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Several Differentiation Formulas of Special Functions. Part VI

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Summary. In this article, we prove a series of differentiation identities [3] involving the secant and cosecant functions and specific combinations of special functions including trigonometric, exponential and logarithmic functions.

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The papers [11], [13], [1], [15], [2], [8], [9], [16], [5], [12], [10], [4], [6], [7], and [14] provide the notation and terminology for this paper.

In this paper x denotes a real number and Z denotes an open subset of $\mathbb{R}$.

One can prove the following propositions:

(1) Suppose $Z \subseteq \mathrm{dom}((\text{the function tan}) \cdot (\text{the function cot}))$. Then

(i) (the function tan) $\cdot$(the function cot) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((\text{the function tan}) \cdot (\text{the function cot}))'_{\restriction Z}(x) = \frac{1}{(\text{the function cos})((\text{the function cot})(x))^{\mathbf{2}}} \cdot -\frac{1}{(\text{the function sin})(x)^{\mathbf{2}}}$.

(2) Suppose $Z \subseteq \mathrm{dom}((\text{the function tan}) \cdot (\text{the function tan}))$. Then

(i) (the function tan) $\cdot$(the function tan) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((\text{the function tan}) \cdot (\text{the function tan}))'_{\restriction Z}(x) = \frac{1}{(\text{the function cos})((\text{the function tan})(x))^{\mathbf{2}}} \cdot \frac{1}{(\text{the function cos})(x)^{\mathbf{2}}}$.

(3) Suppose $Z \subseteq \mathrm{dom}((\text{the function cot}) \cdot (\text{the function cot}))$. Then

(i) (the function cot) $\cdot$(the function cot) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((\text{the function cot}) \cdot (\text{the function cot}))'_{\restriction Z}(x) = \frac{1}{(\text{the function sin})((\text{the function cot})(x))^{\mathbf{2}}} \cdot \frac{1}{(\text{the function sin})(x)^{\mathbf{2}}}$.

(4) Suppose $Z \subseteq \mathrm{dom}((\text{the function cot}) \cdot (\text{the function tan}))$. Then

(i) (the function cot) $\cdot$(the function tan) is differentiable on Z, and

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(ii) for every x such that $x \in Z$ holds ((the function cot) $\cdot$(the function tan))$'_{\restriction Z}(x) = (-\frac{1}{\text{(the function sin)((the function tan)}(x))^{\mathbf{2}}}) \cdot \frac{1}{\text{(the function cos)}(x)^{\mathbf{2}}}$.

(5) Suppose $Z \subseteq$ dom((the function tan)$-$(the function cot)). Then

(i) (the function tan)$-$(the function cot) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function tan)$-$(the function cot))$'_{\restriction Z}(x) = \frac{1}{\text{(the function cos)}(x)^{\mathbf{2}}} + \frac{1}{\text{(the function sin)}(x)^{\mathbf{2}}}$.

(6) Suppose $Z \subseteq$ dom((the function tan)+(the function cot)). Then

(i) (the function tan)+(the function cot) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function tan)+(the function cot))$'_{\restriction Z}(x) = \frac{1}{\text{(the function cos)}(x)^{\mathbf{2}}} - \frac{1}{\text{(the function sin)}(x)^{\mathbf{2}}}$.

(7)(i) (The function sin) $\cdot$(the function sin) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function sin) $\cdot$(the function sin))$'_{\restriction Z}(x) =$ (the function cos)((the function sin)(x)) $\cdot$ (the function cos)(x).

(8)(i) (The function sin) $\cdot$(the function cos) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function sin) $\cdot$(the function cos))$'_{\restriction Z}(x) = -$(the function cos)((the function cos)(x)) $\cdot$ (the function sin) (x).

(9)(i) (The function cos) $\cdot$(the function sin) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function cos) $\cdot$(the function sin))$'_{\restriction Z}(x) = -$(the function sin)((the function sin)(x)) $\cdot$ (the function cos) (x).

(10)(i) (The function cos) $\cdot$(the function cos) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function cos) $\cdot$(the function cos))$'_{\restriction Z}(x) =$ (the function sin)((the function cos)(x)) $\cdot$ (the function sin)(x).

(11) Suppose $Z \subseteq$ dom((the function cos) (the function cot)). Then

(i) (the function cos) (the function cot) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function cos) (the function cot))$'_{\restriction Z}(x) = -$(the function cos)$(x) - \frac{\text{(the function cos)}(x)}{\text{(the function sin)}(x)^{\mathbf{2}}}$.

