# Integrability Formulas. Part I 

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Summary. In this article, we give several differentiation and integrability formulas of special and composite functions including the trigonometric function, and the polynomial function.

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The papers [12], [2], [3], [1], [7], [11], [13], [4], [17], [8], [9], [6], [18], [5], [10], [15], [16], and [14] provide the terminology and notation for this paper.

One can check that there exists a subset of $\mathbb{R}$ which is closed-interval.
For simplicity, we use the following convention: $a, b, x, r$ are real numbers, $n$ is an element of $\mathbb{N}, A$ is a closed-interval subset of $\mathbb{R}, f, g, f_{1}, f_{2}, g_{1}, g_{2}$ are partial functions from $\mathbb{R}$ to $\mathbb{R}$, and $Z$ is an open subset of $\mathbb{R}$.

We now state a number of propositions:
(1) Suppose $Z \subseteq \operatorname{dom}\left(\frac{1}{f_{1}+f_{2}}\right)$ and for every $x$ such that $x \in Z$ holds $f_{1}(x)=1$ and $f_{2}=\square^{2}$. Then $\frac{1}{f_{1}+f_{2}}$ is differentiable on $Z$ and for every $x$ such that $x \in Z$ holds $\left(\frac{1}{f_{1}+f_{2}}\right)^{\prime} Z(x)=-\frac{2 \cdot x}{\left(1+x^{2}\right)^{2}}$.
(2) Suppose that $A \subseteq Z$ and $f=\frac{\frac{1}{g_{1}+g_{2}}}{f_{2}}$ and $f_{2}=$ the function arccot and $Z \subseteq]-1,1\left[\right.$ and $g_{2}=\square^{2}$ and for every $x$ such that $x \in Z$ holds $g_{1}(x)=1$ and $f_{2}(x)>0$ and $Z=\operatorname{dom} f$. Then $\int_{A} f(x) d x=(-($ the function $\ln ) \cdot($ the function $\operatorname{arccot}))(\sup A)-$ $(-($ the function $\ln ) \cdot($ the function arccot) $)(\inf A)$.
(3) Suppose that
(i) $A \subseteq Z$,
(ii) for every $x$ such that $x \in Z$ holds (the function $\exp )(x)<1$ and $f_{1}(x)=1$,
(iii) $Z \subseteq \operatorname{dom}(($ the function arctan $) \cdot($ the function $\exp ))$,
(iv) $Z=\operatorname{dom} f$, and
(v) $f=\frac{\text { the function } \exp }{f_{1}+(\text { the function } \exp )^{2}}$.

Then $\int_{A} f(x) d x=(($ the function arctan $) \cdot($ the function $\exp ))(\sup A)-$ $(($ the function $\arctan ) \cdot($ the function $\exp ))(\inf A)$.
(4) Suppose that
(i) $A \subseteq Z$,
(ii) for every $x$ such that $x \in Z$ holds (the function $\exp )(x)<1$ and $f_{1}(x)=1$,
(iii) $Z \subseteq \operatorname{dom}(($ the function arccot $) \cdot($ the function $\exp ))$,
(iv) $Z=\operatorname{dom} f$, and
(v) $f=\frac{- \text { the function } \exp }{f_{1}+(\text { the function } \exp )^{2}}$.

Then $\int_{A} f(x) d x=(($ the function arccot $) \cdot($ the function $\exp ))(\sup A)-(($ the function arccot) $\cdot($ the function $\exp ))(\inf A)$.
(5) Suppose that
(i) $A \subseteq Z$,
(ii) $Z=\operatorname{dom} f$, and
(iii) $f=($ the function $\exp ) \frac{\text { the function } \sin }{\text { the function cos }}+\frac{\text { the function } \exp }{(\text { the function } \cos )^{2}}$.

Then $\int_{A} f(x) d x=(($ the function $\exp ) \quad($ the function $\tan ))(\sup A)-(($ the function $\exp )($ the function $\tan ))(\inf A)$.
(6) Suppose that
(i) $A \subseteq Z$,
(ii) $Z=\operatorname{dom} f$, and
(iii) $f=($ the function $\exp ) \frac{\text { the function cos }}{\text { the function sin }}-\frac{\text { the function } \exp }{(\text { the function } \sin )^{2}}$.

