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Cayley's Theorem

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Summary. The article formalizes the Cayley's theorem saying that every group G is isomorphic to a subgroup of the symmetric group on G.

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The notation and terminology used in this paper have been introduced in the following papers: [3], [6], [4], [5], [10], [11], [7], [2], [1], [9], and [8].

In this paper X, Y denote sets, G denotes a group, and n denotes a natural number.

Let us consider X. Note that $\emptyset_{X,\emptyset}$ is onto.

Let us observe that every set which is permutational is also functional.

Let us consider X. The functor permutations X is defined as follows:

(Def. 1) $\quad \text{permutations}\, X = \{f : f \text{ ranges over permutations of } X\}$.

Next we state three propositions:

(1) For every set f such that $f \in \text{permutations}\, X$ holds f is a permutation of X.

(2) $\text{permutations}\, X \subseteq X^X$.

(3) $\text{permutations}\, \text{Seg}\, n = \text{the permutations of } n$.

Let us consider X. One can verify that permutations X is non empty and functional.

Let X be a finite set. One can verify that permutations X is finite.

Next we state the proposition

(4) $\text{permutations}\, \emptyset = 1$.

Let us consider X. The functor SymGroup X yields a strict constituted functions multiplicative magma and is defined by:

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(Def. 2) The carrier of SymGroup X = permutations X and for all elements x, y of SymGroup X holds $x \cdot y = (y$ **qua** function$)\ \cdot x$.

One can prove the following proposition

(5) Every element of SymGroup X is a permutation of X.

Let us consider X. Note that SymGroup X is non empty, associative, and group-like.

The following propositions are true:

(6) $\mathbf{1}_{\mathrm{SymGroup}\,X} = \mathrm{id}_X$.

(7) For every element x of SymGroup X holds $x^{-1} = (x$ **qua** function$)\ ^{-1}$.

Let us consider n. One can verify that A_n is constituted functions.

One can prove the following proposition

(8) SymGroup Seg $n = A_n$.

Let X be a finite set. Observe that SymGroup X is finite.

We now state the proposition

(9) SymGroup $\emptyset$ = Trivial-multMagma .

Let us note that SymGroup $\emptyset$ is trivial.

Let us consider X, Y and let p be a function from X into Y. Let us assume that $X \neq \emptyset$ and $Y \neq \emptyset$ and p is bijective. The functor SymGroupsIso p yielding a function from SymGroup X into SymGroup Y is defined by:

(Def. 3) For every element x of SymGroup X holds (SymGroupsIso $p)(x) = p \cdot x \cdot p^{-1}$.

We now state four propositions:

(10) For all non empty sets X, Y and for every function p from X into Y such that p is bijective holds SymGroupsIso p is multiplicative.

(11) For all non empty sets X, Y and for every function p from X into Y such that p is bijective holds SymGroupsIso p is one-to-one.

(12) For all non empty sets X, Y and for every function p from X into Y such that p is bijective holds SymGroupsIso p is onto.

(13) If $X \approx Y$, then SymGroup X and SymGroup Y are isomorphic.

Let us consider G. The functor CayleyIso G yields a function from G into SymGroup (the carrier of G) and is defined as follows:

(Def. 4) For every element g of G holds (CayleyIso $G)(g) = \cdot g$.

Let us consider G. One can verify that CayleyIso G is multiplicative.

Let us consider G. One can verify that CayleyIso G is one-to-one.

One can prove the following proposition

(14) G and Im CayleyIso G are isomorphic.

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