(12) Suppose $Z \subseteq$ dom((the function sin) (the function tan)). Then

(i) (the function sin) (the function tan) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function sin) (the function tan))$'_{\restriction Z}(x) =$ (the function sin)$(x) + \frac{\text{(the function sin)}(x)}{\text{(the function cos)}(x)^{\mathbf{2}}}$.

(13) Suppose $Z \subseteq$ dom((the function sin) (the function cot)). Then

(i) (the function sin) (the function cot) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function sin) (the function cot))$'_{\restriction Z}(x) =$ (the function cos)$(x) \cdot$ (the function cot)$(x) - \frac{1}{\text{(the function sin)}(x)}$.

(14) Suppose $Z \subseteq \mathrm{dom}((\text{the function cos})\ (\text{the function tan}))$. Then
 (i) (the function cos) (the function tan) is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds ((the function cos) (the function tan))$'_{\restriction Z}(x) = -\frac{(\text{the function sin})(x)^{\mathbf{2}}}{(\text{the function cos})(x)} + \frac{1}{(\text{the function cos})(x)}$.

(15) Suppose $Z \subseteq \mathrm{dom}((\text{the function sin})\ (\text{the function cos}))$. Then
 (i) (the function sin) (the function cos) is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds ((the function sin) (the function cos))$'_{\restriction Z}(x) = (\text{the function cos})(x)^{\mathbf{2}} - (\text{the function sin})(x)^{\mathbf{2}}$.

(16) Suppose $Z \subseteq \mathrm{dom}((\text{the function ln})\ (\text{the function sin}))$. Then
 (i) (the function ln) (the function sin) is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds ((the function ln) (the function sin))$'_{\restriction Z}(x) = \frac{(\text{the function sin})(x)}{x} + (\text{the function ln})(x) \cdot (\text{the function cos})(x)$.

(17) Suppose $Z \subseteq \mathrm{dom}((\text{the function ln})\ (\text{the function cos}))$. Then
 (i) (the function ln) (the function cos) is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds ((the function ln) (the function cos))$'_{\restriction Z}(x) = \frac{(\text{the function cos})(x)}{x} - (\text{the function ln})(x) \cdot (\text{the function sin})(x)$.

(18) Suppose $Z \subseteq \mathrm{dom}((\text{the function ln})\ (\text{the function exp}))$. Then
 (i) (the function ln) (the function exp) is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds ((the function ln) (the function exp))$'_{\restriction Z}(x) = \frac{(\text{the function exp})(x)}{x} + (\text{the function ln})(x) \cdot (\text{the function exp})(x)$.

(19) Suppose $Z \subseteq \mathrm{dom}((\text{the function ln})\ \cdot(\text{the function ln}))$ and for every x such that $x \in Z$ holds $x > 0$. Then
 (i) (the function ln) $\cdot$(the function ln) is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds ((the function ln) $\cdot$(the function ln))$'_{\restriction Z}(x) = \frac{1}{(\text{the function ln})(x) \cdot x}$.

(20) Suppose $Z \subseteq \mathrm{dom}((\text{the function exp})\ \cdot(\text{the function exp}))$. Then
 (i) (the function exp) $\cdot$(the function exp) is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds ((the function exp) $\cdot$(the function exp))$'_{\restriction Z}(x) = (\text{the function exp})((\text{the function exp})(x)) \cdot (\text{the function exp})(x)$.

(21) Suppose $Z \subseteq \mathrm{dom}((\text{the function sin})\ \cdot(\text{the function tan}))$. Then
 (i) (the function sin) $\cdot$(the function tan) is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds ((the function sin) $\cdot$(the function tan))$'_{\restriction Z}(x) = \frac{\cos\,(\text{the function tan})(x)}{(\text{the function cos})(x)^{\mathbf{2}}}$.

(22) Suppose $Z \subseteq \mathrm{dom}((\text{the function sin})\ \cdot(\text{the function cot}))$. Then
 (i) (the function sin) $\cdot$(the function cot) is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds ((the function sin) $\cdot$(the function cot))$'_{\restriction Z}(x) = -\frac{\cos\,(\text{the function cot})(x)}{(\text{the function sin})(x)^{\mathbf{2}}}$.

(23) Suppose $Z \subseteq$ dom((the function cos) $\cdot$(the function tan)). Then
 (i) (the function cos) $\cdot$(the function tan) is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds ((the function cos) $\cdot$(the function tan))$'_{\restriction Z}(x) = -\frac{\sin(\text{the function tan})(x)}{(\text{the function cos})(x)^{\mathbf{2}}}$.

