

$$K_{56}^q = K_{23}^q$$

$$K_{66}^q = K_{33}^q$$

Simplifying, by neglecting deformed geometry ($q = 0$), the stiffness matrix $\bar{\boldsymbol{K}}$ of straight beam elasticaly end connected from non- linear analysis reads:

$$\bar{K}_{11} = k_{11}$$

$$\bar{K}_{12} = \frac{1}{\bar{L}}\left(k_{11} + k_{12}\right)$$

$$\bar{K}_{13} = 0$$

$$\bar{K}_{14} = k_{12}$$

$$\bar{K}_{15} = -\bar{K}_{12}$$

$$\bar{K}_{16} = 0$$

$$\bar{K}_{22} = \frac{1}{\bar{L}^2}\left(k_{11} + 2k_{12} + k_{22}\right)$$

$$\bar{K}_{23} = 0$$

$$\bar{K}_{24} = \frac{1}{\bar{L}}\left(k_{11} + k_{12}\right) \tag{33}$$

$$\bar{K}_{25} = -\bar{K}_{22}$$

$$\bar{K}_{26} = 0$$

$$\bar{K}_{33} = k_{33}$$

$$\bar{K}_{34} = 0$$

$$\bar{K}_{35} = 0$$

$$\bar{K}_{36} = -k_{33}$$

$$\overline{K}_{44} = k_{22}$$
$$\overline{K}_{45} = -\overline{K}_{24}$$
$$K^q_{46} = 0$$
$$K^q_{55} = K^q_{22}$$
$$K^q_{56} = 0$$
$$\overline{K}_{66} = k_{33}$$

Considering the case of element with point connecting zones and rigid end connections, the following hold:

$$\theta_{rA} = \theta_{rB} = 0$$

$$k_{11} = \frac{4EI}{L}; k_{22} = \frac{4EI}{L}; \quad k_{12} = k_{21} = \frac{2EI}{L}; k_{33} = \frac{EA}{L}$$

The well-known geometric non linear stiffnes matrix $\boldsymbol{K}_e$ (23) is recovered with the follwing entries:

$$\boldsymbol{K}_e = \begin{bmatrix} \frac{4EI}{L} & \frac{6EI}{L^2} & 0 & \frac{2EI}{L} & -\frac{6EI}{L^2} & 0 \\ & \frac{12EI}{L^3} & 0 & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & 0 \\ & & \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} \\ & & & \frac{4EI}{L} & \frac{6EI}{L^2} & 0 \\ & & & & \frac{12EI}{L^3} & 0 \\ & & & & & \frac{EA}{L} \end{bmatrix} \quad (34)$$

Another simplifying direction of the general form (31) of the stiffness matrix beam may be followed by maintaining the geometric characteristics of ”finite dimensions connected zone” and by converting elastic connections (semi-rigid) into ideal (rigid / pinned) ones.

- The case of semi-rigidly connected element (Figure 3):

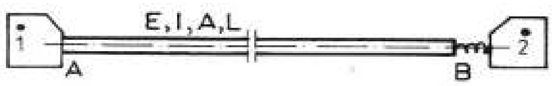

Figure 3. The case of semi-rigidly connected element

With $R_{iA} \to \infty$ it results into:

$$k_{11} = \frac{4EIR_{iB}L + 12(EI)^2}{R_{iB}L^2 + 4EIL}$$
$$k_{12} = k_{21} = \frac{2EILR_{iB}}{R_{iB}L^2 + 4EIL} \quad (35)$$
$$k_{22} = \frac{4EILR_{iB}}{R_{iB}L^2 + 4EIL}$$
$$k_{33} = \frac{EA}{L}$$

Similarly, when $R_{iB} \to \infty$ (semi-rigidly –rigidly connected) results into:

$$k_{11} = \frac{4EILR_{iA}}{R_{iA}L^2 + 4EIL}$$
$$k_{12} = k_{21} = \frac{2EILR_{iA}}{R_{iA}L^2 + 4EIL}$$
$$k_{22} = \frac{4EIR_{iA}L + 12(EI)^2}{R_{iA}L^2 + 4EIL} \quad (36)$$
$$k_{33} = \frac{EA}{L}$$

- The case of rigidly- rigidly connected element (Figure 4):