Then $\int_{A} f(x) d x=(($ the function $\exp ) \quad$ (the function $\left.\cot )\right)(\sup A)-(($ the function $\exp )($ the function $\cot )(\inf A)$.
(7) Suppose that
(i) $A \subseteq Z$,
(ii) for every $x$ such that $x \in Z$ holds $f_{1}(x)=1$,
(iii) $Z \subseteq]-1,1[$,
(iv) $Z=\operatorname{dom} f$, and
(v) $f=($ the function $\exp )$ (the function $\arctan )+\frac{\text { the function } \exp }{f_{1}+\square^{2}}$.

Then $\int_{A} f(x) d x=(($ the function $\exp ))($ the function $\left.\arctan )\right)(\sup A)-(($ the function $\exp )($ the function $\arctan )(\inf A)$.
(8) Suppose that
(i) $A \subseteq Z$,
(ii) for every $x$ such that $x \in Z$ holds $f_{1}(x)=1$,
(iii) $Z \subseteq]-1,1[$,
(iv) $Z=\operatorname{dom} f$, and
(v) $f=$ (the function $\exp$ ) (the function arccot) $-\frac{\text { the function } \exp }{f_{1}+\square^{2}}$.

Then $\int_{A} f(x) d x=(($ the function $\exp )($ the function arccot $))(\sup A)-(($ the function $\exp )($ the function arccot)) (inf $A)$.
(9) $\quad$ Suppose $A \subseteq Z=\operatorname{dom} f$ and $f=(($ the function $\exp ) \cdot($ the function sin $))$ (the function cos). Then $\int_{A} f(x) d x=(($ the function $\exp ) \cdot($ the function $\sin ))(\sup A)-(($ the function $\exp ) \cdot($ the function $\sin ))(\inf A)$.
(10) Suppose $A \subseteq Z=\operatorname{dom} f$ and $f=$ ((the function $\exp ) \cdot$ (the function $\cos )$ ) (the function $\sin$ ).
Then $\int_{A} f(x) d x=(-($ the function $\exp ) \cdot($ the function $\cos ))(\sup A)-$ $(-($ the function $\exp ) \cdot($ the function $\cos ))(\inf A)$.
(11) Suppose $A \subseteq Z$ and for every $x$ such that $x \in Z$ holds $x>0$ and $Z=\operatorname{dom} f$ and $f=\left((\right.$ the function cos) $\cdot($ the function $\ln )) \frac{1}{\mathrm{id} Z}$. Then $\int_{A} f(x) d x=(($ the function $\sin ) \cdot($ the function $\ln ))(\sup A)-(($ the function sin) $\cdot($ the function $\ln ))(\inf A)$.
(12) Suppose $A \subseteq Z$ and for every $x$ such that $x \in Z$ holds $x>0$ and $Z=\operatorname{dom} f$ and $f=(($ the function $\sin ) \cdot($ (the function $\ln ))$ $\frac{1}{\mathrm{id} Z}$. Then $\int_{A} f(x) d x=(-($ the function $\cos ) \cdot($ the function $\ln ))(\sup A)-$ $(-($ the function cos) $\cdot($ the function $\ln ))(\inf A)$.
(13) Suppose $A \subseteq Z=\operatorname{dom} f$ and $f=$ (the function $\exp$ ) ((the function cos) $\cdot($ the function $\exp ))$. Then $\int_{A} f(x) d x=(($ the function $\sin ) \cdot$ (the function $\exp ))(\sup A)-(($ the function $\sin ) \cdot($ the function $\exp ))(\inf A)$.
(14) Suppose $A \subseteq Z=\operatorname{dom} f$ and $f=$ (the function $\exp$ ) ((the function sin) -(the function $\exp$ )).
Then $\int_{A} f(x) d x=(-($ the function $\cos ) \cdot($ the function $\exp ))(\sup A)-$ $(-($ the function $\cos ) \cdot($ the function $\exp ))(\inf A)$.
(15) Suppose that $A \subseteq Z \subseteq \operatorname{dom}\left((\right.$ the function $\left.\ln ) \cdot\left(f_{1}+f_{2}\right)\right)$ and $r \neq 0$ and for every $x$ such that $x \in Z$ holds $g(x)=\frac{x}{r}$ and $g(x)>-1$ and $g(x)<1$ and $f_{1}(x)=1$ and $f_{2}=\left(\square^{2}\right) \cdot g$ and $Z=\operatorname{dom} f$ and $f=$ (the function arctan) $\cdot g$. Then $\int_{A} f(x) d x=\left(\operatorname{id}_{Z}((\right.$ the function arctan $) \cdot g)-\frac{r}{2}(($ the function $\ln )$ $\left.\left.\cdot\left(f_{1}+f_{2}\right)\right)\right)(\sup A)-\left(\operatorname{id}_{Z}((\right.$ the function arctan $) \cdot g)-\frac{r}{2}(($ the function $\ln )$ $\left.\left.\cdot\left(f_{1}+f_{2}\right)\right)\right)(\inf A)$.
(16) Suppose that $A \subseteq Z \subseteq \operatorname{dom}\left((\right.$ the function $\left.\ln ) \cdot\left(f_{1}+f_{2}\right)\right)$ and $r \neq 0$ and for every $x$ such that $x \in Z$ holds $g(x)=\frac{x}{r}$ and $g(x)>-1$ and $g(x)<1$ and $f_{1}(x)=1$ and $f_{2}=\left(\square^{2}\right) \cdot g$ and $Z=\operatorname{dom} f$ and $f=$ (the function arccot) $\cdot g$. Then $\int_{A} f(x) d x=\left(\right.$ id $_{Z}(($ the function arccot $) \cdot g)+\frac{r}{2}(($ the function $\ln )$ $\left.\left.\cdot\left(f_{1}+f_{2}\right)\right)\right)(\sup A)-\left(\operatorname{id}_{Z}((\right.$ the function arccot $) \cdot g)+\frac{r}{2}(($ the function $\ln )$ $\left.\left.\cdot\left(f_{1}+f_{2}\right)\right)\right)(\inf A)$.
(17) Suppose that
(i) $A \subseteq Z$,
(ii) $f=$ (the function arctan) $\cdot f_{1}+\frac{\mathrm{id} z}{r\left(g+f_{1}{ }^{2}\right)}$,
(iii) for every $x$ such that $x \in Z$ holds $g(x)=1$ and $f_{1}(x)=\frac{x}{r}$ and $f_{1}(x)>-1$ and $f_{1}(x)<1$,
(iv) $Z=\operatorname{dom} f$, and
(v) $f$ is continuous on $A$.

