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Some Basic Properties of Some Special Matrices. Part III[1]

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Summary. This article describes definitions of subsymmetric matrix, anti-subsymmetric matrix, central symmetric matrix, symmetry circulant matrix and their basic properties.

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The notation and terminology used here have been introduced in the following papers: [7], [9], [13], [6], [14], [1], [3], [18], [17], [4], [2], [8], [11], [12], [16], [15], [5], and [10].

1. Basic Properties of Subordinate Symmetric Matrices

For simplicity, we use the following convention: n denotes a natural number, K denotes a field, a, b denote elements of K, p, q denote finite sequences of elements of K, and M_1, M_2 denote square matrices over K of dimension n.

Let K be a field, let n be a natural number, and let M be a square matrix over K of dimension n. We say that M is subsymmetric if and only if:

(Def. 1) For all natural numbers i, j, k, l such that $\langle i, j\rangle \in$ the indices of M and $k = (n+1) - j$ and $l = (n+1) - i$ holds $M_{i,j} = M_{k,l}$.

Let us consider n, K, a. Note that $(a)^{n\times n}$ is subsymmetric.

Let us consider n, K. Observe that there exists a square matrix over K of dimension n which is subsymmetric.

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Let us consider n, K and let M be a subsymmetric square matrix over K of dimension n. Note that $-M$ is subsymmetric.

Let us consider n, K and let M_1, M_2 be subsymmetric square matrices over K of dimension n. One can check that $M_1 + M_2$ is subsymmetric.

Let us consider n, K, a and let M be a subsymmetric square matrix over K of dimension n. Note that $a \cdot M$ is subsymmetric.

Let us consider n, K and let M_1, M_2 be subsymmetric square matrices over K of dimension n. One can verify that $M_1 - M_2$ is subsymmetric.

Let us consider n, K and let M be a subsymmetric square matrix over K of dimension n. Observe that M^{T} is subsymmetric.

Let us consider n, K. Observe that every square matrix over K of dimension n which is line circulant is also subsymmetric and every square matrix over K of dimension n which is column circulant is also subsymmetric.

Let K be a field, let n be a natural number, and let M be a square matrix over K of dimension n. We say that M is anti-subsymmetric if and only if:

(Def. 2) For all natural numbers i, j, k, l such that $\langle i, j\rangle \in$ the indices of M and $k = (n+1) - j$ and $l = (n+1) - i$ holds $M_{i,j} = -M_{k,l}$.

Let us consider n, K. One can verify that there exists a square matrix over K of dimension n which is anti-subsymmetric.

The following proposition is true

(1) Let K be a Fanoian field, n, i, j, k, l be natural numbers, and M_1 be a square matrix over K of dimension n. Suppose $\langle i, j\rangle \in$ the indices of M_1 and $i + j = n + 1$ and $k = (n+1) - j$ and $l = (n+1) - i$ and M_1 is anti-subsymmetric. Then $(M_1)_{i,j} = 0_K$.

Let us consider n, K and let M be an anti-subsymmetric square matrix over K of dimension n. Note that $-M$ is anti-subsymmetric.

Let us consider n, K and let M_1, M_2 be anti-subsymmetric square matrices over K of dimension n. Observe that $M_1 + M_2$ is anti-subsymmetric.

Let us consider n, K, a and let M be an anti-subsymmetric square matrix over K of dimension n. One can verify that $a \cdot M$ is anti-subsymmetric.

Let us consider n, K and let M_1, M_2 be anti-subsymmetric square matrices over K of dimension n. One can check that $M_1 - M_2$ is anti-subsymmetric.

Let us consider n, K and let M be an anti-subsymmetric square matrix over K of dimension n. One can verify that M^{T} is anti-subsymmetric.

2. Basic Properties of Central Symmetric Matrices

Let K be a field, let n be a natural number, and let M be a square matrix over K of dimension n. We say that M is central symmetric if and only if:

(Def. 3) For all natural numbers i, j, k, l such that $\langle i, j\rangle \in$ the indices of M and $k = (n+1) - i$ and $l = (n+1) - j$ holds $M_{i,j} = M_{k,l}$.

Let us consider n, K, a. Note that $(a)^{n \times n}$ is central symmetric.

