



# Comparison of Design Wave Heights Determined on the Basis of Long- and Short-term Measurement Data

Marek Szmytkiewicz<sup>1</sup>, Piotr Szmytkiewicz<sup>1</sup>, Tomasz Marcinkowski<sup>2</sup>,

<sup>1</sup>Institute of Hydro-Engineering, Polish Academy of Sciences, ul. Kościarska 7, 80-953 Gdańsk, Poland;

<sup>2</sup>Instytut Morski w Gdańsku, ul. Długi Targ 41/42, 22-100 Gdańsk, Poland

e-mail: P.Szmytkiewicz@ibwpan.gda.pl

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## Abstract

The objective of this study is to determine differences between design wave heights determined on the basis of short- and long-term wave trains. Wave parameters measured over a period of 7.47 years in the vicinity of Coastal Research Station Lubiatowo were used as short-term wave data, while wave parameters determined through the so-called wave reconstruction for a period of 44 years for the same region and depth were used as long-term wave data. The results of the calculations lead to the conclusions the significant wave height distributions obtained for short and long-term wave data are similar.

**Key words:** Wave Measurements, Reconstruction of Waves in the Baltic, Weibull distribution, Gumbel distribution

## 1. Introduction

In recent years, there has been an increasing trend in the construction of new ports and marinas, as well as in the expansion and modernisation of the existing ones. Without going into detail on problems associated with the construction of ports, it should be borne in mind that a well-designed port must provide a safe entry, exit and stay of ships in the harbours even under the worst weather conditions. Therefore, it is crucial that breakwaters are designed in such a way as not to fail under storm conditions.

Regardless of whether breakwaters are built as flexible structures in the form of rock-fill material or as rigid structures (e.g. in the form of caissons), each element of this type of structure has to withstand the impact of storm waves. Without elaborating on formulae used to calculate the stability of breakwaters, one should remember, for example, that the minimum weight of a single block in rock or rubble mound breakwaters that can withstand wave impact is directly proportional to the wave height to the third power ( $G \sim H^3$ ).

It follows from this general formula that the use of underestimated values of the wave height will result in a future failure or even destruction of the breakwater, while

overestimated wave height values will entail an unnecessary redimensioning of the structure and, consequently, excessive costs of construction. Striking a balance between structural safety and a reasonably calculated risk of failure is one of the key issues when designing sea structures.

Breakwaters are usually built for at least several decades, which means they must be able to withstand the impact of all waves that may occur in the future during the port's operation. Therefore, the adoption of correct parameters of the so-called design wave is the key element determining the economic design of safe port structures.

Characteristic features of wind-generated waves (wind waves) are their high variability in time, entirely random nature and unpredictability of their parameters during the lifetime of the structure.

Based on the measured water surface level as a function of time, it is possible to determine characteristic parameters of waves (heights and periods, as well as angles of wave approach) through

- statistical analysis,
- stochastic analysis.

The purpose of statistical analysis is to determine a representative wave with a given probability of occurrence. In other words, the whole measured series of wind waves is replaced by one regular (sinusoidal) wave with a specific height, period and angle of wave approach.

The purpose of stochastic analysis is to determine the energy distribution function for each elementary wave in the measured wind wave series (determination of the so-called spectral density function). The representative wave is the one that carries the most energy.

As a rule, when determining wave loads acting on breakwaters, the parameters of the design wave are determined by statistical analysis. Stochastic analysis is applied mainly to strictly dynamic problems when the vibrations of the structure are analysed. A typical example of when this type of analysis can be used is the calculation of impact on offshore structures, such as drilling platforms.

Regardless of the method used to analyse waves, one should bear in mind that the longer the period of the series measured, the smaller the error of the characteristic parameters of waves determined. The time period for which measurement data are available is critical for the reliability of the design wave determined, which is a statistical extrapolation of empirical data.

The objective of this study was to determine differences between design wave heights determined on the basis of short- and long-term wave trains. Wave parameters (height, period and approach angle) measured over a period of 7.47 years at a depth of about 20 m in the vicinity of Coastal Research Station (CRS) Lubiatowo (Poland) were used as short-term wave data, while wave parameters determined through the so-called wave reconstruction for a period of 44 years for the same region and depth were used as long-term wave data.

## 2. Wave Measurements

Waves in the region of CRS Lubiatowo were measured with a directional waverider buoy Waverider Mk. II and DWR-7 Mk. III (manufactured by the Dutch company Datawell B.V), which measures water surface elevations (used to determine characteristic wave heights and periods) and wave direction. It was mounted every year at a distance of about 2–3 km from the shore at a depth of 18–20 m and at coordinates 54° 50' N, 17° 50' E. The battery-powered waverider buoy performed continuous measurements with a sampling frequency of 2.56 Hz, while statistical-stochastic parameters of waves were determined at least once an hour on the basis of data recordings for at least twenty minutes. The data were automatically transmitted by radio to a receiver in the CRS Lubiatowo laboratory, where raw series of measurements were recorded on a PC, followed by statistical and stochastic analyses and recording the results obtained through these analyses in the form of reports.

Measurements of deep-water wave parameters (height, period and direction) were performed with some irregularity in 1996–2016. The measurement periods, the highest parameters of significant waves and the maximum wave heights recorded in particular years are presented in Table 1.

The highest significant wave recorded in the period 1996–2016 was  $H_{s,\max} = 5.03$  m, and the maximum wave was  $H_{\max} = 7.18$  m.

## 3. Reconstruction of Waves in the Baltic

Measurements of Baltic waves are carried out irregularly and in few places only, so it is impossible to use them for long-term analyses aimed at determining the probability of waves with varying parameters (height, period and approach angle). Therefore, the waves are reconstructed by means of mathematical models. The input data for wave models are wind fields. Older studies were based on the results of wave calculations performed by parametric models for stationary and homogeneous winds. The main simplification of these models was the assumption that winds measured at onshore meteorological stations were representative of the whole Baltic Sea and could be considered as a factor inducing wind waves in this water area.

Since 1997, the Interdisciplinary Centre for Mathematical and Computational Modelling at the University of Warsaw (ICM) has been systematically forecasting weather conditions using the mesoscale atmosphere model UMPL (Unified Model for Poland) that covers the entire Baltic.

