

Inverse Trigonometric Functions Arcsec and Arccosec

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Summary. This article describes definitions of inverse trigonometric functions arcsec and arccosec, as well as their main properties.

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The papers [1], [2], [16], [3], [12], [17], [13], [5], [8], [11], [14], [4], [6], [7], [10], [15], and [9] provide the notation and terminology for this paper.

In this paper x, r denote real numbers.

The following propositions are true:

- (1) $[0, \frac{\pi}{2}] \subseteq \text{dom}(\text{the function sec})$.
- (2) $]\frac{\pi}{2}, \pi] \subseteq \text{dom}(\text{the function sec})$.
- (3) $[-\frac{\pi}{2}, 0[\subseteq \text{dom}(\text{the function cosec})$.
- (4) $]0, \frac{\pi}{2}] \subseteq \text{dom}(\text{the function cosec})$.
- (5) The function sec is differentiable on $]0, \frac{\pi}{2}[$ and for every x such that $x \in]0, \frac{\pi}{2}[$ holds $(\text{the function sec})'(x) = \frac{\sin x}{(\cos x)^2}$.
- (6) The function sec is differentiable on $]\frac{\pi}{2}, \pi[$ and for every x such that $x \in]\frac{\pi}{2}, \pi[$ holds $(\text{the function sec})'(x) = \frac{\sin x}{(\cos x)^2}$.
- (7)(i) The function cosec is differentiable on $]-\frac{\pi}{2}, 0[$, and

- (ii) for every x such that $x \in]-\frac{\pi}{2}, 0[$ holds (the function cosec)'(x) = $-\frac{\cos x}{(\sin x)^2}$.
- (8)(i) The function cosec is differentiable on $]0, \frac{\pi}{2}[$, and
- (ii) for every x such that $x \in]0, \frac{\pi}{2}[$ holds (the function cosec)'(x) = $-\frac{\cos x}{(\sin x)^2}$.
- (9) The function sec is continuous on $]0, \frac{\pi}{2}[$.
- (10) The function sec is continuous on $]\frac{\pi}{2}, \pi[$.
- (11) The function cosec is continuous on $]-\frac{\pi}{2}, 0[$.
- (12) The function cosec is continuous on $]0, \frac{\pi}{2}[$.
- (13) The function sec is increasing on $]0, \frac{\pi}{2}[$.
- (14) The function sec is increasing on $]\frac{\pi}{2}, \pi[$.
- (15) The function cosec is decreasing on $]-\frac{\pi}{2}, 0[$.
- (16) The function cosec is decreasing on $]0, \frac{\pi}{2}[$.
- (17) The function sec is increasing on $[0, \frac{\pi}{2}[$.
- (18) The function sec is increasing on $]\frac{\pi}{2}, \pi]$.
- (19) The function cosec is decreasing on $[-\frac{\pi}{2}, 0[$.
- (20) The function cosec is decreasing on $]0, \frac{\pi}{2}]$.
- (21) (The function sec) $\restriction [0, \frac{\pi}{2}[$ is one-to-one.
- (22) (The function sec) $\restriction]\frac{\pi}{2}, \pi]$ is one-to-one.
- (23) (The function cosec) $\restriction [-\frac{\pi}{2}, 0[$ is one-to-one.
- (24) (The function cosec) $\restriction]0, \frac{\pi}{2}]$ is one-to-one.

One can verify the following observations:

- * (the function sec) $\restriction [0, \frac{\pi}{2}[$ is one-to-one,
- * (the function sec) $\restriction]\frac{\pi}{2}, \pi]$ is one-to-one,
- * (the function cosec) $\restriction [-\frac{\pi}{2}, 0[$ is one-to-one, and
- * (the function cosec) $\restriction]0, \frac{\pi}{2}]$ is one-to-one.

The partial function the 1st part of arcsec from \mathbb{R} to \mathbb{R} is defined as follows:

(Def. 1) The 1st part of arcsec = ((the function sec) $\restriction [0, \frac{\pi}{2}[$) $^{-1}$.

The partial function the 2nd part of arcsec from \mathbb{R} to \mathbb{R} is defined as follows:

(Def. 2) The 2nd part of arcsec = ((the function sec) $\restriction]\frac{\pi}{2}, \pi]$) $^{-1}$.

