

## Original Contributions - Originalbeiträge

Jocelyn Penny Small & Dejan Todorović

# From Cubes to Ribbons: Transformation of an Illusion

## Part 1: The Discovery and Classical Background, by Jocelyn Penny Small

I am a classical archeologist.<sup>1</sup> When teaching an undergraduate class about Roman painting and mosaics, I stood, as usual, at the lectern to the right of the screen and at a severe oblique angle to the slides on the left. In the days of “real” 35 mm slides, I was occasionally surprised by what a projected slide showed. As a result, I developed the habit of checking to make sure that what I am showing is what I am talking about. In this case, when I looked at my PowerPoint slide of a Roman mosaic with perspective cubes, I did not see cubes but rectangular boxes (see Appendix 1) (Figures 1 and 2) At first, I thought I had probably chosen the wrong object; then, I realized that the problem was not my memory or even the slide itself but actually an illusion of an illusion. As my angle of viewing grew more severe (or oblique), the cubes changed to rectangular boxes and further to “ribbons,” as a set of photographs, taken subsequently, of the actual object demonstrates. When I later projected my photograph of the narrow strips on a screen back home, I found that these strips did not turn back into rectangular boxes, much less into cubes, no matter what angle I viewed them from. Hence, the transformation is unidirectional.

The motif first appeared in floor mosaics of second century BC, often as the main decorative motif, and never disappeared from Western art.<sup>2</sup> It was sufficiently popular to have been mentioned by both Vitruvius (7.1.4) and Pliny (Natural History 36.185). They used the term “scutulatum,” which means “lozenge,” and reflected the fact that the “cubes” are not “square” but “rhombi” or diamonds. Unlike the “traditional” Necker cube whose effect derives from its depiction in outline, the classical examples use three different solid colors: very light face (light struck), very dark face (shadow), and “midway” between the first two colors.

The question then arises whether the Greeks and Romans knew about the illusion of the illusion. I suspect that, like us, they did not, because of the way their cubes are viewed. The example I have used was originally a “threshold” mosaic; that is, it decorated the small rectangular area of the floor between the two “posts” of a

<sup>1</sup> I thank Arthur Shapiro for his encouragement and putting me in touch with Dejan Todorović.

<sup>2</sup> Various aspects of the motif, especially in the textual sources, continue to be debated by classicists and classical archaeologists, but these issues do not affect the motif itself as presented here.



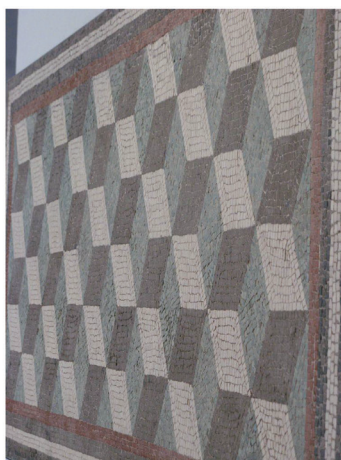
**Fig. 1** Original photo taken head-on of cube mosaic.



**(a)**



**(b)**



**(c)**

**Fig. 2** The “cubes” change form depending on the angle from which they are viewed. All photographs shot in the museum.



(a)



(b)