Then $\int_{A} f(x) d x=\left(\operatorname{id}_{Z}\left((\right.\right.$ the function $\left.\left.\arctan ) \cdot f_{1}\right)\right)(\sup A)-\left(\mathrm{id}_{Z}((\right.$ the function arctan) $\left.\left.\cdot f_{1}\right)\right)(\inf A)$.
(18) Suppose that
(i) $A \subseteq Z$,
(ii) $f=$ (the function arccot) $\cdot f_{1}-\frac{\mathrm{id}_{Z}}{r\left(g+f_{1}{ }^{2}\right)}$,
(iii) for every $x$ such that $x \in Z$ holds $g(x)=1$ and $f_{1}(x)=\frac{x}{r}$ and $f_{1}(x)>-1$ and $f_{1}(x)<1$,
(iv) $Z=\operatorname{dom} f$, and
(v) $f$ is continuous on $A$.

Then $\int_{A} f(x) d x=\left(\mathrm{id}_{Z}\left((\right.\right.$ the function arccot $\left.\left.) \cdot f_{1}\right)\right)(\sup A)-\left(\mathrm{id}_{Z}((\right.$ the function arccot) $\left.\left.\cdot f_{1}\right)\right)(\inf A)$.
(19) Suppose that $A \subseteq Z \subseteq]-1,1[$ and for every $x$ such that $x \in Z$ holds $f_{1}(x)=1$ and $Z=\operatorname{dom} f$ and $Z \subseteq \operatorname{dom}\left(\left(\square^{n}\right) \cdot(\right.$ the function $\left.\arcsin )\right)$ and $1<n$ and $f=\frac{n\left(\left(\square^{n-1}\right) \cdot(\text { (the function arcsin })\right)}{\left(\square^{\frac{1}{2}}\right) \cdot\left(f_{1}-\square^{2}\right)}$. Then $\int_{A} f(x) d x=\left(\left(\square^{n}\right) \cdot\right.$ (the function $\arcsin ))(\sup A)-\left(\left(\square^{n}\right) \cdot(\right.$ the function $\left.\arcsin )\right)(\inf A)$.
(20) Suppose that $A \subseteq Z \subseteq]-1,1[$ and for every $x$ such that $x \in Z$ holds

