THE SHAPE-STRUCTURE RELATION FOR THE LIGHT CONSTRUCTION MEMBRANE TYPE

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ABSTRACT:

In the case of light structures membrane type the form is confused with the structure and vice versa. Thus the analysis process, non-linear type, the one for form finding is also a means of optimizing these structures. To respect the natural principle of minimum it is advisable that the structure's shape is similar to the minimum surface area. The numerical problem solving is based on using finite elements with constant strain of soap film. Based on these considerations, the paper presents aspects of determining the shape of the membrane structure using finite elements of soap film.

1. INTRODUCTION

Light structures membrane type or membrane with cables allows the most varied and complex monumental engineering construction (Kopenetz & Cătărig, 2006; Kopenetz & Pârv, 2014; Kopenetz, & Prada, 2011).

Examples of such structures are shown in the following:

a) Juventus Stadium, Turin, Italy (figure 1) - Structural solution characterized by a roof suspended from steel cables connected by two pillars and the main structural system (consisting of reinforced concrete beams to level + 18.55 m and steel beams up to the external supports of the roof, elevation + 33.00 m).

Figure 1. Juventus Stadium, Turin, Italy (http://www.italia.it/uploads/RTEmagicC_Torino__Juventus_Stadium__www.juventus.com.jpg.jpg) (view at 18 Aug. 2015)

b) Florida Suncoast Dome (Tropicana Field), Saint Petersburg, Florida, USA (figure 2) – "Tensegrity" structure with radial cable systems. The canvas roof is made of fibreglass coated with Teflon.

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Figure 2. Florida Suncoast Dome (Tropicana Field), Saint Petersburg, Florida, USA
(https://upload.wikimedia.org/wikipedia/commons/0/09/Tropicana_field_from_air.JPG) (view at 18 Aug. 2015)

c) Tao-Yuan County Arena, Taoyuan, Taiwan (figure 3) - Membrane Structure having a circular plan with 120*m* in diameter. The membrane made of PTFE (polytetrafluoroethylene) having a long life.

Figure 3. Tao-Yuan County Arena, Taoyuan, Taiwan
(http://www.taiyokogyo.co.jp/img/lgr/mk_371.jpg) (view at 18 Aug. 2015)

d) Hajj Terminal, Jeddah, Saudi Arabia (figure 4) - Structure made of Teflon membrane, covering an area of 105 hectares.

Figure 4. Hajj Terminal, Jeddah, Saudi Arabia
(http://archrecord.construction.com/features/aiaAwards/10_25year Award/1.jpg) (view at 18 Aug. 2015)

Given the flexibility of these structures in general, and structural subassemblies, in particular, the structural design must address two key issues:

- Determining the initial geometry (Kopenetz et al., 2004, 2005; (Kopenetz, 2006).

- Establishing the tensions and surface shape, from pre-stress and static and dynamic loads applied (Kopenetz & Ionescu, 1985).

If for determining the stresses there are relatively many bibliographic alerts, for the form study there are few communications (Bentley, 1999; Woodbury, 2010).

To respect the natural principle of minimum it is advisable that the light structure's shape is similar to the minimum area surface.

The analytical solving of the minimal surface shape is possible only for certain specific contours (coil, askew quadrangle with tips on a regular tetrahedron etc.) (Fox & Kemp, 2009; Burry, 2011).

From the point of view of structural engineering resolutions are based on PLATEAU's problem (Plateau, 1873), that means finding a minimal area surface for a closed space contour (Kopenetz & Cătărig, 2006; Kopenetz & Pârv, 2014).

2. DETERMINATION OF MINIMUM AREA SURFACE

Minimum area surfaces are those surfaces which of all the surfaces they pass through a skew curve, have the smallest area.

If on a closed elastic or rigid contour is considered a soap film with its own negligible weight, due to superficial stress, characteristic to fluid films, these take the form of minimum area surface, corresponding to the contours. In this regard, the minimum area surfaces represent the natural form of existence of films made of soap film, unloaded (Lynn, 1999; Hawking, & Mlodinov, 2010).