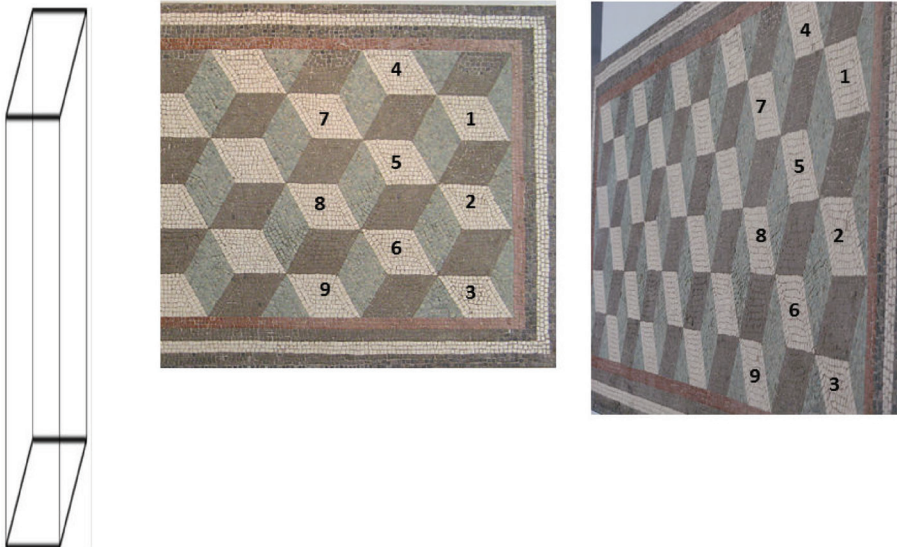
**Fig. 3** (a) Usual view from standing position. (b) Shot squatting down from oblique angle.

doorway. Such a position obviously limits the angles the mosaic can be viewed from. The mosaic, however, has been detached from its original context and has been mounted relatively high up on a wall in the museum. The design appears mainly on floors in antiquity. The other locations also restricted one's viewing. For instance, it is painted on a late Etruscan funerary urn (See Appendix 2). Although we do not know precisely where the urn comes from, enough similar urns have survived in situ for us to know that it would have been unlikely within the confines of a family tomb for the urn to have been seen from different angles. Another example occurs as a decorative pattern in the rectangular area at the bottom of a wall (the dado) in a cubiculum (a small room, often called a "bedroom") of the House of the Griffins, a Roman Republican house on the Palatine Hill in Rome from the first century BC (See Appendix 3).

The Getty Museum in Malibu (the old, original one, now devoted to classical antiquities) replicates the motif in two floor mosaics (Figure 3). The cubes remained cubes, no matter what my viewing angle was, as long as I was standing.

When, however, I crouched as low as I could to the floor for an oblique angle, the illusion appeared. I have not cropped my photographs so that my viewing angle is apparent. Hence, I doubt whether anyone then would have noticed this illusion with the possible exception of the mosaicists.

Euclid (Optics 58), however, implies that a square viewed obliquely becomes a rectangle, when he says, at the end of Optics 58: “For similarly we shall prove what happens, just as in the case of circles.” (Translation from Burton, 1945, p. 372) In Optics 36, he demonstrates why “the wheels of the chariots appear sometimes circular, sometimes distorted.” (Translation from Burton, 1945, p. 367) This comparison of the effect of viewing angles on circles and squares is mentioned in two later problematic texts (Heron, Definitions 135.12 and Ptolemy, Optics 2.72). They simply say that a circle looks like an ellipse and a square looks like a rectangle. It is important to note that all three texts consider only individual circles and squares. The full illusion of the illusion, however, depends on multiple cubes arranged in a single array. One Necker cube, when viewed from an oblique angle, as expected becomes a skinny rectangular box (Figure 4). For a true ribbon effect, however, multiple cubes are necessary, because the new skinny “boxes” or “ribbons” are visually formed by three cubes. For example, cube 5, when seen obliquely, now has two light faces (5 and 8) and two dark sides (5 and 7). The third color is defined by three cubes (5, 7, and 8) and now looks like either the bottom of a new “box” or “empty space” seen through a frame.



**Fig. 4** From left to right: elongated Necker cube, “original” mosaic with cubes numbered, and the illusion showing how these cubes form “ribbons.”