The partial function the 1st part of arccosec from \mathbb{R} to \mathbb{R} is defined by:

(Def. 3) The 1st part of arccosec = ((the function cosec) $\restriction [-\frac{\pi}{2}, 0[$) $^{-1}$.

The partial function the 2nd part of arccosec from \mathbb{R} to \mathbb{R} is defined by:

(Def. 4) The 2nd part of arccosec = ((the function cosec) $\restriction]0, \frac{\pi}{2}]$) $^{-1}$.

Let r be a real number. The functor arcsec₁ r is defined by:

(Def. 5) arcsec₁ r = (the 1st part of arcsec)(r).

The functor arcsec₂ r is defined as follows:

(Def. 6) $\operatorname{arcsec}_2 r = (\text{the 2nd part of arcsec})(r)$.

The functor $\operatorname{arccosec}_1 r$ is defined as follows:

(Def. 7) $\operatorname{arccosec}_1 r = (\text{the 1st part of arccosec})(r)$.

The functor $\operatorname{arccosec}_2 r$ is defined by:

(Def. 8) $\operatorname{arccosec}_2 r = (\text{the 2nd part of arccosec})(r)$.

Let r be a real number. Then $\operatorname{arcsec}_1 r$ is a real number. Then $\operatorname{arcsec}_2 r$ is a real number. Then $\operatorname{arccosec}_1 r$ is a real number. Then $\operatorname{arccosec}_2 r$ is a real number.

We now state four propositions:

- (25) $\operatorname{rng}(\text{the 1st part of arcsec}) = [0, \frac{\pi}{2}[$.
- (26) $\operatorname{rng}(\text{the 2nd part of arcsec}) =]\frac{\pi}{2}, \pi]$.
- (27) $\operatorname{rng}(\text{the 1st part of arccosec}) = [-\frac{\pi}{2}, 0[$.
- (28) $\operatorname{rng}(\text{the 2nd part of arccosec}) =]0, \frac{\pi}{2}]$.

One can check the following observations:

- * the 1st part of arcsec is one-to-one,
- * the 2nd part of arcsec is one-to-one,
- * the 1st part of arccosec is one-to-one, and
- * the 2nd part of arccosec is one-to-one.

Let t_1 be a real number. Then $\sec t_1$ is a real number. Then $\operatorname{cosec} t_1$ is a real number.

We now state a number of propositions:

- (29) $\sin(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$ and $\cos(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$.
- (30) $\sin(-\frac{\pi}{4}) = -\frac{1}{\sqrt{2}}$ and $\cos(-\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$ and $\sin(\frac{3}{4} \cdot \pi) = \frac{1}{\sqrt{2}}$ and $\cos(\frac{3}{4} \cdot \pi) = -\frac{1}{\sqrt{2}}$.
- (31) $\sec 0 = 1$ and $\sec(\frac{\pi}{4}) = \sqrt{2}$ and $\sec(\frac{3}{4} \cdot \pi) = -\sqrt{2}$ and $\sec \pi = -1$.
- (32) $\operatorname{cosec}(-\frac{\pi}{2}) = -1$ and $\operatorname{cosec}(-\frac{\pi}{4}) = -\sqrt{2}$ and $\operatorname{cosec}(\frac{\pi}{4}) = \sqrt{2}$ and $\operatorname{cosec}(\frac{\pi}{2}) = 1$.
- (33) For every set x such that $x \in [0, \frac{\pi}{4}]$ holds $\sec x \in [1, \sqrt{2}]$.
- (34) For every set x such that $x \in [\frac{3}{4} \cdot \pi, \pi]$ holds $\sec x \in [-\sqrt{2}, -1]$.
- (35) For every set x such that $x \in [-\frac{\pi}{2}, -\frac{\pi}{4}]$ holds $\operatorname{cosec} x \in [-\sqrt{2}, -1]$.
- (36) For every set x such that $x \in [\frac{\pi}{4}, \frac{\pi}{2}]$ holds $\operatorname{cosec} x \in [1, \sqrt{2}]$.
- (37) The function \sec is continuous on $[0, \frac{\pi}{2}[$.
- (38) The function \sec is continuous on $] \frac{\pi}{2}, \pi]$.
- (39) The function cosec is continuous on $[-\frac{\pi}{2}, 0[$.
- (40) The function cosec is continuous on $]0, \frac{\pi}{2}]$.
- (41) $\operatorname{rng}((\text{the function sec}) \upharpoonright [0, \frac{\pi}{4}]) = [1, \sqrt{2}]$.
- (42) $\operatorname{rng}((\text{the function sec}) \upharpoonright [\frac{3}{4} \cdot \pi, \pi]) = [-\sqrt{2}, -1]$.