(24) Suppose $Z \subseteq$ dom((the function cos) $\cdot$(the function cot)). Then
 (i) (the function cos) $\cdot$(the function cot) is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds ((the function cos) $\cdot$(the function cot))$'_{\restriction Z}(x) = \frac{\sin(\text{the function cot})(x)}{(\text{the function sin})(x)^{\mathbf{2}}}$.

(25) Suppose $Z \subseteq$ dom((the function sin) ((the function tan)+(the function cot))). Then
 (i) (the function sin) ((the function tan)+(the function cot)) is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds ((the function sin) ((the function tan)+(the function cot)))$'_{\restriction Z}(x)$ = (the function cos)$(x) \cdot$ ((the function tan)(x) + (the function cot)(x)) + (the function sin)$(x) \cdot$ $(\frac{1}{(\text{the function cos})(x)^{\mathbf{2}}} - \frac{1}{(\text{the function sin})(x)^{\mathbf{2}}})$.

(26) Suppose $Z \subseteq$ dom((the function cos) ((the function tan)+(the function cot))). Then
 (i) (the function cos) ((the function tan)+(the function cot)) is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds ((the function cos) ((the function tan)+(the function cot)))$'_{\restriction Z}(x) = -$(the function sin)$(x) \cdot$ ((the function tan)(x) + (the function cot)(x))+(the function cos)$(x)\cdot(\frac{1}{(\text{the function cos})(x)^{\mathbf{2}}} - \frac{1}{(\text{the function sin})(x)^{\mathbf{2}}})$.

(27) Suppose $Z \subseteq$ dom((the function sin) ((the function tan)$-$(the function cot))). Then
 (i) (the function sin) ((the function tan)$-$(the function cot)) is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds ((the function sin) ((the function tan)$-$(the function cot)))$'_{\restriction Z}(x)$ = (the function cos)$(x) \cdot$ ((the function tan)(x) $-$ (the function cot)(x)) + (the function sin)$(x) \cdot$ $(\frac{1}{(\text{the function cos})(x)^{\mathbf{2}}} + \frac{1}{(\text{the function sin})(x)^{\mathbf{2}}})$.

(28) Suppose $Z \subseteq$ dom((the function cos) ((the function tan)$-$(the function cot))). Then
 (i) (the function cos) ((the function tan)$-$(the function cot)) is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds ((the function cos) ((the function tan)$-$(the function cot)))$'_{\restriction Z}(x) = -$(the function sin)$(x) \cdot$ ((the function tan)(x) $-$ (the function cot)(x))+(the function cos)$(x)\cdot(\frac{1}{(\text{the function cos})(x)^{\mathbf{2}}} + \frac{1}{(\text{the function sin})(x)^{\mathbf{2}}})$.

(29) Suppose $Z \subseteq$ dom((the function exp) ((the function tan)+(the function cot))). Then

(i) (the function exp) ((the function tan)+(the function cot)) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function exp) ((the function tan)+(the function cot)))$'_{\restriction Z}(x)$ = (the function exp)(x) $\cdot$ ((the function tan)(x) + (the function cot)(x)) + (the function exp)(x) $\cdot$ $(\frac{1}{(\text{the function cos})(x)^{\mathbf{2}}} - \frac{1}{(\text{the function sin})(x)^{\mathbf{2}}})$.

(30) Suppose $Z \subseteq$ dom((the function exp) ((the function tan)$-$(the function cot))). Then

(i) (the function exp) ((the function tan)$-$(the function cot)) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function exp) ((the function tan)$-$(the function cot)))$'_{\restriction Z}(x)$ = (the function exp)(x) $\cdot$ ((the function tan)(x) $-$ (the function cot)(x)) + (the function exp)(x) $\cdot$ $(\frac{1}{(\text{the function cos})(x)^{\mathbf{2}}} + \frac{1}{(\text{the function sin})(x)^{\mathbf{2}}})$.

(31) Suppose $Z \subseteq$ dom((the function sin) ((the function sin)+(the function cos))). Then

(i) (the function sin) ((the function sin)+(the function cos)) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function sin) ((the function sin)+(the function cos)))$'_{\restriction Z}(x)$ = ((the function cos)$(x)^{\mathbf{2}}$ + $2 \cdot$ (the function sin)(x) $\cdot$ (the function cos)(x)) $-$ (the function sin)$(x)^{\mathbf{2}}$.

(32) Suppose $Z \subseteq$ dom((the function sin) ((the function sin)$-$(the function cos))). Then

(i) (the function sin) ((the function sin)$-$(the function cos)) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function sin) ((the function sin)$-$(the function cos)))$'_{\restriction Z}(x)$ = ((the function sin)$(x)^{\mathbf{2}}$ + $2 \cdot$ (the function sin)(x) $\cdot$ (the function cos)(x)) $-$ (the function cos)$(x)^{\mathbf{2}}$.

