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Cell Petri Net Concepts

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Summary. Based on the Petri net definitions and theorems already formalized in [8], with this article, we developed the concept of "Cell Petri Nets". It is based on [9]. In a cell Petri net we introduce the notions of colors and colored states of a Petri net, connecting mappings for linking two Petri nets, firing rules for transitions, and the synthesis of two or more Petri nets.

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The papers [11], [12], [6], [13], [14], [10], [8], [2], [5], [3], [4], [7], and [1] provide the terminology and notation for this paper.

1. Preliminaries: Thin Cylinder, Locus

Let A be a non empty set, let B be a set, let B_1 be a set, and let y_1 be a function from B_1 into A. Let us assume that $B_1 \subseteq B$. The functor $\mathrm{cylinder}_0(A, B, B_1, y_1)$ yields a non empty subset of A^B and is defined by:

(Def. 1) $\mathrm{cylinder}_0(A, B, B_1, y_1) = \{y : B \to A\colon y \restriction B_1 = y_1\}$.

Let A be a non empty set and let B be a set. A non empty subset of A^B is said to be a thin cylinder of A and B if:

(Def. 2) There exists a subset B_1 of B and there exists a function y_1 from B_1 into A such that B_1 is finite and it $= \mathrm{cylinder}_0(A, B, B_1, y_1)$.

The following propositions are true:

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(1) Let A be a non empty set, B be a set, and D be a thin cylinder of A and B. Then there exists a subset B_1 of B and there exists a function y_1 from B_1 into A such that B_1 is finite and $D = \{y : B \to A\colon y \restriction B_1 = y_1\}$.

(2) Let A_1, A_2 be non empty sets, B be a set, and D_1 be a thin cylinder of A_1 and B. If $A_1 \subseteq A_2$, then there exists a thin cylinder D_2 of A_2 and B such that $D_1 \subseteq D_2$.

Let A be a non empty set and let B be a set. The thin cylinders of A and B constitute a non empty family of subsets of A^B defined by:

(Def. 3) The thin cylinders of A and $B = \{D \subseteq A^B\colon D$ is a thin cylinder of A and $B\}$.

We now state three propositions:

(3) Let A be a non trivial set, B be a set, B_2 be a set, y_2 be a function from B_2 into A, B_3 be a set, and y_3 be a function from B_3 into A. If $B_2 \subseteq B$ and $B_3 \subseteq B$ and $\mathrm{cylinder}_0(A, B, B_2, y_2) = \mathrm{cylinder}_0(A, B, B_3, y_3)$, then $B_2 = B_3$ and $y_2 = y_3$.

(4) Let A_1, A_2 be non empty sets and B_4, B_5 be sets. Suppose $A_1 \subseteq A_2$ and $B_4 \subseteq B_5$. Then there exists a function F from the thin cylinders of A_1 and B_4 into the thin cylinders of A_2 and B_5 such that for every set x if $x \in$ the thin cylinders of A_1 and B_4, then there exists a subset B_1 of B_4 and there exists a function y_2 from B_1 into A_1 and there exists a function y_3 from B_1 into A_2 such that B_1 is finite and $y_2 = y_3$ and $x = \mathrm{cylinder}_0(A_1, B_4, B_1, y_2)$ and $F(x) = \mathrm{cylinder}_0(A_2, B_5, B_1, y_3)$.

(5) Let A_1, A_2 be non empty sets and B_4, B_5 be sets. Then there exists a function G from the thin cylinders of A_2 and B_5 into the thin cylinders of A_1 and B_4 such that for every set x if $x \in$ the thin cylinders of A_2 and B_5, then there exists a subset B_3 of B_5 and there exists a subset B_2 of B_4 and there exists a function y_2 from B_2 into A_1 and there exists a function y_3 from B_3 into A_2 such that B_2 is finite and B_3 is finite and $B_2 = B_4 \cap B_3 \cap {y_3}^{-1}(A_1)$ and $y_2 = y_3 \restriction B_2$ and $x = \mathrm{cylinder}_0(A_2, B_5, B_3, y_3)$ and $G(x) = \mathrm{cylinder}_0(A_1, B_4, B_2, y_2)$.

Let A_1, A_2 be non trivial sets and let B_4, B_5 be sets. Let us assume that there exist sets x, y such that $x \neq y$ and x, $y \in A_1$ and $A_1 \subseteq A_2$ and $B_4 \subseteq B_5$. The functor $\mathrm{Extcylinders}(A_1, B_4, A_2, B_5)$ yielding a function from the thin cylinders of A_1 and B_4 into the thin cylinders of A_2 and B_5 is defined by the condition (Def. 4).

(Def. 4) Let x be a set. Suppose $x \in$ the thin cylinders of A_1 and B_4. Then there exists a subset B_1 of B_4 and there exists a function y_2 from B_1 into A_1 and there exists a function y_3 from B_1 into A_2 such that B_1 is finite and $y_2 = y_3$ and $x = \mathrm{cylinder}_0(A_1, B_4, B_1, y_2)$ and $(\mathrm{Extcylinders}(A_1, B_4, A_2, B_5))(x) = \mathrm{cylinder}_0(A_2, B_5, B_1, y_3)$.