The minimum area surface is determined in two conditions:
a) The first condition is related to the mean curvature:

$$H = \frac{1}{2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \qquad (1)$$

This curvature must be equal to zero in any point of the surface, which is equivalent to the condition $R_1 = -R_2$, where R_1 and R_2 are the main radii of the curvature.

b) The second condition is related to stress, meaning that they must be constant at any point and in after each direction.

The approach of this condition, regarding the size of the minimum area surface, is experimental and numerical and it is followed by analytical methods only for particular cases.

The experimental determination of minimum area surface shape sums up to finding the coordinates and curvatures in all the points of the area, through stereo-photogrammetry methods or even normal shooting from different angles. The numerical study, both for finding minimum area surface and for determining the shape from different loads, appeals to the special finite element with uniform stress σ^t .

By discrete modelling of the fluid film surface, the membrane structure with an infinite number of degrees of freedom is replaced with a system with a finite number of degrees of freedom. This system represents a set of bidimensional membrane type elements, using isoparametric finite elements of indefinite shape, linked in knots (Kwinter, 2007; Rubinstein & Firstenberg, 1999).

The finite elements used are Zienkiewicz-Irons type, the geometry and the allowed distribution of the displacement being presented using the same functions of the form and interpolation.

The implemented method of calculation is based on the theory presented in references (Kopenetz & Ionescu, 1985; Kopenetz, et al., 2005¸Kopenetz & Cătărig, 2005; Kopenetz & Pârv, 2014; Kopenetz & Prada, 2011), considering only the linear-elastic response of the material.

Newton-Raphson iterative method is used in order to solve the stability issues which are not influenced by the type of the finite element. The differential equations of movement are solved using the Newark and Wilson methods.

The initial shape is set by the software in case of membrane type structures for which the initial shape is very important. The software uses a finite element having a constant tension as a pattern for the material of the surface structure. If there are *no* external loads, the minimum area is assumed as initial shape.

SUM01 calculation program developed to solve this problem is using efficiently an iterative process (Kopenetz & Ionescu, 1985; Kopenetz & Cătărig, 2006; Kopenetz & Pârv, 2014; Kopenetz & Prada, 2011).

3. NUMERICAL EXEMPLES

a) In order to determine the surface shape of a minimum area for the membrane type structure in figure 5 using two variants, let's consider point 12 fixed. The minimum surface area is calculated considering first the constant stress in the membrane equals 5 daN/mm, and then 20 daN/mm.

After analysing the results (table 1), we notice that as expected identical ordinates are obtained.

If point 12 is free, the minimum area is obtained as the horizontal plan, again an expected physical result.

STARTING AREAS

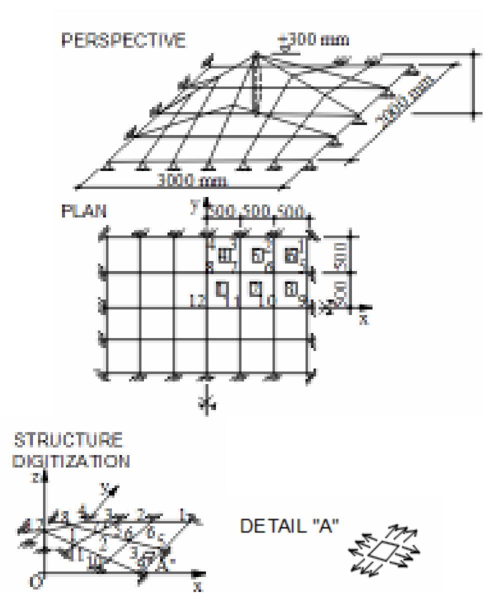

Figure 5. Shape of a minimum area for membrane structure

Material: Soap Film

Point	Coordinates					
	Starting			After 10 iterations with t_0=5 and t_0=20 daN/mm		
	x	Y	z	x	y	z
1	1500	1000	0	1500	1000	0
2	1000	1000	0	1000	1000	0
3	500	1000	0	500	1000	0
4	0	1000	0	0	1000	0
5	1500	500	0	1500	500	0
6	1000	500	50	985,56	498	20,66
7	500	500	100	486,78	490,45	57,20
8	0	500	150	0	437,33	75,44
9	1500	0	0	1500	0	0
10	1000	0	100	975,92	0	29,38
11	500	0	200	422,2	0	84,55
12	0	0	300	0	0	300

