FORMALIZED MATHEMATICS
Vol. 22, No. 1, Pages 29–35, 2014
DOI: 10.2478/forma-2014-0004

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The Formalization of Decision-Free Petri Net

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Summary. In this article we formalize the definition of Decision-Free Petri Net (DFPN) presented in [19]. Then we formalize the concept of directed path and directed circuit nets in Petri nets to prove properties of DFPN. We also present the definition of firing transitions and transition sequences with natural numbers marking that always check whether transition is enabled or not and after firing it only removes the available tokens (i.e., it does not remove from zero number of tokens). At the end of this article, we show that the total number of tokens in a circuit of decision-free Petri net always remains the same after firing any sequences of the transition.

MSC: 68Q60 68Q85 03B35

Keywords: specification and verification of discrete systems; Petri net;

MML identifier: PETRI_DF, version: 8.1.02 5.22.1199

The notation and terminology used in this paper have been introduced in the following articles: [1], [4], [17], [14], [8], [5], [6], [15], [12], [3], [9], [10], [20], [11], [13], [18], and [7].

1. Preliminaries

From now on N denotes a place/transition net structure, P denotes a Petri net, and i denotes a natural number.

[1]This work was supported in part by JSPS Kakenhi grant number 2230028502.
[2]This work was supported in part by the University of Bialystok grant BST225 *Database of mathematical texts checked by computer*.

ISSN 1426–2630(Print), 1898-9934(Online)

Now we state the propositions:

(1) Let us consider natural numbers x, y and a finite sequence f. Suppose

(i) $f_{\restriction 1}$ is one-to-one, and

(ii) $1 < x \leqslant \operatorname{len} f$, and

(iii) $1 < y \leqslant \operatorname{len} f$, and

(iv) $f(x) = f(y)$.

Then $x = y$.

(2) Let us consider a non empty set D and a non empty finite sequence f of elements of D. If f is circular, then $f(1) = f(\operatorname{len} f)$.

Let D be a non empty set and a, b be elements of D. Let us observe that $\langle a, b, a \rangle$ is circular as a finite sequence of elements of D.

Now we state the proposition:

(3) Let us consider objects a, b. If $a \neq b$, then $\langle a, b, a \rangle$ is almost one-to-one.

Let X be a set, Y be a non empty set, P_1 be a finite subset of X, and M_1 be a function from X into Y.

An enumeration of M_1 and P_1 is a finite sequence of elements of Y and is defined by

(Def. 1) (i) $\operatorname{len} it = \operatorname{len}$ the enumeration of P_1 and for every i such that $i \in \operatorname{dom} it$ holds $it(i) = M_1$(the enumeration of $P_1(i)$), **if** P_1 is not empty,

(ii) $it = \varepsilon_Y$, **otherwise**.

The functor PN_0 yielding a Petri net is defined by the term

(Def. 2) $\langle \{0\}, \{1\}, \Omega_{\{1\}}(\{0\}), \Omega_{\{0\}}(\{1\}) \rangle$.

Let us consider N. We introduce the places and transitions of N as a synonym of Elements(N).

Let us consider P. Let us note that the places and transitions of P is non empty.

In the sequel f_1 denotes a finite sequence of elements of the places and transitions of P.

Let us consider P and f_1. The functors: the places of f_1 and the transitions of f_1 yielding finite subsets of P are defined by terms,

(Def. 3) $\{p$, where p is a place of $P : p \in \operatorname{rng} f_1\}$,

(Def. 4) $\{t$, where t is a transition of $P : t \in \operatorname{rng} f_1\}$,

respectively.

2. The Number of Tokens in a Circuit

Let us consider N. The markings of N yielding a non empty set of functions from the carrier of N to $\mathbb{N}$ is defined by the term

(Def. 5) $\mathbb{N}^{\alpha}$, where α is the carrier of N.

A marking of N is an element of the markings of N. Let P_1 be a finite subset of N and M_1 be a marking of N. The number of tokens of P_1 and M_1 yielding an element of $\mathbb{N}$ is defined by the term

(Def. 6) $\sum$ the enumeration of M_1 and P_1.