Let A_1 be a non empty set, let A_2 be a non trivial set, and let B_4, B_5 be sets. Let us assume that $A_1 \subseteq A_2$ and $B_4 \subseteq B_5$. The functor $\mathrm{Ristcylinders}(A_1, B_4, A_2, B_5)$ yields a function from the thin cylinders of A_2 and B_5 into the thin cylinders of A_1 and B_4 and is defined by the condition (Def. 5).

(Def. 5) Let x be a set. Suppose $x \in$ the thin cylinders of A_2 and B_5. Then there exists a subset B_3 of B_5 and there exists a subset B_2 of B_4 and there exists a function y_2 from B_2 into A_1 and there exists a function y_3 from B_3 into A_2 such that B_2 is finite and B_3 is finite and $B_2 = B_4 \cap B_3 \cap y_3{}^{-1}(A_1)$ and $y_2 = y_3 \restriction B_2$ and $x = \mathrm{cylinder}_0(A_2, B_5, B_3, y_3)$ and $(\mathrm{Ristcylinders}(A_1, B_4, A_2, B_5))(x) = \mathrm{cylinder}_0(A_1, B_4, B_2, y_2)$.

Let A be a non trivial set, let B be a set, and let D be a thin cylinder of A and B. The functor $\mathrm{loc}\, D$ yielding a finite subset of B is defined by the condition (Def. 6).

(Def. 6) There exists a subset B_1 of B and there exists a function y_1 from B_1 into A such that B_1 is finite and $D = \{y : B \to A\colon y \restriction B_1 = y_1\}$ and $\mathrm{loc}\, D = B_1$.

2. Colored Petri Nets

Let A_1, A_2 be non trivial sets, let B_4, B_5 be sets, let C_1, C_2 be non trivial sets, let D_1, D_2 be sets, and let F be a function from the thin cylinders of A_1 and B_4 into the thin cylinders of C_1 and D_1. The functor $\mathrm{CylinderFunc}(A_1, B_4, A_2, B_5, C_1, D_1, C_2, D_2, F)$ yielding a function from the thin cylinders of A_2 and B_5 into the thin cylinders of C_2 and D_2 is defined as follows:

(Def. 7) $\mathrm{CylinderFunc}(A_1, B_4, A_2, B_5, C_1, D_1, C_2, D_2, F) = \mathrm{Extcylinders}(C_1, D_1, C_2, D_2) \cdot F \cdot \mathrm{Ristcylinders}(A_1, B_4, A_2, B_5)$.

We consider colored place/transition net structures as extensions of place/transition net structure as systems

$\langle$ places, transitions, S-T arcs, T-S arcs, a colored set, a firing-rule $\rangle$,

where the places and the transitions constitute non empty sets, the S-T arcs constitute a non empty relation between the places and the transitions, the T-S arcs constitute a non empty relation between the transitions and the places, the colored set is a non empty finite set, and the firing-rule is a function.

Let C_3 be a colored place/transition net structure and let t_0 be a transition of C_3. We say that t_0 is outbound if and only if:

(Def. 8) $\overline{\{t_0\}} = \emptyset$.

Let C_4 be a colored place/transition net structure. The functor $\mathrm{Outbds}\, C_4$ yielding a subset of the transitions of C_4 is defined by:

(Def. 9) $\mathrm{Outbds}\, C_4 = \{x; x$ ranges over transitions of C_4: x is outbound$\}$.

Let C_3 be a colored place/transition net structure. We say that C_3 is colored-PT-net-like if and only if the conditions (Def. 10) are satisfied.

(Def. 10)(i) dom (the firing-rule of C_3) $\subseteq$ (the transitions of C_3) $\setminus$ Outbds C_3, and

(ii) for every transition t of C_3 such that $t \in$ dom (the firing-rule of C_3) there exists a non empty subset C_5 of the colored set of C_3 and there exists a subset I of ${}^*\{t\}$ and there exists a subset O of $\overline{\{t\}}$ such that (the firing-rule of C_3)(t) is a function from the thin cylinders of C_5 and I into the thin cylinders of C_5 and O.

We now state two propositions:

(6) Let C_3 be a colored place/transition net structure and t be a transition of C_3. Suppose C_3 is colored-PT-net-like and $t \in$ dom (the firing-rule of C_3). Then there exists a non empty subset C_5 of the colored set of C_3 and there exists a subset I of ${}^*\{t\}$ and there exists a subset O of $\overline{\{t\}}$ such that (the firing-rule of C_3)(t) is a function from the thin cylinders of C_5 and I into the thin cylinders of C_5 and O.

(7) Let C_4, C_6 be colored place/transition net structures, t_1 be a transition of C_4, and t_2 be a transition of C_6. Suppose that

(i) the places of $C_4 \subseteq$ the places of C_6,

(ii) the transitions of $C_4 \subseteq$ the transitions of C_6,

(iii) the S-T arcs of $C_4 \subseteq$ the S-T arcs of C_6,

(iv) the T-S arcs of $C_4 \subseteq$ the T-S arcs of C_6, and

(v) $t_1 = t_2$.