Table 1. Coordinates x, y, z after 10 iterations

b) In this example, the study of form is proposed for a bearing structure with pre-stressed membranes with arches, suitable for gyms (figure 6).

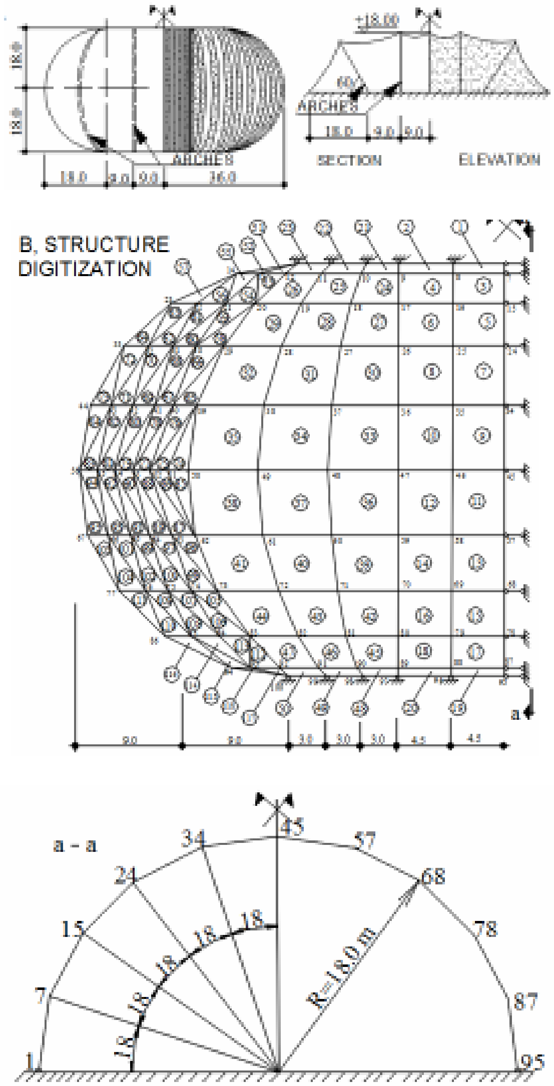

Figure 6. Prestressed membranes with arches suitable for gyms

The results are presented in figure 7 and table 2:

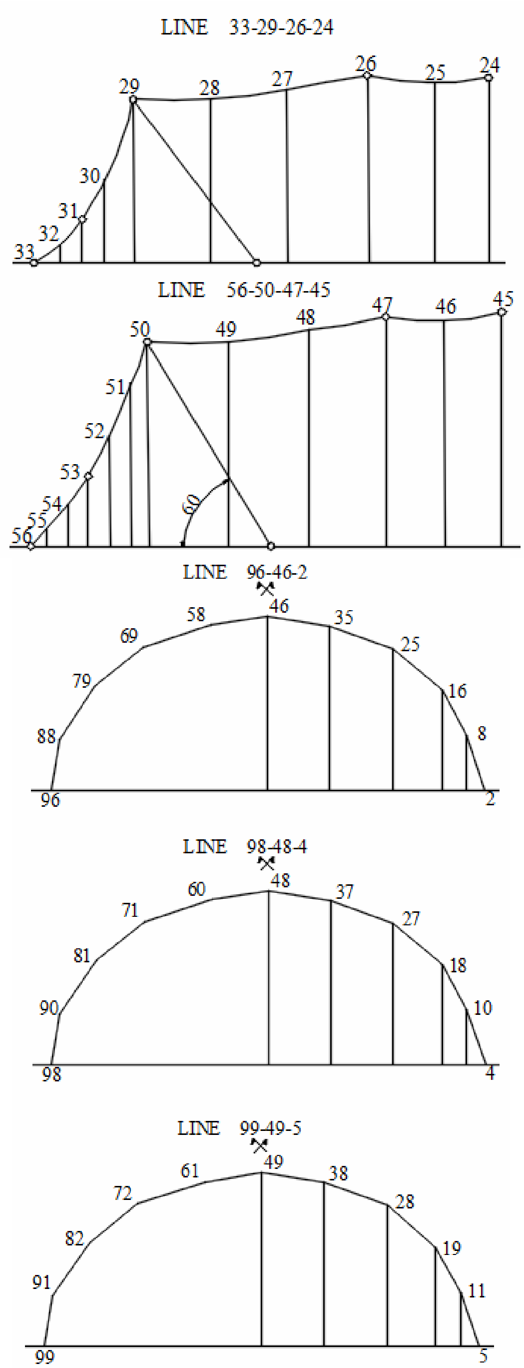

Figure 7. Shape result for prestressed membrane with arches suitable for gyms

Material: Soap Film

Point	Coordinates after 5 iterations [m]		
	x	y	z
1.	36,000	18,000	0
2.	31,500	18,000	0
3.	27,000	18,000	0
4.	24,000	18,000	0
5.	21,000	18,000	0

6.	18,000	18,000	0
7.	36,000	17,120	5,560
8.	33,115	16,820	5,513
9.	30,292	17,120	5,560
10.	27,423	16,618	5,203
11.	24,669	16,696	4,939
12.	22,077	17,120	4,830
13.	17,351	16,279	1,830
14.	12,440	17,120	0
15.	36,000	14,560	10,580
16.	34,110	14,447	10,483
17.	32,292	14,560	10,580
18.	29,714	14,134	9,820
19.	27,428	14,203	9,332
20.	25,495	14,560	9,160
21.	19,552	13,390	4,877
22.	13,472	13,628	1,965
23.	7,420	14,560	0
24.	36,000	10,580	14,560
25.	34,678	10,535	14,488
26.	33,412	10,580	14,560
27.	31,072	10,267	13,510
28.	29,135	10,352	12,890
29.	27,495	10,580	12,610
30.	22,019	9,798	8,091
31.	16,000	9,541	4,394
32.	9,747	9,874	1,801
33.	3,440	10,580	0
34.	36,000	5,560	17,120
35.	34,955	5,543	17,062
36.	33,955	5,560	17,120
37.	31,753	5,393	15,879
38.	30,027	5,447	15,195
39.	28,660	5,560	14,830
40.	23,756	5,220	10,551
41.	18,436	5,012	6,817
42.	12,760	5,007	3,837
43.	6,860	5,199	1,581
44.	0,880	5,560	0
45.	36,000	0	18,000
46.	35,031	0,001	17,946
47.	34,103	0	18,000
48.	31,933	0,002	16,690
49.	30,247	0,001	15,981
50.	28,850	0	15,580
51.	24,734	-0,003	11,939
52.	20,352	-0,006	8,460
53.	15,576	-0,005	5,495
54.	10,511	0,010	3,105
55.	5,278	0,007	1,306
56.	0	0	0

Table 2. Coordinates x, y, z, after 5 iterations

c) The next example presents the study of the shape for a circular hall with membrane structure in three versions:
- Mechanical stress.
- Mneumatic stress.
- Pneumatic stress for cable supported structures.

The results are presented in figure 8, table 3, figure 9, table 4, figure 10 and table 5.