Therefore, it has also become possible to calculate wave motion in the Baltic Sea on the basis of pressure fields recorded over this water area. Nowadays, such forecasts are made using mathematical models based on the energy balance equation. The most popular model of this type, commonly used worldwide, is the WAM4 model (Wave Modelling 1988, Blomgren et al 2001). The wind fields over the Baltic Sea used as input data came from the regional atmospheric model REMO (Regional Climate Model, Feser et al 2001).

**Table 1.** Parameters of deep-water significant waves (height  $H_s$ , period  $T_p$ , direction  $\theta_{az}$ ), maximum wave heights  $H_{\max}$  and measurement periods in 1996–2016

No.	Year	Maximum annual significant waves	Maximum annual wave heights $H_{\max}$	The range of data
1	1996	$H_s = 4.00$ m; $T_p = 8.33$ s; $\theta_{az} = 318.6^\circ$	5.68 m	November, December – partial, total ~ 35 days
2	1997	$H_s = 3.76$ m; $T_p = 7.69$ s; $\theta_{az} = 303.1^\circ$	5.07 m	April, May – partial, September – partial, October–December – complete, total ~ 138 days
3	1998	$H_s = 4.36$ m; $T_p = 11.76$ s; $\theta_{az} = 22.90^\circ$	7.08 m	January – partial, February–September – complete, October–December – partial, total ~ 338 days
4	1999	$H_s = 3.72$ m; $T_p = 9.09$ s; $\theta_{az} = 294.8^\circ$	6.20 m	January – partial, February–August – complete, September – almost complete, October – fragmentary, total ~ 250 days
5	2001	$H_s = 3.60$ m; $T_p = 9.09$ s; $\theta_{az} = 359.7^\circ$	5.21 m	February – partial, March – complete, April–September – fragmentary, total ~ 97 days
6	2002	$H_s = 1.82$ m; $T_p = 6.25$ s; $\theta_{az} = 287.4^\circ$	2.76 m	September–November – fragmentary, total ~ 20 days
7	2003	$H_s = 3.70$ m; $T_p = 7.69$ s; $\theta_{az} = 314.9^\circ$	7.06 m	August – partial, September–October – complete, November – partial, total ~ 98 days
8	2006	$H_s = 5.03$ m; $T_p = 10.0$ s; $\theta_{az} = 358.6^\circ$	7.18 m	September – fragmentary, October–December – complete, total ~ 104 days
9	2007	$H_s = 4.10$ m; $T_p = 10.0$ s; $\theta_{az} = 310.0^\circ$	4.70 m	January–August – complete, September – fragmentary, total ~ 248 days
10	2011	$H_s = 3.32$ m; $T_p = 8.33$ s; $\theta_{az} = 293.9^\circ$	4.33 m	June – partial, July – complete, August – almost complete, September–December complete, total ~ 201 days
11	2012	$H_s = 4.03$ m; $T_p = 10.0$ s; $\theta_{az} = 357.2^\circ$	5.69 m	January – complete, February – fragmentary, June–December – complete, total ~ 254 days
12	2013	$H_s = 4.77$ m; $T_p = 9.09$ s; $\theta_{az} = 326.3^\circ$	4.93 m	January–March – complete, April – partial, May–December – complete, total ~ 345 days
13	2014	$H_s = 3.73$ m; $T_p = 10.0$ s; $\theta_{az} = 351.6^\circ$	4.99 m	January – complete, February – partial, May–December – complete, total ~ 297 days
14	2015	$H_s = 4.25$ m;	5.71 m $T_p = 10.0$ s; $\theta_{az} = 299.5^\circ$	January–February – complete, March – partial, July–December – complete, total ~ 262 days
15	2016	$H_s = 3.06$ m; $T_p = 9.09$ s; $\theta_{az} = 288.3^\circ$	4.39 m	January – complete, February – partial, total ~ 38 days
The total number of measurement days				2,725 days $\approx$ 7.47 years

REMO data were available in the form of one-hour time series in a rectangular grid with spherical coordinates covering Europe and the north-eastern Atlantic, with a resolution of  $0.5^\circ$  both in longitudinal and latitudinal directions. For the purpose of wave modelling, a subset covering the Baltic Sea was selected from the database.

The basic equation of the WAM4 model is the so-called wave action balance equation, which allows for energy transfer from wind to the sea, whitecapping, bottom friction and resonance interactions of wave components. The model also includes data on the ice cover. Ice reduces the wave generation area, and this affects wave motion, even at large distances from places covered by ice. The spatial resolution of the computational grid was  $5' \times 5'$  (about 9 km) (Cieřlikiewicz and Herman 2001, Cieřlikiewicz and Paplińska-Swerpel 2008).

The results of calculations by this model made it possible to reconstruct wave motion in the Baltic Sea for a period of 44 years (1958–2001). Spatially interpolated wind fields were fed into the wave model with a one-hour time step, and then interpolated inside the model with a time step of 300 s. As a result of these calculations, representative wave parameters were obtained for each subsequent hour, i.e. significant wave heights  $H_s$ , periods  $T_p$  and azimuths of wave approach angles  $A_z$  in individual nodes of the numerical grid. In total, sets containing  $24 \times 365 \times 44$  numerical values were obtained for each forecast point.

The highest significant wave height determined from wave reconstruction for the period 1958–2001 was  $H_{s,\max} = 6.51$  m.

A comparison of wave modelling results and *in situ* wave measurement results with satellite measurements has shown that the model well approximates the actual wave motion and can be used for the analysis of wave climate in the Baltic Sea (Paplińska and Reda 2001, Cieřlikiewicz and Paplińska-Swerpel 2008). The compliance between the model and measurements was analysed using data collected during 2.5 years by many wave buoys deployed in various parts of the Baltic Sea. In total, more than 17,000 data records were analysed. The mean value of the correlation coefficient between the measured and modelled values was 0.82. The maximum difference in the significant wave height measured and modelled was about  $\pm 2.7$  m. These differences were due mainly to the fact that the actual storm occurred later or sooner than the storm modelled. On the other hand, the difference between the mean values ( $\overline{H}_{s,\text{meas}} - \overline{H}_{s,\text{cal}}$ ) during the whole analysed period was only 1.1 cm.