Then ${}^*\{t_1\} \subseteq {}^*\{t_2\}$ and $\overline{\{t_1\}} \subseteq \overline{\{t_2\}}$.

One can verify that there exists a colored place/transition net structure which is strict and colored-PT-net-like.

A colored place/transition net is a colored-PT-net-like colored place/transition net structure.

3. Color Counts of CPNT

Let C_4, C_6 be colored place/transition net structures. We say that C_4 misses C_6 if and only if:

(Def. 11) (The places of C_4) $\cap$ (the places of C_6) $= \emptyset$ and (the transitions of C_4) $\cap$ (the transitions of C_6) $= \emptyset$.

Let us note that the predicate C_4 misses C_6 is symmetric.

4. Colored States of CPNT

Let C_4 be a colored place/transition net structure and let C_6 be a colored place/transition net structure. Connecting mapping of C_4 and C_6 is defined by the condition (Def. 12).

(Def. 12) There exists a function O_{12} from Outbds C_4 into the places of C_6 and there exists a function O_{21} from Outbds C_6 into the places of C_4 such that it $= \langle O_{12}, O_{21} \rangle$.

5. Outbound Transitions of CPNT

Let C_4, C_6 be colored place/transition nets and let O be a connecting mapping of C_4 and C_6. Connecting firing rule of C_4, C_6, and O is defined by the condition (Def. 13).

(Def. 13) There exist functions q_{12}, q_{21} and there exists a function O_{12} from Outbds C_4 into the places of C_6 and there exists a function O_{21} from Outbds C_6 into the places of C_4 such that

(i) $O = \langle O_{12}, O_{21} \rangle$,

(ii) $\operatorname{dom} q_{12} = \text{Outbds}\, C_4$,

(iii) $\operatorname{dom} q_{21} = \text{Outbds}\, C_6$,

(iv) for every transition t_3 of C_4 such that t_3 is outbound holds $q_{12}(t_3)$ is a function from the thin cylinders of the colored set of C_4 and $^*\{t_3\}$ into the thin cylinders of the colored set of C_4 and $O_{12}{}^\circ t_3$,

(v) for every transition t_4 of C_6 such that t_4 is outbound holds $q_{21}(t_4)$ is a function from the thin cylinders of the colored set of C_6 and $^*\{t_4\}$ into the thin cylinders of the colored set of C_6 and $O_{21}{}^\circ t_4$, and

(vi) it $= \langle q_{12}, q_{21} \rangle$.

6. Connecting Mapping for CPNT1, CPNT2

Let C_4, C_6 be colored place/transition nets, let O be a connecting mapping of C_4 and C_6, and let q be a connecting firing rule of C_4, C_6, and O. Let us assume that C_4 misses C_6. The functor synthesis(C_4, C_6, O, q) yielding a strict colored place/transition net is defined by the condition (Def. 14).

(Def. 14) There exist functions q_{12}, q_{21} and there exists a function O_{12} from Outbds C_4 into the places of C_6 and there exists a function O_{21} from Outbds C_6 into the places of C_4 such that
$O = \langle O_{12}, O_{21} \rangle$ and $\operatorname{dom} q_{12} = \text{Outbds}\, C_4$ and $\operatorname{dom} q_{21} = \text{Outbds}\, C_6$ and for every transition t_3 of C_4 such that t_3 is outbound holds $q_{12}(t_3)$ is a function from the thin cylinders of the colored set of C_4 and $^*\{t_3\}$ into the thin cylinders of the colored set of C_4 and $O_{12}{}^\circ t_3$ and for every transition t_4 of C_6 such that t_4 is outbound holds $q_{21}(t_4)$ is a function from the thin cylinders of the colored set of C_6 and $^*\{t_4\}$ into the thin cylinders of the colored set of C_6 and $O_{21}{}^\circ t_4$ and $q = \langle q_{12}, q_{21} \rangle$ and the places of synthesis$(C_4, C_6, O, q) =$ (the places of C_4) $\cup$ (the places of C_6) and the

transitions of synthesis(C_4, C_6, O, q) = (the transitions of C_4)$\cup$(the transitions of C_6) and the S-T arcs of synthesis(C_4, C_6, O, q) = (the S-T arcs of C_4)$\cup$(the S-T arcs of C_6) and the T-S arcs of synthesis(C_4, C_6, O, q) = (the T-S arcs of C_4) $\cup$ (the T-S arcs of C_6) $\cup O_{12} \cup O_{21}$ and the colored set of synthesis(C_4, C_6, O, q) = (the colored set of C_4) $\cup$ (the colored set of C_6) and the firing-rule of synthesis(C_4, C_6, O, q) = (the firing-rule of C_4)$+\cdot$(the firing-rule of C_6)$+\cdot q_{12} + \cdot q_{21}$.

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