Let us consider n, K. One can verify that there exists a square matrix over K of dimension n which is central symmetric.

Let us consider n, K and let M be a central symmetric square matrix over K of dimension n. One can verify that $-M$ is central symmetric.

Let us consider n, K and let M_1, M_2 be central symmetric square matrices over K of dimension n. One can verify that $M_1 + M_2$ is central symmetric.

Let us consider n, K, a and let M be a central symmetric square matrix over K of dimension n. Note that $a \cdot M$ is central symmetric.

Let us consider n, K and let M_1, M_2 be central symmetric square matrices over K of dimension n. Observe that $M_1 - M_2$ is central symmetric.

Let us consider n, K and let M be a central symmetric square matrix over K of dimension n. Observe that M^{T} is central symmetric.

Let us consider n, K. Note that every square matrix over K of dimension n which is symmetric and subsymmetric is also central symmetric.

3. Basic Properties of Symmetric Circulant Matrices

Let K be a set, let M be a matrix over K, and let p be a finite sequence. We say that M is symmetry circulant about p if and only if the conditions (Def. 4) are satisfied.

(Def. 4)(i) $\operatorname{len} p = \operatorname{width} M$,

(ii) for all natural numbers i, j such that $\langle i, j \rangle \in$ the indices of M and $i + j \neq \operatorname{len} p + 1$ holds $M_{i,j} = p(((i + j) - 1) \bmod \operatorname{len} p)$, and

(iii) for all natural numbers i, j such that $\langle i, j \rangle \in$ the indices of M and $i + j = \operatorname{len} p + 1$ holds $M_{i,j} = p(\operatorname{len} p)$.

The following propositions are true:

(2) $(a)^{n \times n}$ is symmetry circulant about $n \mapsto a$.

(3) If M_1 is symmetry circulant about p, then $a \cdot M_1$ is symmetry circulant about $a \cdot p$.

(4) If M_1 is symmetry circulant about p, then $-M_1$ is symmetry circulant about $-p$.

(5) If M_1 is symmetry circulant about p and M_2 is symmetry circulant about q, then $M_1 + M_2$ is symmetry circulant about $p + q$.

Let K be a set and let M be a matrix over K. We say that M is symmetry circulant if and only if:

(Def. 5) There exists a finite sequence p of elements of K such that $\operatorname{len} p = \operatorname{width} M$ and M is symmetry circulant about p.

Let K be a non empty set and let p be a finite sequence of elements of K. We say that p is first symmetry of circulant if and only if:

(Def. 6) There exists a square matrix over K of dimension $\operatorname{len} p$ which is symmetry circulant about p.

Let K be a non empty set and let p be a finite sequence of elements of K. Let us assume that p is first symmetry of circulant. The functor $\mathrm{SCirc}\, p$ yielding a square matrix over K of dimension $\operatorname{len} p$ is defined as follows:

(Def. 7) $\mathrm{SCirc}\, p$ is symmetry circulant about p.

Let us consider n, K, a. Note that $(a)^{n\times n}$ is symmetry circulant.

Let us consider n, K. Note that there exists a square matrix over K of dimension n which is symmetry circulant.

In the sequel D is a non empty set, t is a finite sequence of elements of D, and A is a square matrix over D of dimension n.

We now state the proposition

(6) Let p be a finite sequence of elements of D. Suppose $0 < n$ and A is symmetry circulant about p. Then A^{T} is symmetry circulant about p.

Let us consider n, K, a and let M_1 be a symmetry circulant square matrix over K of dimension n. Note that $a \cdot M_1$ is symmetry circulant.

Let us consider n, K and let M_1, M_2 be symmetry circulant square matrices over K of dimension n. Note that $M_1 + M_2$ is symmetry circulant.

Let us consider n, K and let M_1 be a symmetry circulant square matrix over K of dimension n. Note that $-M_1$ is symmetry circulant.

Let us consider n, K and let M_1, M_2 be symmetry circulant square matrices over K of dimension n. Observe that $M_1 - M_2$ is symmetry circulant.

The following propositions are true:

(7) If A is symmetry circulant and $n > 0$, then A^{T} is symmetry circulant.

(8) If p is first symmetry of circulant, then $-p$ is first symmetry of circulant.