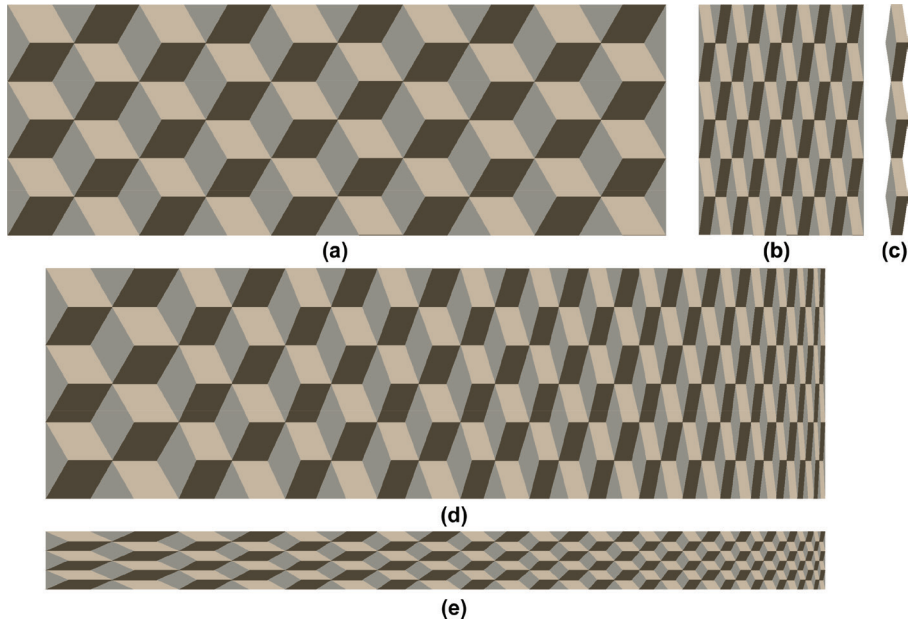


## Part 2: Some Variants of the Phenomenon, by Dejan Todorović

There are a number of phenomena in which the change of the angle of view changes some aspect of the perceived scene. However, the “boxes-to-ribbons” effect discovered by Small does not quite fit into any traditional category and seems to be a genuinely novel perceptual phenomenon. The effect is clearly different from the “hollow face” (Hill & Johnston, 2007), “reverspective” (Dobias & Papathomas, 2014) and related phenomena, because such effects involve 3D objects rather than flat mosaics. It is also different from the effects induced by changes in observation points of pictures of complex scenes, which involve prominent changes in perceived directions of virtual 3D space (Todorović, 2009). The effect is in part similar to phenomena observed in anamorphic images (Topper, 2000), in that the initial perceived change of cubes into rectangular boxes, due to the change in the viewing angle, can be described as geometrical distortion; however, the further metamorphosis of boxes into thin ribbons involves a more radical structural change that cannot be described as just a distortion of initially perceived objects. Rather, visual Gestalten of one type (a closely packed array of cubes) is transformed into visual Gestalten of a different type (chains of ribbons on common background).

The change in viewpoint that results in the appearance of ribbons in Figure 2 induces a perspective transformation of the image of the mosaic. However, this effect can also be produced by simple scaling of a dimension of the image, as demonstrated in Figure 5. Figure 5a presents a simplified, artless version of the mosaic in Figure 1. The three constituent quadrilateral elements forming the repeated hexagonal subfigures in this image have the same shapes (referred to as lozenges or diamonds or rhombuses, with opposite angles of 60 and 120 degrees) but different orientations and gray shades. Such designs are called rhombille tessellations in the geometry literature and Beaunis cubes in the older perception literature (Wade, Campbell, Ross, & Lingelbach, 2010). They constitute isometric perspective images of arrays of cubes, three sides of which are visible. Such images usually evoke impressions of cubes whose spatial arrangement is such that the sides of three neighboring cubes form cube-shaped concave indentations. However, occasionally, the impression may invert, such that the concave portions become regular convex, protruding cubes and the original cubes become the new concave portions. In such cases, the impression of the direction of illumination is also inverted. Such perceptual reversals have been known and studied for a long time (see Wade, 2004; Price, 1968). Although percepts of 3D cubes are predominant, some observers also report impressions of flat hexagonal tilings.

Figure 5b presents the result of compressing Figure 5a to one-quarter of its extent along the horizontal dimension, causing the original regular hexagons to be squeezed. In particular, the light gray and the dark gray lozenges are



**Fig. 5** A mosaic and its transformations. (a) Basic image, evoking impressions of cubes. (b) Horizontal compression of (a), evoking impressions of ribbons. (c) An extract from (b), evoking impressions of short boxes. (d) Gradual horizontal compression of (a), evoking an impression of gradual transformation of boxes into ribbons and further into texture. (e) Vertical compression of (d), evoking impressions of very flat boxes or ribbons.