$$
\begin{aligned}
& f_{1}(x)=1 \text { and } Z \subseteq \operatorname{dom}\left(\left(\square^{n}\right) \cdot(\text { the function arccos })\right) \text { and } Z=\operatorname{dom} f \\
& \text { and } 1<n \text { and } f=\frac{n\left(\left(\square^{n-1}\right) \cdot(\text { the function arccos) })\right.}{\left(\square^{\frac{1}{2}}\right) \cdot\left(f_{1}-\square^{2}\right)} \text {. Then } \int_{A} f(x) d x= \\
& \left(-\left(\square^{n}\right) \cdot(\text { the function arccos })\right)(\sup A)-\left(-\left(\square^{n}\right) \cdot(\text { the function } \arccos )\right) \\
& (\inf A) \text {. }
\end{aligned}
$$

(21) Suppose $A \subseteq Z$ and for every $x$ such that $x \in Z$ holds $f_{1}(x)=1$ and $Z \subseteq]-1,1[$ and $Z=\operatorname{dom} f$ and $f=$ (the function $\arcsin )+\frac{\mathrm{id}_{Z}}{\left(\square^{\frac{1}{2}}\right) \cdot\left(f_{1}-\square^{2}\right)}$. Then $\int_{A} f(x) d x=\left(\mathrm{id}_{Z}(\right.$ the function $\left.\arcsin )\right)(\sup A)-\left(\mathrm{id}_{Z}\right.$ (the function $\arcsin ))(\inf A)$.
(22) Suppose $A \subseteq Z$ and for every $x$ such that $x \in Z$ holds $f_{1}(x)=1$ and $Z \subseteq]-1,1[$ and $Z=\operatorname{dom} f$ and $f=$ (the function $\arccos )-\frac{\mathrm{id} Z}{\left(\square^{\frac{1}{2}}\right) \cdot\left(f_{1}-\square^{2}\right)}$. Then $\int_{A} f(x) d x=\left(\operatorname{id}_{Z}(\right.$ the function $\left.\arccos )\right)(\sup A)-\left(\operatorname{id}_{Z}\right.$ (the function $\arccos ))(\inf A)$.
(23) Suppose that
(i) $A \subseteq Z$,
(ii) $Z \subseteq]-1,1[$,
(iii) for every $x$ such that $x \in Z$ holds $f_{1}(x)=a \cdot x+b$ and $f_{2}(x)=1$,
(iv) $Z=\operatorname{dom} f$, and
(v) $f=a$ (the function $\arcsin )+\frac{f_{1}}{\left(\square^{\frac{1}{2}}\right) \cdot\left(f_{2}-\square^{2}\right)}$.

Then $\int_{A} f(x) d x=\left(f_{1}\right.$ (the function $\left.\left.\arcsin \right)\right)(\sup A)-\left(f_{1}\right.$ (the function $\arcsin )(\inf A)$.
(24) Suppose that
(i) $A \subseteq Z$,
(ii) $Z \subseteq]-1,1[$,
(iii) for every $x$ such that $x \in Z$ holds $f_{1}(x)=a \cdot x+b$ and $f_{2}(x)=1$,
(iv) $Z=\operatorname{dom} f$, and
(v) $f=a$ (the function $\arccos )-\frac{f_{1}}{\left(\square^{\frac{1}{2}}\right) \cdot\left(f_{2}-\square^{2}\right)}$.

Then $\int_{A} f(x) d x=\left(f_{1}\right.$ (the function $\left.\left.\arccos \right)\right)(\sup A)-\left(f_{1}\right.$ (the function $\arccos ))(\inf A)$.
(25) Suppose that
(i) $A \subseteq Z$,
(ii) for every $x$ such that $x \in Z$ holds $g(x)=1$ and $f_{1}(x)=\frac{x}{a}$ and $f_{1}(x)>-1$ and $f_{1}(x)<1$,
(iii) $Z=\operatorname{dom} f$,
(iv) $f$ is continuous on $A$, and
(v) $\quad f=($ the function $\arcsin ) \cdot f_{1}+\frac{\mathrm{id}_{Z}}{a\left(\left(\square^{\frac{1}{2}}\right) \cdot\left(g-f_{1}{ }^{2}\right)\right)}$.