$$(43) \quad \text{rng}((\text{the function cosec}) \upharpoonright [-\frac{\pi}{2}, -\frac{\pi}{4}]) = [-\sqrt{2}, -1].$$

$$(44) \quad \text{rng}((\text{the function cosec}) \upharpoonright [\frac{\pi}{4}, \frac{\pi}{2}]) = [1, \sqrt{2}].$$

$$(45) \quad [1, \sqrt{2}] \subseteq \text{dom}(\text{the 1st part of arcsec}).$$

$$(46) \quad [-\sqrt{2}, -1] \subseteq \text{dom}(\text{the 2nd part of arcsec}).$$

$$(47) \quad [-\sqrt{2}, -1] \subseteq \text{dom}(\text{the 1st part of arccosec}).$$

$$(48) \quad [1, \sqrt{2}] \subseteq \text{dom}(\text{the 2nd part of arccosec}).$$

One can check the following observations:

- * (the function sec) $\upharpoonright [0, \frac{\pi}{4}]$ is one-to-one,
- * (the function sec) $\upharpoonright [\frac{3}{4} \cdot \pi, \pi]$ is one-to-one,
- * (the function cosec) $\upharpoonright [-\frac{\pi}{2}, -\frac{\pi}{4}]$ is one-to-one, and
- * (the function cosec) $\upharpoonright [\frac{\pi}{4}, \frac{\pi}{2}]$ is one-to-one.

One can prove the following propositions:

$$(49) \quad (\text{The 1st part of arcsec}) \upharpoonright [1, \sqrt{2}] = ((\text{the function sec}) \upharpoonright [0, \frac{\pi}{4}])^{-1}.$$

$$(50) \quad (\text{The 2nd part of arcsec}) \upharpoonright [-\sqrt{2}, -1] = ((\text{the function sec}) \upharpoonright [\frac{3}{4} \cdot \pi, \pi])^{-1}.$$

$$(51) \quad (\text{The 1st part of arccosec}) \upharpoonright [-\sqrt{2}, -1] = ((\text{the function cosec}) \upharpoonright [-\frac{\pi}{2}, -\frac{\pi}{4}])^{-1}.$$

$$(52) \quad (\text{The 2nd part of arccosec}) \upharpoonright [1, \sqrt{2}] = ((\text{the function cosec}) \upharpoonright [\frac{\pi}{4}, \frac{\pi}{2}])^{-1}.$$

$$(53) \quad ((\text{The function sec}) \upharpoonright [0, \frac{\pi}{4}] \text{ qua function}) \cdot ((\text{the 1st part of arcsec}) \upharpoonright [1, \sqrt{2}]) = \text{id}_{[1, \sqrt{2}]}.$$

$$(54) \quad ((\text{The function sec}) \upharpoonright [\frac{3}{4} \cdot \pi, \pi] \text{ qua function}) \cdot ((\text{the 2nd part of arcsec}) \upharpoonright [-\sqrt{2}, -1]) = \text{id}_{[-\sqrt{2}, -1]}.$$

$$(55) \quad ((\text{The function cosec}) \upharpoonright [-\frac{\pi}{2}, -\frac{\pi}{4}] \text{ qua function}) \cdot ((\text{the 1st part of arccosec}) \upharpoonright [-\sqrt{2}, -1]) = \text{id}_{[-\sqrt{2}, -1]}.$$

$$(56) \quad ((\text{The function cosec}) \upharpoonright [\frac{\pi}{4}, \frac{\pi}{2}] \text{ qua function}) \cdot ((\text{the 2nd part of arccosec}) \upharpoonright [1, \sqrt{2}]) = \text{id}_{[1, \sqrt{2}]}.$$

$$(57) \quad ((\text{The function sec}) \upharpoonright [0, \frac{\pi}{4}]) \cdot ((\text{the 1st part of arcsec}) \upharpoonright [1, \sqrt{2}]) = \text{id}_{[1, \sqrt{2}]}.$$

$$(58) \quad ((\text{The function sec}) \upharpoonright [\frac{3}{4} \cdot \pi, \pi]) \cdot ((\text{the 2nd part of arcsec}) \upharpoonright [-\sqrt{2}, -1]) = \text{id}_{[-\sqrt{2}, -1]}.$$