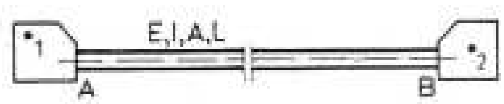

Figure 4. The case of rigidly- rigidly connected element

Conditioning $R_{iA} \to \infty; R_{iB} \to \infty$, results into:

$$k_{11} = \frac{4EI}{L}$$
$$k_{12} = k_{21} = \frac{2EI}{L} \quad (37)$$
$$k_{22} = \frac{4EI}{L}$$
$$k_{33} = \frac{EA}{L}$$

- The case of semi-rigidly- pinned connected element (Figure 5):

Figure 5. The case of semi-rigidly- pinned connected element

With $R_{iB} = 0$, (23), it results into:

$$k_{11} = \frac{3EIR_{iA}}{R_{iA}L + 4EI}$$
$$k_{12} = k_{21} = 0 \qquad (38)$$
$$k_{22} = 0$$
$$k_{33} = \frac{EA}{L}$$

Similarly, when $R_{iA} = 0$ (semi-rigidly- pinned connected) leads to:

$$k_{11} = 0$$
$$k_{12} = k_{21} = 0$$
$$k_{22} = \frac{3EIR_{iA}}{R_{iA}L + 4EI} \qquad (39)$$
$$k_{33} = \frac{EA}{L}$$

- The case of Rigidly- pinned connected element (Figure 6):

Figure 6. The case of Rigidly- pinned connected element

Substituting $R_{iA} \to \infty; R_{iB} = 0$ in (23), it results into:

$$k_{11} = \frac{3EI}{L}$$
$$k_{12} = k_{21} = 0$$
$$k_{22} = 0$$
$$k_{33} = \frac{EA}{L} \qquad (40)$$

Respectively- for rigid –pinned case:

$$k_{11} = 0$$
$$k_{12} = k_{21} = 0$$
$$k_{22} = \frac{3EI}{L}$$
$$k_{33} = \frac{EA}{L} \qquad (41)$$

- The case of pinned-pinned connected element $R_{iA} = 0; R_{iB} = 0$ (Figure 7):

Figure 7. The case of pinned-pinned connected element

$$k_{11} = 0$$
$$k_{12} = k_{21} = 0$$
$$k_{22} = 0$$
$$k_{33} = \frac{EA}{L} \qquad (42)$$

By replacing relations (35) - (42) in the stiffness matrix general form (31) one can obtain its new form. The vector ***q*** of degrees of freedom therefore, the stiffness coeficients K_{ij} are definned at ends 1 and 2 of the element. Consequently, in the case of a straight bar connected in finite dimension zones with pinned A and B, the coeficients K_{ij}- bending moments- are not necessarily 0. They become 0 if the obtained relationship will be customised by neglecting finite dimensions of the connected zones (when element ends 1 and 2 coincide with A respectively, B).

Alternative forms of these particular matrixes are to be found in the literature reffering to elastically ends connected with or without finite dimensions zones. The case of geometric linear element semi rigidly connected at on or both ist ends is largely reported in the literature by using a single parameter to define the semirigidity of connections (Bjorhovde et al.), (Jones et al., 1983), (Moldovan, 1997), (Moldovan, 2005). The stiffness geometric non linear matrix for semi- rigidly point connected ends as well as as rigidly connected and finite dimensions nodes may be found in (Lui et al.,1987), (Kim et al., 1996).

4. CONCLUDING REMARKS

The elastic state of structural element semi-rigidly connected into finite dimension zone has been studied via geometrically non-linear stiffness approach. The traditional six degrees of freedom of planar analysis have been allotted to the element. Presented analysis follows the pattern of independently conferring if the three fundamental vectorial relations: static equilibrium, kinematic compatibility and material constitutivity. Static equilibrium and kinematic compatibility relations take into account finite dimensions zone, semi-rigid connectivity at element ends, while material constitutivity is associated with a simple linear elastic element. The validity of principle of virtual work (via virtual displacements) is emphasized in its elementary form. The most general forms of stiffness matrix is, gradually, reduced to its traditional simplest form associated to point rigid / pinned geometrically linear stiffness matrix. Obtained element level stiffness matrix allows the transition from element to structural level by usual technique of general displacement method.

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