To determine wave parameters in the Lubiatowo area, the present study analysed wave datasets for forecast points located in this water region as close to the shore as possible, obtained from the reconstruction of waves in the Baltic over a period of 44 years. The above criterion was met by one point located approximately 10 km offshore at a depth of about 22 m. The geographic coordinates of this point were as follows: N  $17^\circ 51' 02, 16''$ , E  $54^\circ 54' 28, 44''$ .

#### 4. Comparison of Measured and Calculated Significant Wave Heights

A comparison of significant wave heights based on measurements with the corresponding values obtained from the reconstruction of waves in the Lubiatowo region was possible only for a relatively short period of 858 days, which corresponds to approximately 2.35 years in the period 1996–2001.

Figures 1a and 1b compare measured and calculated significant wave heights for selected autumn-winter (Fig. 1a) and summer (Fig. 1b) periods. It follows from these figures that wave heights determined by two completely different methods are similar both for the storm season and for moderate waves. The magnitude of the relative error was determined from formula (1), and its values are presented in Table 2.

$$\sigma = \frac{1}{N} \sum_{i=1}^N \left( 1 - \frac{H_{i_{cal}}}{H_{i_{meas}}} \right). \quad (1)$$

**Table 2.** Magnitude of the relative error between significant wave heights obtained by wave buoy measurements ( $H_{meas}$ ) and determined through wave reconstruction ( $H_{cal}$ )

Year	Error magnitude $\sigma$	Number of events
1996	-0.017	691
1997	-0.078	2539
1998	-0.0522	6458
1999	-0.1616	5090
2001	-0.151	2368
1996–2001	-0.087	17146

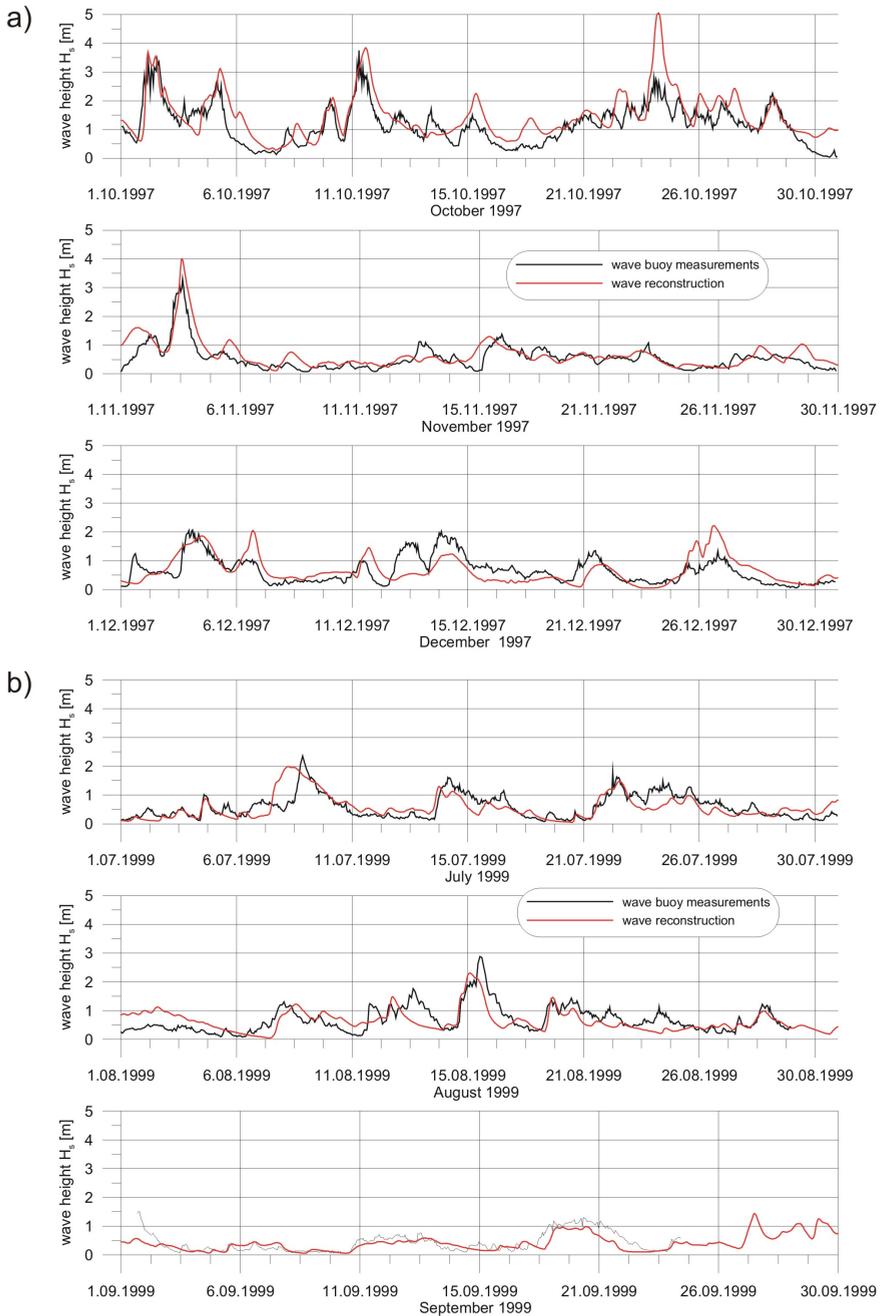
The numerical values presented in Table 2 indicate that

- events during which the significant wave heights determined through wave reconstruction ( $H_{cal}$ ) were higher than those determined through direct measurements ( $H_{meas}$ ) prevailed in the whole period considered,
- the mean relative error for the entire period did not exceed 10%,
- the main cause of the errors (Fig. 2) were time shifts between the actual and calculated storm seasons.

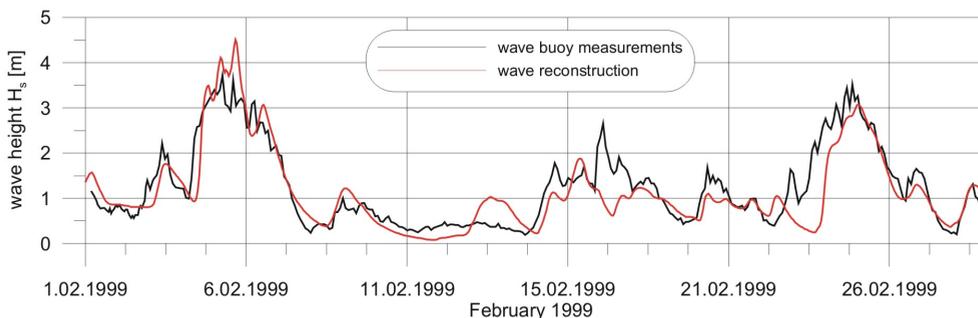
The starting point for wave reconstruction were hand-drawn isobars on Baltic maps. They served as the basis for the determination of the velocity and direction of winds occurring over the sea, which in turn were the boundary conditions for the calculation of waves in this water region. Given this procedure, the conformity obtained between the measured wave heights and their calculated counterparts should be considered as at least good.