Then $\int_{A} f(x) d x=\left(\operatorname{id}_{Z}\left((\right.\right.$ the function $\left.\left.\arcsin ) \cdot f_{1}\right)\right)(\sup A)-\left(\mathrm{id}_{Z}((\right.$ the function $\left.\arcsin ) \cdot f_{1}\right)(\inf A)$.
(26) Suppose that
(i) $A \subseteq Z$,
(ii) for every $x$ such that $x \in Z$ holds $g(x)=1$ and $f_{1}(x)=\frac{x}{a}$ and $f_{1}(x)>-1$ and $f_{1}(x)<1$,
(iii) $Z=\operatorname{dom} f$,
(iv) $f$ is continuous on $A$, and
(v) $f=($ the function $\arccos ) \cdot f_{1}-\frac{\mathrm{id}_{Z}}{a\left(\left(\square^{\frac{1}{2}}\right) \cdot\left(g-f_{1}{ }^{2}\right)\right)}$.

Then $\int_{A} f(x) d x=\left(\operatorname{id}_{Z}\left((\right.\right.$ the function arccos $\left.\left.) \cdot f_{1}\right)\right)(\sup A)-\left(\operatorname{id}_{Z}((\right.$ the function $\left.\left.\arccos ) \cdot f_{1}\right)\right)(\inf A)$.
(27) Suppose $A \subseteq Z$ and $f=\frac{n\left(\left(\square^{n-1}\right) \cdot(\text { the function } \sin )\right)}{\left(\square^{n+1}\right) \cdot(\text { the function } \cos )}$ and $1 \leq n$ and $Z \subseteq$ $\operatorname{dom}\left(\left(\square^{n}\right) \cdot(\right.$ the function $\left.\tan )\right)$ and $Z=\operatorname{dom} f$. Then $\int_{A} f(x) d x=\left(\left(\square^{n}\right) \cdot\right.$ (the function $\tan ))(\sup A)-\left(\left(\square^{n}\right) \cdot(\right.$ the function $\left.\tan )\right)(\inf A)$.
(28) Suppose $A \subseteq Z$ and $f=\frac{n\left(\left(\square^{n-1}\right) \cdot(\text { the function } \cos )\right)}{\left(\square^{n+1}\right) \cdot(\text { the function } \sin )}$ and $1 \leq n$ and $Z \subseteq \operatorname{dom}\left(\left(\square^{n}\right) \cdot(\right.$ the function $\left.\cot )\right)$ and $Z=\operatorname{dom} f$. Then $\int_{A} f(x) d x=$ $\left(-\left(\square^{n}\right) \cdot(\right.$ the function $\left.\cot )\right)(\sup A)-\left(-\left(\square^{n}\right) \cdot(\right.$ the function $\left.\cot )\right)(\inf A)$.
(29) Suppose that
(i) $A \subseteq Z$,
(ii) $Z \subseteq \operatorname{dom}\left((\right.$ the function $\left.\tan ) \cdot f_{1}\right)$,
(iii) $f=\frac{\left((\text { the function } \sin ) \cdot f_{1}\right)^{2}}{\left((\text { the function } \cos ) \cdot f_{1}\right)^{2}}$,
(iv) for every $x$ such that $x \in Z$ holds $f_{1}(x)=a \cdot x$ and $a \neq 0$, and
(v) $Z=\operatorname{dom} f$.

Then $\int_{A} f(x) d x=\left(\frac{1}{a}\left((\right.\right.$ the function $\left.\left.\tan ) \cdot f_{1}\right)-\mathrm{id}_{Z}\right)(\sup A)-\left(\frac{1}{a}((\right.$ the function $\left.\left.\tan ) \cdot f_{1}\right)-\operatorname{id}_{Z}\right)(\inf A)$.
(30) Suppose that
(i) $A \subseteq Z$,
(ii) $Z \subseteq \operatorname{dom}\left((\right.$ the function cot $\left.) \cdot f_{1}\right)$,
(iii) $f=\frac{\left((\text { the function } \cos ) \cdot f_{1}\right)^{2}}{\left.(\text { (the function } \sin ) \cdot f_{1}\right)^{2}}$,
(iv) for every $x$ such that $x \in Z$ holds $f_{1}(x)=a \cdot x$ and $a \neq 0$, and
(v) $Z=\operatorname{dom} f$.