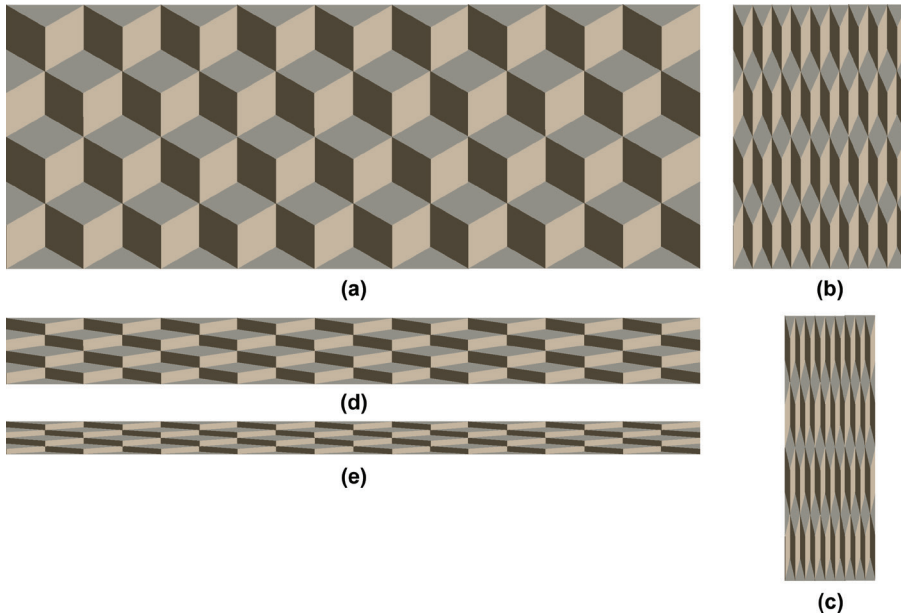
transformed into elongated oblique parallelograms of alternating, near vertical slant, whereas the medium gray lozenges remain rhombuses, but with the sharp angles becoming more sharper and the obtuse angles becoming more obtuse. What is the perceptual outcome of this simple geometrical manipulation? One would have expected that a squeezed array of cubes would look like an array of squeezed cubes. Instead, the cubes have disappeared from view and are replaced, in part, by zigzagging narrow ribbons formed by the joined light and dark squeezed lozenges, similar as in Figure 2. The remaining medium gray lozenges have lost their status as the individual third sides of the cubes and have instead become portions of a larger visual unit, the medium gray background. Thus, the original configuration of closely packed objects is transformed into a figures-on-ground scene, with ribbons as figures and the ground partly occluded by them, visible in between them, and continuing amodally behind them. The ribbons form a 3D configuration and are not seen as lying flat on the ground. However, an isolated column of squeezed hexagons, such as in Figure 5c, does look like a bunch of stacked squeezed cubes, in which the medium gray lozenges are not grouped together but have retained their separate identities as third sides of cubes.

Figure 5d was produced by concatenating increasingly squeezed segments of the original cube display, similar to a perspective transformation, but only along the horizontal dimension. Its appearance changes in interesting ways from left to right. Near the left end, the image looks like a packed array of cubes, progressively turning into ever narrower rectangular boxes. This percept gives way to a ribbons-on-ground impression, but the precise switching point is elusive and the region between definite boxes and definite ribbons tends to be bistable, turning from boxes to ribbons and back. Still further along, near the right end, the figure-ground distinction tends to be lost and gives way to the appearance of a multi-shaded texture.

Figure 5e is produced by compressing Figure 5d to one-quarter of its extent along the vertical dimension. The portion of this figure near its left end conveys the impression of wide zigzagging ribbons, although boxes can also be seen there; boxes become dominant in the middle portion, and the rightmost portion looks like a texture.