#### 5. Determination of the Design Wave

The starting point to determine parameters of the design wave is the knowledge of characteristic wave parameters specified for a deep-water forecast point situated in



**Fig. 1.** Comparison of significant wave heights  $H_s$  determined by measurements and reconstructed by the WAM4 model for the autumn and winter of 1999 (a) and for the summer of 1999 (b)



**Fig. 2.** The main cause of disagreements: a time shift between the real and reconstructed storms. Significant wave heights  $H_s$  measured and reconstructed for February 1999

the foreground of the planned structure. These characteristic wave parameters are usually significant wave heights  $H_s$  and peak periods of the spectral density function  $T_p$ . These parameters can be determined as follows:

- directly from the measured long wave trains in a given water area,
- by calculations based on long-term wind data for a given water area.

Usually the designed lifetime of the structure significantly exceeds the period covered by measurement data. As a result, the determination of wave parameters characteristic of the design storm involves extrapolation of these parameters from a finite set of measured or determined predictive models of wave trains.

The process of extrapolation used to predict wave heights and to determine the parameters of design waves can be divided into two steps:

- extrapolation leading to the determination of a representative wave height (usually a significant wave) characteristic of a design storm with a given return period from a statistical set containing known wave heights and forming a series of independent variables,
- extrapolation leading to the determination of a specific design wave height with the required probability of occurrence in the design storm.

Maritime structures are usually designed for a specific period of service life, for example, several dozen years. This means that they must be able to withstand the impact of waves occurring during a storm with a return period equal to the intended service life of the structure. The probability of such a storm is defined as follows:

$$P [\%] = \frac{1}{T_R} \cdot 100\%, \quad (2)$$

where:  $T_R$  – defined service life of the structure expressed in years.

In Poland, the design return period of storms  $T_R$  for the dimensioning of maritime structures is adopted in accordance with the Regulation of the Minister of Transport and Marine Economy published in Journal of Laws, No. 101 of 6 August 1998. For example, according to this regulation, 100 years should be allowed for as the design

storm return period in the case of breakwaters protecting port entrances, quays and piers.

Depending on the type of structure, the design wave heights are assumed either as averaged values from a defined range of the highest waves occurring during the design storm, or as maximum waves with a given probability of occurrence during this storm.

According to both the Shore Protection Manual (1984) and the Coastal Engineering Manual (2003), the adopted representative design wave height should depend on the rigidity of a given structure. For an entirely rigid marine structure (e.g. caisson breakwaters), which may fail permanently under extreme hydrodynamic loads, the design wave height should be assumed as an average of 1% of the highest waves in the design storm ( $H_{1\%}$ ). For flexible structures (e.g. rubble-mound breakwaters), it is recommended to assume the wave height in the range of  $H_s - H_{10\%}$ . For semi-rigid structures, the design wave height should be between  $H_{10\%}$  and  $H_{1\%}$ , where  $H_{1\%}$  and  $H_{10\%}$  are the averages of 1% and 10% of the highest waves in the design storm, respectively.

The starting point for the determination of statistical or stochastic wave parameters is the availability of long-term wave series measured at specific points of the sea. In Europe, continuous wave size recording started after the construction of drilling platforms in open waters of the North Sea and then the Norwegian Sea. In Poland, wave measurements are carried out mainly at the Coastal Research Station (CRS) in Lubiatowo, which belongs to the Institute of Hydro-Engineering, Polish Academy of Sciences in Gdańsk. In recent years, it has become increasingly common for specialized companies, such as BMT Argoss in Europe or NOAA (National Oceanic and Atmospheric Administration) in the United States, to monitor waves on the surface of seas and oceans by means of satellite techniques. Oceanographic measurements, including wave measurements with the use of wave buoys anchored in various sea areas, are also increasingly common.

Nevertheless, despite the undeniable progress in measurement techniques, the lack of long-term wave data is often a problem for the construction of new ports or expansion of the existing ones.

When long-term wave records are not available, the wave climate can be reconstructed by the so-called predictive models, which calculate characteristic parameters of waves, i.e. their height, period and approach angle, on the basis of data on wind velocity, directions and fetch. All these models are based on available information about wind parameters. In Europe, meteorological measurements have been carried out for at least 100 years.

In general, two basic methods of wave forecasting can be distinguished:

- The standard method determines wave parameters on the basis of data on wind directions and wind velocity obtained from a single measurement point, usually located on land (as close to the shore as possible).

- Operational wave forecasts, which are currently being developed using global numerical models, calculate wave parameters in each node of the numerical grid of a given water area on the basis of pressure distributions over the entire sea area determined from synoptic maps.

The standard methods belong to the group of older predictive models. They adopt wind velocity and wind directions measured at a given meteorological station as representative of the entire sea region. The computational equations are based on the search for empirical relationships between wind velocity ( $W$ ), the fetch ( $X$ ) and duration ( $t$ ) of wind impact and the wave parameters (height  $H$  and wave period  $T$ ). A number of such formulas can be found in the literature, e.g. CEM (2003), Hasselmann et al (1976), Young and Verhagen (1996), valid for specific water areas, for which they have been derived and verified. In Poland, the method of Kryłow et al (1976) is generally used to forecast average wind wave parameters at a deep-water location.

The spectral method of Kryłow is one of the few methods of wind wave forecasting that has been verified in nature through measurements. Its basic assumption is that the wave energy generated at a given point at sea is the total value of energy arriving at this point from the sector ( $\pm\pi/2$ ) with respect to the wind direction observed at a weather forecast point. One of the advantages of this method is the fact that it takes into account the effect of shoreline variation and the effect of depth change along individual rays of spectral component propagation on calculated wave parameters.