Then $\int_{A} f(x) d x=\left(\left(-\frac{1}{a}\right)\left((\right.\right.$ the function $\left.\left.\cot ) \cdot f_{1}\right)-\operatorname{id}_{Z}\right)(\sup A)-\left(\left(-\frac{1}{a}\right)((\right.$ the function $\left.\left.\cot ) \cdot f_{1}\right)-\operatorname{id}_{Z}\right)(\inf A)$.
(31) Suppose that
(i) $A \subseteq Z$,
(ii) for every $x$ such that $x \in Z$ holds $f_{1}(x)=a \cdot x+b$,
(iii) $Z=\operatorname{dom} f$, and
(iv) $f=a \frac{\text { the function } \sin }{\text { the function } \cos }+\frac{f_{1}}{(\text { the function } \cos )^{2}}$.

Then $\int_{A} f(x) d x=\left(f_{1}(\right.$ the function tan $\left.)\right)(\sup A)-\left(f_{1}(\right.$ the function $\left.\tan )\right)(\inf A)$.
(32) Suppose that
(i) $A \subseteq Z$,
(ii) for every $x$ such that $x \in Z$ holds $f_{1}(x)=a \cdot x+b$,
(iii) $Z=\operatorname{dom} f$, and
(iv) $f=a \frac{\text { the function } \cos }{\text { the function } \sin }-\frac{f_{1}}{(\text { the function } \sin )^{2}}$.

Then $\int_{A} f(x) d x=\left(f_{1}(\right.$ the function $\left.\cot )\right)(\sup A)-\left(f_{1}(\right.$ the function $\left.\cot )\right)(\inf A)$.
(33) Suppose that
(i) $A \subseteq Z$,
(ii) for every $x$ such that $x \in Z$ holds $f(x)=\frac{(\text { the function } \sin )(x)^{2}}{(\text { the function } \cos )(x)^{2}}$,
(iii) $Z \subseteq \operatorname{dom}\left((\right.$ the function $\left.\tan )-\mathrm{id}_{Z}\right)$,
(iv) $Z=\operatorname{dom} f$, and
(v) $f$ is continuous on $A$.

Then $\int_{A} f(x) d x=\left((\right.$ the function $\left.\tan )-\mathrm{id}_{Z}\right)(\sup A)-(($ the function $\left.\tan )-\mathrm{id}_{Z}\right)(\inf A)$.
(34) Suppose that
(i) $A \subseteq Z$,
(ii) for every $x$ such that $x \in Z$ holds $f(x)=\frac{(\text { the function } \cos )(x)^{2}}{(\text { the function } \sin )(x)^{2}}$,
(iii) $Z \subseteq \operatorname{dom}\left(-\right.$ the function $\left.\cot -\mathrm{id}_{Z}\right)$,
(iv) $Z=\operatorname{dom} f$, and
(v) $f$ is continuous on $A$.

Then $\int_{A} f(x) d x=\left(-\right.$ the function $\left.\cot -\mathrm{id}_{Z}\right)(\sup A)-(-$ the function $\cot -$ $\left.\operatorname{id}_{Z}\right)(\inf A)$.
(35) Suppose that
(i) $A \subseteq Z$,
(ii) for every $x$ such that $x \in Z$ holds $f(x)=\frac{1}{x \cdot\left(1+(\text { the function } \ln )(x)^{2}\right)}$ and (the function $\ln )(x)>-1$ and (the function $\ln )(x)<1$,
(iii) $\quad Z \subseteq \operatorname{dom}(($ the function arctan $) \cdot($ the function $\ln ))$,
(iv) $Z=\operatorname{dom} f$, and
(v) $f$ is continuous on $A$.