(33) Suppose $Z \subseteq$ dom((the function cos) ((the function sin)$-$(the function cos))). Then

(i) (the function cos) ((the function sin)$-$(the function cos)) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function cos) ((the function sin)$-$(the function cos)))$'_{\restriction Z}(x)$ = ((the function cos)$(x)^{\mathbf{2}}$ + $2 \cdot$ (the function sin)(x) $\cdot$ (the function cos)(x)) $-$ (the function sin)$(x)^{\mathbf{2}}$.

(34) Suppose $Z \subseteq$ dom((the function cos) ((the function sin)+(the function cos))). Then

(i) (the function cos) ((the function sin)+(the function cos)) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function cos) ((the function sin)+(the function cos)))$'_{\restriction Z}(x)$ = (the function cos)$(x)^{\mathbf{2}} - 2 \cdot$ (the function sin)$(x) \cdot$ (the function cos)(x) − (the function sin)$(x)^{\mathbf{2}}$.

(35) Suppose $Z \subseteq$ dom((the function sin) $\cdot$((the function tan)+(the function cot))). Then

(i) (the function sin) $\cdot$((the function tan)+(the function cot)) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function sin) $\cdot$((the function tan)+(the function cot)))$'_{\restriction Z}(x)$ = (the function cos)((the function tan)(x) + (the function cot)$(x)) \cdot (\frac{1}{\text{(the function cos)}(x)^{\mathbf{2}}} - \frac{1}{\text{(the function sin)}(x)^{\mathbf{2}}})$.

(36) Suppose $Z \subseteq$ dom((the function sin) $\cdot$((the function tan)−(the function cot))). Then

(i) (the function sin) $\cdot$((the function tan)−(the function cot)) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function sin) $\cdot$((the function tan)−(the function cot)))$'_{\restriction Z}(x)$ = (the function cos)((the function tan)(x) − (the function cot)$(x)) \cdot (\frac{1}{\text{(the function cos)}(x)^{\mathbf{2}}} + \frac{1}{\text{(the function sin)}(x)^{\mathbf{2}}})$.

(37) Suppose $Z \subseteq$ dom((the function cos) $\cdot$((the function tan)−(the function cot))). Then

(i) (the function cos) $\cdot$((the function tan)−(the function cot)) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function cos) $\cdot$((the function tan)−(the function cot)))$'_{\restriction Z}(x)$ = −(the function sin)((the function tan)(x) − (the function cot)$(x)) \cdot (\frac{1}{\text{(the function cos)}(x)^{\mathbf{2}}} + \frac{1}{\text{(the function sin)}(x)^{\mathbf{2}}})$.

(38) Suppose $Z \subseteq$ dom((the function cos) $\cdot$((the function tan)+(the function cot))). Then

(i) (the function cos) $\cdot$((the function tan)+(the function cot)) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function cos) $\cdot$((the function tan)+(the function cot)))$'_{\restriction Z}(x)$ = −(the function sin)((the function tan) (x) + (the function cot)$(x)) \cdot (\frac{1}{\text{(the function cos)}(x)^{\mathbf{2}}} - \frac{1}{\text{(the function sin)}(x)^{\mathbf{2}}})$.

(39) Suppose $Z \subseteq$ dom((the function exp) $\cdot$((the function tan)+(the function cot))). Then

(i) (the function exp) $\cdot$((the function tan)+(the function cot)) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function exp) $\cdot$((the function tan)+(the function cot)))$'_{\restriction Z}(x)$ = (the function exp)((the function tan)(x) + (the function cot)$(x)) \cdot (\frac{1}{\text{(the function cos)}(x)^{\mathbf{2}}} - \frac{1}{\text{(the function sin)}(x)^{\mathbf{2}}})$.

(40) Suppose $Z \subseteq$ dom((the function exp) $\cdot$((the function tan)−(the function cot))). Then

(i) (the function exp) $\cdot$((the function tan)$-$(the function cot)) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function exp) $\cdot$((the function tan)$-$(the function cot)))$'_{\restriction Z}(x)$ = (the function exp)((the function tan)(x) $-$ (the function cot)(x)) $\cdot$ $(\frac{1}{(\text{the function cos})(x)^{2}} + \frac{1}{(\text{the function sin})(x)^{2}})$.

(41) Suppose $Z \subseteq \mathrm{dom}(\frac{(\text{the function tan})-(\text{the function cot})}{\text{the function exp}})$. Then

(i) $\frac{(\text{the function tan})-(\text{the function cot})}{\text{the function exp}}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\frac{(\text{the function tan})-(\text{the function cot})}{\text{the function exp}})'_{\restriction Z}(x) = \frac{((\frac{1}{(\text{the function cos})(x)^{2}}+\frac{1}{(\text{the function sin})(x)^{2}})-(\text{the function tan})(x))+(\text{the function cot})(x)}{(\text{the function exp})(x)}$.