Current operational wave forecasts are based mainly on the energy balance equation:

$$\frac{\partial E}{\partial t} + \frac{\partial(C_g E)}{\partial x} = E_w + Diss, \quad (3)$$

where:

- $E$  – wave energy per unit area of water surface,
- $C_g$  – group velocity of the wave,
- $E_w$  – energy transmitted from the atmosphere to the sea per unit of time,
- $Diss$  – dissipation of wave energy.

Calculations of wave field parameters for a given area of water based on the above relation are usually carried out using the numerical wave model WAM4.

A series of waves determined directly from measurements or calculated from a predictive model is usually short. One of the basic problems is therefore the reliability of extrapolating these data over longer periods of time. If storms in this series of waves were particularly strong or, on the contrary, if moderate wave conditions prevailed, extrapolation of these data leads, in the former case, to overestimation or, in the latter, to underestimation of the extreme waves. The reliability of the wave heights thus determined increases with the increasing number of wave observation years.

The basic condition for determining the parameters of the design wave is the assumption that the available set of waves is statistically independent. In order to satisfy this condition, only one maximum wave height from each storm should be used in the

analysis of wave data collected over the years. This ensures full statistical independence of wave parameters, because it is physically impossible that, for example, the wave height in a given storm depends on the wave height in the previous storm. On the other hand, it is completely arbitrary to adopt a lower limit of the wave height for storm conditions, the so-called threshold wave height. There is no guidance as to which of the adopted minimum limits is the “best.”. By reducing the value of this threshold wave height, we increase the number of events, which is useful for statistical analysis. On the other hand, such a set contains a significant number of events during which wave heights are relatively small, which may reduce the calculated height of the design wave.

Detailed procedures for determining the design wave parameters are presented, for example, in papers by Kamphuis (2010) and Marcinkowski and Szmytkiewicz (2010).

Gumbel and Weibull distributions are the most commonly used statistical distributions for the southern Baltic. They provide good agreement in describing extreme events and are used to determine the theoretical function of the probability of exceedance.

The Gumbel distribution has the following form:

$$p(H) = \exp\left(-\exp\left(-\frac{H_n - a}{b}\right)\right), \quad (4)$$

where:  $a, b$  – numerical coefficients.

Linearization of the above equation results in

$$Y = A + B \cdot H, \quad (5)$$

where:

$$Y = -\ln\left(\frac{1}{\ln p(H)}\right), \quad A = -\frac{a}{b}, \quad B = \frac{1}{b}.$$

The values of the coefficients  $A$  and  $B$  are determined by the method of least squares.

This results in the following formula for the wave height with a given probability of occurrence:

$$H_{\max,n} = b \cdot \ln\left[\frac{1}{\ln\left(\frac{1}{p_n}\right)}\right] + a, \quad (6)$$

where:  $p_n = 1 - P$ ,  $P$  – probability of occurrence, e.g. for  $P = 0.01$  (once every 100 years)  $p_n = 1 - 0.01 = 0.99$ .

The Weibull distribution improves on the Gumbel distribution by introducing a third parameter, which ensures a better fit of the function to real values. The distribution has the following form:

$$P = 1 - \exp\left(-\left(\frac{H_n - \gamma}{\beta}\right)^\alpha\right), \quad (7)$$

where:  $\alpha, \beta, \gamma$  – numerical coefficients.

Linearization of (7) results in the following formula:

$$Y = \left[ \ln \left( \frac{1}{1-P} \right) \right]^{1/\alpha}; \quad X = H; \quad A = \frac{1}{\beta}; \quad B = -\frac{\gamma}{\beta}. \quad (8)$$

Similarly to the Gumbel distribution, the values of  $A$  and  $B$  are determined by the least squares method.

To determine the wave height for a given probability of occurrence, equation (7) is transformed to the following form:

$$H = \beta \cdot \left[ \ln \left( \frac{1}{(1-P)} \right) \right]^{1/\alpha} + \gamma. \quad (9)$$

When using this distribution to describe the maximum wave heights, one should remember that the third parameter, i.e. the coefficient  $\alpha$ , cannot be determined directly from the linear regression. Its value is set *a priori*, and then it is checked whether wave heights are arranged along a straight line. If not, the value of this coefficient is changed, and the calculations are repeated until a satisfactory result is obtained.

It should be emphasized that an ordinary assessment of the goodness of fit of the measuring points (wave heights) to the approximation line on the basis of regression coefficients may be insufficient. As recommended by Kamphuis (2010), the so-called “visual inspection” should be performed, i.e. it should be assessed how the measuring points are located in the vicinity of a straight line, and what value of the coefficient  $\alpha$  in the Weibull distribution should be adopted to obtain points corresponding to the highest wave heights located as close as possible to the straight line.

Periods of time covered by wave data are usually much shorter than required to determine the design wave height with a specific return period. In such cases, the British Standard Code (1984) recommends the following formula to determine the probability of occurrence of a design storm:

$$p_n = 1 - \frac{T_0}{T_R(1+n)}, \quad (10)$$

where:  $T_0$  – the number of years covered by the available observational data,  $n$  – the amount of wave data.

Kamphuis (2010), however, determines the probability of the design storm occurrence from a slightly modified formula (10):

$$p_n = 1 - \frac{T_0}{T_R \cdot n}. \quad (11)$$

With a large amount of wave data, both relationships result in basically the same values  $p_n$ .

Once the probability of a storm is determined from equation (10) or (11), the corresponding wave height is calculated from the selected statistical distribution, for example, that of Weibull or Gumbel.

## 6. Design Wave Determined from Long- and Short-term Measurement Data

Short-term wave data consisted of deep-water wave parameters measured (with some irregularity) in 1996–2016 at a distance of approximately 2–3 km from the shore and a depth of 18–20 m in the coastal zone of the Lubiatowo region. Altogether, the measurements were conducted for 7.47 years (2,725 days), and  $2,725 \times 24 = 65,400$  hours of wave series were recorded.

Wave parameters determined from the reconstruction of wave fields for a period of 44 years (1958–2001) at the forecast point located about 10 km offshore at a depth of about 22 m near CRS Lubiatowo were used as long-term wave data. In total,  $24 \times 365 \times 44 = 385,440$  hours of wave series were recorded.