Then $\int_{A} f(x) d x=(($ the function arctan $) \cdot($ the function $\ln ))(\sup A)-(($ the function $\arctan ) \cdot($ the function $\ln ))(\inf A)$.
(36) Suppose that
(i) $A \subseteq Z$,
(ii) for every $x$ such that $x \in Z$ holds $f(x)=-\frac{1}{x \cdot\left(1+(\text { the function } \ln )(x)^{2}\right)}$ and (the function $\ln )(x)>-1$ and (the function $\ln )(x)<1$,
(iii) $\quad Z \subseteq \operatorname{dom}(($ the function arccot $) \cdot($ the function $\ln ))$,
(iv) $Z=\operatorname{dom} f$, and
(v) $f$ is continuous on $A$.

Then $\int_{A} f(x) d x=(($ the function arccot $) \cdot($ the function $\ln ))(\sup A)-(($ the function arccot) $\cdot($ the function $\ln ))(\inf A)$.
(37) Suppose that
(i) $A \subseteq Z$,
(ii) for every $x$ such that $x \in Z$ holds $f(x)=\frac{a}{\sqrt{1-(a \cdot x+b)^{2}}}$ and $f_{1}(x)=a \cdot x+b$ and $f_{1}(x)>-1$ and $f_{1}(x)<1$,
(iii) $Z \subseteq \operatorname{dom}\left((\right.$ the function $\left.\arcsin ) \cdot f_{1}\right)$,
(iv) $Z=\operatorname{dom} f$, and
(v) $f$ is continuous on $A$.

Then $\int_{A} f(x) d x=\left((\right.$ the function $\left.\arcsin ) \cdot f_{1}\right)(\sup A)-(($ the function arcsin $)$ - $\left.f_{1}\right)(\inf A)$.
(38) Suppose that
(i) $A \subseteq Z$,
(ii) for every $x$ such that $x \in Z$ holds $f(x)=\frac{a}{\sqrt{1-(a \cdot x+b)^{2}}}$ and $f_{1}(x)=a \cdot x+b$ and $f_{1}(x)>-1$ and $f_{1}(x)<1$,
(iii) $Z \subseteq \operatorname{dom}\left((\right.$ the function $\left.\arccos ) \cdot f_{1}\right)$,
(iv) $Z=\operatorname{dom} f$, and
(v) $f$ is continuous on $A$.

Then $\int_{A} f(x) d x=\left(-(\right.$ the function $\left.\arccos ) \cdot f_{1}\right)(\sup A)-(-($ the function $\left.\arccos ) \cdot f_{1}\right)(\inf A)$.
(39) Suppose that $A \subseteq Z$ and $f_{1}=g-f_{2}$ and $f_{2}=\square^{2}$ and for every $x$ such that $x \in Z$ holds $f(x)=x \cdot\left(1-x^{2}\right)^{-\frac{1}{2}}$ and $g(x)=1$ and $f_{1}(x)>0$ and $Z \subseteq \operatorname{dom}\left(\left(\square^{\frac{1}{2}}\right) \cdot f_{1}\right)$ and $Z=\operatorname{dom} f$ and $f$ is continuous on $A$. Then $\int_{A} f(x) d x=$
$\left(-\left(\square^{\frac{1}{2}}\right) \cdot f_{1}\right)(\sup A)-\left(-\left(\square^{\frac{1}{2}}\right) \cdot f_{1}\right)(\inf A)$.
(40) Suppose that $A \subseteq Z$ and $g=f_{1}-f_{2}$ and $f_{2}=\square^{2}$ and for every $x$ such that $x \in Z$ holds $f(x)=x \cdot\left(a^{2}-x^{2}\right)^{-\frac{1}{2}}$ and $f_{1}(x)=a^{2}$ and $g(x)>0$ and $Z \subseteq \operatorname{dom}\left(\left(\square^{\frac{1}{2}}\right) \cdot g\right)$ and $Z=\operatorname{dom} f$ and $f$ is continuous on $A$. Then $\int_{A} f(x) d x=$ $\left(-\left(\square^{\frac{1}{2}}\right) \cdot g\right)(\sup A)-\left(-\left(\square^{\frac{1}{2}}\right) \cdot g\right)(\inf A)$.
(41) Suppose that
(i) $A \subseteq Z$,
(ii) $n>0$,
(iii) for every $x$ such that $x \in Z$ holds $f(x)=\frac{(\text { the function } \cos )(x)}{(\text { the function } \sin )(x)^{n+1}}$ and (the function $\sin )(x) \neq 0$,
(iv) $Z \subseteq \operatorname{dom}\left(\left(\square^{n}\right) \cdot \frac{1}{\text { the function sin }}\right)$,
(v) $Z=\operatorname{dom} f$, and
(vi) $f$ is continuous on $A$.