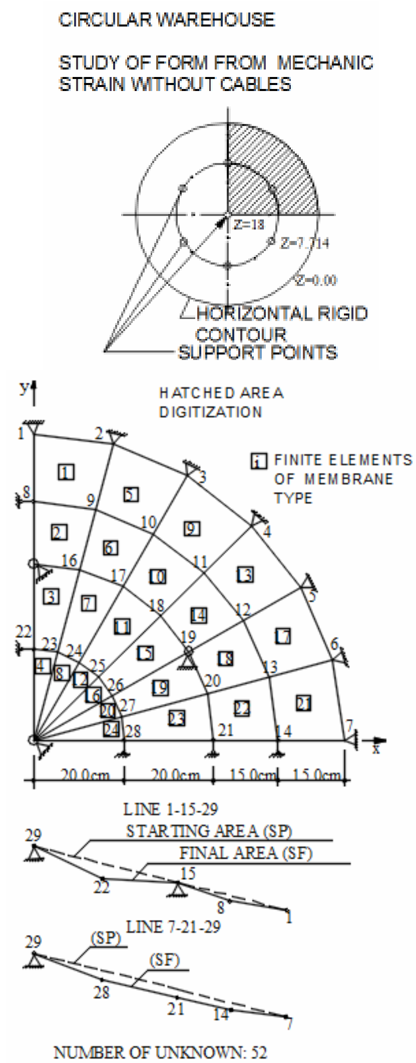

Figure 8. Shape for circular hall with membrane. Mechanical stress

Material: Soap film with stress T=1,0 daN/cm

Point	Coordinates after 8 iterations [cm]		
	x	y	z
8.	0	54,390	2,731
9.	14,109	52,824	2,739
10.	27,306	47,293	2,579
11.	38,688	38,615	2,742
12.	47,102	27,195	2,730
13.	52,797	14,209	2,739
14.	54,611	0	2,579
15.	0	40,000	7,714
16.	10,074	38,047	5,881
17.	19,615	33,972	5,473
18.	27,910	27,736	5,883
19.	34,640	20,000	7,714
20.	37,984	10,312	5,882
21.	39,229	0	5,472
22.	0	18,521	8,841
23.	4,691	17,632	9,174
24.	9,095	15,752	9,219

25.	12,923	12,873	9,175
26.	16,039	9,260	8,841
27.	17,614	4,759	9,174
28.	18,190	0	9,219
29.	0	0	18,000