Both of the above data sets were used to determine the significant wave height in the design storm. The starting point for these calculations was to create statistical distributions in which the variables considered are independent of each other. In order to preserve this condition, only one maximum significant wave height was used in the analysis of wave data measured over the years at CRS Lubiatowo and determined through the reconstruction of waves during each individual storm.

On the other hand, it was completely arbitrary to adopt the lower limit of wave heights defining storm conditions, the so-called threshold wave height. In the calculations performed, the lower limit of the significant wave height in the storm was  $H_s > 1.0, 2.0$  and  $3.0$  m. The number of storms, depending on the adopted threshold wave height, for both measured (wave buoy) and calculated (wave reconstruction) data is presented in Table 3.

**Table 3.** The number of storms depending on the adopted lower threshold of the significant wave height during storm conditions

Lower limit of wave height under storm conditions	The number of storms	
	Wave buoy	Wave reconstruction
$H_s > 1.0$ m	315	1693
$H_s > 2.0$ m	141	712
$H_s > 3.0$ m	46	270

Calculations of significant wave heights with a defined probability of occurrence were performed using Weibull and Gumbel statistical distributions.

Fig. 3a–3c compare significant wave heights calculated using the Gumbel statistical distribution based on the reconstruction of waves in 1958–2001 and measured in 1996–2001 for the three adopted lower wave height thresholds during storm conditions. The significant wave heights calculated with the Weibull statistical distribution for the same wave data and threshold values are presented in Fig. 4a–4c.

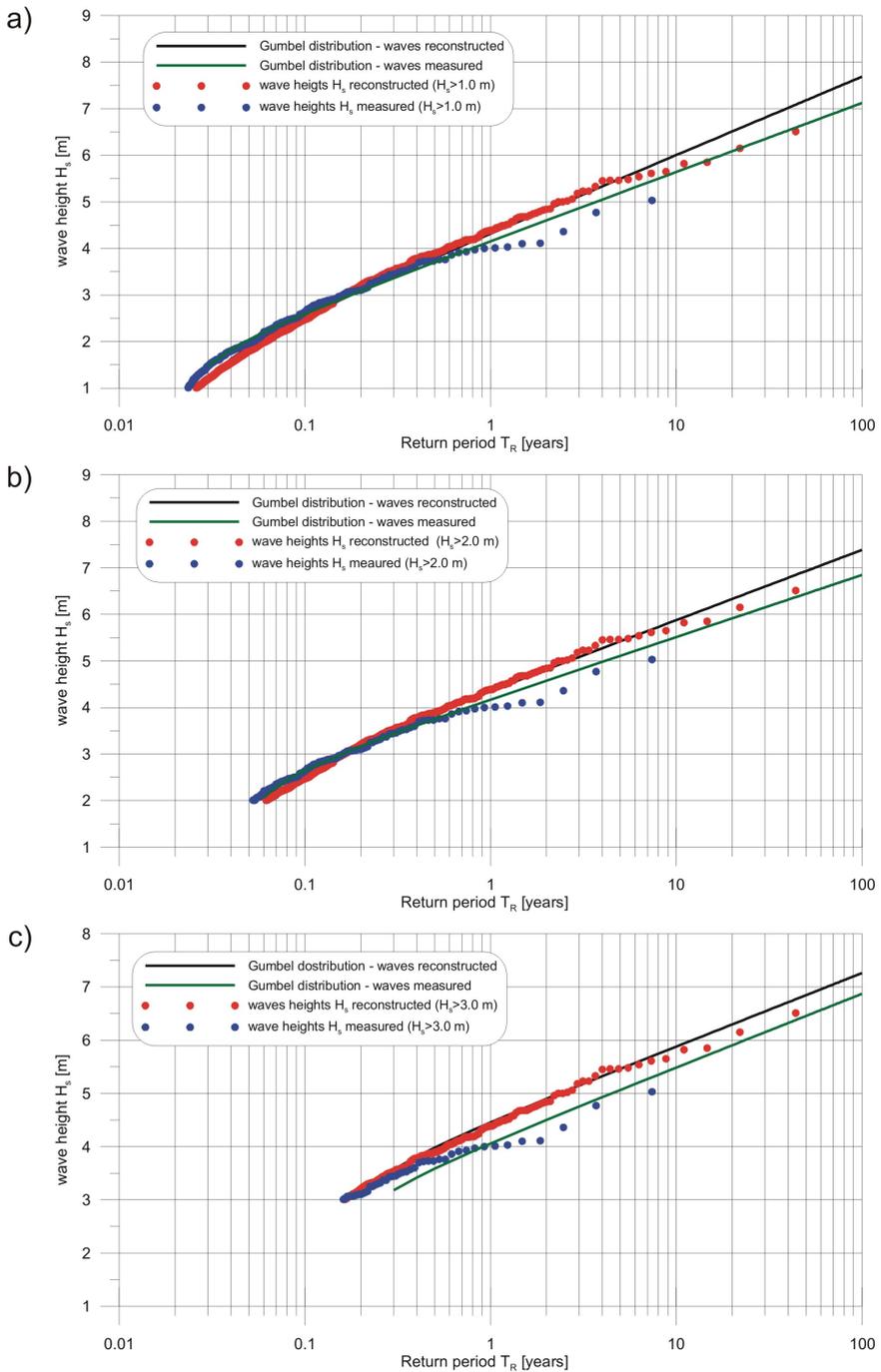
Analysis of these figures shows that:

- regardless of the calculation formula applied (Gumbel or Weibull) and regardless of the adopted threshold value ( $H_s > 1.0, 2.0$  or  $3.0$  m), the significant wave height distributions determined for data derived from the reconstruction of wind-generated waves have slightly higher values compared to those obtained from the measurements,
- despite the considerable difference in the duration of the wave series derived from wave buoy measurements (ca. 7.4 years) and wave reconstruction (44 years), the wave height distributions are similar,
- the Weibull distribution provides a better fit between measured and calculated wave heights and, in particular, a better fit for lower probabilities of occurrence,
- to obtain higher wave heights in the Weibull distribution for small probabilities of occurrence, the exponent  $\alpha$  should be in the range 0.8–0.9, and to reduce the wave height, it should be in the range 1.1–1.2.