Then $\int_{A} f(x) d x=\left(\left(-\frac{1}{n}\right)\left(\left(\square^{n}\right) \cdot \frac{1}{\text { the function } \sin }\right)\right)(\sup A)-\left(\left(-\frac{1}{n}\right)\left(\left(\square^{n}\right)\right.\right.$. $\left.\left.\frac{1}{\text { the function } \sin }\right)\right)(\inf A)$.
(42) Suppose that
(i) $A \subseteq Z$,
(ii) $n>0$,
(iii) for every $x$ such that $x \in Z$ holds $f(x)=\frac{(\text { the function } \sin )(x)}{\left(\text { the function cos) }(x)^{n+1}\right.}$ and (the function $\cos )(x) \neq 0$,
(iv) $Z \subseteq \operatorname{dom}\left(\left(\square^{n}\right) \cdot \frac{1}{\text { the function cos }}\right)$,
(v) $Z=\operatorname{dom} f$, and
(vi) $f$ is continuous on $A$.

Then $\int_{A} f(x) d x=\left(\frac{1}{n}\left(\left(\square^{n}\right) \cdot \frac{1}{\text { the function } \cos }\right)\right)(\sup A)-\left(\frac{1}{n}\left(\left(\square^{n}\right)\right.\right.$. $\left.\frac{1}{\text { the function } \cos }\right)(\inf A)$.
(43) Suppose that $A \subseteq Z$ and $f=\frac{\frac{1}{g_{1}+g_{2}}}{f_{2}}$ and $f_{2}=$ the function arccot and $Z \subseteq]-1,1\left[\right.$ and $g_{2}=\square^{2}$ and for every $x$ such that $x \in Z$ holds $f(x)=\frac{1}{\left(1+x^{2}\right) \cdot(\text { the function arccot)(x) }}$ and $g_{1}(x)=1$ and $f_{2}(x)>0$ and $Z=$ $\operatorname{dom} f$. Then $\int_{A} f(x) d x=(-($ the function $\ln ) \cdot($ the function $\operatorname{arccot}))(\sup A)-$ $(-($ the function $\ln ) \cdot($ the function arccot $))(\inf A)$.
(44) Suppose that
(i) $A \subseteq Z$,
(ii) $Z \subseteq]-1,1[$,
(iii) for every $x$ such that $x \in Z$ holds (the function $\arcsin$ ) $(x)>0$ and $f_{1}(x)=1$,
(iv) $Z \subseteq \operatorname{dom}(($ the function $\ln ) \cdot($ the function $\arcsin ))$,
(v) $Z=\operatorname{dom} f$, and
(vi) $\quad f=\frac{1}{\left(\left(\square^{\frac{1}{2}}\right) \cdot\left(f_{1}-\square^{2}\right)\right) \text { (the function arcsin) }}$.

Then $\int_{A} f(x) d x=(($ the function $\ln ) \cdot($ the function $\arcsin ))(\sup A)-(($ the function $\ln ) \cdot($ the function $\arcsin ))(\inf A)$.
(45) Suppose that
(i) $A \subseteq Z$,
(ii) $Z \subseteq]-1,1[$,
(iii) for every $x$ such that $x \in Z$ holds $f_{1}(x)=1$ and (the function $\left.\arccos \right)(x)>0$,
(iv) $Z \subseteq \operatorname{dom}(($ the function $\ln ) \cdot($ the function arccos $))$,
(v) $Z=\operatorname{dom} f$, and
(vi) $\quad f=\frac{1}{\left(\left(\square^{\frac{1}{2}}\right) \cdot\left(f_{1}-\square^{2}\right)\right) \text { (the function arccos) }}$.

Then $\int_{A} f(x) d x=(-($ the function $\ln ) \cdot($ the function $\arccos ))(\sup A)-$
$(-($ the function $\ln ) \cdot($ the function $\arccos ))(\inf A)$.

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