Table 3. Coordinates after 8 iterations with stress T=1,0 daN/cm

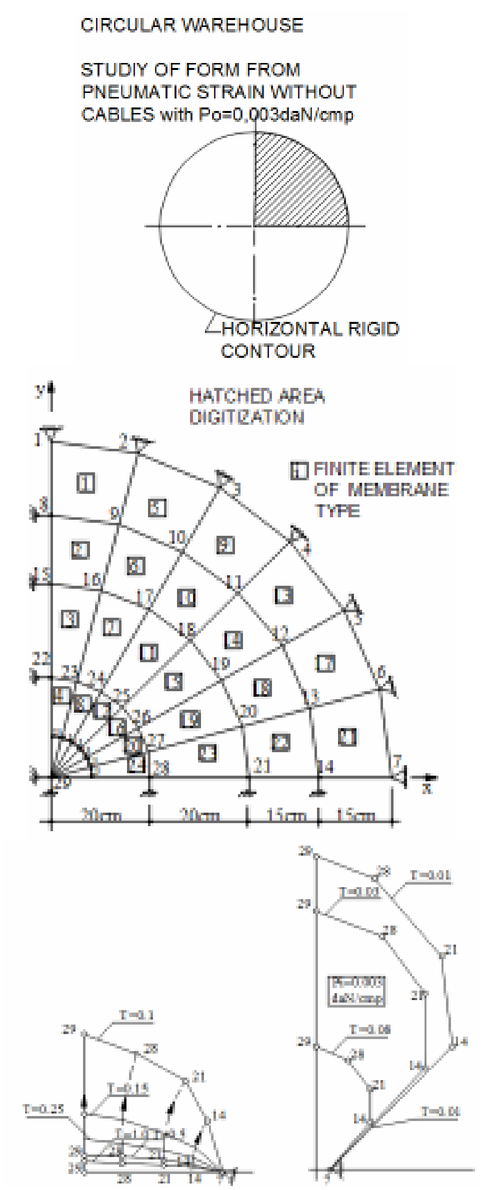

Figure 9. Shape for a circular hall with membrane structure. Pneumatic stress

Point	Coordinates after 8 iterations [cm]		
	x	Y	Z
12	47,439	27,390	1,413
	47,628	27,499	2,828
	48,032	27,733	5,828
	48,930	28,251	10,54
	54,918	31,709	25,84
	251,43	145,18	231,76
	515,83	297,86	518,47
	587,86	339,44	624,27
	587,01	338,96	623,13
19	34,275	19,789	2,483
	34,548	19,947	4,981
	35,126	20,281	10,270
	36,304	20,961	19,009
	43,378	25,046	44,968
	245,44	141,72	413,37
	497,44	287,22	904,47
	551,86	318,65	1131,01
	551,54	318,46	1129,71
21	39,578	0	2,483
	39,894	0	4,981
	40,561	0	10,270
	41,992	0	19,009
	50,089	0	44,968
	283,41	0	413,37
	574,39	0	904,48
	637,23	0	1131,02
	636,86	0	1129,72
26	16,832	9,718	3,394
	17,074	9,858	6,830
	17,607	10,166	14,106
	18,643	10,764	25,971
	23,811	13,748	60,763
	151,30	87,35	560,51
	295,31	170,70	1201,39
	287,44	165,96	1493,88
	287,47	165,98	1493,27
28	19,437	0	3,394
	19,716	0	6,830
	20,331	0	14,106
	21,528	0	25,971
	27,495	0	60,763
	174,71	0	560,51
	341,01	0	1201,38
	331,92	0	1493,87
	331,96	0	1493,2
29	0	0	3,765
	0	0	7,594
	0	0	15,734
	0	0	29,028
	0	0	68,127
	0	0	633,35
	0	0	1335,13
	0	0	1618,0
	0	0	1618,0

Table 4. Points coordinates x,y,z [cm]

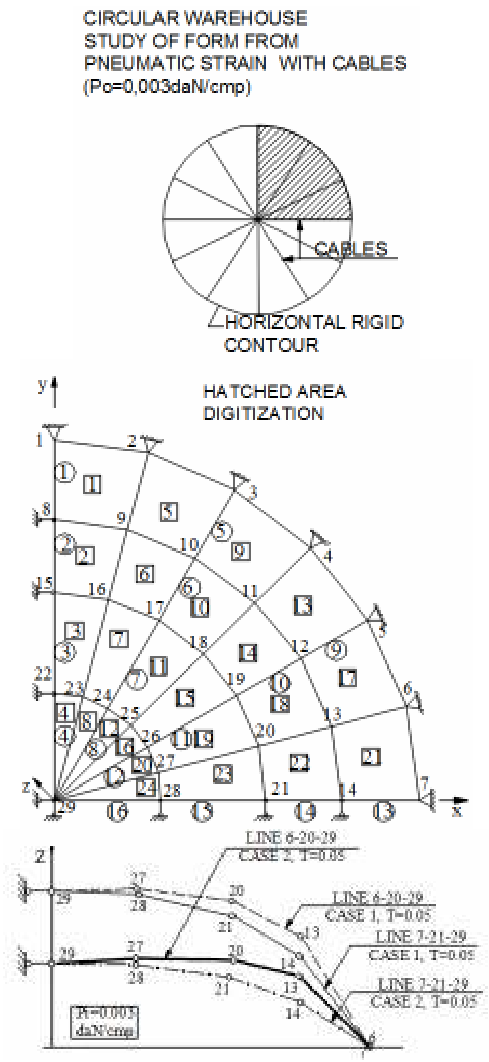

Figure 10. Shape for a circular hall with membrane structure. Pneumatic stress for situation when there are stabilization cables

CASE 1: Initial length $L_0\ (IJ)_1$ of the elements of cable

Cable	$L_0\ (IJ)_1$
1	25,000
2	17,493
3	20,881
4	20,000
5	25,000
6	17,493
7	20,881
8	20,000