All designs in Figure 5 are composed of adjoining tri-part hexagons but with different width-to-height ratios. In Figure 5a, this ratio is equal to 1.15 (since the width of these hexagons is larger than their height by 15%). In Figure 5b, because of the fourfold horizontal compression, the ratio is four times smaller. In Figures 5d and 5e, the ratio decreases from left to right; in Figure 5d, it starts at a value of 1.15; and in Figure 5e, because of fourfold vertical compression, it starts with a four times larger value. As geometrical designs, all these figures are ambiguous, because all of them could be depictions of either boxes or ribbons. However, our visual system seems to interpret figures with width-to-height ratios within a broad range around a central value (here 1.15) as boxes and figures with larger or smaller ratios as ribbons, with intermediate values inducing bistability.

The preceding descriptions and numbers are subject to some qualifications. For example, whether a hexagonal configuration with a given width-to-height ratio evokes the appearance of ribbons or boxes depends not only on that ratio but also on the context in which it is embedded and the perception of its parts; for example, whereas the medium gray lozenges are perceived as parts of the common background in Figure 5b, they are perceived as third sides of boxes in Figure 5c. Furthermore, both the type of the geometrical transformation of a design and the type of perceived objects depend on the relation between the orientation of the design and the direction of its compression. To illustrate this, Figure 6a presents a mosaic similar to Figure 5a, but with the basic hexagonal subfigures turned by 90 degrees; here the width-to-height ratio equals 0.86. Like Figure 5a, Figure 6a conveys the impression of a field of cubes but with a different orientation. When the horizontal extent of this figure is compressed to one-quarter, the result is Figure 6b, which looks rather different from the corresponding Figure 5b. This

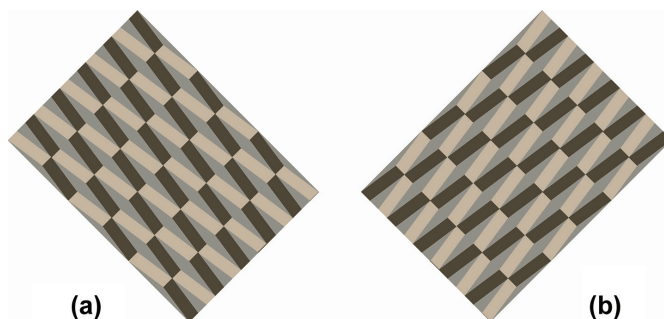


**Fig. 6** Another mosaic and its transformations. (a) Basic image, evoking impressions of cubes. (b) Horizontal compression of (a), evoking impressions of boxes or ribbons. (c) Horizontal compression of (b), evoking impressions of ribbons. (d) Vertical compression of (a), evoking impressions of flat boxes. (e) Vertical compression of (d), evoking an impression of gradual transformation of ribbons into boxes.

figure is ambiguous, in that one can either see somewhat odd, elongated boxes, or dark-and-light zigzagging ribbons on a medium shade background, but these ribbons are of a different kind than in Figure 5b. When this design is squeezed once more, as in Figure 6c, the boxes are hard to see and the ribbons dominate. When the vertical extent of Figure 6a is compressed to one-quarter, the result is Figure 6d, which conveys the impression of an array of short boxes. When it is squeezed once more, as in Figure 6e, one starts to see ribbons, similar to in Figure 5b, but in a different orientation. The designs in Figure 6 can be described in a similar way as in Figure 5: hexagons with a width-to-height ratio within a wide range around a central value (here 0.86) are seen as boxes, whereas designs with much higher or much smaller ratios are seen as ribbons.