(9) If p is first symmetry of circulant, then $\mathrm{SCirc}(-p) = -\mathrm{SCirc}\, p$.

(10) Suppose p is first symmetry of circulant and q is first symmetry of circulant and $\operatorname{len} p = \operatorname{len} q$. Then $p + q$ is first symmetry of circulant.

(11) If $\operatorname{len} p = \operatorname{len} q$ and p is first symmetry of circulant and q is first symmetry of circulant, then $\mathrm{SCirc}(p + q) = \mathrm{SCirc}\, p + \mathrm{SCirc}\, q$.

(12) If p is first symmetry of circulant, then $a \cdot p$ is first symmetry of circulant.

(13) If p is first symmetry of circulant, then $\mathrm{SCirc}(a \cdot p) = a \cdot \mathrm{SCirc}\, p$.

(14) If p is first symmetry of circulant, then $a \cdot \mathrm{SCirc}\, p + b \cdot \mathrm{SCirc}\, p = \mathrm{SCirc}((a + b) \cdot p)$.

(15) If p is first symmetry of circulant and q is first symmetry of circulant and $\operatorname{len} p = \operatorname{len} q$, then $a \cdot \mathrm{SCirc}\, p + a \cdot \mathrm{SCirc}\, q = \mathrm{SCirc}(a \cdot (p + q))$.

(16) Suppose p is first symmetry of circulant and q is first symmetry of circulant and $\operatorname{len} p = \operatorname{len} q$. Then $a \cdot \mathrm{SCirc}\, p + b \cdot \mathrm{SCirc}\, q = \mathrm{SCirc}(a \cdot p + b \cdot q)$.

(17) If M_1 is symmetry circulant, then ${M_1}^{\mathrm{T}} = M_1$.

Let us consider n, K. Note that every square matrix over K of dimension n which is symmetry circulant is also symmetric.

References

[1] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Formalized Mathematics*, 1(**1**):107–114, 1990.
[2] Czesław Byliński. Binary operations. *Formalized Mathematics*, 1(**1**):175–180, 1990.
[3] Czesław Byliński. Finite sequences and tuples of elements of a non-empty sets. *Formalized Mathematics*, 1(**3**):529–536, 1990.
[4] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.
[5] Czesław Byliński. Partial functions. *Formalized Mathematics*, 1(**2**):357–367, 1990.
[6] Czesław Byliński. Some basic properties of sets. *Formalized Mathematics*, 1(**1**):47–53, 1990.
[7] Katarzyna Jankowska. Matrices. Abelian group of matrices. *Formalized Mathematics*, 2(**4**):475–480, 1991.
[8] Katarzyna Jankowska. Transpose matrices and groups of permutations. *Formalized Mathematics*, 2(**5**):711–717, 1991.
[9] Eugeniusz Kusak, Wojciech Leończuk, and Michał Muzalewski. Abelian groups, fields and vector spaces. *Formalized Mathematics*, 1(**2**):335–342, 1990.
[10] Karol Pąk. Basic properties of the rank of matrices over a field. *Formalized Mathematics*, 15(**4**):199–211, 2007, doi:10.2478/v10037-007-0024-5.
[11] Andrzej Trybulec. Binary operations applied to functions. *Formalized Mathematics*, 1(**2**):329–334, 1990.
[12] Michał J. Trybulec. Integers. *Formalized Mathematics*, 1(**3**):501–505, 1990.
[13] Wojciech A. Trybulec. Groups. *Formalized Mathematics*, 1(**5**):821–827, 1990.
[14] Zinaida Trybulec. Properties of subsets. *Formalized Mathematics*, 1(**1**):67–71, 1990.
[15] Edmund Woronowicz. Relations and their basic properties. *Formalized Mathematics*, 1(**1**):73–83, 1990.
[16] Xiaopeng Yue, Xiquan Liang, and Zhongpin Sun. Some properties of some special matrices. *Formalized Mathematics*, 13(**4**):541–547, 2005.
[17] Katarzyna Zawadzka. The sum and product of finite sequences of elements of a field. *Formalized Mathematics*, 3(**2**):205–211, 1992.
[18] Katarzyna Zawadzka. The product and the determinant of matrices with entries in a field. *Formalized Mathematics*, 4(**1**):1–8, 1993.

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