Point	Coordinates after 8 iterations [cm] For T=0,025		
	x	Y	Z
1	0	70	0
11	46,876	46,876	30,038
12	48,071	27,702	20,441
13	63,912	17,056	29,945
14	55,390	0	20,331
18	29,694	29,694	35,793
19	35,085	20,194	29,541
20	40,543	10,791	35,634
21	40,406	0	29,409
25	15,024	15,024	35,398
26	17,381	9,995	34,140
27	20,548	5,433	35,328
28	20,018	0	34,064
29	0	0	34,760
Point	Coordinates after 8 iterations [cm] For T=0,05		
	x	Y	Z
1	0	70	0
11	39,026	39,026	24,780
12	47,657	27,497	20,078
13	53,267	14,298	24,738
14	54,952	0	19,996
18	26,971	26,971	32,296
19	34,880	20,115	29,539
20	36,818	9,865	32,242
21	40,210	0	29,446
25	14,931	14,931	35,251
26	17,347	9,999	34,837
27	20,394	5,455	35,219
28	20,001	0	34,789
29	0	0	35,686
Point	Coordinates after 8 iterations [*cm*] for T=0,1		
	x	y	z
1	0	70	0
11	35,807	35,807	23,029
12	46,825	27,036	19,286
13	48,919	13,122	22,999
14	54,048	0	19,259
18	25,727	25,727	30,931
19	34,382	19,847	29,278
20	35,137	9,429	30,916
21	39,681	0	29,248
25	14,745	14,745	35,951
26	17,261	9,964	36,038
27	20,138	5,397	35,943
28	19,924	0	36,024
29	0	0	37,840

Table 5. Points coordinates: x,y,z [cm]

CASE 2: Initial length $L_0\ (IJ)_2$ of the elements of cable

Cable	$L_0\ (IJ)_2$
1	18,0278
2	15,8114
3	20,2237
4	20,0998
5	18,0278
6	15,8114
7	20,2237
8	20,0998

Point	Coordinates after 8 iterations [cm] for T=0,025		
	x	y	z
1	0	70	0
11	40,260	40,260	23,488
12	47,645	27,455	10,120
13	54,967	14,648	23,227
14	54,881	0	9,891
18	29,282	29,282	23,982
19	34,872	20,056	15,968
20	39,954	10,581	23,679

21	40,125	0	19,667
25	19,888	19,888	20,954
26	17,503	10,046	19,224
27	27,194	7,237	20,757
28	20,137	0	19,032
29	0	0	19,600
Point	Coordinates after 8 iterations [cm] For T=0,05		
	x	y	z
1	0	70	0
11	34,548	34,548	17,021
12	47,508	27,403	9,875
13	47,156	12,709	16,892
14	54,754	0	9,682
18	25,422	25,422	19,456
19	34,794	20,048	15,873
20	34,701	9,341	19,322
21	40,071	0	15,623
25	18,956	18,956	19,699
26	17,474	10,056	19,330
27	25,903	6,995	19,588
28	20,126	0	19,187
29	0	0	19,867
Point	Coordinates after 8 iterations [cm] for T=0,1		
	x	Y	Z
1	0	70	0
11	31,984	31,984	15,042
12	47,292	27,310	9,463
13	43,753	11,735	14,941
14	54,556	0	9,339
18	24,293	24,293	17,819
19	34,655	20,013	15,635
20	33,189	8,934	17,748
21	39,965	0	15,478
25	18,922	18,922	19,060
26	17,419	10,064	19,448
27	25,871	6,938	19,003
28	20,096	0	19,371
29	0	0	20,425

Table 5. Points coordinates: x,y,z [cm]

4. CONCLUSIONS

- In order to cover large areas (ths m^2) membrane type light structures are suitable design options.

- n case of such structures, the shape is the structure and the structure coincides with the shape, so any solution that resolves the optimal and stable shape has a practical importance.

- This paper presents a numerical procedure for finding the shape by using finite elements with constant stress.

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