Additional effects of the role of the orientation of these designs can be noted when some of them are rotated by 90 degrees. For example, when Figure 5b is rotated, one does not see what one might expect to see, that is, ribbons turned by 90 degrees, but rather an array of short boxes, very similar to what is seen in Figure 6d. Conversely, when Figure 6d is rotated, boxes tend to turn into ribbons. It is not gravitational but retinal orientation that is crucial in this curious effect, as demonstrated in Figure 7. Figures 7a and 7b are identical to Figure 5b, except for



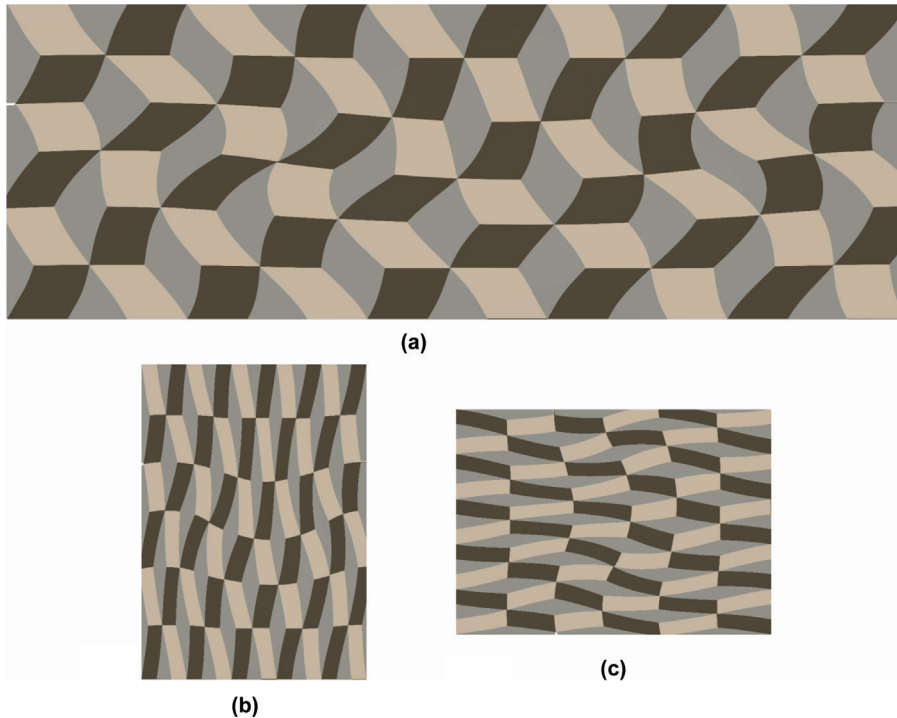


**Fig. 7** Same designs as in Figure 5b but tilted in two orientations. Impressions of boxes or ribbons depend on the orientation of the head of the observer, as explained in the text.

orientation. If observers tilt their head toward their left shoulder, so that the axis of the head is parallel to the long sides of Figure 7a, ribbons will dominantly be seen in Figure 7a and short boxes in Figure 7b. On the other hand, if they tilt the head in a similar manner toward the right shoulder, aligning its axis with the long sides of Figure 7b, the percepts of boxes and ribbons will tend to switch. With some concentration and effort of will, either percept may appear in either figure, but the alternate percepts are difficult to obtain and are fleeting, whereas the ones described earlier are more stable and appear spontaneously.

What is the explanation of the various phenomena demonstrated in this paper? One possible account would be based on our experience with regular box-shaped constructions. We have encountered such objects from the outside and the inside very often and in various formats in our environment, but mostly within a certain range of a) ratios of dimensions of their sides and b) spatial orientations. Designs involving ratios and orientations that fall outside of these ranges may not be able to evoke the corresponding impressions of boxes, and the ribbons-on-ground percept may be the best possible solution that the visual system would be able to come up with in such cases.

One problem with this account is illustrated in Figure 8. Figure 8a is a geometrical transformation of Figure 5a in which most lozenges are distorted, with their edges having undergone various changes in length, orientation, and shape. This image evokes the impression of an irregular array of curved boxes involving nonplanar dihedral and trihedral spatial angles, none of their sides being constituted by flat rectangles of uniform size. The point is that we are not likely to have encountered many such shapes in our environment. Nevertheless, when subjected to the same transformations as shown in Figure 5a, Figure 8a exhibits analogous effects. For example, Figure 8b is a horizontally squeezed version of Figure 8a, and it is seen as depicting an array of curved ribbons. However, when it is turned by 90 degrees, as in Figure 8c, one does not see