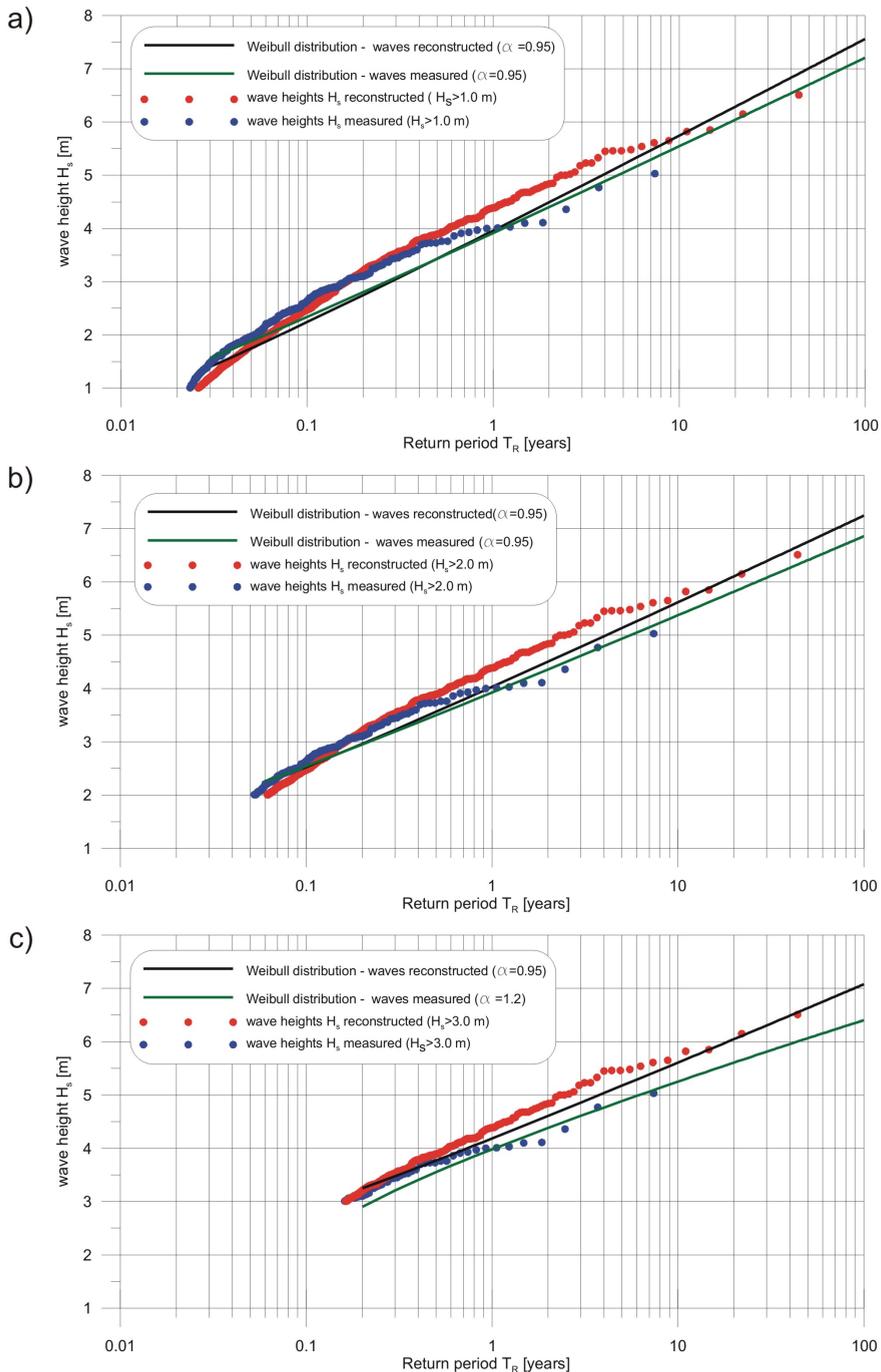
Tables 4 and 5 compare the significant wave heights  $H_s$  calculated for the return periods  $T_R = 1, 2, 5, 10, 25, 50$  and 100 years from the wave data obtained by wave buoy measurements and from the reconstruction of waves described by Gumbel and Weibull distributions, respectively.

It follows from the results of the calculations presented in the above tables that

- the Gumbel distribution of significant wave heights with a return period of 1–100 years for data derived from the reconstruction of waves are about 4–9% higher compared to data obtained by wave buoy measurements,
- the Weibull distribution of significant wave heights with a return period of 1–100 years for data derived from the reconstruction of waves are about 3–10% higher compared to data obtained by wave buoy measurements,
- both Gumbel and Weibull distributions of significant wave heights depend on the adopted threshold value ( $H_s > 1.0, 2.0$  and  $3.0$  m),
- the highest calculated significant wave heights for both short-term (7.4 years) and long-term (44 years) data were obtained for the significant wave height threshold  $H_s > 1.0$  m,
- for return periods  $T_R = 50$  and 100 years, the significant wave heights calculated for  $H_s > 1.0$  m were higher than those determined for  $H_s > 2.0$  and  $3.0$  m:
  - for significant wave heights described by the Gumbel distribution and data derived from wave reconstruction, by 4–6%,
  - for significant wave heights described by the Weibull distribution and data derived from wave reconstruction, by 4–8%.



**Fig. 3.** Comparison of significant wave heights  $H_s > 1.0$  m (a),  $H_s > 2.0$  m (b) and  $H_s > 2.0$  m (c) described by the Gumbel statistical distribution in the Lubiatowo region at a depth of about 20 m, based on the reconstruction of waves for 1958–2001 and measured in 1996–2016



**Fig. 4.** Comparison of significant wave heights  $H_s > 1.0$  m (a),  $H_s > 2.0$  m (b) and  $H_s > 3.0$  m (c) described by the Weibull statistical distribution in the Lubiatowo region at a depth of about 20 m, based on the reconstruction of waves for 1958–2001 and measured in 1996–2016

**Table 4.** Comparison of the calculated significant wave heights  $H_s$  described by the Gumbel distribution with a return period of 1–100 years for three threshold values of wave data obtained through wave buoy measurements and reconstruction of waves