3. Decision-Free Petri Net Concept and Properties of Circuits in Petri Nets

Let I be a Petri net. We say that I is decision-free-like if and only if

(Def. 7) Let us consider a place s of I. Then

- (i) there exists a transition t of I such that $\langle t, s\rangle \in$ the T-S arcs of I, and
- (ii) for every transitions t_1, t_2 of I such that $\langle t_1, s\rangle$, $\langle t_2, s\rangle \in$ the T-S arcs of I holds $t_1 = t_2$, and
- (iii) there exists a transition t of I such that $\langle s, t\rangle \in$ the S-T arcs of I, and
- (iv) for every transitions t_1, t_2 of I such that $\langle s, t_1\rangle$, $\langle s, t_2\rangle \in$ the S-T arcs of I holds $t_1 = t_2$.

Let us consider P. Let I be a finite sequence of elements of the places and transitions of P. We say that I is directed path if and only if

(Def. 8)
- (i) $\operatorname{len} I \geqslant 3$, and
- (ii) $\operatorname{len} I \bmod 2 = 1$, and
- (iii) for every i such that $i \bmod 2 = 1$ and $i + 1 < \operatorname{len} I$ holds $\langle I(i), I(i+1)\rangle \in$ the S-T arcs of P and $\langle I(i+1), I(i+2)\rangle \in$ the T-S arcs of P, and
- (iv) $I(\operatorname{len} I) \in$ the carrier of P.

Now we state the proposition:

(4) Let us consider a finite sequence f_1 of elements of the places and transitions of PN_0. Suppose $f_1 = \langle 0, 1, 0\rangle$. Then f_1 is directed path. Proof: f_1 is directed path by [2, (13)], [4, (45)]. □

Let us consider P. Observe that every finite sequence of elements of the places and transitions of P which is directed path is also non empty.

Let I be a Petri net. We say that I has directed path if and only if

(Def. 9) There exists a finite sequence f_1 of elements of the places and transitions of I such that f_1 is directed path.

Let us consider P. We say that P has directed circuit if and only if

(Def. 10) There exists f_1 such that f_1 is directed path, circular, and almost one-to-one.

One can verify that PN_0 is decision-free-like and Petri-like and has directed circuit and there exists a Petri net which is Petri-like and decision-free-like and has directed circuit and every Petri net which has directed circuit has also directed path and there exists a Petri net which has directed path.

Let D_1 be a Petri net with directed path. Let us note that there exists a finite sequence of elements of the places and transitions of D_1 which is directed path.

From now on D_1 denotes a Petri net with directed path and d denotes a directed path finite sequence of elements of the places and transitions of D_1.

Now we state the propositions:

(5) $\langle d(1), d(2)\rangle \in$ the S-T arcs of D_1.

(6) $\langle d(\mathrm{len}\, d - 1), d(\mathrm{len}\, d)\rangle \in$ the T-S arcs of D_1.

From now on D_1 denotes a Petri-like Petri net with directed path and d denotes a directed path finite sequence of elements of the places and transitions of D_1.

Now we state the proposition:

(7) If $d(i) \in$ the places of d and $i \in \mathrm{dom}\, d$, then $i \bmod 2 = 1$. PROOF: Consider p being a place of D_1 such that $p = d(i)$ and $p \in \mathrm{rng}\, d$. $i \bmod 2 = 1$ by [2, (21)], [16, (25)], [7, (87)]. □

Let us assume that $d(i) \in$ the transitions of d and $i \in \mathrm{dom}\, d$. Now we state the propositions:

(8) $i \bmod 2 = 0$. PROOF: $\langle d(\mathrm{len}\, d - 1), d(\mathrm{len}\, d)\rangle \in$ the T-S arcs of D_1. Consider t being a transition of D_1 such that $t = d(i)$ and $t \in \mathrm{rng}\, d$. $i \neq \mathrm{len}\, d$ by [7, (87)]. $i + 1 \neq \mathrm{len}\, d$ by [7, (87)], [2, (11)], [16, (25)], [5, (3)]. □

(9) (i) $\langle d(i - 1), d(i)\rangle \in$ the S-T arcs of D_1, and

(ii) $\langle d(i), d(i + 1)\rangle \in$ the T-S arcs of D_1.