**Fig. 8** A curvilinear mosaic and its transformations. (a) A “swirled” version of Figure 5a, evoking impressions of curved boxes. (b) Horizontal compression of (a), evoking impressions of curved ribbons. (c) Same as (b) but turned 90 degrees, evoking impressions of curved slabs.

ribbons but distorted 3D slabs. Rotating these figures by 45 degrees, similar to in Figure 7, and turning the head in the described manner result in analogous percepts. These effects can hardly be based on explicit experience with such objects, although perhaps experience with rectangular boxes may generalize to their distorted counterparts. Which mechanisms are at work in the 3D interpretations of this class of images, what spatial parameters affect its appearance and how they affect it, and to what extent and in which manner experience may play a role remain to be established.

### Summary

In Part 1 Small describes her discovery that an array of depicted cubes produces another and completely different illusion from that of a single cube. When a group of such cubes are viewed at an angle, they turn into rectangular boxes, and as the angle gets more severe, they become narrow ribbons. The illusion works only in one direction. In Part 2, Todorović manipulates the image to demonstrate various transformations and offers an explanation of how and why they work the way they do.

**Keywords:** Roman Mosaic, Isometric Cubes, Perceptual Transformation.

## Von Würfeln zu Bändern: Die Wandlung einer Illusion

### Zusammenfassung

In Teil 1 beschreibt Small ihre Entdeckung, dass eine Reihe von dargestellten Würfeln eine andere und völlig verschiedene Illusion erzeugt als die eines einzelnen Würfels. Wenn eine Gruppe solcher Würfel aus einem schrägen Winkel betrachtet wird, verwandeln sie sich in rechteckige Kästchen und werden, wenn der Winkel noch schräger wird, zu schmalen Bändern. Die Illusion funktioniert nur in einer Richtung. In Teil 2 manipuliert Todorović das Bild, um verschiedene Transformationen zu demonstrieren und bietet eine Erklärung an, wie und warum sie so funktionieren, wie sie es tun.

**Schlüsselwörter:** Römisches Mosaik, Isometrische Würfel, Wahrnehmungstransformation.

This work was supported by Grant ON179033 from the Serbian Ministry of Science to D.T.

### Appendix to Part 1: Objects Discussed with the Motif

Note that the motif itself is classified in Balmelle et al. (2002), Part 1, p. 39 with pl. 212a, as a “triaxial pattern of trichrome cubes.”

1. Threshold mosaic in opus tessellatum with perspective cubes. From Rome, Via Nomentana, Casale San Basilio, villa rustica, tablinum I. Now Rome, Museo Nazionale Romano, Palazzo Massimo alle Terme Inv. 125524. Limestone. 78 cm × 184 cm. Date: 100-50 BC. The cubes are placed within a rectangular border consisting of four bands of color (beginning with the outermost band: dark gray, white, dark gray, and red). The cubes are white, slate blue, and dark gray. Paris and Di Carcina (2012), page 49 No. 4.2, with bibliography, and page 220 bottom figure. Moorman and Swinkels (1983), p. 256 No. 23.3. Guimier-Sorbets (1994), p. 23 fig. 11.
2. Late Etruscan funerary urn from Chiusi, now in Florence, Museo Archeologico 5543. Painted terracotta. Date: 125-100 BC. Minto (1941), with pl. XL fig. 1. Pairault (1972), p. 38 with pl. 5. Moorman and Swinkels (1983), p. 251 No. 10 with bibliography and p. 244 fig. 14.3.
3. Rome, Palatine Hill, House of the Griffins, cubiculum IV, in situ. Fresco. Date: 100-75 BC. Moorman and Swinkels (1983), p. 256 No. 23.2 with bibliography under No. 23.1. Mazzoleni and Pappalardo (2004), pp. 67, 74-76.