$$(59) \quad ((\text{The function cosec}) \upharpoonright [-\frac{\pi}{2}, -\frac{\pi}{4}]) \cdot ((\text{the 1st part of arccosec}) \upharpoonright [-\sqrt{2}, -1]) = \text{id}_{[-\sqrt{2}, -1]}.$$

$$(60) \quad ((\text{The function cosec}) \upharpoonright [\frac{\pi}{4}, \frac{\pi}{2}]) \cdot ((\text{the 2nd part of arccosec}) \upharpoonright [1, \sqrt{2}]) = \text{id}_{[1, \sqrt{2}]}.$$

$$(61) \quad (\text{The 1st part of arcsec} \text{ qua function}) \cdot ((\text{the function sec}) \upharpoonright [0, \frac{\pi}{2}]) = \text{id}_{[0, \frac{\pi}{2}]}$$

$$(62) \quad (\text{The 2nd part of arcsec} \text{ qua function}) \cdot ((\text{the function sec}) \upharpoonright [\frac{\pi}{2}, \pi]) = \text{id}_{[\frac{\pi}{2}, \pi]}.$$

$$(63) \quad (\text{The 1st part of arccosec} \text{ qua function}) \cdot ((\text{the function cosec}) \upharpoonright [-\frac{\pi}{2}, 0]) = \text{id}_{[-\frac{\pi}{2}, 0]}.$$

- (64) (The 2nd part of $\operatorname{arccosec}$ **qua** function) $\cdot ((\text{the function cosec}) \upharpoonright [0, \frac{\pi}{2}]) = \operatorname{id}_{]0, \frac{\pi}{2}[}$.
- (65) (The 1st part of arcsec) $\cdot ((\text{the function sec}) \upharpoonright [0, \frac{\pi}{2}[) = \operatorname{id}_{[0, \frac{\pi}{2}[}$.
- (66) (The 2nd part of arcsec) $\cdot ((\text{the function sec}) \upharpoonright]\frac{\pi}{2}, \pi]) = \operatorname{id}_{]\frac{\pi}{2}, \pi]}$.
- (67) (The 1st part of $\operatorname{arccosec}$) $\cdot ((\text{the function cosec}) \upharpoonright [-\frac{\pi}{2}, 0[) = \operatorname{id}_{[-\frac{\pi}{2}, 0[}$.
- (68) (The 2nd part of $\operatorname{arccosec}$) $\cdot ((\text{the function cosec}) \upharpoonright [0, \frac{\pi}{2}]) = \operatorname{id}_{[0, \frac{\pi}{2}]}$.
- (69) If $0 \leq r < \frac{\pi}{2}$, then $\operatorname{arcsec}_1 \sec r = r$.
- (70) If $\frac{\pi}{2} < r \leq \pi$, then $\operatorname{arcsec}_2 \sec r = r$.
- (71) If $-\frac{\pi}{2} \leq r < 0$, then $\operatorname{arccosec}_1 \operatorname{cosec} r = r$.
- (72) If $0 < r \leq \frac{\pi}{2}$, then $\operatorname{arccosec}_2 \operatorname{cosec} r = r$.
- (73) $\operatorname{arcsec}_1 1 = 0$ and $\operatorname{arcsec}_1 \sqrt{2} = \frac{\pi}{4}$.
- (74) $\operatorname{arcsec}_2(-\sqrt{2}) = \frac{3}{4} \cdot \pi$ and $\operatorname{arcsec}_2(-1) = \pi$.
- (75) $\operatorname{arccosec}_1(-1) = -\frac{\pi}{2}$ and $\operatorname{arccosec}_1(-\sqrt{2}) = -\frac{\pi}{4}$.