PROOF: $\langle d(\mathrm{len}\, d - 1), d(\mathrm{len}\, d)\rangle \in$ the T-S arcs of D_1. Consider t being a transition of D_1 such that $t = d(i)$ and $t \in \mathrm{rng}\, d$. $i \neq \mathrm{len}\, d$ by [7, (87)]. □

Now we state the proposition:

(10) Suppose $d(i) \in$ the places of d and $1 < i < \mathrm{len}\, d$. Then

(i) $\langle d(i - 2), d(i - 1)\rangle \in$ the S-T arcs of D_1, and

(ii) $\langle d(i - 1), d(i)\rangle \in$ the T-S arcs of D_1, and

(iii) $\langle d(i), d(i + 1)\rangle \in$ the S-T arcs of D_1, and

(iv) $\langle d(i+1), d(i+2)\rangle \in$ the T-S arcs of D_1, and

(v) $3 \leqslant i$.

PROOF: $i \bmod 2 = 1$. $\langle d(\operatorname{len} d - 1), d(\operatorname{len} d)\rangle \in$ the T-S arcs of D_1. $\langle d(1), d(2)\rangle \in$ the S-T arcs of D_1. Consider p being a place of D_1 such that $p = d(i)$ and $p \in \operatorname{rng} d$. $i + 1 \neq \operatorname{len} d$ by [7, (87)]. $2 \neq i$ by [7, (87)]. $\square$

4. Firable and Firing Conditions for Transitions and Transition Sequences with Natural Marking

From now on M_1 denotes a marking of P, t denotes a transition of P, and Q, Q_1 denote finite sequences of elements of the carrier' of P.

Let us consider P, M_1, and t. We say that t is firable at M_1 if and only if

(Def. 11) Let us consider a natural number m. If $m \in {M_1}^{\circ}({}^*\{t\})$, then $m > 0$.

The functor Firing(t, M_1) yielding a marking of P is defined by

(Def. 12) (i) for every place s of P, if $s \in {}^*\{t\}$ and $s \notin \overline{\{t\}}$, then $it(s) = M_1(s)-1$ and if $s \in \overline{\{t\}}$ and $s \notin {}^*\{t\}$, then $it(s) = M_1(s) + 1$ and if $s \in {}^*\{t\}$ and $s \in \overline{\{t\}}$ or $s \notin {}^*\{t\}$ and $s \notin \overline{\{t\}}$, then $it(s) = M_1(s)$, **if** t is firable at M_1,

(ii) $it = M_1$, **otherwise**.

Let us consider Q. We say that Q is firable at M_1 if and only if

(Def. 13) (i) $Q = \emptyset$, or

(ii) there exists a finite sequence M of elements of the markings of P such that $\operatorname{len} Q = \operatorname{len} M$ and Q_1 is firable at M_1 and $M_1 = \text{Firing}(Q_1, M_1)$ and for every i such that $i < \operatorname{len} Q$ and $i > 0$ holds Q_{i+1} is firable at M_i and $M_{i+1} = \text{Firing}(Q_{i+1}, M_i)$.

The functor Firing(Q, M_1) yielding a marking of P is defined by

(Def. 14) (i) $it = M_1$, **if** $Q = \emptyset$,

(ii) there exists a finite sequence M of elements of the markings of P such that $\operatorname{len} Q = \operatorname{len} M$ and $it = M_{\operatorname{len} M}$ and $M_1 = \text{Firing}(Q_1, M_1)$ and for every i such that $i < \operatorname{len} Q$ and $i > 0$ holds $M_{i+1} = \text{Firing}(Q_{i+1}, M_i)$, **otherwise**.

Now we state the propositions:

(11) $\text{Firing}(t, M_1) = \text{Firing}(\langle t\rangle, M_1)$.

(12) t is firable at M_1 if and only if $\langle t\rangle$ is firable at M_1.

(13) $\text{Firing}(Q \frown Q_1, M_1) = \text{Firing}(Q_1, \text{Firing}(Q, M_1))$.

(14) If $Q \frown Q_1$ is firable at M_1, then Q_1 is firable at $\text{Firing}(Q, M_1)$ and Q is firable at M_1.