Wave height lower limit under storm conditions	Return period [years]	Significant wave heights $H_s$ [m]		Calculated:measured wave height ratio
		Measured by wave buoy	Calculated from wave reconstruction	
$H_s > 1.0$ m	1	4.15	4.31	increase by 3.9%
	2	4.60	4.83	increase by 5.0%
	5	5.19	5.50	increase by 6.0%
	10	5.64	6.00	increase by 6.4%
	25	6.23	6.67	increase by 7.1%
	50	6.68	7.18	increase by 7.5%
	100	7.12	7.69	increase by 8.0%
$H_s > 2.0$ m	1	4.17	4.35	increase by 4.3%
	2	4.58	4.81	increase by 5.0%
	5	5.11	5.42	increase by 6.1%
	10	5.51	5.88	increase by 6.7%
	25	6.04	6.48	increase by 7.3%
	50	6.44	6.90	increase by 7.1%
	100	6.85	7.38	increase by 7.7%
$H_s > 3.0$ m	1	4.06	4.45	increase by 9.6%
	2	4.50	4.90	increase by 8.9%
	5	5.07	5.46	increase by 7.7%
	10	5.49	5.88	increase by 7.1%
	25	6.04	6.43	increase by 6.5%
	50	6.46	6.85	increase by 6.0%
	100	6.87	7.26	increase by 5.7%

## 7. Summary

One of the main phases of designing any marine structure is to determine the parameters of a characteristic wave, referred to as a design wave, based on which the maximum impacts (loads) acting on the structure with a specific lifetime are determined.

At present, in Poland, data from the reconstruction of waves in the Baltic over a period of 44 years (1958–2001) are increasingly used to determine the design wave. This means that for a planned construction/expansion of ports or an offshore structure (e.g. a wind farm) in a specific Baltic coastal zone, a forecast point or points in the nearest proximity of that water area are selected from the existing data. Then, the parameters of the design wave are determined for the 44-year wave train available for a given point.

Given this procedure, questions have been raised about the reliability of thus determined wave heights. Deep-water wave measurements carried out irregularly in 1996–2016 (for about 7.4 years in total) in the region of CRS Lubiatowo provided

**Table 5.** Comparison of the calculated significant wave heights  $H_s$  described by the Weibull distribution with a return period of 1–100 years for three threshold values of wave data obtained through wave buoy measurements and reconstruction of waves

Wave height lower limit under storm conditions	Return period [years]	Significant wave heights $H_s$ [m]		Calculated:measured wave height ratio
		Measured by wave buoy	Calculated from wave reconstruction	
$H_s > 1.0$ m		$\alpha = 0.95$	$\alpha = 0.95$	
	1	3.91	3.96	increase by 1.2%
	2	4.40	4.49	increase by 2.0%
	5	5.05	5.20	increase by 3.0%
	10	5.43	5.74	increase by 5.7%
	25	6.20	6.46	increase by 4.2%
	50	6.70	7.01	increase by 4.6%
100	7.21	7.56	increase by 4.9%	
$H_s > 2.0$ m		$\alpha = 0.95$	$\alpha = 0.95$	
	1	3.93	4.03	increase by 2.5%
	2	4.36	4.50	increase by 3.2%
	5	4.93	5.14	increase by 4.3%
	10	5.37	5.62	increase by 4.7%
	25	5.96	6.26	increase by 5.0%
	50	6.41	6.75	increase by 5.3%
100	6.86	7.25	increase by 5.7%	
$H_s > 3.0$ m		$\alpha = 1.20$	$\alpha = 0.95$	
	1	3.98	4.18	increase by 5.0%
	2	4.38	4.61	increase by 5.3%
	5	4.89	5.17	increase by 5.7%
	10	5.25	5.61	increase by 6.9%
	25	5.72	6.19	increase by 8.2%
	50	6.07	6.51	increase by 7.2%
100	6.40	7.08	increase by 10.6%	

a relatively large set of actually measured parameters of wind-generated waves. The comparison of design wave heights determined from these short- and long-term wave trains made it possible to consider the parameters determination of the wave based on data derived from forecast points (wave reconstruction) as reliable for the coastal zone of the southern Baltic.

Comparisons of significant wave heights based on the buoy-measured wave parameters in the Lubiato region with those determined from wave reconstruction for the same water region show that:

- the course of significant wave heights in the entire period of 7.4 years determined from the measurements and calculations was similar and was characterised by a certain time shift of the actual storm season in relation to the calculated one,

- events during which the significant wave heights determined through wave reconstruction ( $H_{cal}$ ) were higher than those determined through direct measurements ( $H_{meas}$ ) prevailed in the whole period considered,
- the average value of the relative error between the measured and calculated wave heights did not exceed 10% for the whole period.

The first step to determine the parameters of the design wave was to create a statistically independent set of maximum waves. For this purpose, the analysis of short- and long-term wave data considered only one maximum significant wave height from each storm. In the calculations performed, the lower limit of the significant wave height during a storm was assumed to be  $H_s > 1.0$  or  $2.0$  or  $3.0$  m. The sets of maximum waves obtained were then approximated with two statistical distributions describing extreme events, namely, Gumbel and Weibull distributions.

The results of the calculations lead to the following conclusions:

- the significant wave height distributions obtained for short- (about 7.4 years) and long-term (44 years) wave data are similar,
- regardless of the approximation of the maximum wave sets by Gumbel and Weibull statistical distributions and regardless of the adopted threshold value ( $H_s > 1.0, 2.0$  and  $3.0$  m), the significant wave height distributions determined for data derived from the reconstruction of wind-generated waves have slightly higher values compared to those obtained by measurements,
- the Gumbel distribution of significant wave heights with a return period of 1–100 years for data derived from the reconstruction of waves are about 4–9% higher compared to data obtained by wave buoy measurements,
- the Weibull distribution of significant wave heights with a return period of 1–100 years for data derived from the reconstruction of waves are about 3–10% higher compared to data obtained by wave buoy measurements,
- the Weibull distribution provides a better fit between the actual and calculated wave heights and, in particular, a better fit for lower probabilities of occurrence,
- Gumbel and Weibull distributions of significant wave heights depend on the adopted threshold value ( $H_s > 1.0, 2.0$  or  $3.0$  m),
- the greatest calculated significant wave heights for both short-term (7.4 years) and long-term (44 years) data were obtained for the significant wave height threshold  $H_s > 1.0$  m, while the lowest for  $H_s > 3.0$  m,
- for return periods  $T_R = 50$  and 100 years, the significant wave heights calculated for  $H_s > 1.0$  m were higher than those determined for  $H_s > 2.0$  and  $3.0$  m by 4–7% on average.

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