(42) Suppose $Z \subseteq \mathrm{dom}(\frac{(\text{the function tan})+(\text{the function cot})}{\text{the function exp}})$. Then

(i) $\frac{(\text{the function tan})+(\text{the function cot})}{\text{the function exp}}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\frac{(\text{the function tan})+(\text{the function cot})}{\text{the function exp}})'_{\restriction Z}(x) = \frac{\frac{1}{(\text{the function cos})(x)^{2}}-\frac{1}{(\text{the function sin})(x)^{2}}-(\text{the function tan})(x)-(\text{the function cot})(x)}{(\text{the function exp})(x)}$.

(43) Suppose $Z \subseteq \mathrm{dom}$((the function sin) $\cdot$ sec). Then

(i) (the function sin) $\cdot$ sec is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function sin) $\cdot$ sec)$'_{\restriction Z}(x) = \frac{(\text{the function cos})((\text{sec})(x))\cdot(\text{the function sin})(x)}{(\text{the function cos})(x)^{2}}$.

(44) Suppose $Z \subseteq \mathrm{dom}$((the function cos) $\cdot$ sec). Then

(i) (the function cos) $\cdot$ sec is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function cos) $\cdot$ sec)$'_{\restriction Z}(x) = -\frac{(\text{the function sin})((\text{sec})(x))\cdot(\text{the function sin})(x)}{(\text{the function cos})(x)^{2}}$.

(45) Suppose $Z \subseteq \mathrm{dom}$((the function sin) $\cdot$ cosec). Then

(i) (the function sin) $\cdot$ cosec is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function sin) $\cdot$ cosec)$'_{\restriction Z}(x) = -\frac{(\text{the function cos})((\text{cosec})(x))\cdot(\text{the function cos})(x)}{(\text{the function sin})(x)^{2}}$.

(46) Suppose $Z \subseteq \mathrm{dom}$((the function cos) $\cdot$ cosec). Then

(i) (the function cos) $\cdot$ cosec is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function cos) $\cdot$ cosec)$'_{\restriction Z}(x) = \frac{(\text{the function sin})((\text{cosec})(x))\cdot(\text{the function cos})(x)}{(\text{the function sin})(x)^{2}}$.

References

[1] Grzegorz Bancerek. The ordinal numbers. *Formalized Mathematics*, 1(**1**):91–96, 1990.

[2] Czesław Byliński. Partial functions. *Formalized Mathematics*, 1(**2**):357–367, 1990.

[3] Fritz Chemnitius. *Differentiation und Integration ausgewählter Beispiele*. VEB Verlag Technik, Berlin, 1956.

[4] Krzysztof Hryniewiecki. Basic properties of real numbers. *Formalized Mathematics*, 1(**1**):35–40, 1990.

[5] Jarosław Kotowicz. Partial functions from a domain to the set of real numbers. *Formalized Mathematics*, 1(**4**):703–709, 1990.
[6] Jarosław Kotowicz. Real sequences and basic operations on them. *Formalized Mathematics*, 1(**2**):269–272, 1990.
[7] Konrad Raczkowski and Paweł Sadowski. Real function differentiability. *Formalized Mathematics*, 1(**4**):797–801, 1990.
[8] Konrad Raczkowski and Paweł Sadowski. Topological properties of subsets in real numbers. *Formalized Mathematics*, 1(**4**):777–780, 1990.
[9] Yasunari Shidama. The Taylor expansions. *Formalized Mathematics*, 12(**2**):195–200, 2004.
[10] Andrzej Trybulec. Subsets of complex numbers. *To appear in Formalized Mathematics*.
[11] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.
[12] Andrzej Trybulec and Czesław Byliński. Some properties of real numbers. *Formalized Mathematics*, 1(**3**):445–449, 1990.
[13] Zinaida Trybulec. Properties of subsets. *Formalized Mathematics*, 1(**1**):67–71, 1990.
[14] Peng Wang and Bo Li. Several differentiation formulas of special functions. Part V. *Formalized Mathematics*, 15(**3**):73–79, 2007.
[15] Edmund Woronowicz. Relations defined on sets. *Formalized Mathematics*, 1(**1**):181–186, 1990.
[16] Yuguang Yang and Yasunari Shidama. Trigonometric functions and existence of circle ratio. *Formalized Mathematics*, 7(**2**):255–263, 1998.

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