### References

#### Part 1 and Part 2

- Balmelle, C., Blanchard-Lemée, M., Christophe, J., Darmon, J.-P., Guimier-Sorbets, A.-M., Lavagne, H., Prudhomme, R., & Stern, H. (2002). *Le décor géométrique de la mosaïque romaine* (2 Vols). Paris, France: Picard.
- Burton, H. E. (Trans.). (1945). The optics of Euclid. *Journal of the Optical Society of America*, 35, 357–372.
- Dobias, J. J., & Papathomas, T. V. (2014). The role of linear perspective cues on the perceived depth magnitude of the surfaces they are painted on: proper-, reverse-, and flat-perspective paintings. *Perception*, 43, 989–1000.

- Guimier-Sorbets, A.-M. (1994). Mosaïques et dallages dans le monde grec aux époques classique et hellénistique. In P. Johnson, R. Ling, & D. J. Smith (Eds.), *Fifth International Colloquium on Ancient Mosaics. Bath, England 5–12, 1987. Journal of Roman Archaeology Supplement 9.1* (pp. 13–25). Portsmouth, RI: Journal of Roman Archaeology.
- Hill, H. C., & Johnston, A. (2007). The hollow-face illusion: Object specific knowledge, general assumptions or properties of the stimulus. *Perception*, 36(2), 199–223.
- Mazzoleni, D., & Pappalardo, U. (2004). *Domus. Wall painting in the Roman house*. Los Angeles, CA: Getty Publications.
- Minto, A. (1941). Di un motivo decorativo ad opus scutulatum dipinto in una urnetta fittile chiusina. *Studi Etruschi*, 15, 385–389.
- Moorman, E. M., & Swinkels, L. J. F. (1983). Lozenges in perspective. In A. Barbet (Ed.), *La peinture murale romaine dans les provinces de l'Empire: [actes des] journées d'étude de Paris, 23–25 septembre 1982. British Archaeological Reports – International Series 165* (pp. 239–262). Oxford, UK: Archaeopress.
- Pairault, F.-H. (1972). *Recherches sur quelques séries d'urnes de Volterra à représentations mythologiques. Collection de l'École Française de Rome* (Vol. 12). Paris, France: Editions de Boccard.
- Paris, R., & Di Sarcina, M. T. (2012). *Museo Nazionale Romano – Palazzo Massimo alle Terme – I mosaici*. Milan, Italy: Mondadori Electa S.p.A.
- Price, J. R. (1968). Studies of reversible perspective: a methodological review. *Behavior Research Methods & Instrumentation*, 1(3), 102–106.
- Todorović, D., (2009). The effect of the observer vantage point on perceived distortions in linear perspective images. *Attention, Perception & Psychophysics*, 71, 183–193.
- Topper, D. (2000). On anamorphosis: Setting some things straight. *Leonardo*, 33(2), 115–124.
- Wade, N. J. (2004). Good figures. *Perception*, 33(2), 127–134.
- Wade, N. J., Campbell, R. N., Ross, H. E., & Lingelbach, B. (2010). Necker in scotch perspective. *Perception*, 39, 1–4.

**Jocelyn Penny Small** (b. 1945), Professor Emerita in the Department of Art History, Rutgers University, New Brunswick, NJ, USA, is a classical archeologist who has focused on classical iconography and Etruscan art for much of her career. She was the director of the U.S. Center of the Lexicon Iconographicum Mythologiae Classicae for twenty-one years. Her interest in cognitive science began when working on the interplay between literacy and memory for her book *Wax Tablets of the Mind* (Routledge, 1997). Currently, she is working on illusionism in Greek and Roman art.

**Address:** 890 West End Avenue, Apt. 4C, New York, NY 10025-3520, USA.

**E-mail:** jpennysmall@gmail.com

**Dejan Todorović** (b. 1950) is a professor in the Department of Psychology and a member of the Laboratory of Experimental Psychology at the University of Belgrade, Serbia. His research interests include experimental studies, mathematical analyses, and computational models of perception of lightness, space, shape, pictures, and faces. He is a co-editor, with Arthur Shapiro, of *Oxford Compendium of Visual Illusions* (Oxford University Press, 2017).

**Address:** Department of Psychology, University of Belgrade, Čika Ljubina 18-20, 11000 Belgrade, Serbia.

**E-mail:** dtodorov@f.bg.ac.rs