5. The Theorem Stating that the Number of Tokens in a Circuit Remains the Same After any Firing Sequences

Now we state the proposition:

(15) Let us consider a Petri-like decision-free-like Petri net D_1 with directed path, a directed path finite sequence d of elements of the places and transitions of D_1, and a transition t of D_1. Suppose

(i) d is circular, and

(ii) there exists a place p_1 of D_1 such that $p_1 \in$ the places of d and $\langle p_1, t\rangle \in$ the S-T arcs of D_1 or $\langle t, p_1\rangle \in$ the T-S arcs of D_1.

Then $t \in$ the transitions of d. The theorem is a consequence of (7), (5), (6), and (2).

A decision-free Petri net is a Petri-like decision-free-like Petri net with directed circuit. Let D_1 be a Petri net with directed circuit. Observe that there exists a finite sequence of elements of the places and transitions of D_1 which is directed path, circular, and almost one-to-one.

A circuit of places and transitions of D_1 is a directed path circular almost one-to-one finite sequence of elements of the places and transitions of D_1. Now we state the propositions:

(16) Let us consider a decision-free Petri net D_1, a circuit d of places and transitions of D_1, a marking M_1 of D_1, and a transition t of D_1. Then the number of tokens of the places of d and $M_1 =$ the number of tokens of the places of d and $\mathrm{Firing}(t, M_1)$. The theorem is a consequence of (6), (5), (8), (2), (9), (1), (10), and (15).

(17) Let us consider a decision-free Petri net D_1, a circuit d of places and transitions of D_1, a marking M_1 of D_1, and a finite sequence Q of elements of the carrier' of D_1. Then the number of tokens of the places of d and $M_1 =$ the number of tokens of the places of d and $\mathrm{Firing}(Q, M_1)$. The theorem is a consequence of (16).

References

[1] Grzegorz Bancerek. Cardinal numbers. *Formalized Mathematics*, 1(**2**):377–382, 1990.

[2] Grzegorz Bancerek. The fundamental properties of natural numbers. *Formalized Mathematics*, 1(**1**):41–46, 1990.

[3] Grzegorz Bancerek. The ordinal numbers. *Formalized Mathematics*, 1(**1**):91–96, 1990.

[4] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Formalized Mathematics*, 1(**1**):107–114, 1990.

[5] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.

[6] Czesław Byliński. Functions from a set to a set. *Formalized Mathematics*, 1(**1**):153–164, 1990.

[7] Czesław Byliński. Some basic properties of sets. *Formalized Mathematics*, 1(**1**):47–53, 1990.

[8] Agata Darmochwał. Finite sets. *Formalized Mathematics*, 1(**1**):165–167, 1990.
[9] Pauline N. Kawamoto, Yasushi Fuwa, and Yatsuka Nakamura. Basic Petri net concepts. *Formalized Mathematics*, 3(**2**):183–187, 1992.
[10] Andrzej Kondracki. Basic properties of rational numbers. *Formalized Mathematics*, 1(**5**): 841–845, 1990.
[11] Jarosław Kotowicz. Functions and finite sequences of real numbers. *Formalized Mathematics*, 3(**2**):275–278, 1992.
[12] Robert Milewski. Subsequences of almost, weakly and poorly one-to-one finite sequences. *Formalized Mathematics*, 13(**2**):227–233, 2005.
[13] Karol Pąk. Continuity of barycentric coordinates in Euclidean topological spaces. *Formalized Mathematics*, 19(**3**):139–144, 2011. doi:10.2478/v10037-011-0022-5.
[14] Andrzej Trybulec. On the decomposition of finite sequences. *Formalized Mathematics*, 5 (**3**):317–322, 1996.
[15] Michał J. Trybulec. Integers. *Formalized Mathematics*, 1(**3**):501–505, 1990.
[16] Wojciech A. Trybulec. Non-contiguous substrings and one-to-one finite sequences. *Formalized Mathematics*, 1(**3**):569–573, 1990.
[17] Wojciech A. Trybulec. Pigeon hole principle. *Formalized Mathematics*, 1(**3**):575–579, 1990.
[18] Zinaida Trybulec. Properties of subsets. *Formalized Mathematics*, 1(**1**):67–71, 1990.
[19] Jiacun Wang. *Timed Petri Nets, Theory and Application*. Kluwer Academic Publishers, 1998.
[20] Edmund Woronowicz. Relations and their basic properties. *Formalized Mathematics*, 1 (**1**):73–83, 1990.

Received March 31, 2014