- (76) $\operatorname{arccosec}_2 \sqrt{2} = \frac{\pi}{4}$ and $\operatorname{arccosec}_2 1 = \frac{\pi}{2}$.
- (77) The 1st part of arcsec is increasing on $(\text{the function sec})^\circ [0, \frac{\pi}{2}[$.
- (78) The 2nd part of arcsec is increasing on $(\text{the function sec})^\circ]\frac{\pi}{2}, \pi]$.
- (79) The 1st part of $\operatorname{arccosec}$ is decreasing on $(\text{the function cosec})^\circ [-\frac{\pi}{2}, 0[$.
- (80) The 2nd part of $\operatorname{arccosec}$ is decreasing on $(\text{the function cosec})^\circ [0, \frac{\pi}{2}]$.
- (81) The 1st part of arcsec is increasing on $[1, \sqrt{2}]$.
- (82) The 2nd part of arcsec is increasing on $[-\sqrt{2}, -1]$.
- (83) The 1st part of $\operatorname{arccosec}$ is decreasing on $[-\sqrt{2}, -1]$.
- (84) The 2nd part of $\operatorname{arccosec}$ is decreasing on $[1, \sqrt{2}]$.
- (85) For every set x such that $x \in [1, \sqrt{2}]$ holds $\operatorname{arcsec}_1 x \in [0, \frac{\pi}{4}]$.
- (86) For every set x such that $x \in [-\sqrt{2}, -1]$ holds $\operatorname{arcsec}_2 x \in [\frac{3}{4} \cdot \pi, \pi]$.
- (87) For every set x such that $x \in [-\sqrt{2}, -1]$ holds $\operatorname{arccosec}_1 x \in [-\frac{\pi}{2}, -\frac{\pi}{4}]$.
- (88) For every set x such that $x \in [1, \sqrt{2}]$ holds $\operatorname{arccosec}_2 x \in [\frac{\pi}{4}, \frac{\pi}{2}]$.
- (89) If $1 \leq r \leq \sqrt{2}$, then $\sec \operatorname{arcsec}_1 r = r$.
- (90) If $-\sqrt{2} \leq r \leq -1$, then $\sec \operatorname{arcsec}_2 r = r$.
- (91) If $-\sqrt{2} \leq r \leq -1$, then $\operatorname{cosec} \operatorname{arccosec}_1 r = r$.
- (92) If $1 \leq r \leq \sqrt{2}$, then $\operatorname{cosec} \operatorname{arccosec}_2 r = r$.
- (93) The 1st part of arcsec is continuous on $[1, \sqrt{2}]$.
- (94) The 2nd part of arcsec is continuous on $[-\sqrt{2}, -1]$.
- (95) The 1st part of $\operatorname{arccosec}$ is continuous on $[-\sqrt{2}, -1]$.
- (96) The 2nd part of $\operatorname{arccosec}$ is continuous on $[1, \sqrt{2}]$.
- (97) $\operatorname{rng}((\text{the 1st part of arcsec}) \upharpoonright [1, \sqrt{2}]) = [0, \frac{\pi}{4}]$.
- (98) $\operatorname{rng}((\text{the 2nd part of arcsec}) \upharpoonright [-\sqrt{2}, -1]) = [\frac{3}{4} \cdot \pi, \pi]$.
- (99) $\operatorname{rng}((\text{the 1st part of arccosec}) \upharpoonright [-\sqrt{2}, -1]) = [-\frac{\pi}{2}, -\frac{\pi}{4}]$.

- (100) $\text{rng}((\text{the 2nd part of arc cosec}) \upharpoonright [1, \sqrt{2}]) = [\frac{\pi}{4}, \frac{\pi}{2}]$.
- (101) If $1 \leq r \leq \sqrt{2}$ and $\text{arcsec}_1 r = 0$, then $r = 1$ and if $1 \leq r \leq \sqrt{2}$ and $\text{arcsec}_1 r = \frac{\pi}{4}$, then $r = \sqrt{2}$.
- (102) If $-\sqrt{2} \leq r \leq -1$ and $\text{arcsec}_2 r = \frac{3}{4} \cdot \pi$, then $r = -\sqrt{2}$ and if $-\sqrt{2} \leq r \leq -1$ and $\text{arcsec}_2 r = \pi$, then $r = -1$.
- (103) If $-\sqrt{2} \leq r \leq -1$ and $\text{arccosec}_1 r = -\frac{\pi}{2}$, then $r = -1$ and if $-\sqrt{2} \leq r \leq -1$ and $\text{arccosec}_1 r = -\frac{\pi}{4}$, then $r = -\sqrt{2}$.
- (104) If $1 \leq r \leq \sqrt{2}$ and $\text{arccosec}_2 r = \frac{\pi}{4}$, then $r = \sqrt{2}$ and if $1 \leq r \leq \sqrt{2}$ and $\text{arccosec}_2 r = \frac{\pi}{2}$, then $r = 1$.
- (105) If $1 \leq r \leq \sqrt{2}$, then $0 \leq \text{arcsec}_1 r \leq \frac{\pi}{4}$.
- (106) If $-\sqrt{2} \leq r \leq -1$, then $\frac{3}{4} \cdot \pi \leq \text{arcsec}_2 r \leq \pi$.
- (107) If $-\sqrt{2} \leq r \leq -1$, then $-\frac{\pi}{2} \leq \text{arccosec}_1 r \leq -\frac{\pi}{4}$.
- (108) If $1 \leq r \leq \sqrt{2}$, then $\frac{\pi}{4} \leq \text{arccosec}_2 r \leq \frac{\pi}{2}$.
- (109) If $1 < r < \sqrt{2}$, then $0 < \text{arcsec}_1 r < \frac{\pi}{4}$.
- (110) If $-\sqrt{2} < r < -1$, then $\frac{3}{4} \cdot \pi < \text{arcsec}_2 r < \pi$.
- (111) If $-\sqrt{2} < r < -1$, then $-\frac{\pi}{2} < \text{arccosec}_1 r < -\frac{\pi}{4}$.
- (112) If $1 < r < \sqrt{2}$, then $\frac{\pi}{4} < \text{arccosec}_2 r < \frac{\pi}{2}$.
- (113) If $1 \leq r \leq \sqrt{2}$, then $\sin \text{arcsec}_1 r = \frac{\sqrt{r^2-1}}{r}$ and $\cos \text{arcsec}_1 r = \frac{1}{r}$.
- (114) If $-\sqrt{2} \leq r \leq -1$, then $\sin \text{arcsec}_2 r = -\frac{\sqrt{r^2-1}}{r}$ and $\cos \text{arcsec}_2 r = \frac{1}{r}$.
- (115) If $-\sqrt{2} \leq r \leq -1$, then $\sin \text{arccosec}_1 r = \frac{1}{r}$ and $\cos \text{arccosec}_1 r = -\frac{\sqrt{r^2-1}}{r}$.
- (116) If $1 \leq r \leq \sqrt{2}$, then $\sin \text{arccosec}_2 r = \frac{1}{r}$ and $\cos \text{arccosec}_2 r = \frac{\sqrt{r^2-1}}{r}$.
- (117) If $1 < r < \sqrt{2}$, then $\text{cosec arcsec}_1 r = \frac{r}{\sqrt{r^2-1}}$.
- (118) If $-\sqrt{2} < r < -1$, then $\text{cosec arcsec}_2 r = -\frac{r}{\sqrt{r^2-1}}$.
- (119) If $-\sqrt{2} < r < -1$, then $\sec \text{arccosec}_1 r = -\frac{r}{\sqrt{r^2-1}}$.
- (120) If $1 < r < \sqrt{2}$, then $\sec \text{arccosec}_2 r = \frac{r}{\sqrt{r^2-1}}$.
- (121) The 1st part of arcsec is differentiable on $(\text{the function sec})^\circ]0, \frac{\pi}{2}[$.
- (122) The 2nd part of arcsec is differentiable on $(\text{the function sec})^\circ]\frac{\pi}{2}, \pi[$.
- (123) The 1st part of arccosec is differentiable on $(\text{the function cosec})^\circ]-\frac{\pi}{2}, 0[$.
- (124) The 2nd part of arccosec is differentiable on $(\text{the function cosec})^\circ]0, \frac{\pi}{2}[$.
- (125) $(\text{The function sec})^\circ]0, \frac{\pi}{2}[$ is open.
- (126) $(\text{The function sec})^\circ]\frac{\pi}{2}, \pi[$ is open.
- (127) $(\text{The function cosec})^\circ]-\frac{\pi}{2}, 0[$ is open.
- (128) $(\text{The function cosec})^\circ]0, \frac{\pi}{2}[$ is open.
- (129) The 1st part of arcsec is continuous on $(\text{the function sec})^\circ]0, \frac{\pi}{2}[$.
- (130) The 2nd part of arcsec is continuous on $(\text{the function sec})^\circ]\frac{\pi}{2}, \pi[$.

- (131) The 1st part of $\operatorname{arccosec}$ is continuous on $(\text{the function cosec})^{-1}[-\frac{\pi}{2}, 0[$.
 (132) The 2nd part of $\operatorname{arccosec}$ is continuous on $(\text{the function cosec})^{-1}]0, \frac{\pi}{2}[$.

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